Stochastic gravitational-wave backgrounds with LISA and beyond: Challenges and Opportunities

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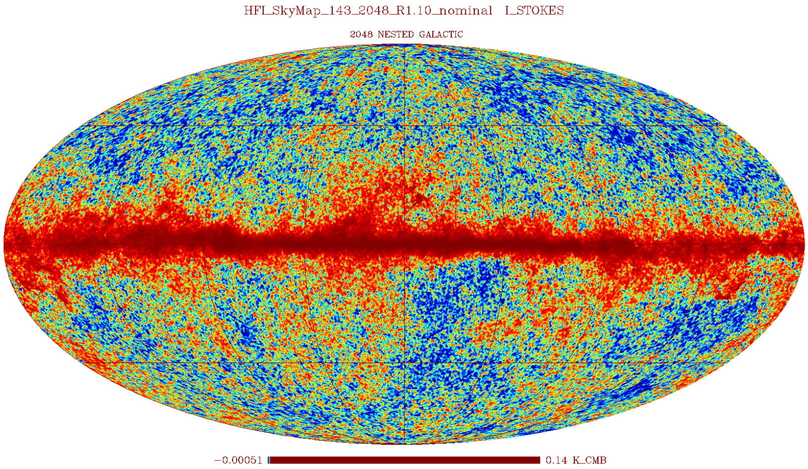
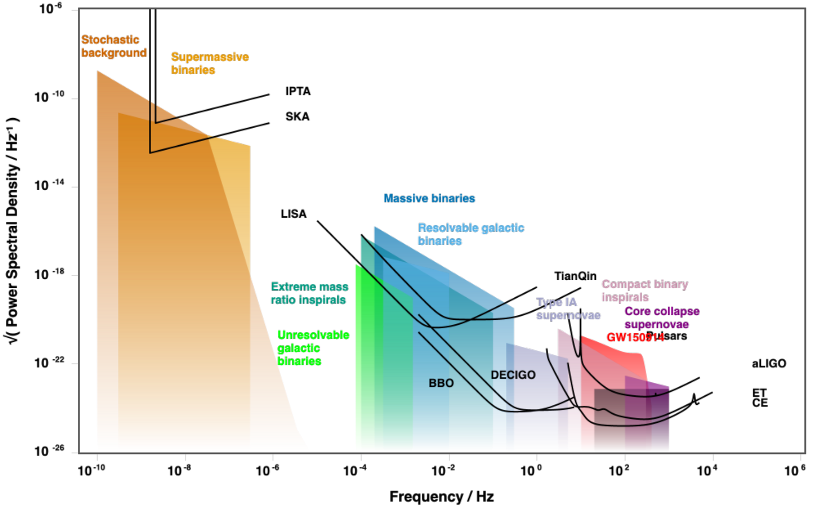
Stavanger, 6$^{\text{th}}$ June, 2023

Overview

Background


HFI_SkyMap_143_2048_R1.10_nominal I_STOKES
2048 NESTED GALACTIC
−0.00051
0.14 K_CMB

Observational landscape



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What is a Stochastic signal?

$$h_{ab}(\vec{x},t) = \sum_A \int_{-\infty}^{+\infty} df \int_\Omega d\Omega_{\hat{k}}\, e^{-i2\pi\left[f(t-\hat{k}\cdot\vec{x})+\phi_i(f,\hat{k})\right]}\, \tilde{h}_A(f,\hat{k})\, e^A_{ab}(\hat{k})\,.$$

- **Resolved**: The signal is correlated either *temporally* or *spatially* (frequency and/or direction).
- The signal is coherent and can be distinguished from random noise by "averaging" data (linear in strain $\tilde{h}$).

Credit: *Wikipedia CC BY-SA 2.0*

$$\langle \tilde{h} \rangle_T \neq 0, \quad \langle \tilde{n} \rangle_T = 0\,.$$

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- **Stochastic**: Limit where phase is uncorrelated between frequencies and/or directions e.g. due to incoherent superposition of sources or generation by random field.
- The signal is incoherent and cannot be distinguished from noise at linear level.

$$\langle \tilde{h}\tilde{h}^{\star} \rangle_T \sim P_h, \quad \langle \tilde{n}\tilde{n}^{\star} \rangle_T \sim S_n .$$

