

AUDIBLE GRAVITATIONAL ECHOES OF NEW PHYSICS

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10th LISA Cosmology Working Group Workshop - Uis



UNIÃO EUROPEIA
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CIDMA

CENTRO DE I&D EM MATEMÁTICA E APLICAÇÕES
CENTER FOR R&D IN MATHEMATICS AND
APPLICATIONS

The SM is a tremendously successful theory that explains
"boringly" well most its predictions!

However, it fails to...

- Explain neutrino masses ← Today's focus
- Explain dark matter
- Explain CP violation and matter/anti-matter asymmetry
- Explain the observed flavour structure - Flavour Problem

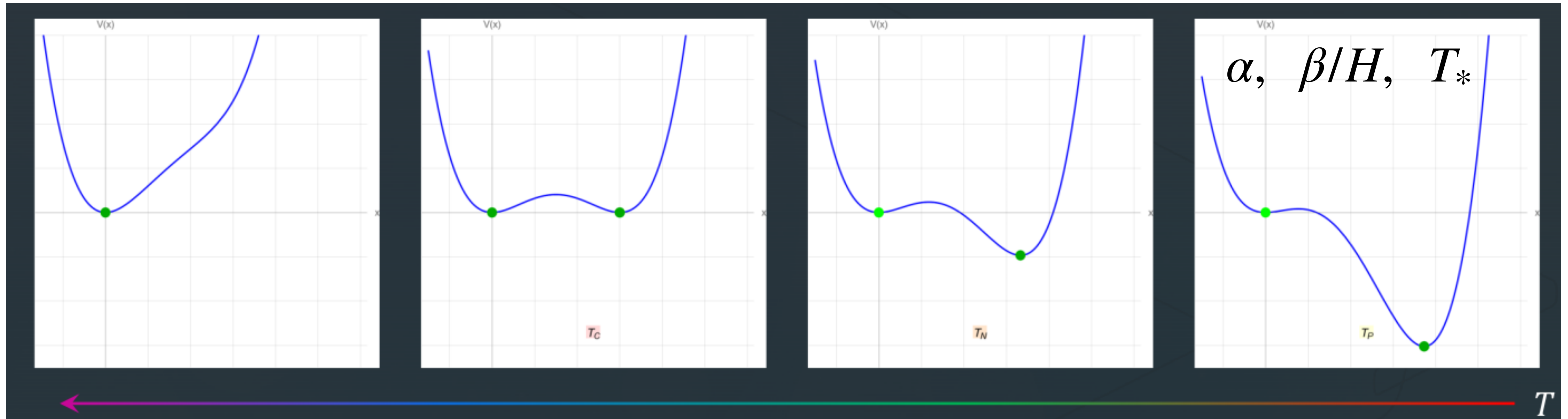
SGWB

- ✓ Superposition of unresolved astrophysical sources
- ✓ Cosmological origin
 - Inflation
 - Topological defects
 - Phase transitions

SGWB as a gravitational probe to New Physics, in combination with, or beyond colliders' reach

First order phase transition (FOPT)

(Illustration)



Credit: Marco Finetti

We use the templates for **SW peak** in [Caprini et al. JCAP 03 (2020) 024]

See Marek's talk for details on phase transitions

Scenario 1: Neutrino masses from lepton number symmetry breaking

[2304.02399] ADDAZI, MARCIANÒ, APM, PASECHNIK, VIANA, YANG

Which seesaw model?

	L^i	ν_R^i	S^i	σ	H	Model
U(1) _L	1	1	×	-2	0	T1S
	1	1	0	-1	0	IS
	1	1	-1	2	0	EIS

- $v_\sigma \gg v_h$ for the T1S; **beyond LISA**
- $v_\sigma \gg v_h$ and/or $\Lambda \ll v_h$ for the IS; **beyond LISA**
- $v_\sigma \sim v_h$ and $\Lambda \gg v_h$ for the EIS. **Well motivated for LISA range**

$$m_\nu^{\text{T1S}} \approx \frac{1}{\sqrt{2}} \frac{y_\nu^2}{y_\sigma} \frac{v_h^2}{v_\sigma},$$

$$m_\nu^{\text{IS}} \approx \frac{y_\nu^2}{y_\sigma^2} \frac{\Lambda v_h^2}{v_\sigma^2},$$

$$m_\nu^{\text{EIS}} \approx \frac{y_\nu^2 y_\sigma}{2\sqrt{2}} \frac{v_h^2 v_\sigma}{\Lambda^2}$$

Neutrino sector revisited

$$\mathcal{L}_\nu^{\text{EIS}} = y_\nu^{ij} \bar{L}_i \tilde{H} \nu_{Rj} + y_\sigma^{ij} \bar{S}_i^c S_j \sigma + y_\sigma^{\prime ij} \bar{\nu}_{Ri}^c \nu_{Rj} \sigma^* + \Lambda^{ij} \bar{\nu}_{Ri}^c S_j + \text{h.c.} \quad M_\nu^{\text{EIS}} = \begin{pmatrix} 0 & \frac{v_h}{\sqrt{2}} \mathbf{y}_\nu & 0 \\ \frac{v_h}{\sqrt{2}} \mathbf{y}_\nu & \frac{v_\sigma}{\sqrt{2}} \mathbf{y}'_\sigma & \Lambda \\ 0 & \Lambda & \frac{v_\sigma}{\sqrt{2}} \mathbf{y}_\sigma \end{pmatrix}$$

✓ EFT approach

$$m_\nu^{\text{EIS}} \approx \frac{\mathbf{y}_\nu^2 \mathbf{y}_\sigma}{2\sqrt{2}} \frac{v_h^2 v_\sigma}{\Lambda^2}$$

3 light active neutrinos

$$m_{N^\pm} \approx \Lambda \pm \frac{v_\sigma}{2\sqrt{2}} (\mathbf{y}_\sigma + \mathbf{y}'_\sigma)$$

6 heavy neutrinos

Use normal ordering masses as input to obtain

$$y_\sigma^i = 2\sqrt{2} \frac{m_{\nu_i} \Lambda^2}{v_h^2 v_\sigma y_{\nu_i}^2}$$

$$V_0(H, \sigma) = V_{\text{SM}}(H) + V_{4\text{D}}(H, \sigma) + V_{6\text{D}}(H, \sigma) + V_{\text{soft}}(\sigma)$$

$$V_{\text{SM}}(H) = \mu_h^2 H^\dagger H + \lambda_h (H^\dagger H)^2,$$

$$V_{4\text{D}}(H, \sigma) = \mu_\sigma^2 \sigma^\dagger \sigma + \lambda_\sigma (\sigma^\dagger \sigma)^2 + \lambda_{\sigma h} H^\dagger H \sigma^\dagger \sigma,$$

$$V_{6\text{D}}(H, \sigma) = \frac{\delta_0}{\Lambda^2} (H^\dagger H)^3 + \frac{\delta_2}{\Lambda^2} (H^\dagger H)^2 \sigma^\dagger \sigma + \frac{\delta_4}{\Lambda^2} H^\dagger H (\sigma^\dagger \sigma)^2 + \frac{\delta_6}{\Lambda^2} (\sigma^\dagger \sigma)^3,$$

$$V_{\text{soft}}(\sigma) = \frac{1}{2} \mu_b^2 (\sigma^2 + \sigma^{*2}).$$

$$\frac{\delta_i}{\Lambda^2} v_\sigma^2 < 4\pi$$

10 TeV < Λ < 1000 TeV \longrightarrow heavy neutrino mass scale

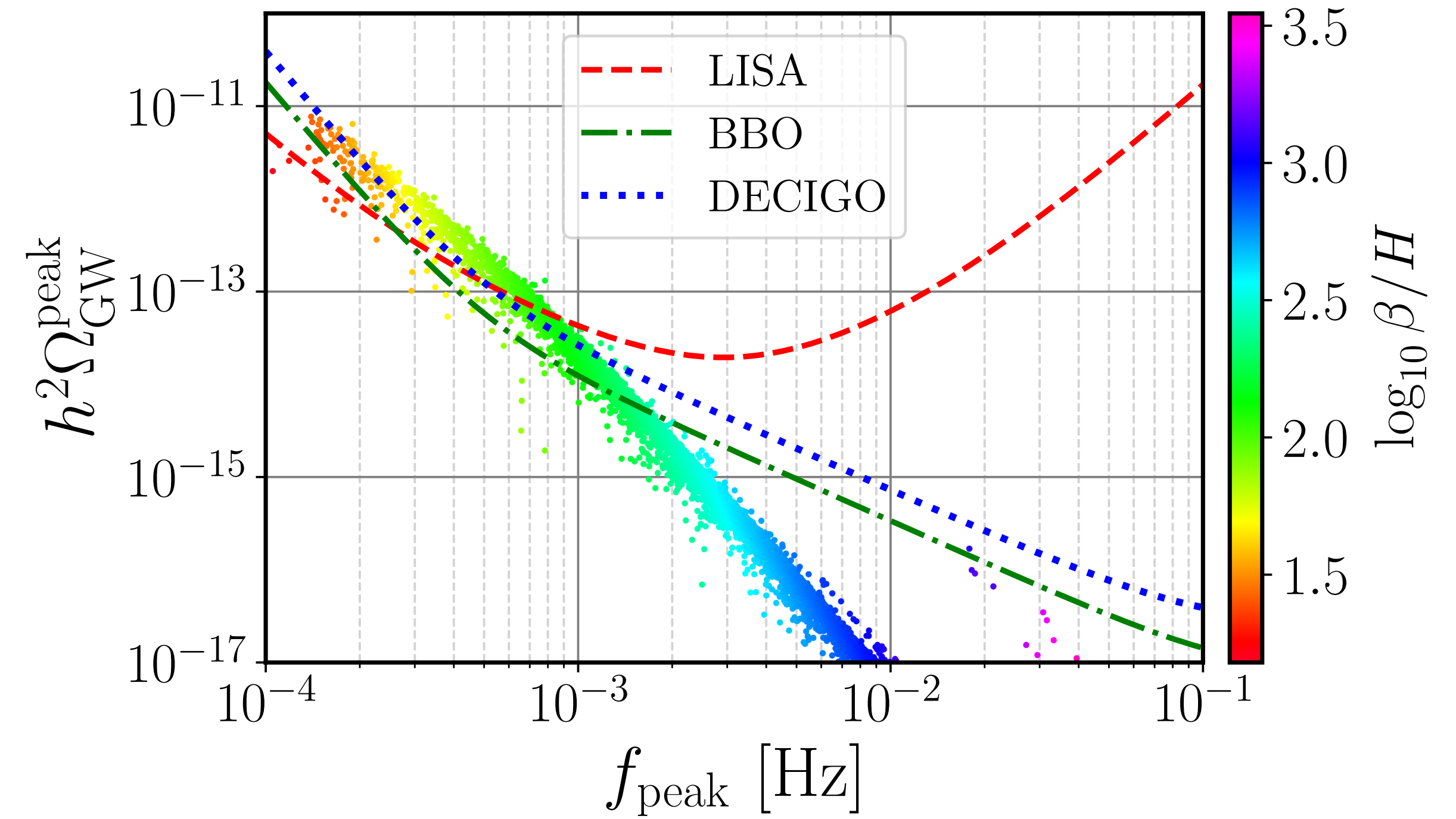
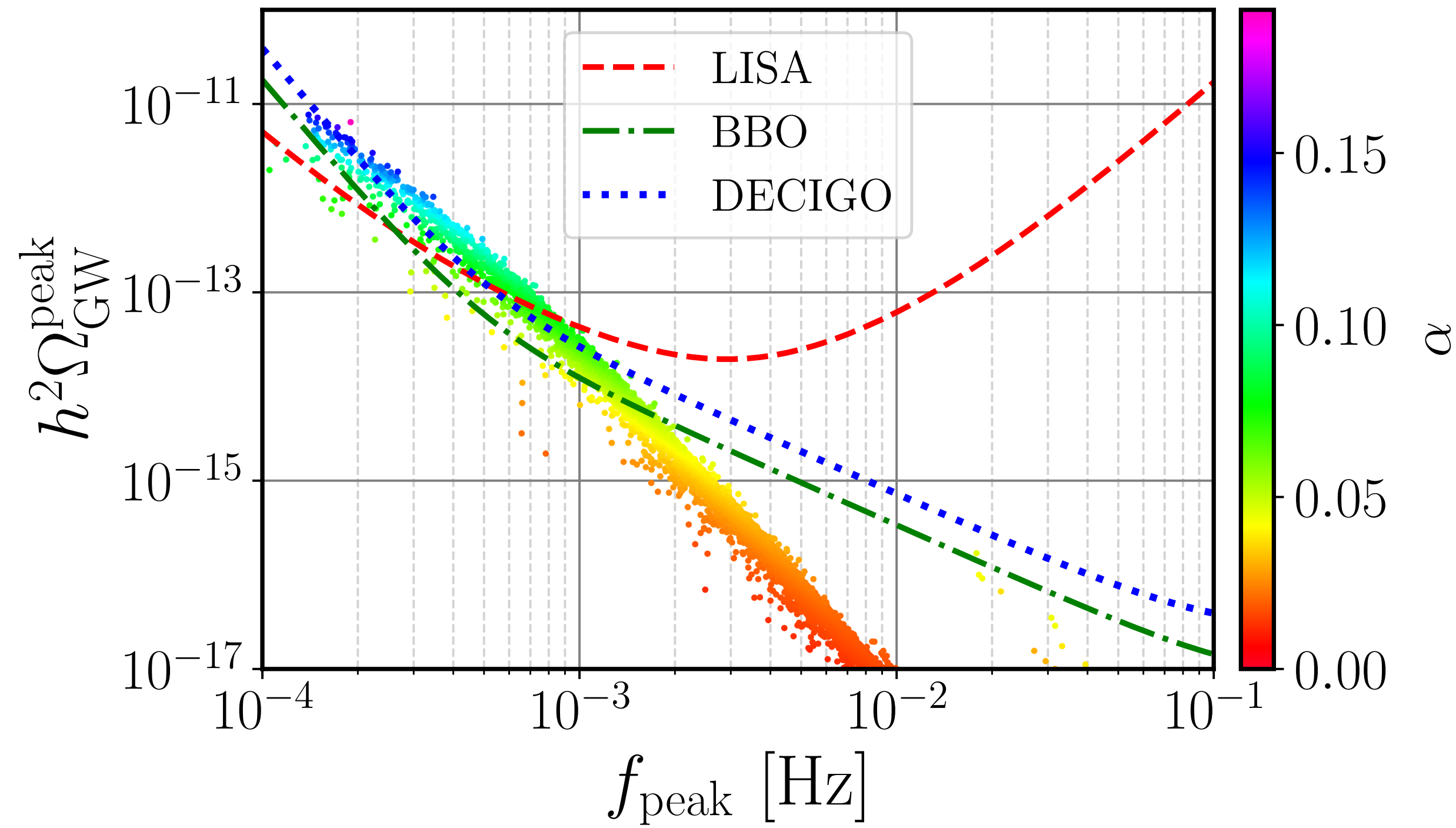
δ_2 and δ_4 allow co-existence of $\Gamma_{\text{Higgs}}^{\text{invisible}}$ and SFOPTs

Results

Parameter	Range	Distribution
m_{h_2}	[60, 1000] GeV	linear
m_J	[10^{-10} eV, 100 keV]	exponential
m_{ν_1}	[10^{-6} , 10^{-1}] eV	exponential
$\text{Br}(h_1 \rightarrow JJ)$	[10^{-15} , 0.18]	exponential
$\sin(\alpha_h)$	$\pm[0, 0.24]$	linear
v_σ	[100, 1000] GeV	linear
Λ	[10, 1000] TeV	exponential
$\frac{\delta_0 v_h^2}{2\Lambda^2}$	$\pm[10^{-10}, 4\pi]$	exponential
$\frac{\delta_2 \max(v_h^2, v_\sigma^2)}{2\Lambda^2}$	$\pm[10^{-10}, 4\pi]$	exponential
$\frac{\delta_4 v_\sigma^2}{2\Lambda^2}$	$\pm[10^{-10}, 4\pi]$	exponential

Results

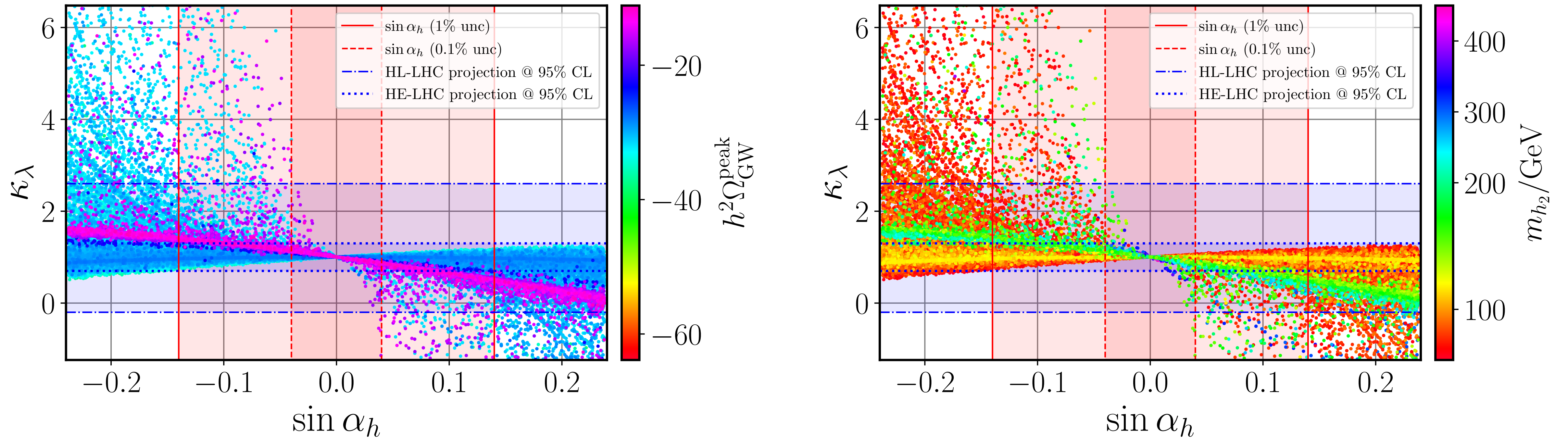
$$\log_{10}(h^2\Omega_{\text{GW}}^{\text{peak}}) \propto -2 \log_{10} f_{\text{peak}} + \log_{10} F(\alpha, T_*)$$