Statistical properties

- Incoherent signal: fully stochastic backgrounds hold no phase information in strain h.
- Usually assumed to be stationary, and statistically isotropic;

$$\langle h(t,\hat{k}) h^{\star}(t+\Delta t,\hat{k}')\rangle \sim \delta^{(3)}(\hat{k}-\hat{k}')\, H(\Delta t)\,,$$
$$\Updownarrow$$
$$\langle h(f,\hat{k}) h^{\star}(f',\hat{k}')\rangle \sim \delta(f-f')\,\delta^{(3)}(\hat{k}-\hat{k}')\, P_h(f)\,.$$

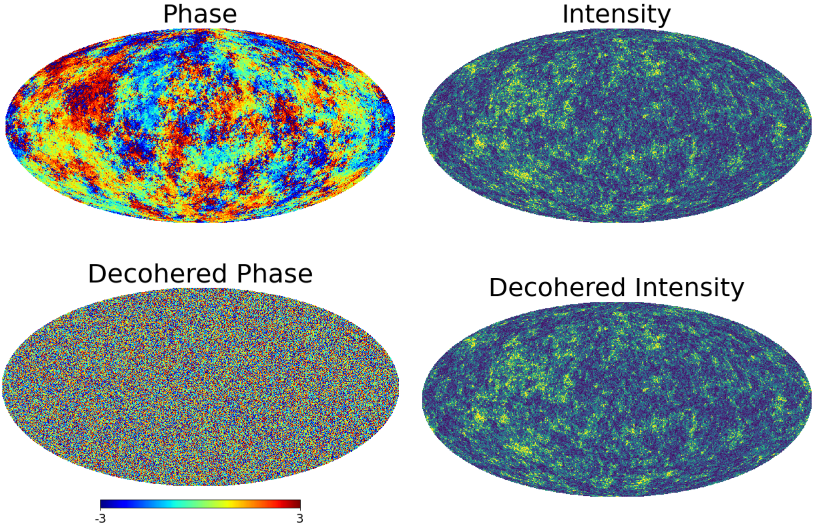
- These assumptions are very important ones for methods aimed at characterising and separating SGWBs.
- Note that statistical isotropy does not imply lack of angular correlations. The strain intensity (power) can be anisotropic and have non-trivial angular correlations

$$\langle h(f,\hat{k}) h^{\star}(f',\hat{k}')\rangle \sim \delta(f-f')\,\delta^{(3)}(\hat{k}-\hat{k}')\, P_h(f,\hat{k})\,,$$
$$\langle P_h(f,\hat{k}) P_h(f,\hat{k}')\rangle = \frac{1}{4\pi}\sum_{\ell}(2\ell+1)C_{\ell}(f)\mathcal{P}_{\ell}(\hat{k}\cdot\hat{k}')\,,$$

One more possibility...

- Only *one way* to generate a *diffuse* background with (temporal and/or angular) coherency i.e. $\langle \phi(f, \hat{k}) \phi(f', \hat{k}') \rangle \not\propto \delta(f - f') \, \delta^{(3)}(\hat{k} - \hat{k}')$.
- GWs that have spent time outside the horizon. These will be squeezed (zero-momentum) and then start oscillating (and travelling) coherently in all directions as they re-enter the horizon.
- Unique signature of inflationary background which would lead to standing waves [Grishchuk & Sazhin 1975].
- Interferometers can distinguish between standing and travelling waves [CC & Magueijo 2018].
- Density perturbations destroy all coherence [Bartolo et al 2019, Margalit, CC, & Pieroni 2020] $\rightarrow$ no unique signature due to coherent $\vec{k}$ and $-\vec{k}$ modes.

Scalar modes are annoying foregrounds...



Non-Gaussianity

- Decoherence, or randomisation of phase correlations, affects what kind of non-Gaussianity can be observed using GWs.
- Any non-Gaussian correlations in the strain field is wiped out by the propagation through a perturbed universe eg.

$$\langle h(\vec{k}_1)h(\vec{k}_2)h(\vec{k}_3)\rangle \to 0\,.$$

- Only three-point correlations of the GW *intensity* will carry information (angular correlations) [Bartolo et al. 2019, 2020].

$$\langle P_h(\vec{k}_1)P_h(\vec{k}_2)P_h(\vec{k}_3)\rangle$$

- Mining non-Gaussianity will require spectral *and* angular resolution.
- Valuable to constrain *all* generation scenarios including astrophysical sources, cosmological phase transitions, topological defects, etc.
- ...but scalar perturbations are a foreground → tensor non-Gaussianity "polluted" by scalar non-Gaussianity. Use GWs to constrain $f_{\rm NL}$?!

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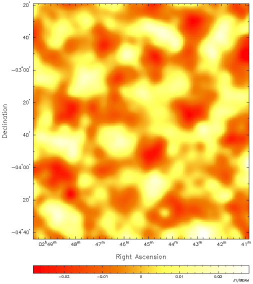
- e.g. CMB radio interferometry; very successful spectral rejection of compact radio source signals but spatial (angular) rejection impossible because of sparse Fourier (uv) coverage.
- GW interferometry; excellent spectral resolution and baseline but low angular resolution (in "intensity"-mode).
- *Not to be confused with* localisation resolution which uses time phase information to reconstruct angular position of coherent compact sources.

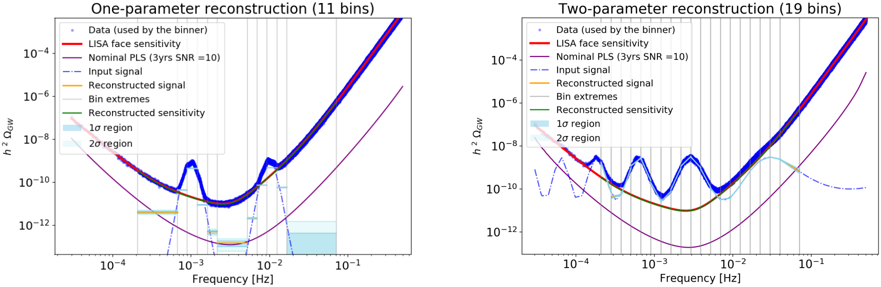
Photon Interferometry *vs* GW Interferemetry



Cosmic Background Imager, Caltech, NSF

Spectral characterisation

- Coherent detectors make very good spectrometers.



Caprini at al. 2019

- ...as long as several real-world effects are taken care of...

Spectral characterisation - Challenges

- Non-stationarity in signal and noise. When does $\langle X(f)X^\star(f')\rangle \to \delta(f-f')$?
  - Noise: well-known problem, complicates estimation of noise and timescales.
  - Signal: when does a superposition of signals become sufficiently "stochastic"? Complicates directional searches.
- Resolved source (time and angular) removal: Great feature of GW signal but will leave non-trivial residuals in the time-domain. e.g. LISA will see *at least* a few high SNR$>>1$ events per hour. All stochastic timestream will contain residuals *plus* significant non-stochastic contribution from SNR~ 1 signal.
- This will complicate the spectral analysis by degrading spectral resolution and make noise estimation harder.
- *cf* CMB analysis; time-domain gaps, cosmic ray hits, noise non-stationarities, glitches, etc.

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- ...null or Sagnac channels do change this...

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  The data $\tilde{d}$ (in frequency domain) can be expressed as

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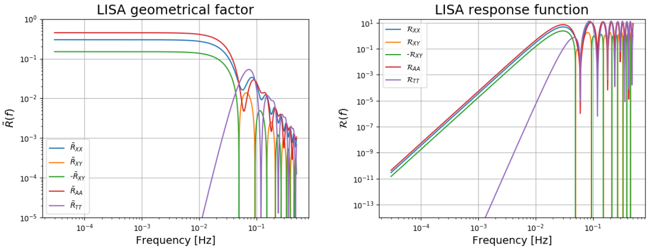
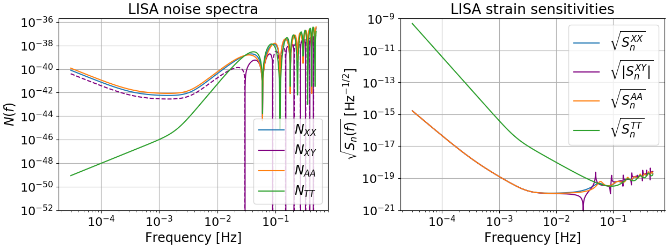
- Angular characterisation will be a crucial step in noise estimation.

LISA Reponse and Noise

After angular integration we get:



by combining noise and response we get the the strain (bottom right):



LISA sky response

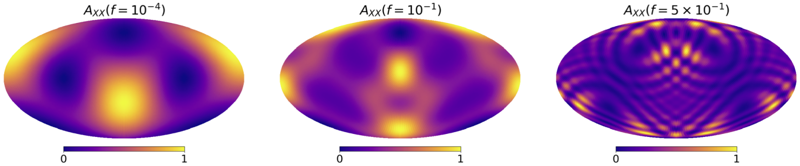
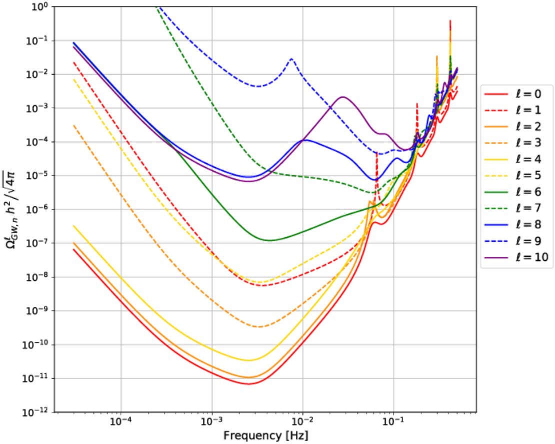