Scan using *CosmoTransitions*

[Comp. Phys. Commun. 183, 2006 (2012)]

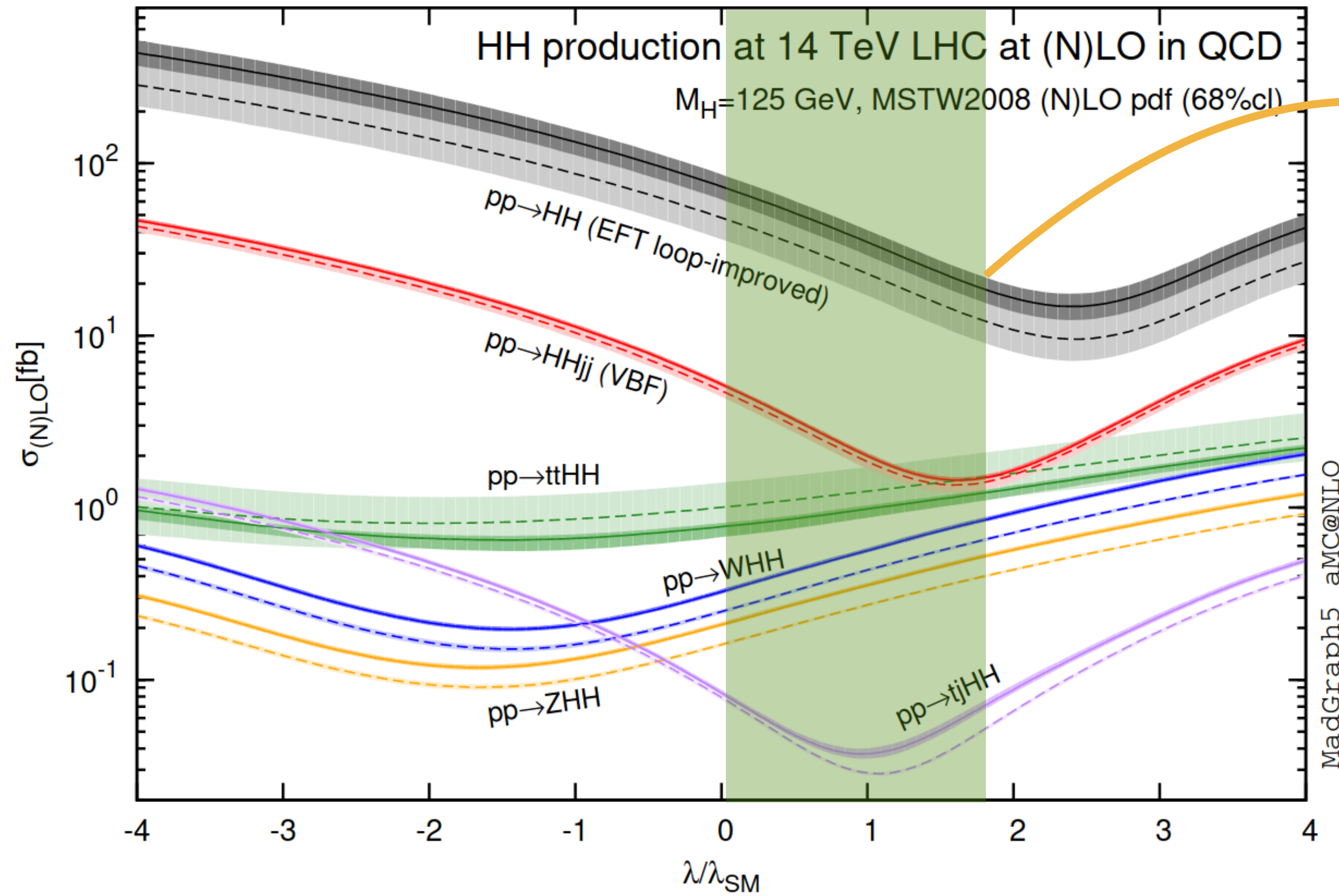
Trilinear Higgs coupling, scalar mixing angle and CP-even scalar mass



$$\kappa_\lambda \equiv \lambda_{h_1 h_1 h_1} / \lambda_{\text{SM}}, \quad \lambda_{\text{SM}} = 3m_{h_1}^2 / v_h$$

- **Magenta band (LISA) / green band** favour $0 < \kappa_\lambda < 2$ and $m_{h_2} \approx (200 \pm 50) \text{ GeV}$
- Illustrates the potential interplay between collider and SGWB interplay

Di-Higgs production



Region compatible with observable SFOPTs in the 6D Majoron model

Phys.Lett.B 732 (2014) 142-149

Scenario 2: Neutrino masses with colour restoration at low temperature

[WORK IN PROGRESS] BERTENSTAM, EKSTEDT, FINETTI, APM, PASECHNIK, VATELLIS

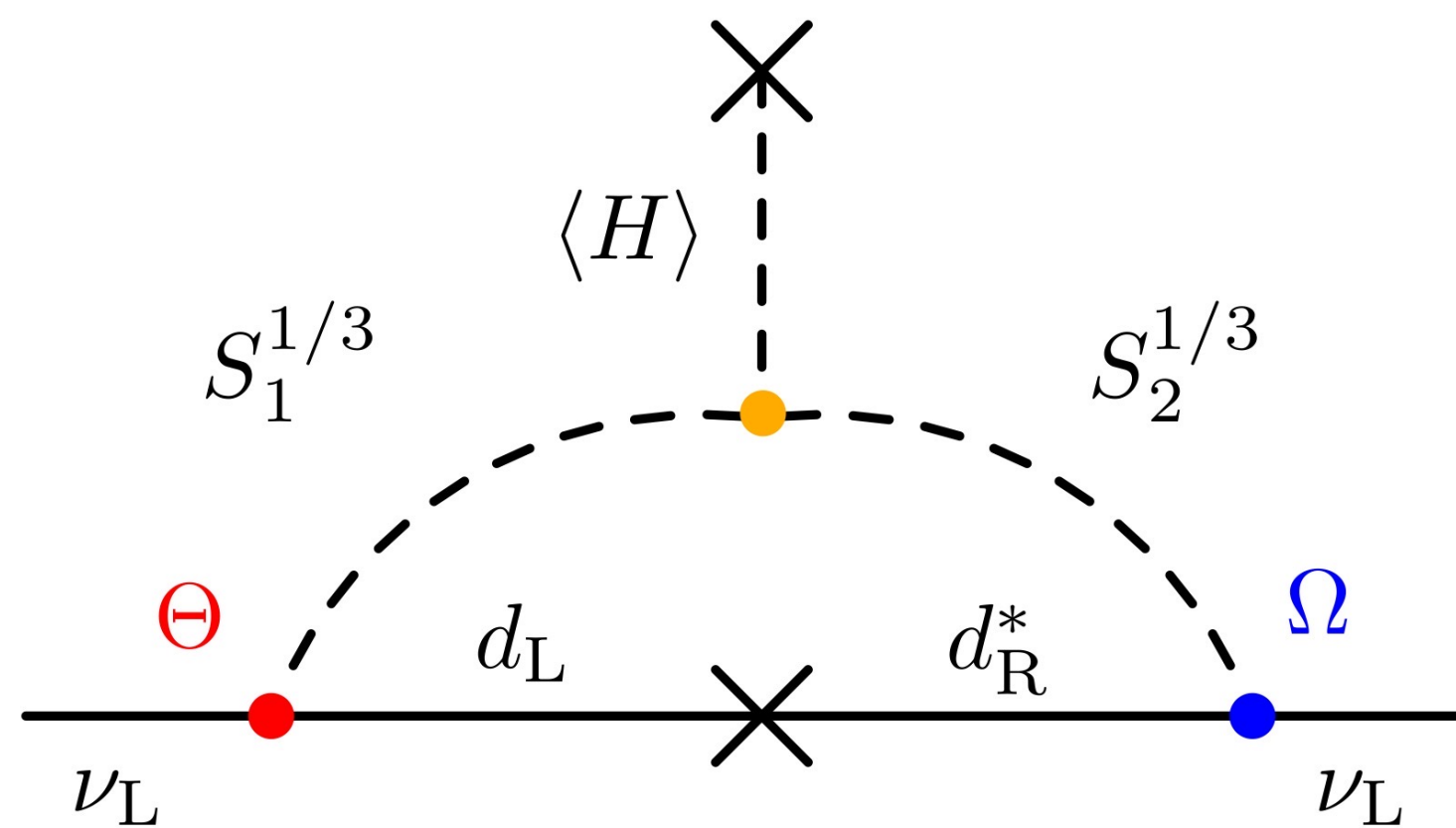
Another possibility for neutrino masses

$$\mathcal{L}_Y = \Theta_{ij} \bar{Q}_j^c L_i S + \Omega_{ij} \bar{L}_i d_j R^\dagger + \Upsilon_{ij} \bar{u}_j e_i S^\dagger + \text{h.c.}$$

$$S \sim (\bar{\mathbf{3}}, \mathbf{1})_{1/3} \quad R \sim (\bar{\mathbf{3}}, \mathbf{2})_{1/6}$$

- **And an exhaustive flavour analysis**

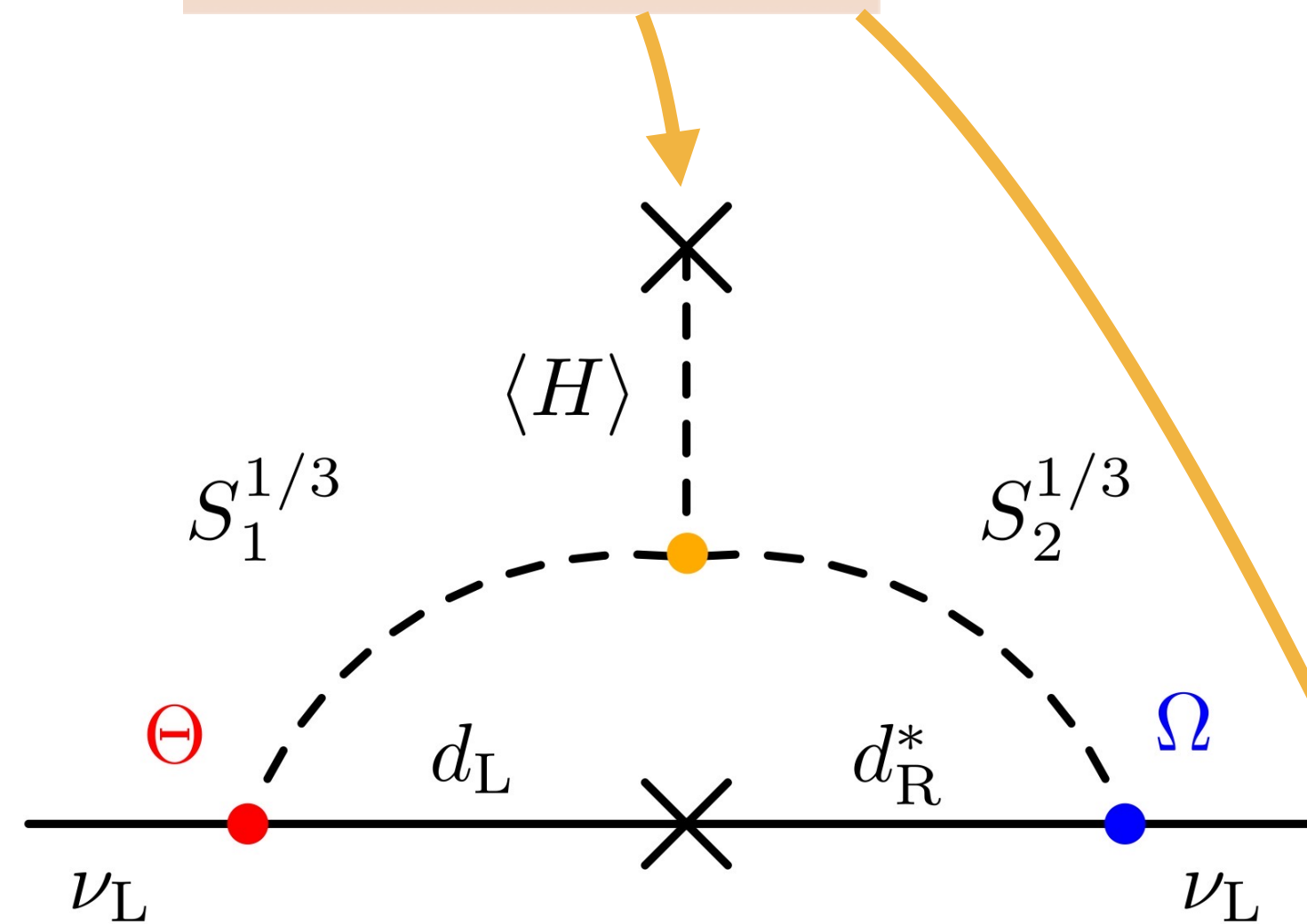
[Gonçalves, APM, Pasechnik, Porod, 2206.01674]



$$(M_\nu)_{ij} = \frac{3}{16\pi^2(m_{S_2^{1/3}}^2 - m_{S_1^{1/3}}^2)} \frac{va_1}{\sqrt{2}} \ln \left(\frac{m_{S_2^{1/3}}^2}{m_{S_1^{1/3}}^2} \right) \sum_{m,a} (m_d)_a V_{am} (\Theta_{im} \Omega_{ja} + \Theta_{jm} \Omega_{ia}),$$

Another possibility for neutrino masses

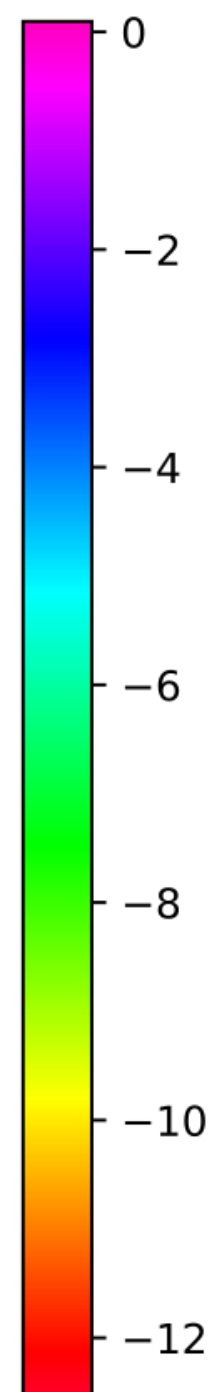
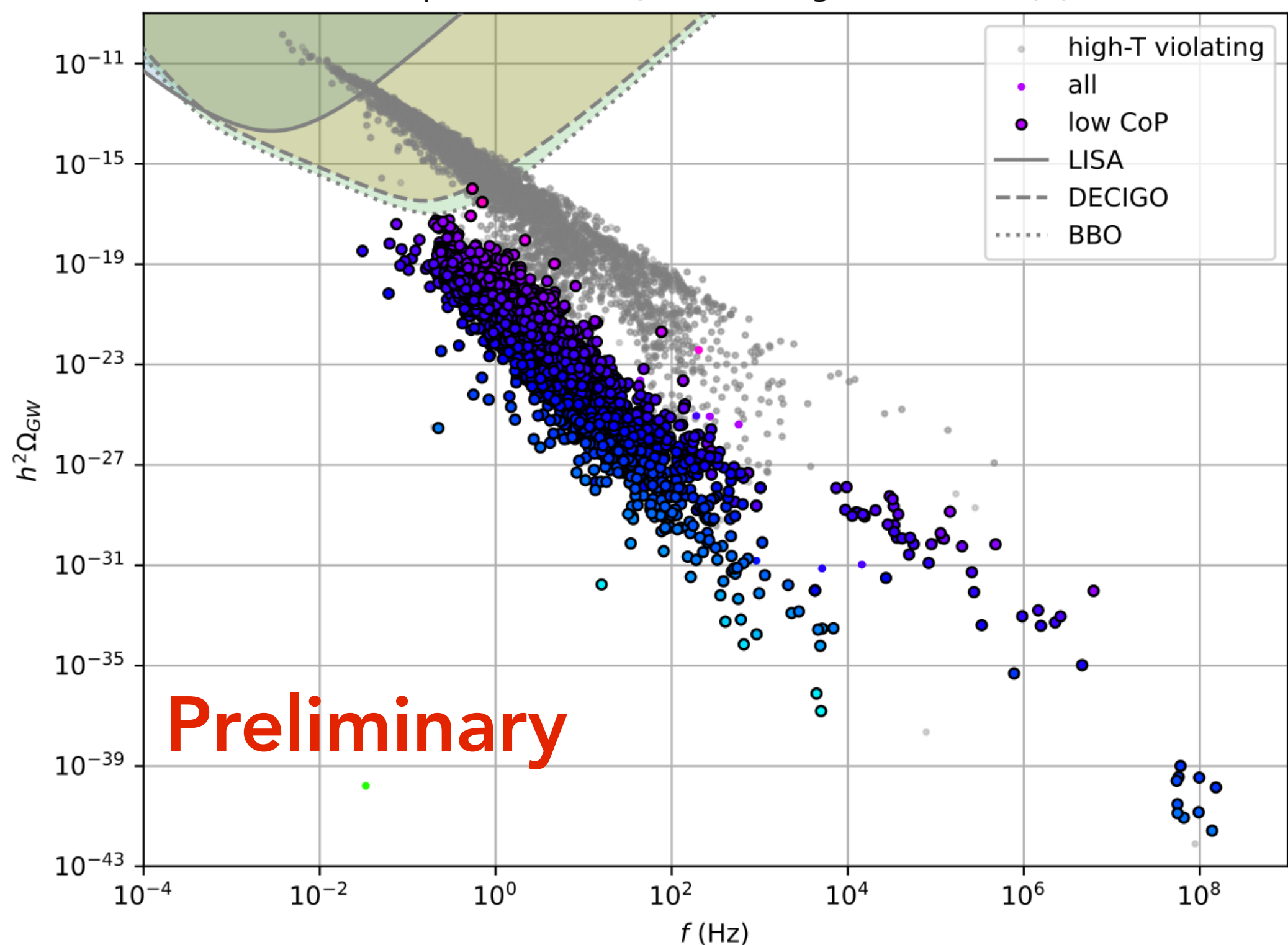
$$V \supset -\mu^2|H|^2 + \mu_S^2|S|^2 + \mu_R^2|R|^2 + \lambda(H^\dagger H)^2 + g_{HR}(H^\dagger H)(R^\dagger R) + g'_{HR}(H^\dagger R)(R^\dagger H) + g_{HS}(H^\dagger H)(S^\dagger S) + (a_1 RSH^\dagger + \text{h.c.}) .$$