FIG. 2. Normalised auto-correlated response of TDI channel X, $\boldsymbol{A}_{XX}$, at time $t = 0$ and in the Solar System Baycentre (SSB) reference frame, at frequencies $f = 10^{-4}$ Hz, $f = 10^{-1}$ Hz, $f = 5 \times 10^{-1}$ Hz from left to right respectively.

- At peak sensitivity frequencies the "beam" has low structure $\ell_{\max} \lesssim 8$.
- The beam rotates around the triangle axis and along the Earth's orbit (fills in *very limited* angular phase information *m*).
- A lot of phase information is not sampled → missing sky modes.

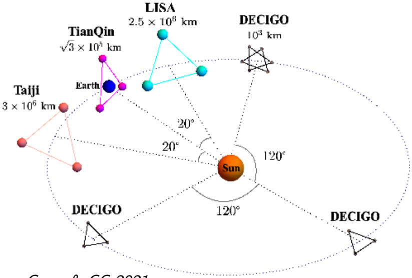
LISA Anisotropies

- LISA: Expected sensitivity to anisotropy multipoles in intensity.

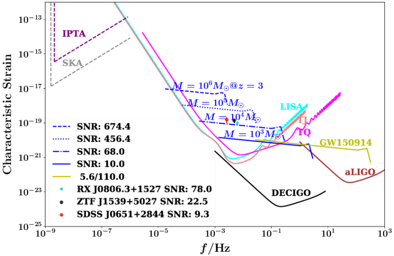


Bartolo at al. 2022

High-ℓ SGWB from space?



Gong & CC 2021



- Beat the fL/c factor by introducing long-baseline interferometry in space.
- Concurrent missions: LISA, TianQin, Taiji?

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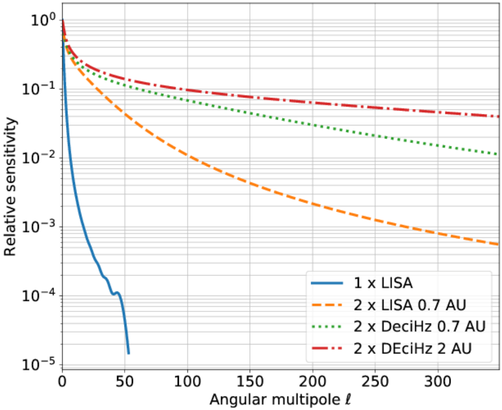
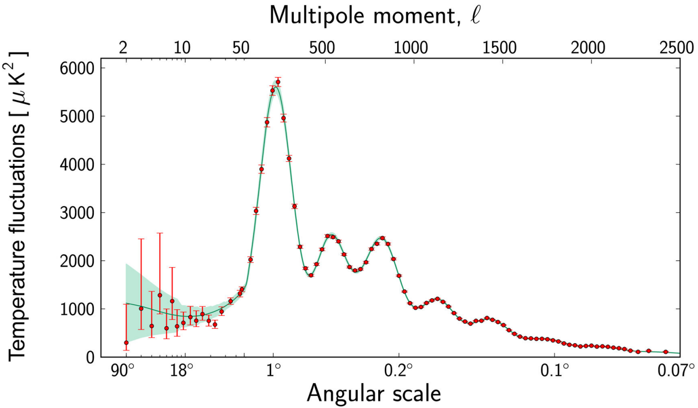


Baker et al. 2021

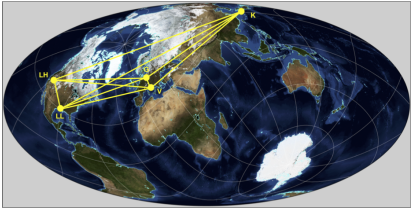
- Resolution dramatically improves with long baseline in space.
- Reminder: this is intensity (angular) resolution.

Cosmic Variance - a new problem?