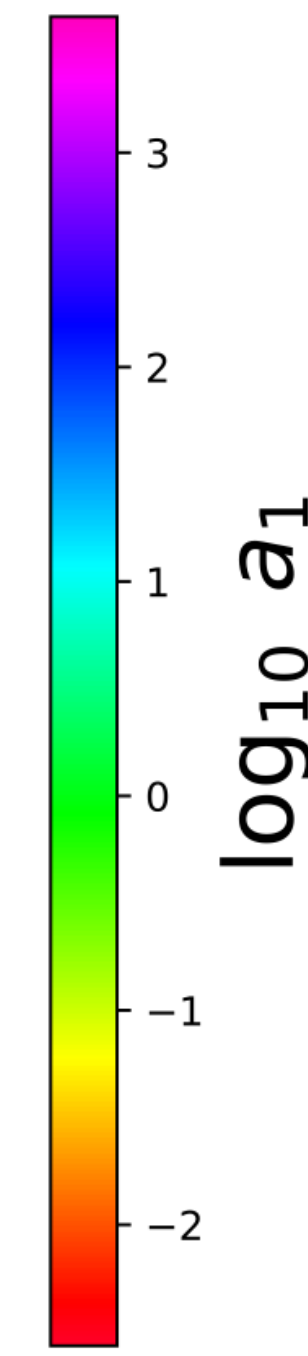
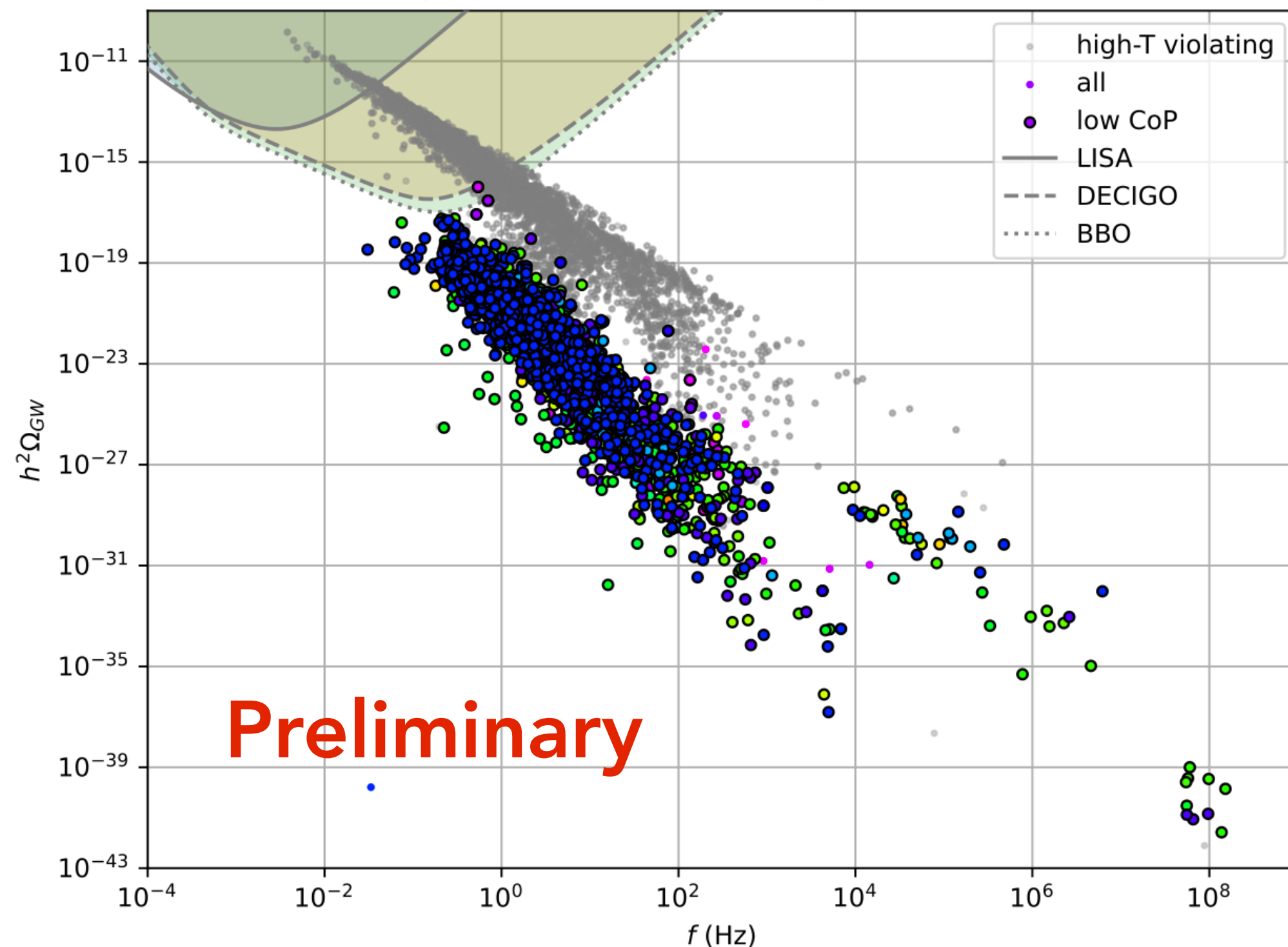
- ✓ Consider the possibility of LQ VEVs at **finite T**
- ✓ Classify all possible FOPTs and determine SGWB

$$(M_\nu)_{ij} = \frac{3}{16\pi^2(m_{S_2^{1/3}}^2 - m_{S_1^{1/3}}^2)} \frac{va_1}{\sqrt{2}} \ln \left(\frac{m_{S_2^{1/3}}^2}{m_{S_1^{1/3}}^2} \right) \sum_{m,a} (m_d)_a V_{am} (\Theta_{im} \Omega_{ja} + \Theta_{jm} \Omega_{ia}) ,$$

GW peaks from LQ model - high-T checked (α)



GW peaks from LQ model - high-T checked (a_1)



DRalgo + hacked CosmoTransitions

[Ekstedt, Schicho, Tenkanen, 2205.08815]

- Viable FOPTs (CoP)

$$(0, \phi_s, 0) \rightarrow (\phi_h, 0, 0) : 3872$$

$$(\phi_h, \phi_s, \phi_r) \rightarrow (\phi'_h, 0, 0) : 13$$

Needs 2 LQs $\leftarrow 1 \lesssim a_1 / \text{GeV} \lesssim 1000$

BBO?

- Low T phase

Colour restoration + EW broken

Colour restoration

Take home message

- Neutrino mass models require BSM physics
- **LISA + future GW detectors can help uncovering its nature**
- **Combination with collider observables for further insights: new scalars (singlet, coloured,...) trilinear couplings, mixing angles**

An aerial photograph of a university campus. The campus features several large, multi-story brick buildings with flat roofs. In the foreground, there is a large parking lot filled with cars, a road with a roundabout, and some green spaces with trees. In the background, a large body of water, possibly a bay or a large lake, stretches across the horizon under a clear blue sky. The text "THANK YOU" is overlaid in the center of the image in a large, white, sans-serif font.

THANK YOU

Current and future experimental facilities will offer new **multi-messenger** channels to search for **New Physics**

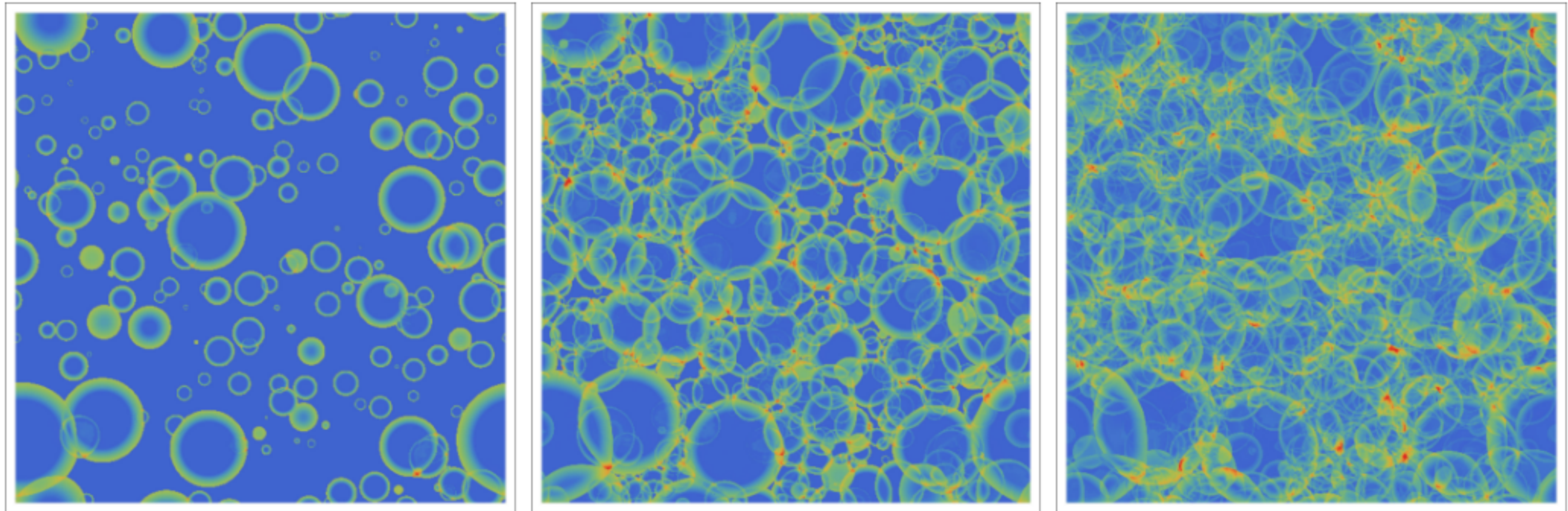
LHC and future colliders

LISA and future GW observatories → SGWB

Basics of Phase Transitions

(Illustration)

✓ First order phase transition (FOPT) example



Credit: JCAP04(2021)014, Jinno, Konstantin, Rubira

FOPTs



The larger the potential energy difference between the true and the false vacuum, the **stronger** the PT

Strength of the PT quantified as:

$$\alpha = \frac{1}{\rho_\gamma} \left[V_i - V_f - \frac{T_*}{4} \left(\frac{\partial V_i}{\partial T} - \frac{\partial V_f}{\partial T} \right) \right]$$

$$\rho_\gamma = g_* \frac{\pi^2}{30} T_*^4$$

Duration of the PT quantified as:

$$\frac{\beta}{H} = T_* \frac{\partial}{\partial T} \left(\frac{\hat{S}_3}{T} \right) \Big|_{T_*}$$

Euclidean action:

$$\hat{S}_3(\hat{\phi}, T) = 4\pi \int_0^\infty dr r^2 \left\{ \frac{1}{2} \left(\frac{d\hat{\phi}}{dr} \right)^2 + V_{\text{eff}}(\hat{\phi}, T) \right\}$$

$$V_{\text{eff}}(T) = V_0 + V_{\text{CW}}^{(1)} + \Delta V(T) + V_{\text{ct}}$$

$\alpha, \beta/H, T_*$ \longrightarrow

calculated from a certain BSM theory, used as inputs to obtain the GW power spectrum

$$h^2 \Omega_{\text{GW}} = h^2 \Omega_{\text{GW}}^{\text{peak}} \left(\frac{4}{7}\right)^{-\frac{7}{2}} \left(\frac{f}{f_{\text{peak}}}\right)^3 \left[1 + \frac{3}{4} \left(\frac{f}{f_{\text{peak}}}\right)\right]^{-\frac{7}{2}}$$

Peak amplitude

Spectral function

$$h^2 \Omega_{\text{GW}}^{\text{peak}}(f_{\text{peak}}) = 7.835 \times 10^{-17} f_{\text{peak}}^{-2} \left(\frac{100}{g_*}\right)^{2/3} \left(\frac{T_*}{100}\right)^2 \frac{K^{\frac{3}{2}}}{c_s} \quad \text{for } H\tau_{\text{sh}} = \frac{2}{\sqrt{3}} \frac{HR}{K^{1/2}} < 1$$

$$h^2 \Omega_{\text{GW}}^{\text{peak}}(f_{\text{peak}}) = 7.835 \times 10^{-17} f_{\text{peak}}^{-2} \left(\frac{100}{g_*}\right)^{2/3} \left(\frac{T_*}{100}\right)^2 \frac{K^2}{c_s^2} \quad \text{for } H\tau_{\text{sh}} = \frac{2}{\sqrt{3}} \frac{HR}{K^{1/2}} \simeq 1,$$

$$f_{\text{peak}} = 26 \times 10^{-6} \left(\frac{1}{HR}\right) \left(\frac{T_*}{100}\right) \left(\frac{g_*}{100 \text{ GeV}}\right)^{\frac{1}{6}} \text{ Hz}$$

$$HR = \frac{H}{\beta} (8\pi)^{\frac{1}{3}} \max(v_b, c_s)$$

$$K = \frac{\kappa\alpha}{1 + \alpha}$$

We use the templates for SW peak in [Caprini et al. JCAP 03 (2020) 024]

Inverted equations

$$\lambda_{\sigma h} = \frac{\tan(2\alpha_h) (M_{hh}^2 - M_{\sigma\sigma}^2)}{2v_h v_\sigma} - \frac{\delta_2 v_h^2 + \delta_4 v_\sigma^2}{\Lambda^2},$$

$$\lambda_\sigma = - \frac{2A(\text{Br})v_h^3 v_\sigma \csc(\alpha_h) + \Lambda^2 \sec(2\alpha_h) (M_{\sigma\sigma}^2 - M_{hh}^2) + \Lambda^2 (-M_{hh}^2 + M_{\sigma\sigma}^2 - 2M_{\sigma\sigma}^2 v_\sigma)}{4\Lambda^2 (v_\sigma - 1) v_\sigma^2} + \frac{\delta_4 v_h^2}{2\Lambda^2},$$

$$\lambda_h = \frac{1}{2} \left(\frac{M_{hh}^2}{v_h^2} - \frac{3\delta_0 v_h^2 + \delta_2 v_\sigma^2}{\Lambda^2} \right),$$

$$\delta_6 = \frac{2A(\text{Br})v_h^3 v_\sigma \csc(\alpha_h) - \Lambda^2 (\sec(2\alpha_h) (M_{hh}^2 - M_{\sigma\sigma}^2) + M_{hh}^2 + M_{\sigma\sigma}^2)}{6(v_\sigma - 1)v_\sigma^4},$$

$$A(\text{Br}) \equiv \pm 4\sqrt{2\pi} \left(1 - 4\frac{m_J^2}{m_h^2}\right) m_h^{3/2} \frac{\Lambda^2}{v_h^3} \sqrt{\frac{\text{Br}(h \rightarrow JJ)\Gamma(h \rightarrow \text{SM})}{[1 - \text{Br}(h \rightarrow JJ)](m_h^2 - 4m_J^2)}}.$$

$$M_{hh,\sigma\sigma}^2 = \frac{1}{2} [m_{h_1}^2 + m_{h_2}^2 \pm (m_{h_1}^2 - m_{h_2}^2) \cos(2\alpha_h)] \quad \text{and} \quad M_{\sigma h}^2 = \frac{1}{2} (m_{h_1}^2 - m_{h_2}^2) \sin(2\alpha_h)$$

Phenomenological inputs

Invisible Higgs decays limit : $\text{Br}(h \rightarrow JJ) < 0.18$ Used as input
[Phys. Rev. D 105 (2022) 9 092007]

Scalar mixing angle limit: $|\sin \alpha_h| < 0.23$ Used as input
[Papaefstathiou, Robens, White, 2207.00043]

Also used as inputs: $m_{h_1} = 125.09 \text{ GeV}, m_{h_2}, m_J, v_h, v_\sigma, \Lambda, \delta_2, \delta_4$

$$\lambda_{JJh_1}^{(0)} = \frac{v_h}{\Lambda^2} \left[(v_h^2 \delta_2 + v_\sigma^2 \delta_4 + \Lambda^2 \lambda_{\sigma h}) \cos \alpha_h + v_\sigma (v_h^2 \delta_4 + 3v_\sigma^2 \delta_6 + 2\Lambda^2 \lambda_\sigma) \sin \alpha_h \right]$$

Which seesaw model?

	L^i	ν_R^i	S^i	σ	H	Model
$U(1)_L$	1	1	\times	-2	0	T1S
	1	1	0	-1	0	IS
	1	1	-1	2	0	EIS

$$M_\nu^{\text{T1S}} = \begin{pmatrix} 0 & \frac{v_h}{\sqrt{2}} \mathbf{y}_\nu \\ \frac{v_h}{\sqrt{2}} \mathbf{y}_\nu & \frac{v_\sigma}{\sqrt{2}} \mathbf{y}_\sigma \end{pmatrix}, \quad M_\nu^{\text{IS}} = \begin{pmatrix} 0 & \frac{v_h}{\sqrt{2}} \mathbf{y}_\nu & 0 \\ \frac{v_h}{\sqrt{2}} \mathbf{y}_\nu & 0 & \frac{v_\sigma}{\sqrt{2}} \mathbf{y}_\sigma \\ 0 & \frac{v_\sigma}{\sqrt{2}} \mathbf{y}_\sigma & \Lambda \end{pmatrix}, \quad M_\nu^{\text{EIS}} = \begin{pmatrix} 0 & \frac{v_h}{\sqrt{2}} \mathbf{y}_\nu & 0 \\ \frac{v_h}{\sqrt{2}} \mathbf{y}_\nu & \frac{v_\sigma}{\sqrt{2}} \mathbf{y}'_\sigma & \Lambda \\ 0 & \Lambda & \frac{v_\sigma}{\sqrt{2}} \mathbf{y}_\sigma \end{pmatrix}.$$

$$m_\nu^{\text{T1S}} \approx \frac{1}{\sqrt{2}} \frac{\mathbf{y}_\nu^2}{\mathbf{y}_\sigma} \frac{v_h^2}{v_\sigma}, \quad m_\nu^{\text{IS}} \approx \frac{\mathbf{y}_\nu^2}{\mathbf{y}_\sigma^2} \frac{\Lambda v_h^2}{v_\sigma^2}, \quad m_\nu^{\text{EIS}} \approx \frac{\mathbf{y}_\nu^2 \mathbf{y}_\sigma}{2\sqrt{2}} \frac{v_h^2 v_\sigma}{\Lambda^2}$$

Thermal effective potential

$$V_{\text{eff}}(T) = V_0 + V_{\text{CW}}^{(1)} + \Delta V(T) + V_{\text{ct}}$$

$$V_{\text{CW}}^{(1)} = \sum_i (-1)^{F_i} n_i \frac{m_i^4(\phi_\alpha)}{64\pi^2} \left(\log \left[\frac{m_i^2(\phi_\alpha)}{Q^2} \right] - c_i \right)$$

$$\Delta V(T) = \frac{T^4}{2\pi^2} \left\{ \sum_b n_b J_B \left[\frac{m_b^2(\phi_\alpha)}{T^2} \right] - \sum_f n_f J_F \left[\frac{m_f^2(\phi_\alpha)}{T^2} \right] \right\}$$

$$m_i^2 \rightarrow m_i^2 + c_i T^2$$

$$\left\langle \frac{\partial V_{\text{ct}}}{\partial \phi_\alpha} \right\rangle = \left\langle -\frac{\partial V_{\text{CW}}^{(1)}}{\partial \phi_\alpha} \right\rangle \quad \left\langle \frac{\partial^2 V_{\text{ct}}}{\partial \phi_\alpha \partial \phi_\beta} \right\rangle = \left\langle -\frac{\partial^2 V_{\text{CW}}^{(1)}}{\partial \phi_\alpha \partial \phi_\beta} \right\rangle$$

$$n_s = 6, \quad n_{A_L} = 1$$

$$n_W = 6, \quad n_Z = 3, \quad n_\gamma = 2$$

$$n_{u,d,c,s,t,b} = 12, \quad n_{e,\mu,\tau} = 4, \quad n_{\nu_{1,2,3}} = n_{N_{1,2,3}^\pm} = 2$$

$$J_{B/F}(y^2) = \int_0^\infty dx x^2 \log \left(1 \mp \exp[-\sqrt{x^2 + y^2}] \right)$$

Counterterms are fixed such that the \rightarrow **T=0 minimum conditions and physical masses are preserved at 1-loop**

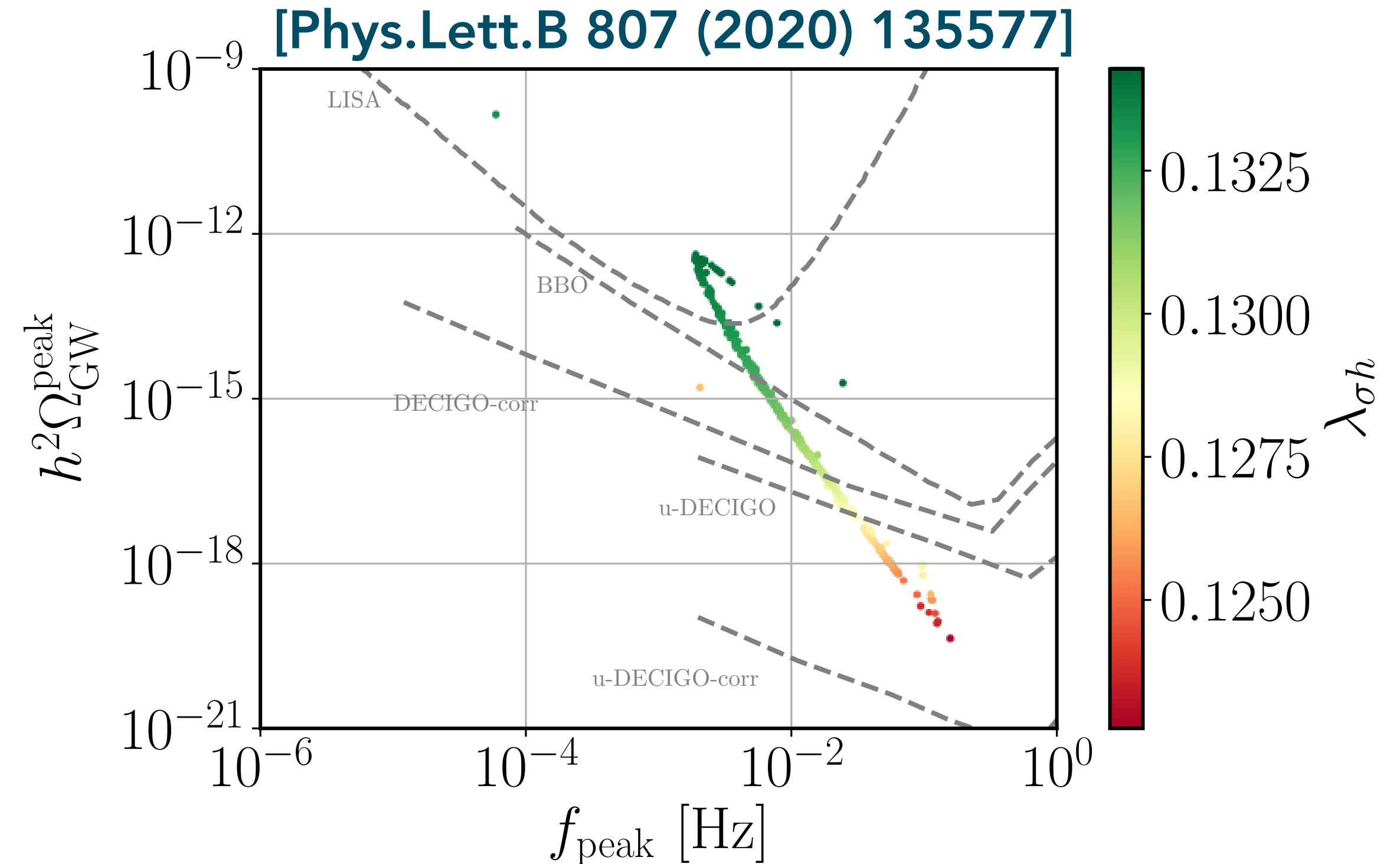
Minimal scalar sector

$$V_{\text{SM}}(H) = \mu_h^2 H^\dagger H + \lambda_h (H^\dagger H)^2,$$

$$V(H, \sigma) = \mu_\sigma^2 \sigma^\dagger \sigma + \lambda_\sigma (\sigma^\dagger \sigma)^2 + \lambda_{\sigma h} H^\dagger H \sigma^\dagger \sigma,$$

$$V_{\text{soft}}(\sigma) = \frac{1}{2} \mu_b^2 (\sigma^2 + \sigma^{*2}).$$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \omega_1 + i\omega_2 \\ \phi_h + h + i\eta \end{pmatrix}, \quad \sigma = \frac{1}{\sqrt{2}} (\phi_\sigma + h' + iJ)$$



✓ The portal coupling size that induces SFOPTs is too large for invisible Higgs decays

✓ Only viable for Majoron $O(100 \text{ GeV} - 1 \text{ TeV})$

Minimal scalar sector

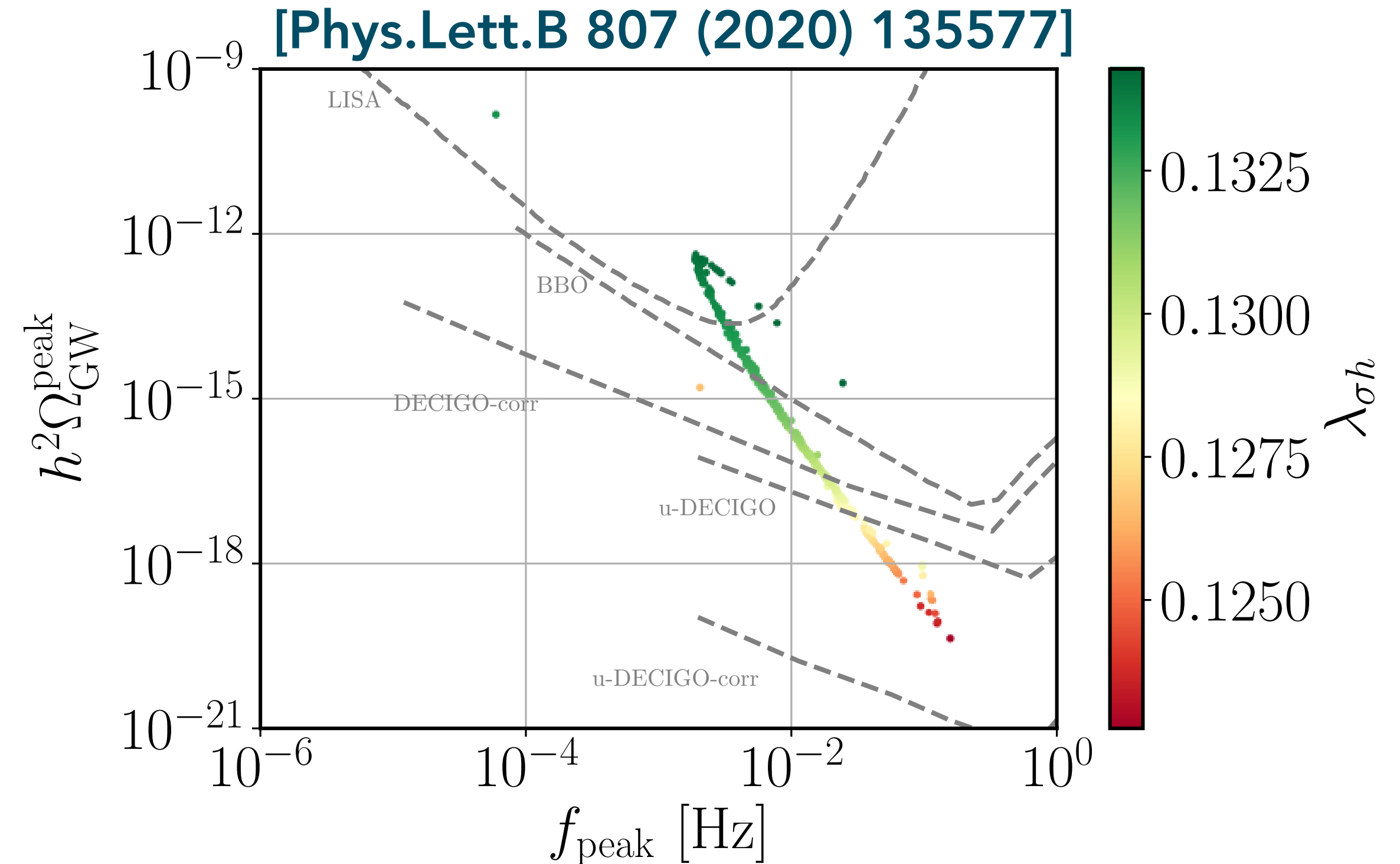
$$\text{Br}(h_1 \rightarrow JJ) = \frac{\Gamma(h_1 \rightarrow JJ)}{\Gamma(h_1 \rightarrow JJ) + \Gamma(h_1 \rightarrow \text{SM})} < 0.18$$

[CMS - Phys. Rev. D 105 (2022) 9 092007]

$$\Gamma(h_1 \rightarrow JJ) = \frac{1}{32\pi} \frac{\left(\lambda_{JJh_1}^{(0)}\right)^2}{m_{h_1}} \sqrt{1 - 4 \frac{m_J^2}{m_{h_1}^2}}$$

$$\lambda_{JJh_1}^{(0)} = \frac{1}{2} v_h \lambda_{\sigma h} \cos \alpha_h$$

$$\lambda_{\sigma h} \lesssim \mathcal{O}(0.01)$$



✓ The portal coupling size that induces SFOPTs is too large for invisible Higgs decays

✓ Only viable for Majoron $\mathcal{O}(100 \text{ GeV} - 1 \text{ TeV})$

Neutrino sector revisited

$$\mathcal{L}_\nu^{\text{EIS}} = y_\nu^{ij} \bar{L}_i \tilde{H} \nu_{Rj} + y_\sigma^{ij} \bar{S}_i^c S_j \sigma + y_\sigma^{\prime ij} \bar{\nu}_{Ri}^c \nu_{Rj} \sigma^* + \Lambda^{ij} \bar{\nu}_{Ri}^c S_j + \text{h.c.}$$

$$M_\nu^{\text{EIS}} = \begin{pmatrix} 0 & \frac{v_h}{\sqrt{2}} \mathbf{y}_\nu & 0 \\ \frac{v_h}{\sqrt{2}} \mathbf{y}_\nu & \frac{v_\sigma}{\sqrt{2}} \mathbf{y}'_\sigma & \Lambda \\ 0 & \Lambda & \frac{v_\sigma}{\sqrt{2}} \mathbf{y}_\sigma \end{pmatrix}$$

$$m_\nu^{\text{EIS}} \approx \frac{\mathbf{y}_\nu^2 \mathbf{y}_\sigma}{2\sqrt{2}} \frac{v_h^2 v_\sigma}{\Lambda^2}$$

3 light active neutrinos

$$m_{N^\pm} \approx \Lambda \pm \frac{v_\sigma}{2\sqrt{2}} (\mathbf{y}_\sigma + \mathbf{y}'_\sigma)$$

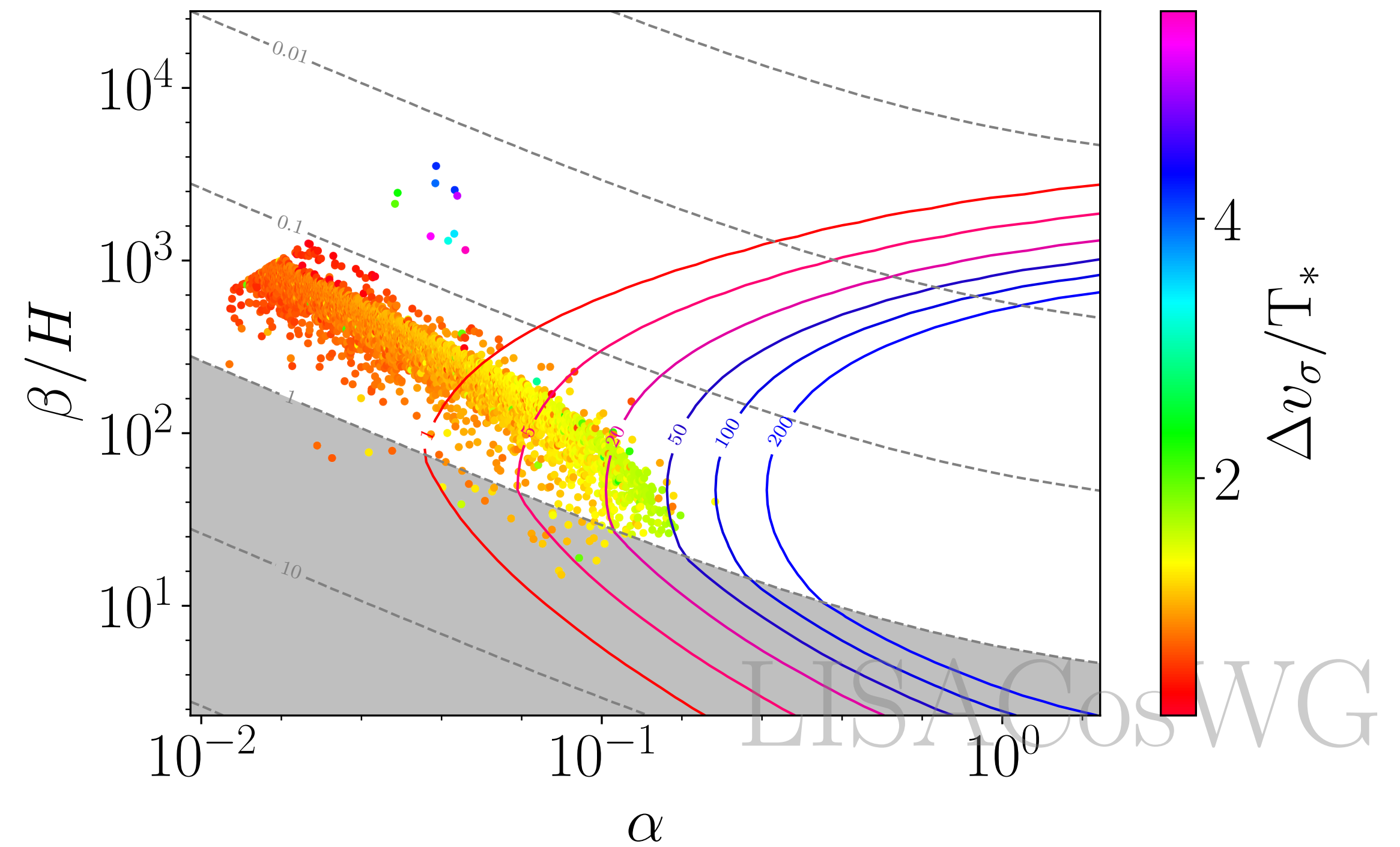
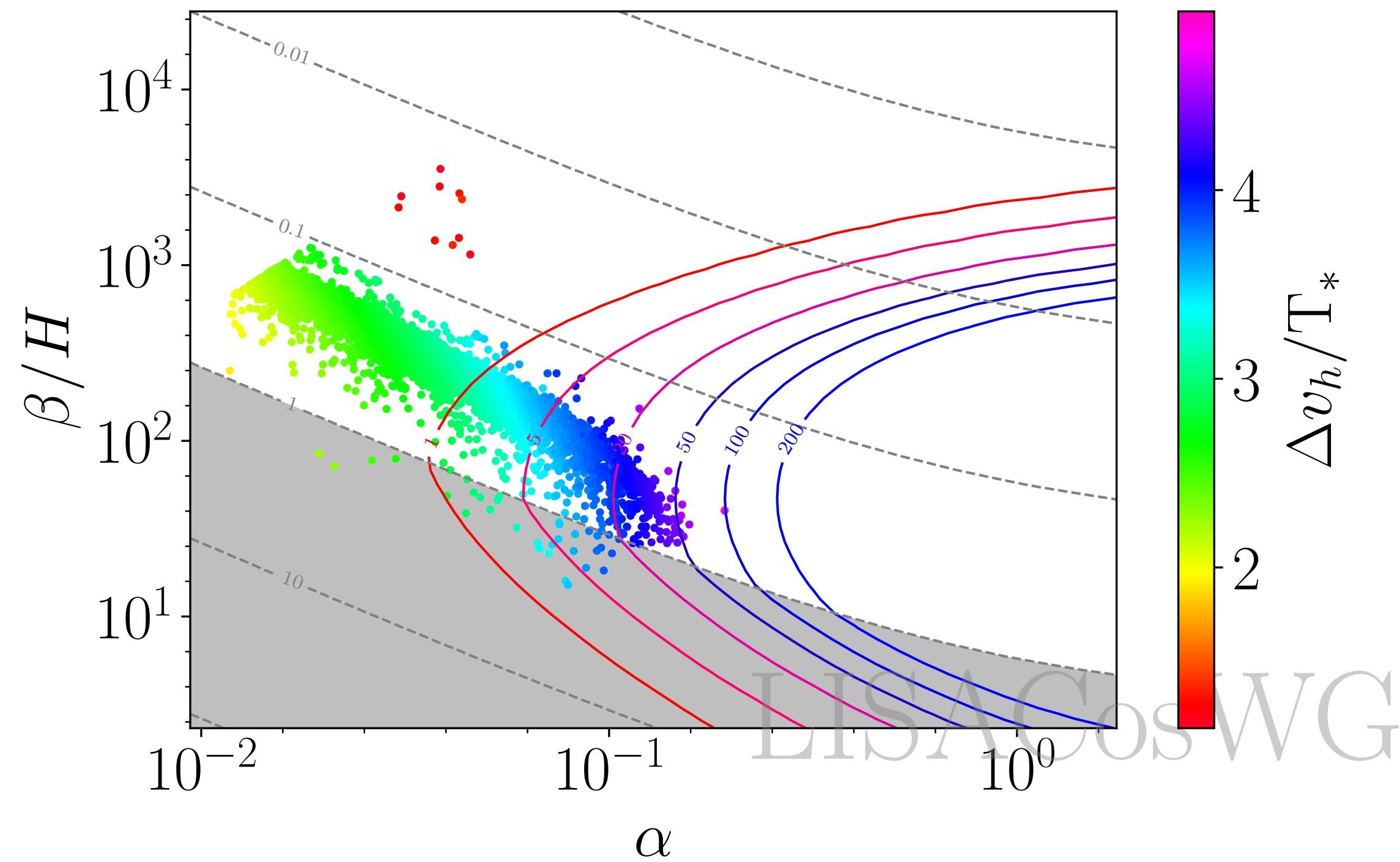
6 heavy neutrinos

Use normal ordering masses as input to obtain

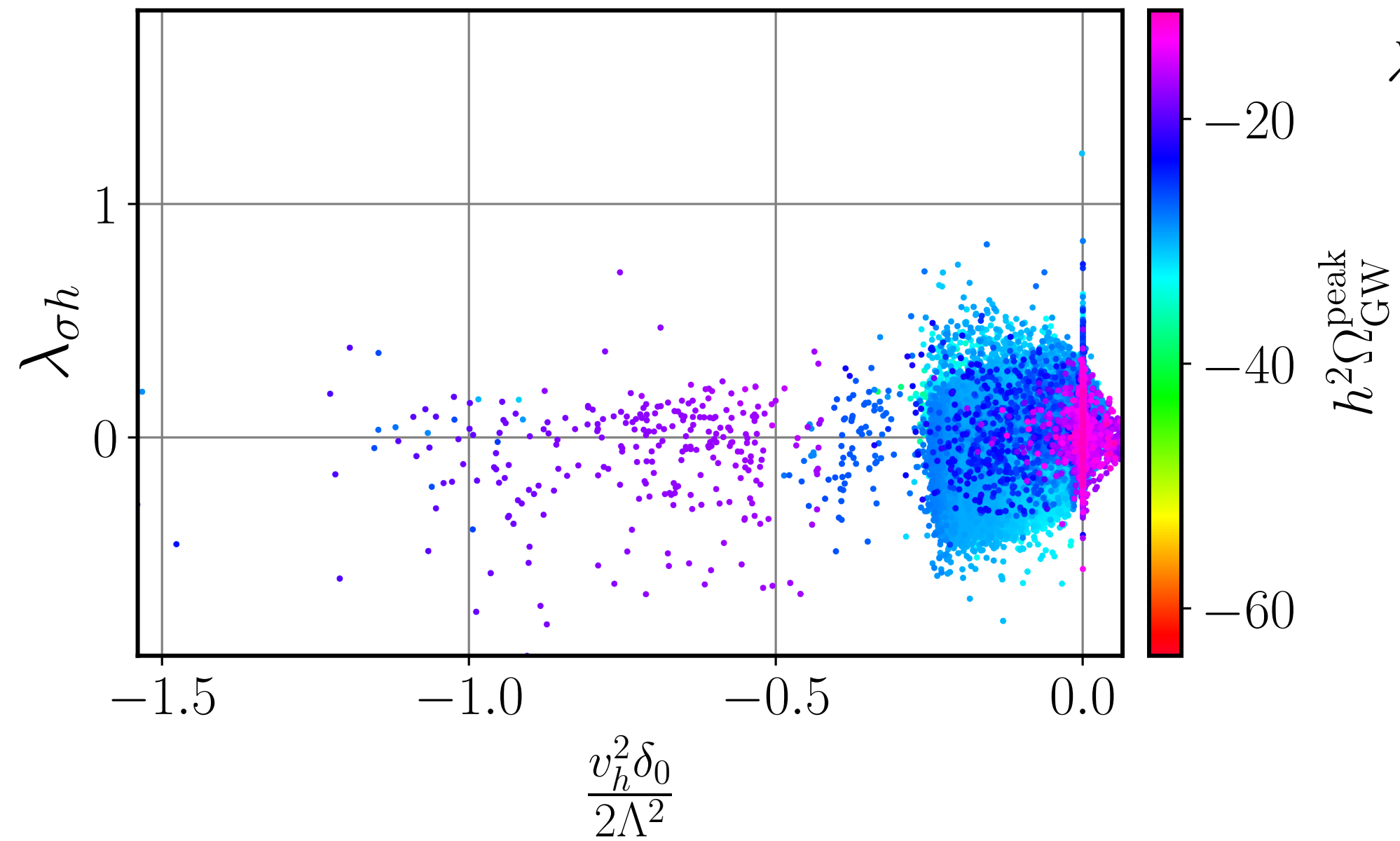
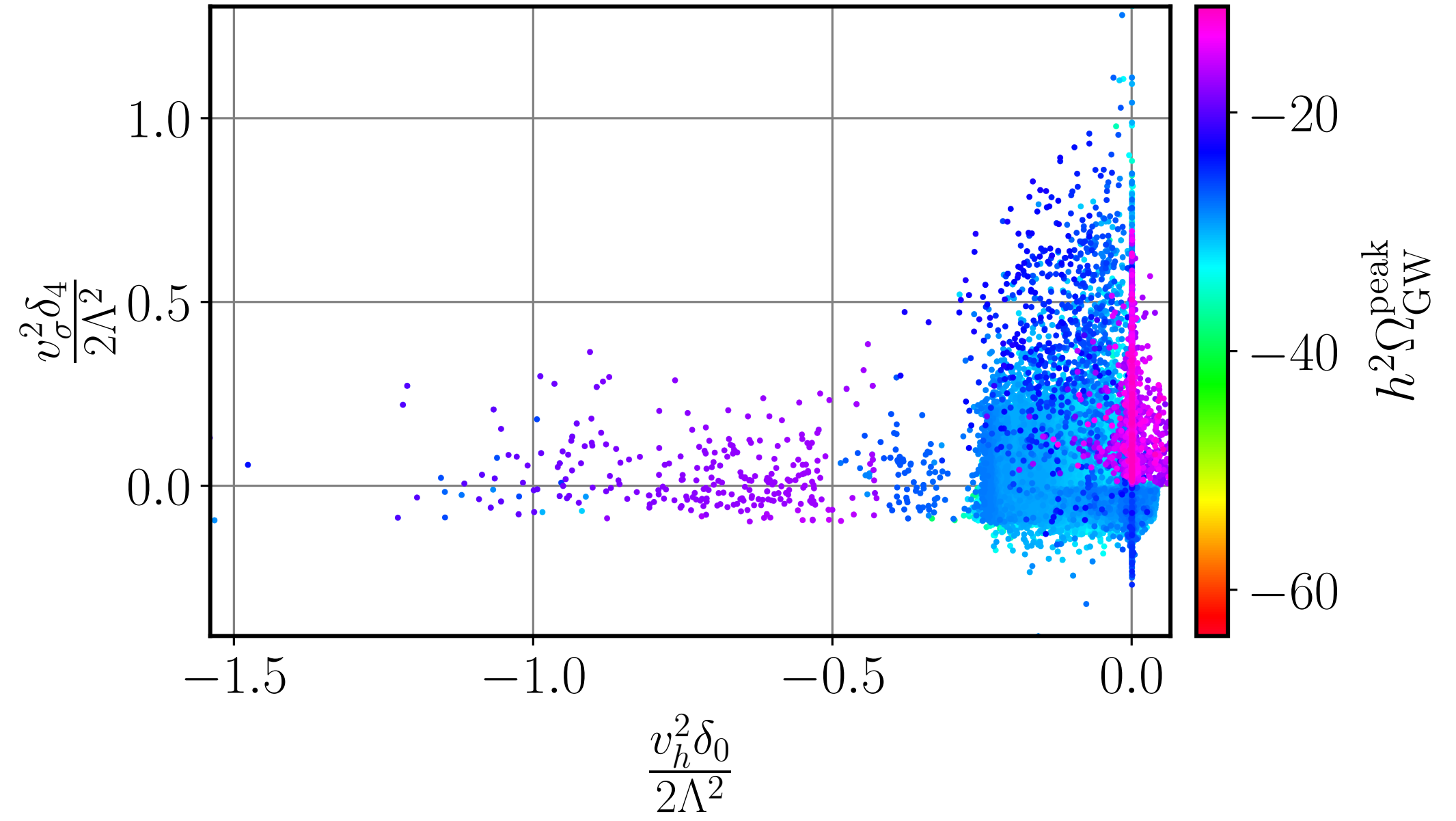
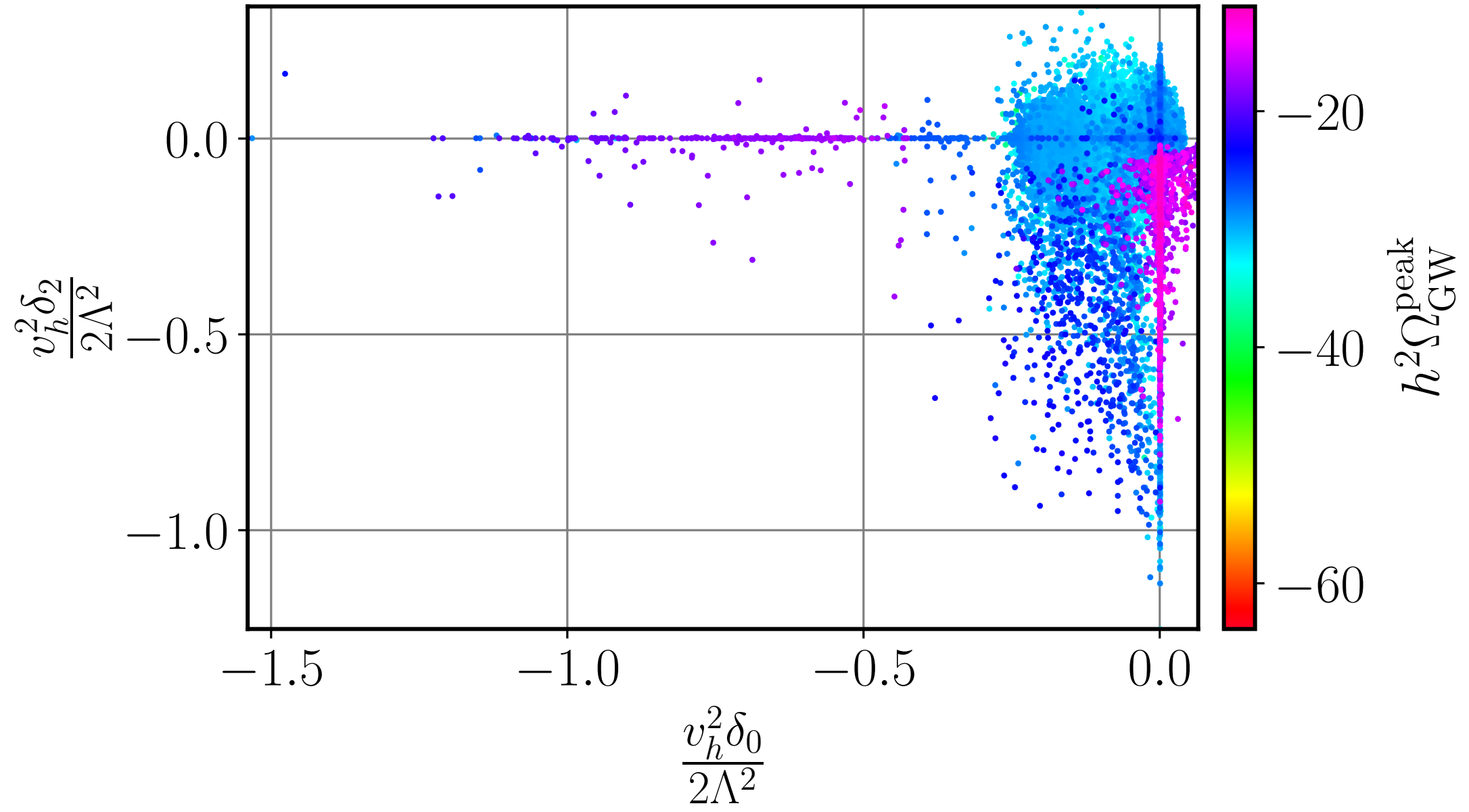
$$y_\sigma^i = 2\sqrt{2} \frac{m_{\nu_i} \Lambda^2}{v_h^2 v_\sigma y_{\nu_i}^2}$$

$$\Delta v_\phi = |v_\phi^f - v_\phi^i|, \quad \phi = h, \sigma$$

Used PTPlot for SNR [JCAP 2003 (2020) 024]

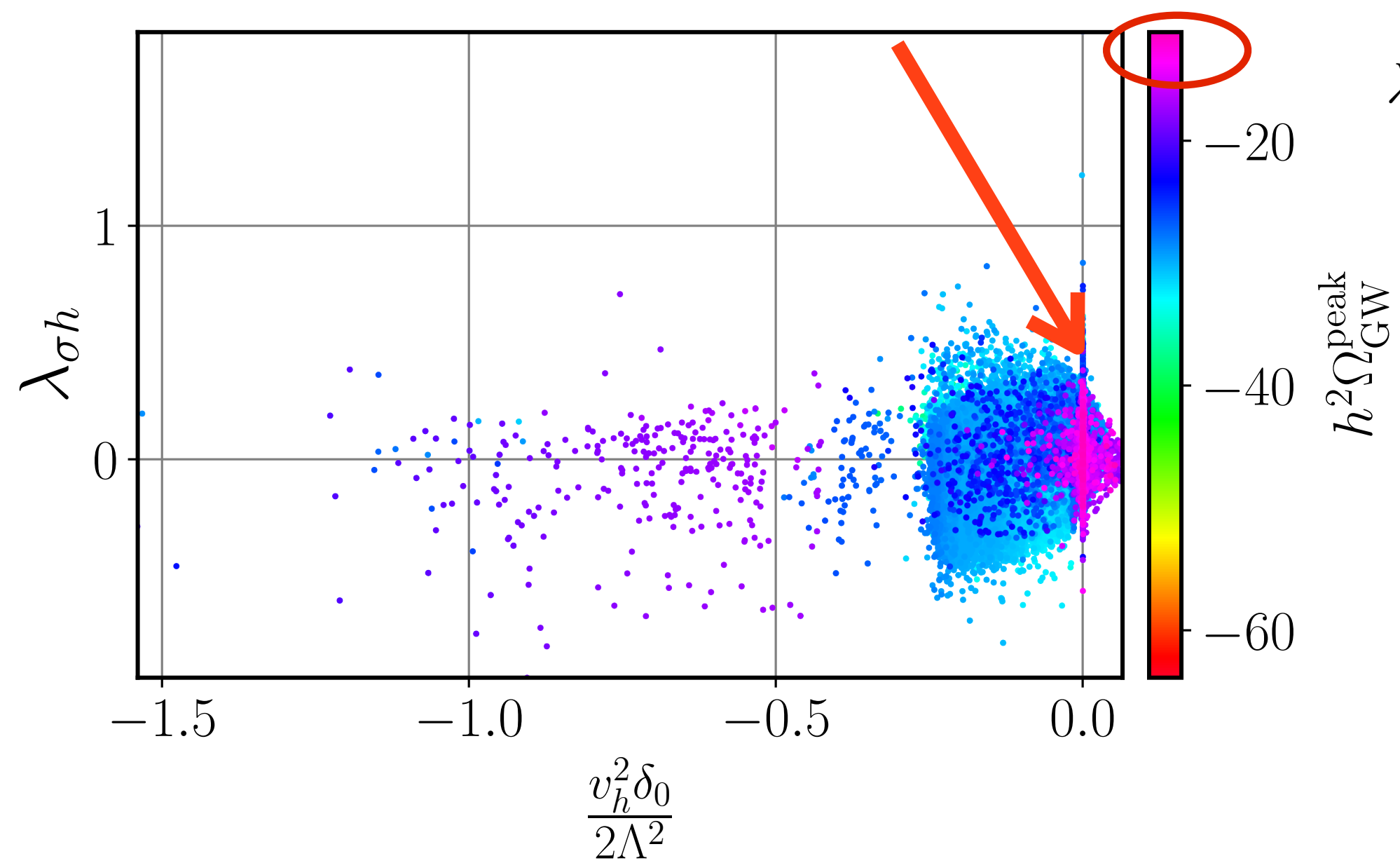
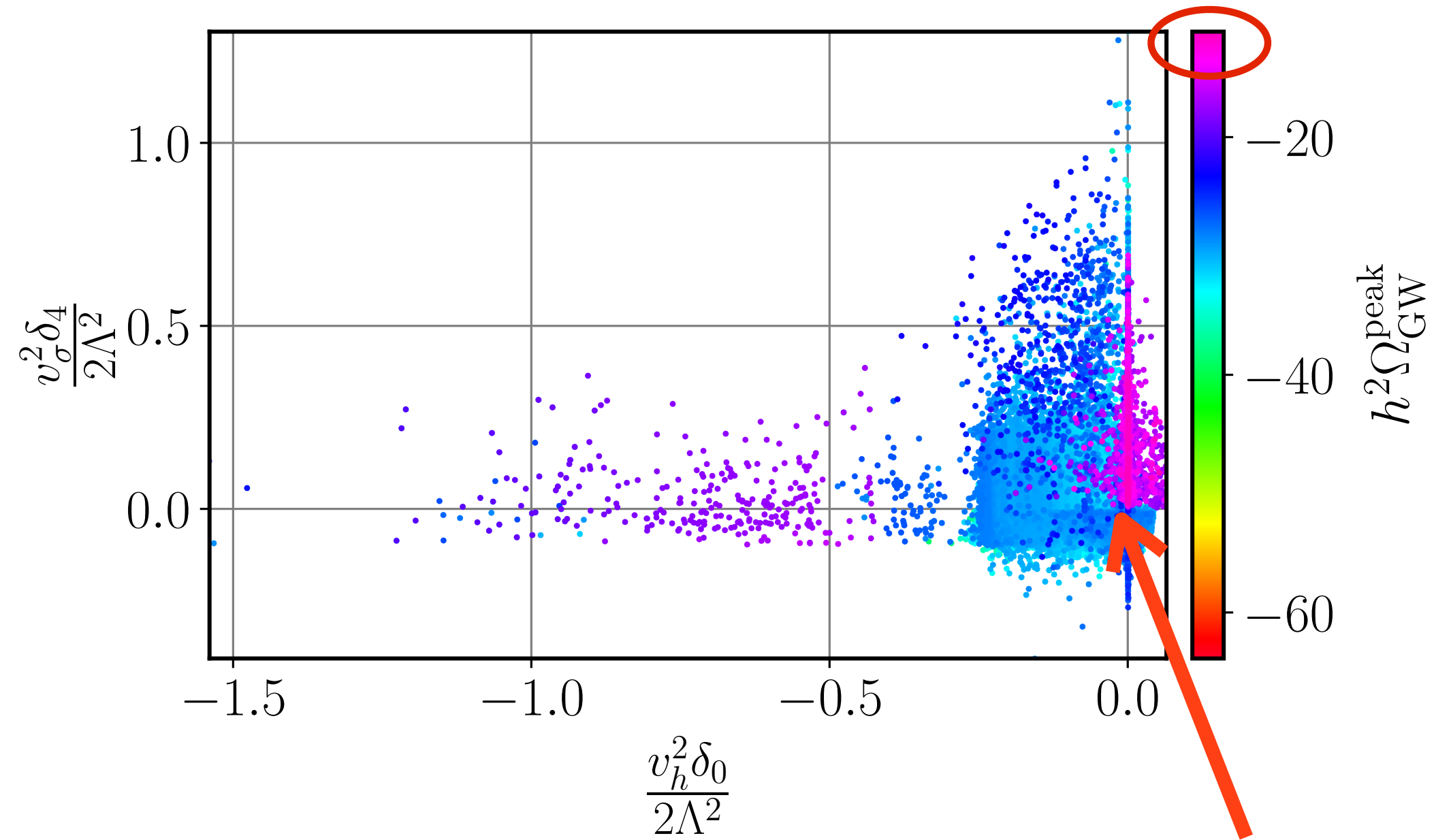
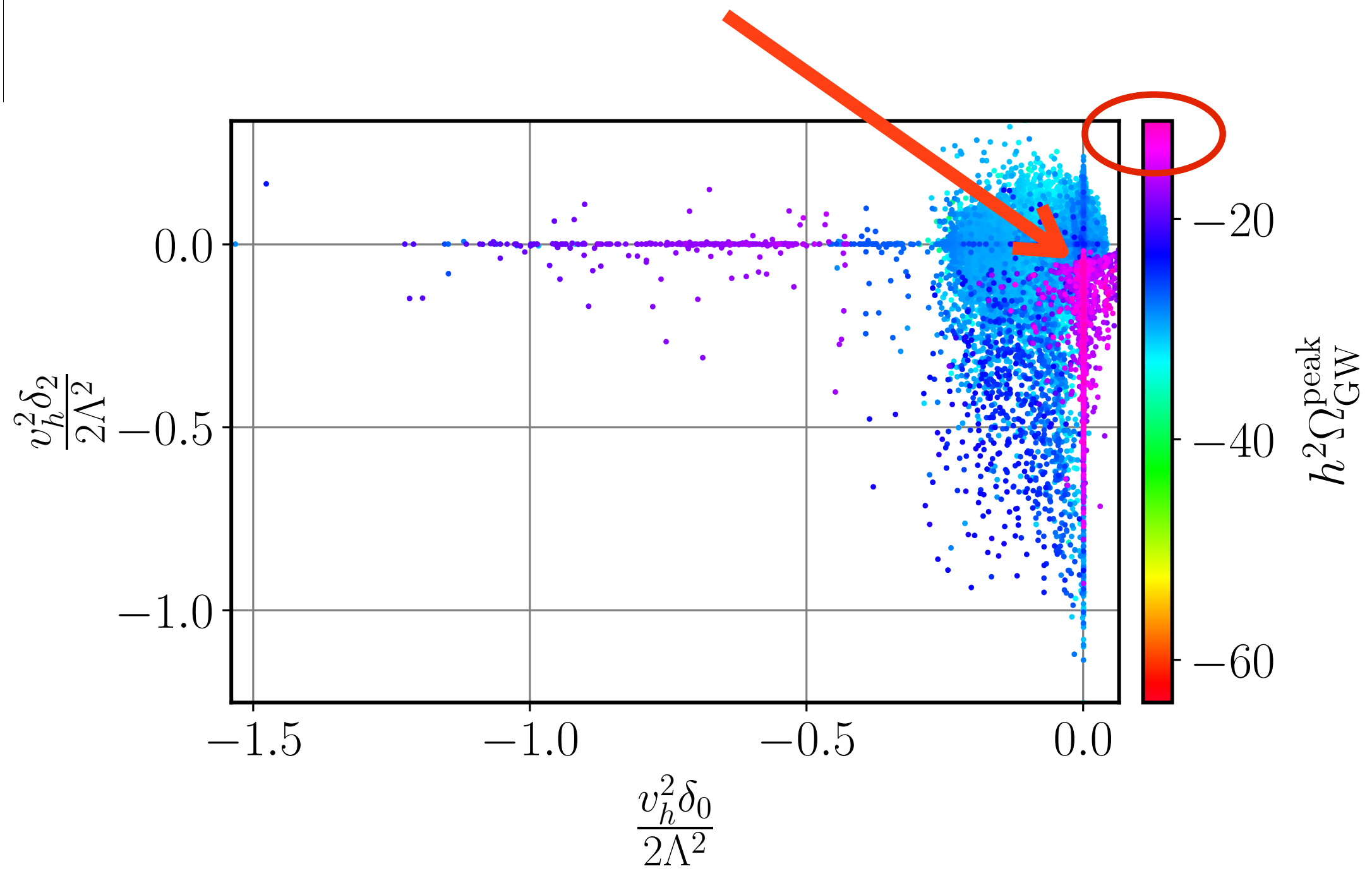


Both order parameters must be large for observable SGWB



$$\lambda_{JJh_1}^{(0)} = \frac{v_h}{\Lambda^2} [(v_h^2 \delta_2 + v_\sigma^2 \delta_4 + \Lambda^2 \lambda_{\sigma h}) \cos \alpha_h + v_\sigma (v_h^2 \delta_4 + 3v_\sigma^2 \delta_6 + 2\Lambda^2 \lambda_\sigma) \sin \alpha_h]$$

$< \mathcal{O}(0.01)$



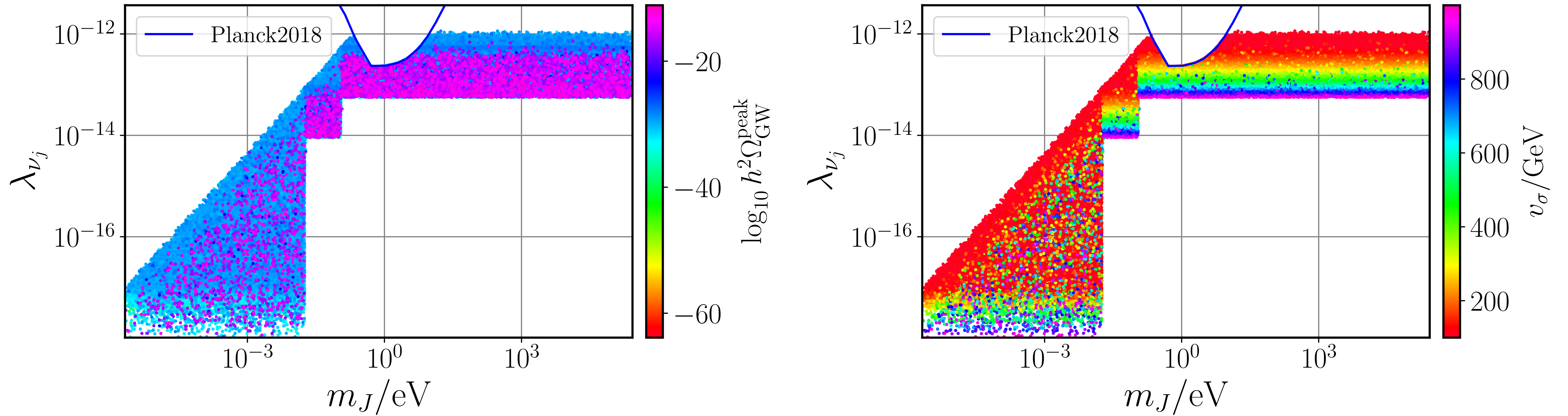
$$\lambda_{JJh_1}^{(0)} = \frac{v_h}{\Lambda^2} [(v_h^2 \delta_2 + v_\sigma^2 \delta_4 + \Lambda^2 \lambda_{\sigma h}) \cos \alpha_h + v_\sigma (v_h^2 \delta_4 + 3v_\sigma^2 \delta_6 + 2\Lambda^2 \lambda_\sigma) \sin \alpha_h]$$

$< \mathcal{O}(0.01)$

✓ **LISA region favours small δ_0**

CMB constraints

[Planck Collaboration, 1807.06209, 1907.12875]



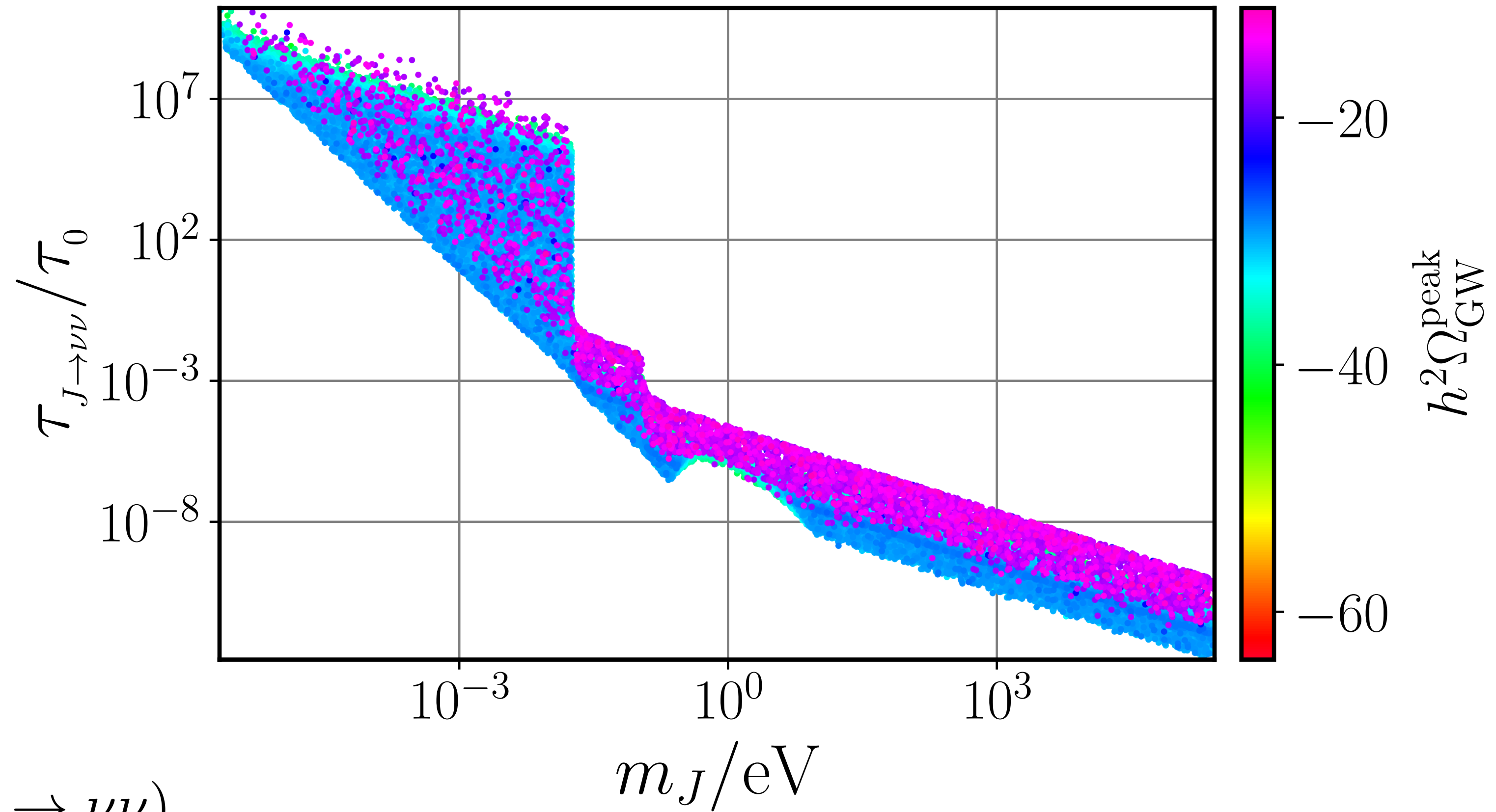
✓ **Planck2018 marginally constrains magenta band (LISA)**

[Escudero, White, EPJC 80 (2020) 4 294]

$$\mathcal{L} = \frac{i}{2} \lambda_{\nu_j} J \bar{\nu}_j \gamma_5 \nu_j$$

$$\lambda_{\nu_j} \equiv m_j / v_\sigma$$

Decaying Majorons



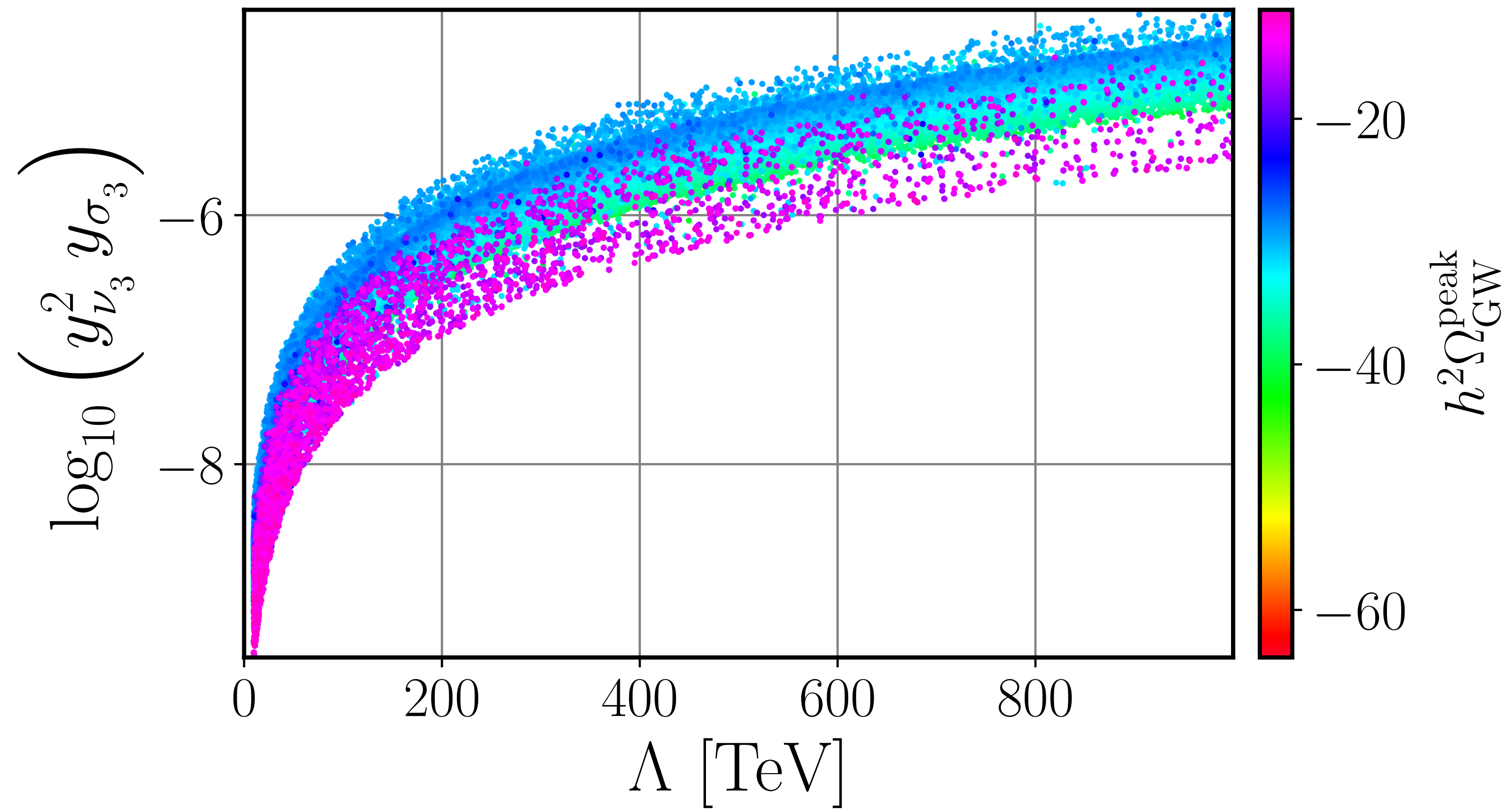
$$\tau_{J \rightarrow \nu\nu} = \Gamma^{-1}(J \rightarrow \nu\nu)$$

$$\Gamma(J \rightarrow \nu\nu) = \frac{m_J}{16\pi v_\sigma^2} \sum_i \left(m_{\nu_i}^2 \sqrt{1 - \frac{4m_{\nu_i}^2}{m_J^2}} \right)$$

$$\tau_0 = 13.787 \text{ Gyr}$$

[Nikolic, Kulkani, Pradler, EPJC 82 (2022) 7 650]

Seesaw effect vs FOPTs



$$m_{\nu}^{\text{EIS}} \approx \frac{y_{\nu}^2 y_{\sigma}}{2\sqrt{2}} \frac{v_h^2 v_{\sigma}}{\Lambda^2}$$

$$m_{N^{\pm}} \approx \Lambda \pm \frac{v_{\sigma}}{2\sqrt{2}} (y_{\sigma} + y'_{\sigma})$$

Two LQ model

SM + Singlet leptoquark + Doublet leptoquark

$$S_1 \sim (\bar{\mathbf{3}}, \mathbf{1})_{1/3}$$

$$\tilde{R}_2 \sim (\mathbf{3}, \mathbf{2})_{1/6}$$

This field content has an UV inspiration...

$[\text{SU}(3)]^3 \times \text{SU}(2)_F \times \text{U}(1)_F \longrightarrow$ Flavoured Trinification

[APM, Pasechnik, Porod, Eur. Phys. J. C 80, (2020) 12, 1162]

$$L = \begin{pmatrix} H & \ell_L \\ \ell_R & \phi \end{pmatrix} \quad Q_L = \begin{pmatrix} q_L & D_L \\ \tilde{R}_2 & \end{pmatrix} \quad Q_R = \begin{pmatrix} q_R^c & D_R^c \\ S_1 & \end{pmatrix}^T$$

This FT contains an emergent \mathbb{Z}_2 B-parity

$$\mathbb{P}_B = (-1)^{3B+2S}$$

	L	\tilde{L}	Q_L	\tilde{Q}_L	Q_R	\tilde{Q}_R
P_B	-	+	+	-	+	-

\tilde{R}_2

S_1

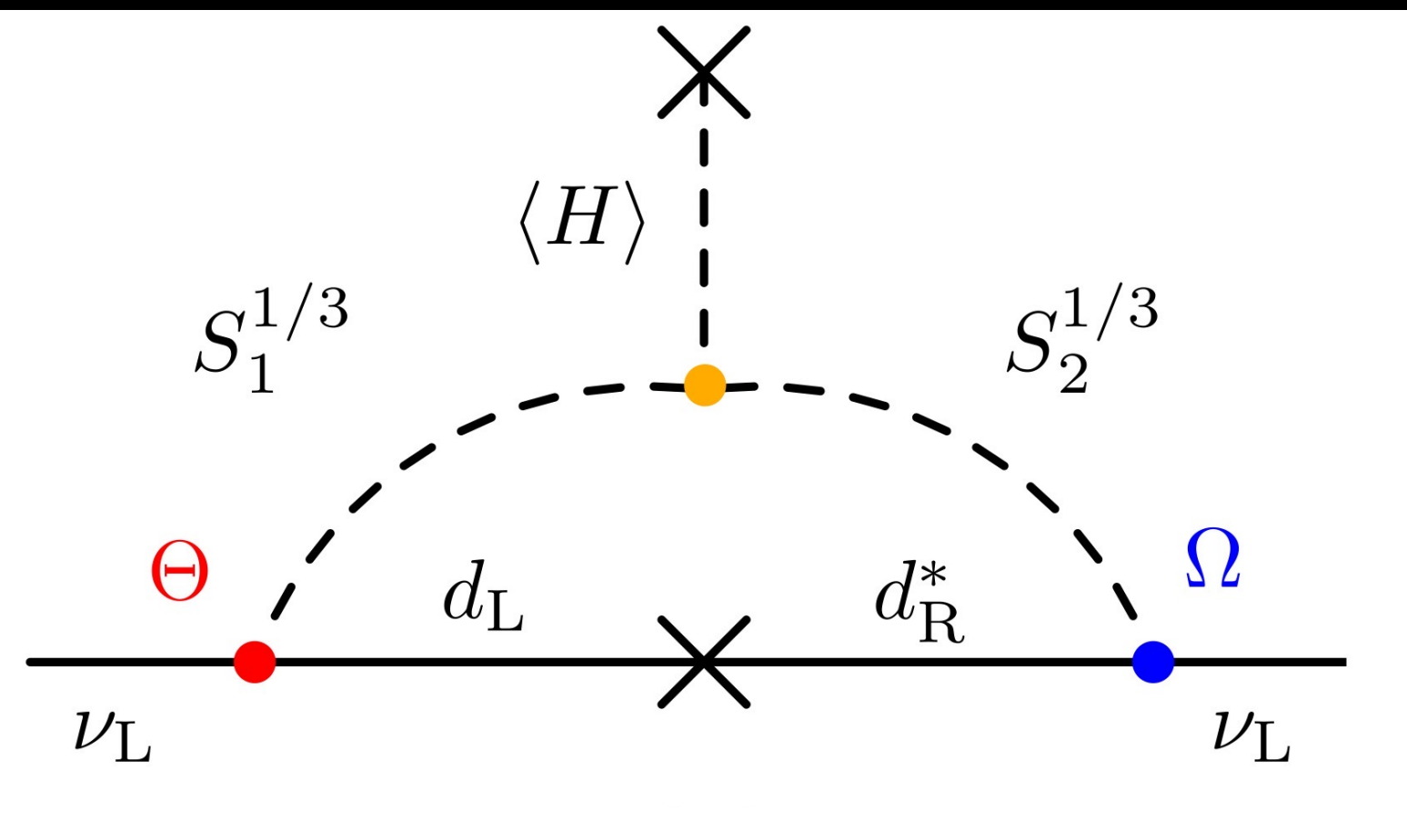
- Forbids di-quark interactions
- Only allows leptoquark interactions

$$\begin{matrix} - & + & - & - & - & + \\ L & Q_L & \tilde{Q}_R & + & L & \tilde{Q}_L & Q_R \end{matrix}$$

- Proton is stable

Neutrino Masses

$$\mathcal{L}_Y = \Theta_{ij} \bar{Q}_j^c L_i S + \Omega_{ij} \bar{L}_i d_j R^\dagger + \Upsilon_{ij} \bar{u}_j e_i S^\dagger + \text{h.c.}$$



- **And an exhaustive flavour analysis**
[Gonçalves, APM, Pasechnik, Porod, 2206.01674]

- [40] I. Doršner, S. Fajfer, and N. Košnik, Eur. Phys. J. C **77**, 417 (2017), 1701.08322.
- [41] D. Aristizabal Sierra, M. Hirsch, and S. G. Kovalenko, Phys. Rev. D **77**, 055011 (2008), 0710.5699.
- [42] D. Zhang, JHEP **07**, 069 (2021), 2105.08670.
- [43] H. Päs and E. Schumacher, Phys. Rev. D **92**, 114025 (2015), 1510.08757.
- [44] Y. Cai, J. Herrero-García, M. A. Schmidt, A. Vicente, and R. R. Volkas, Front. in Phys. **5**, 63 (2017), 1706.08524

$$(M_\nu)_{ij} = \frac{3}{16\pi^2(m_{S_2^{1/3}}^2 - m_{S_1^{1/3}}^2)} \frac{va_1}{\sqrt{2}} \ln \left(\frac{m_{S_2^{1/3}}^2}{m_{S_1^{1/3}}^2} \right) \sum_{m,a} (m_d)_a V_{am} (\Theta_{im} \Omega_{ja} + \Theta_{jm} \Omega_{ia}),$$

Scalar sector

- LQ scalar potential

$$\begin{aligned} V_{LQ} = & \frac{1}{2} (\mu_H H^\dagger H + \mu_S S^\dagger S + \mu_R R^\dagger R) \\ & + \frac{1}{4} (\lambda_H (H^\dagger H)^2 + \lambda_S (S^\dagger S)^2 + \lambda_R (R^\dagger R)^2) \\ & + \frac{1}{4} (g_{HS} (H^\dagger H) (S^\dagger S) + g_{HR} (H^\dagger H) (R^\dagger R) + g'_{HR} (H^\dagger R) (R^\dagger H) + g_{RS} (R^\dagger R) (S^\dagger S)) \\ & + c_3 R^\dagger S H \end{aligned}$$

- ✓ Consider the possibility of LQ VEVs at **finite T**
- ✓ Classify all possible FOPTs and determine SGWB

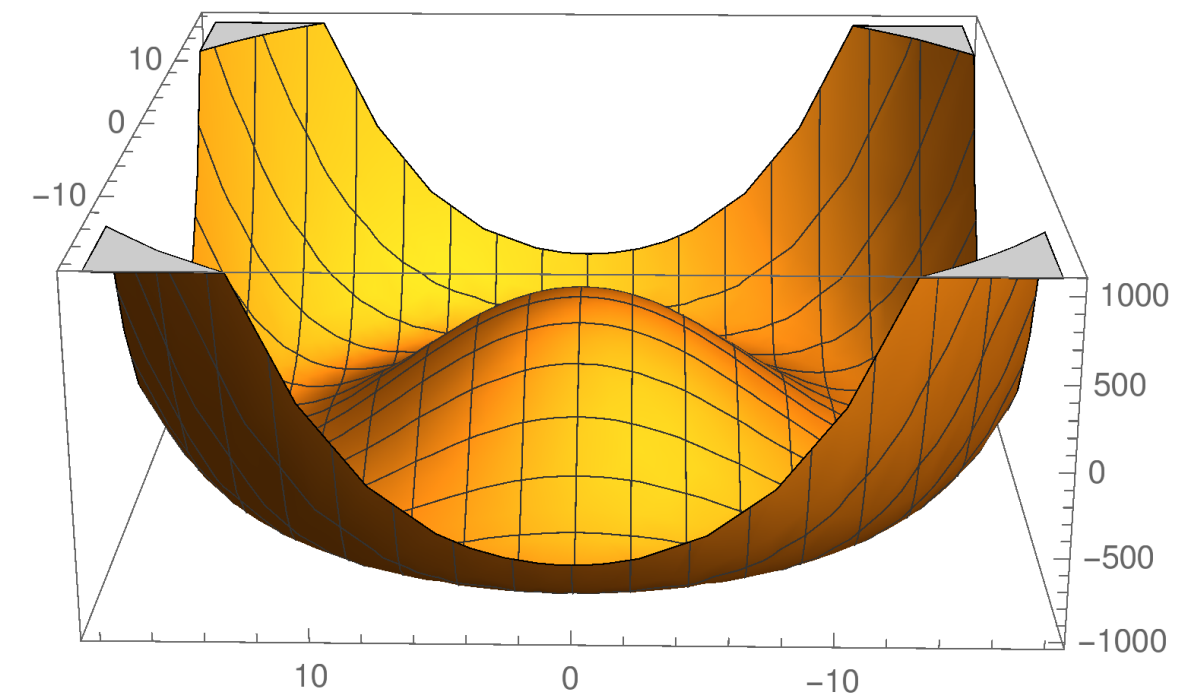
Basics of Phase Transitions

(Illustration)

Consider the scalar potential:

$$V(\phi) = \mu^2 \phi^* \phi + \lambda (\phi^* \phi)^2$$

$$\mu^2 < 0 \text{ and } \lambda > 0$$

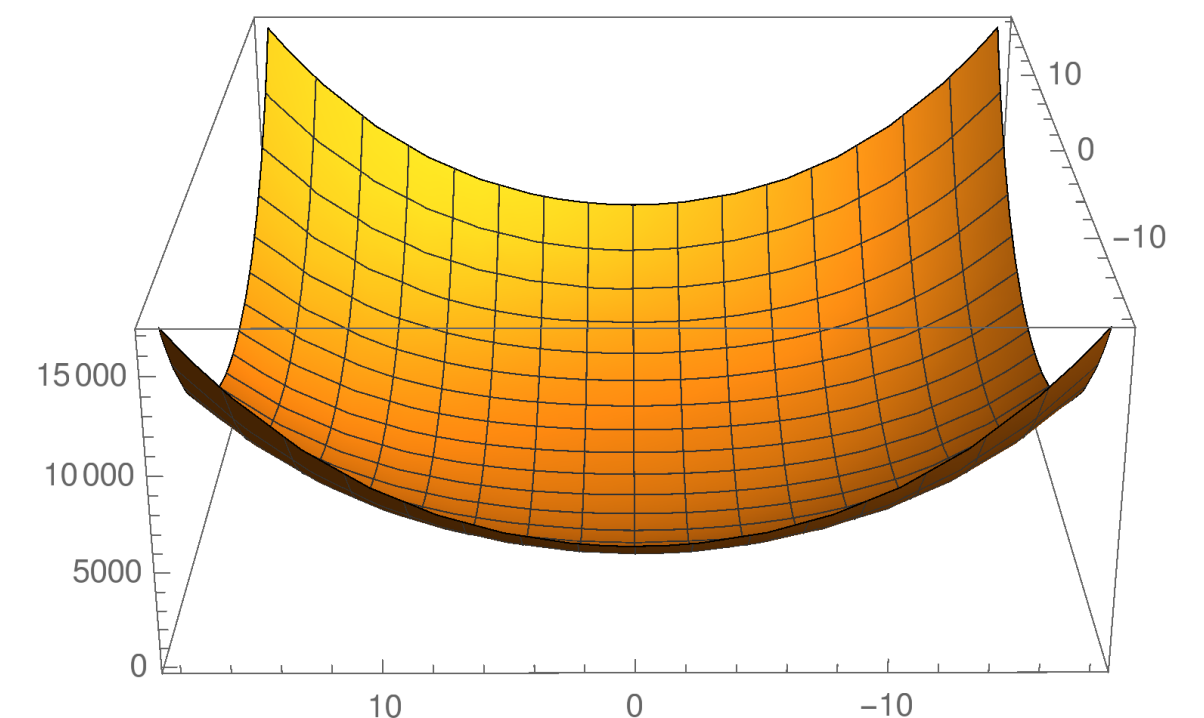


Add thermal corrections:

$$V(\phi, T) = (\mu^2 + C_\phi T^2) \phi^* \phi + \lambda (\phi^* \phi)^2$$

For $C_\phi > 0$, after a certain $T > 0$, $\mu_{eff} \equiv \mu^2 + C_\phi T^2 > 0$

Restored symmetry at high T



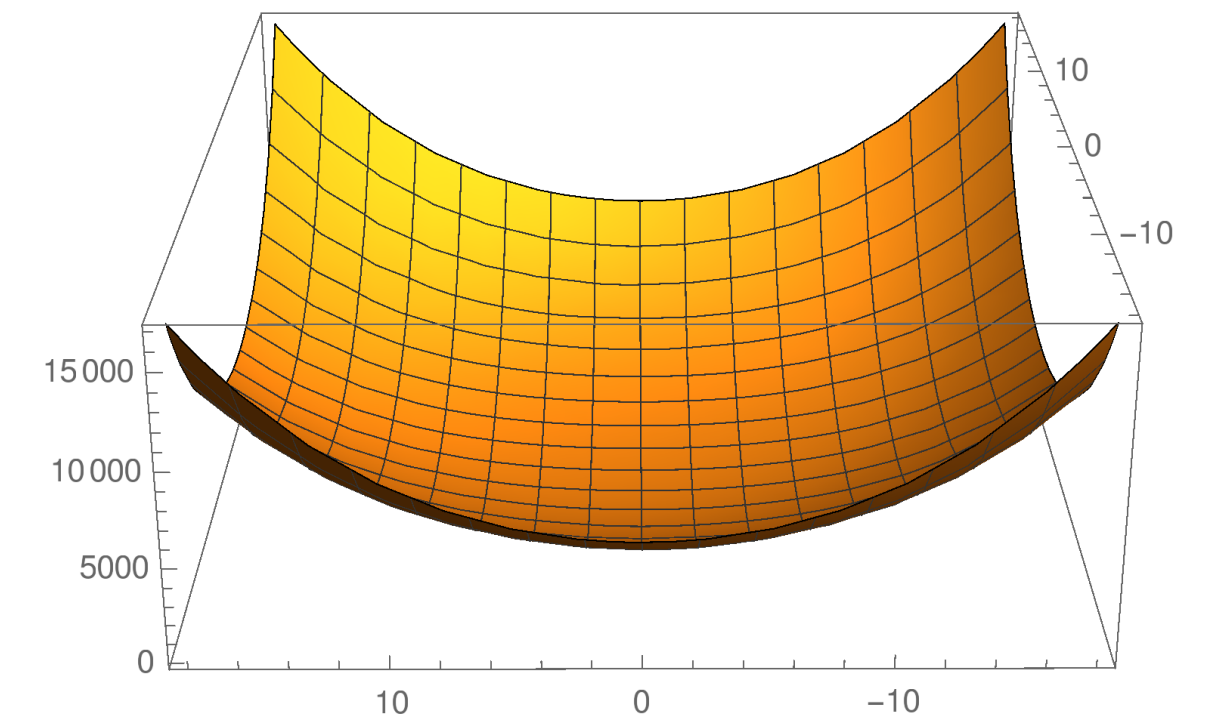
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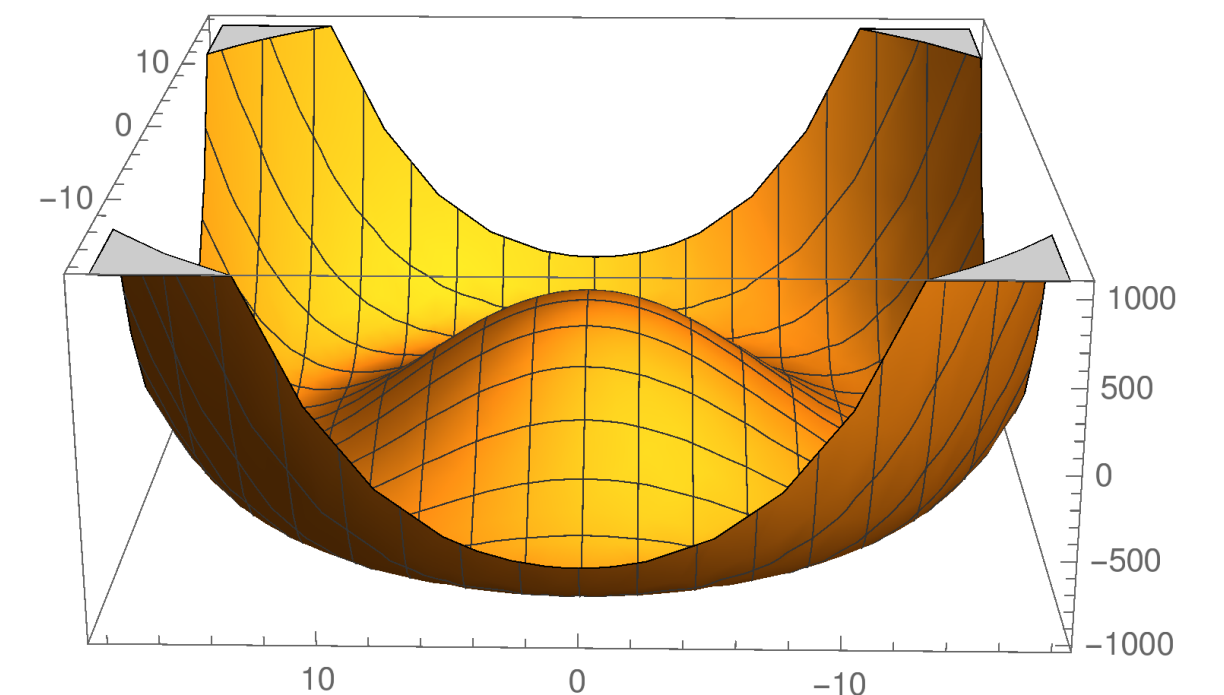


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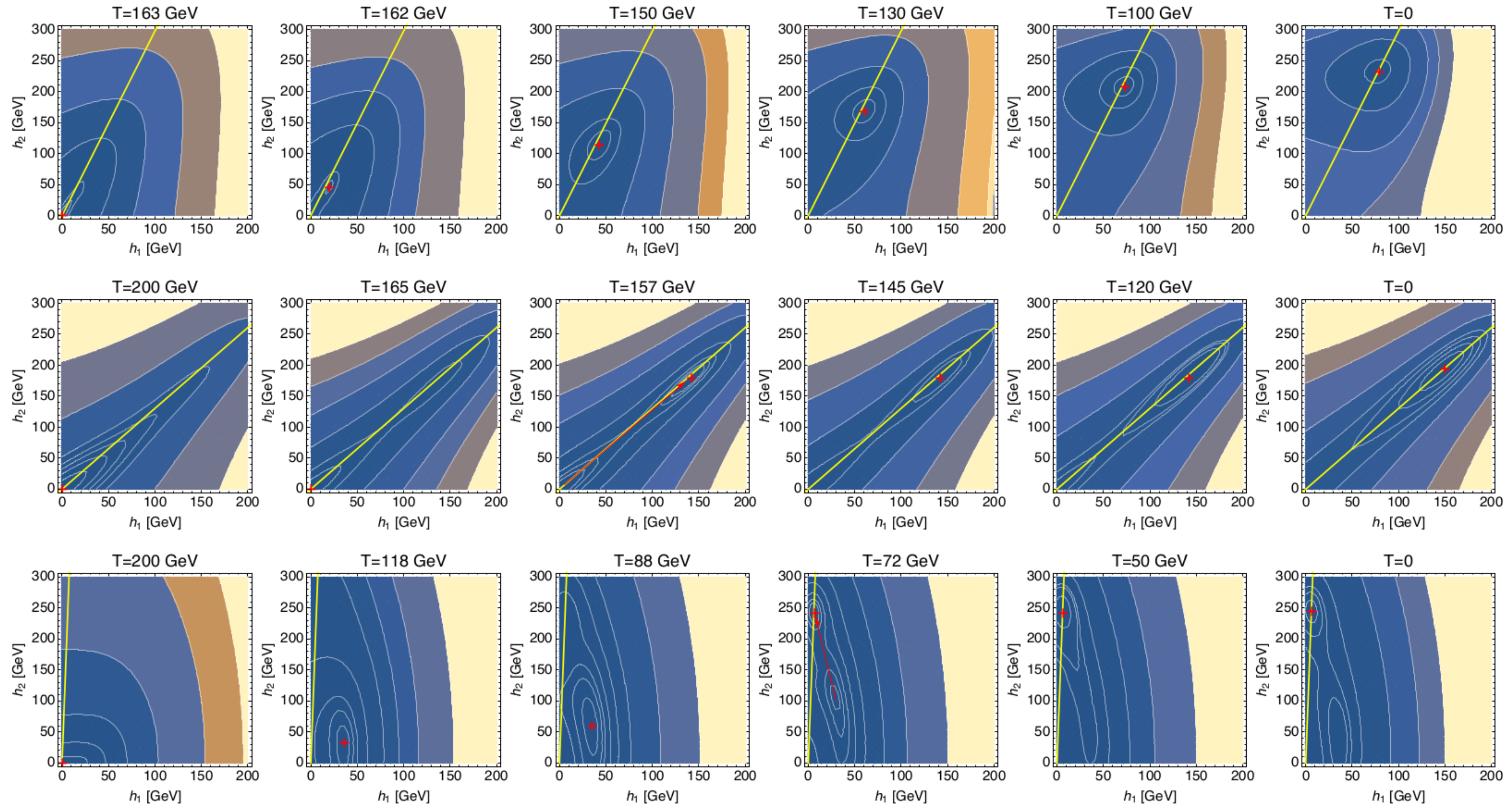
For $C_\phi < 0$, after a certain $T > 0$, $\mu_{eff} \equiv \mu^2 + C_\phi T^2 < 0$

Broken symmetry at high T



If a **multi-Higgs** theory contains multiple vacua, phase transitions can take place:

$$V_{\text{BSM}}(h_1, h_2, T)$$



JHEP 05 (2018) 151

Minimization

$$\left\langle \frac{\partial V_0}{\partial \phi_\alpha} \right\rangle_{\text{vac}} = 0, \quad \langle \phi_h \rangle_{\text{vac}} \equiv v_h \simeq 246 \text{ GeV}, \quad \langle \phi_\sigma \rangle_{\text{vac}} \equiv v_\sigma,$$

$$\mu_h^2 = -v_h^2 \lambda_h - \frac{1}{2} v_\sigma^2 \lambda_{\sigma h} - \frac{1}{2} \frac{v_h^2 v_\sigma^2 \delta_2}{\Lambda^2} - \frac{1}{4} \frac{v_\sigma^4 \delta_4}{\Lambda^2},$$

$$\mu_\sigma^2 = -v_\sigma^2 \lambda_\sigma - \mu_b^2 - \frac{1}{2} v_h^2 \lambda_{\sigma h} - \frac{1}{4} \frac{v_h^4 \delta_2}{\Lambda^2} - \frac{1}{2} \frac{v_h^2 v_\sigma^2 \delta_4}{\Lambda^2} - \frac{3}{4} \frac{v_\sigma^4 \delta_6}{\Lambda^2}.$$

Scalar mass spectrum

$$M^2 = \begin{pmatrix} M_{hh}^2 & M_{\sigma h}^2 \\ M_{\sigma h}^2 & M_{\sigma\sigma}^2 \end{pmatrix}$$

$$M_{hh}^2 = 2v_h^2\lambda_h + \frac{v_h^2 v_\sigma^2 \delta_2}{\Lambda^2}, \quad M_{\sigma\sigma}^2 = 2v_\sigma^2\lambda_\sigma + \frac{v_h^2 v_\sigma^2 \delta_4}{\Lambda^2} + \frac{3v_\sigma^4 \delta_6}{\Lambda^2}, \quad M_{\sigma h}^2 = v_h v_\sigma \lambda_{\sigma h} + \frac{v_h^3 v_\sigma \delta_2}{\Lambda^2} + \frac{v_h v_\sigma^3 \delta_4}{\Lambda^2}.$$

$$\mathbf{m}^2 = O^\dagger_i{}^m M_{mn}^2 O^n{}_j = \begin{pmatrix} m_{h_1}^2 & 0 \\ 0 & m_{h_2}^2 \end{pmatrix}, \quad \text{with} \quad \mathbf{O} = \begin{pmatrix} \cos \alpha_h & \sin \alpha_h \\ -\sin \alpha_h & \cos \alpha_h \end{pmatrix}, \quad \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \mathbf{O} \begin{pmatrix} h \\ h' \end{pmatrix}.$$

$$m_\theta^2 = -2\mu_b^2,$$

Thermal mass resummation

At high-T thermal 1-loop effects overpower the tree-level T=0 potential

Breaks down fixed-order perturbation theory and large T/m ratios must be resummed

Done by introducing Daisy corrections in the effective potential

$$m_i^2 \rightarrow m_i^2 + c_i T^2$$

$$m_i^2 \rightarrow m_i^2 + c_i T^2$$

$$c_h = \frac{3}{16}g^2 + \frac{1}{16}g'^2 + \frac{1}{2}\lambda_h + \frac{1}{12}\lambda_{\sigma h} + \frac{1}{4}(y_t^2 + y_b^2 + y_c^2 + y_s^2 + y_u^2 + y_d^2) + \frac{1}{12}(y_\tau^2 + y_\mu^2 + y_e^2) + \frac{1}{24}K_\nu + K_\Lambda^h,$$

$$c_\sigma = \frac{1}{3}\lambda_\sigma + \frac{1}{6}\lambda_{\sigma h} + \frac{1}{24}K_\sigma + K_\Lambda^\sigma,$$

$$K_\nu = \sum_{i=1}^3 y_{\nu_i}^{\text{eff}} \quad \text{with} \quad y_{\nu_i}^{\text{eff}} = \frac{\phi_h \phi_\sigma}{2} \frac{y_{\nu_i}^2 y_{\sigma_i}}{\Lambda^2} \quad \text{and} \quad m_{\nu_i}(\phi_h) = \frac{\phi_h}{\sqrt{2}} y_{\nu_i}^{\text{eff}}$$

$$K_\sigma = \sum_{i=1}^3 y_{\sigma_i}^2 \quad K_\Lambda^h = \frac{\phi_h^2 + \phi_\sigma^2}{4\Lambda^2} \delta_2 + \frac{\phi_\sigma^2}{6\Lambda^2} \delta_4 \quad K_\Lambda^\sigma = \frac{\phi_h^2}{4\Lambda^2} \delta_2 + \frac{\phi_h^2}{6\Lambda^2} \delta_4 + \frac{\phi_\sigma^2}{2\Lambda^2} \delta_4 + \frac{9\phi_\sigma^2}{4\Lambda^2} \delta_6.$$

And for gauge bosons...

$$M_{\text{gauge}}^2(\phi_h; T) = M_{\text{gauge}}^2(\phi_h) + \frac{11}{6} T^2 \begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & 0 \\ 0 & 0 & 0 & g'^2 \end{pmatrix}$$

$$m_{W_L}^2(\phi_h; T) = m_W^2(\phi_h) + \frac{11}{6} g^2 T^2,$$

$$m_{Z_L, A_L}^2(\phi_h; T) = \frac{1}{2} m_Z^2(\phi_h) + \frac{11}{12} (g^2 + g'^2) T^2 \pm \mathcal{D},$$

$$\mathcal{D}^2 = \left(\frac{1}{2} m_Z^2(\phi_h) + \frac{11}{12} (g^2 + g'^2) T^2 \right)^2 - \frac{11}{12} g^2 g'^2 T^2 \left(\phi_h^2 + \frac{11}{3} T^2 \right)$$