Planck, ESA

Cosmic Variance - GW interferometry

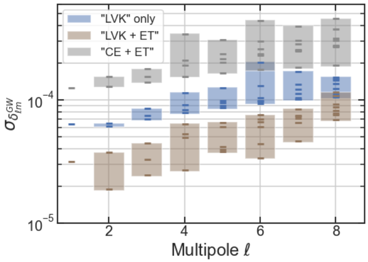


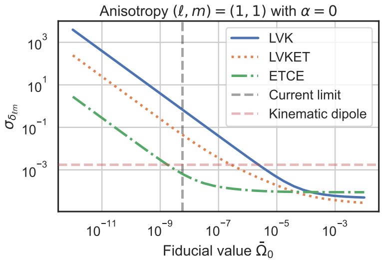
- Growing number of baselines over the next decade.
- Iterative improvement in sensitivity.
- Einstein Telescope (mid 30s?)

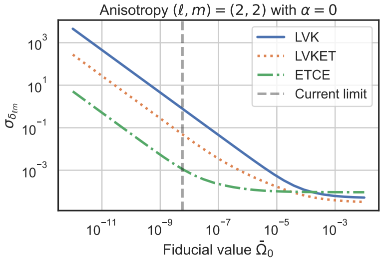
Mentasti, CC, & Peloso [2301.08074, 2304.06640]

- Consider zero-noise limit.
- Interferometers - covariance of multipoles is not diagonal, despite full-sky coverage.
  - Overlapping and finite frequency coverage.
  - Non-compact beam.
- Calculate "SNR" of anisotropies when variance is dominated by monopole.
- "How well can we measure $a_{\ell m}$'s in signal dominated limit?

Cosmic Variance - GW interferometry

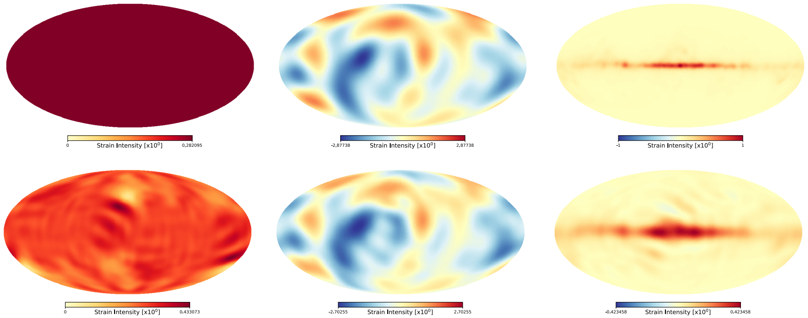






[Mentasti, CC, & Peloso, 2301.08074, 2304.06640]

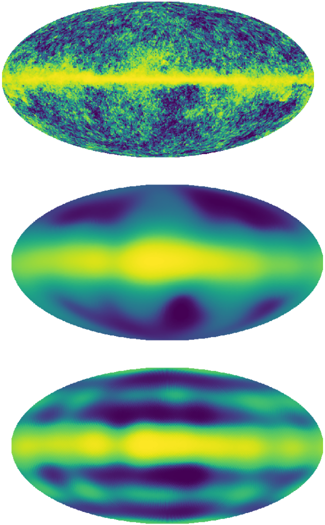
Map-making?



Renzini & CC 2019

- Yes, but only if $\delta_{\ell m}^{\rm GW} > 10^{-2}$.
- ...and assuming stationarity! (see e.g. Capurri et al. 2103.12037).

LISA "map"?



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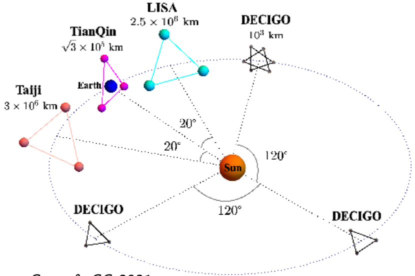
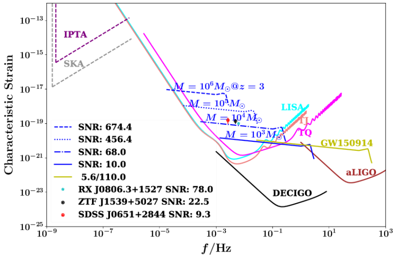


Gong & CC 2021



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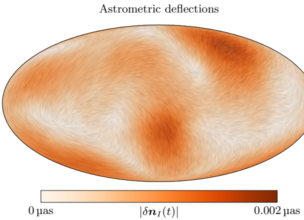
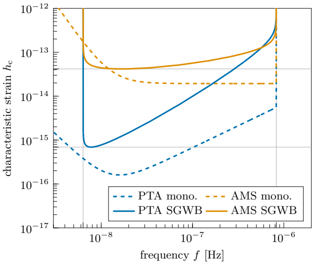
Astrometry

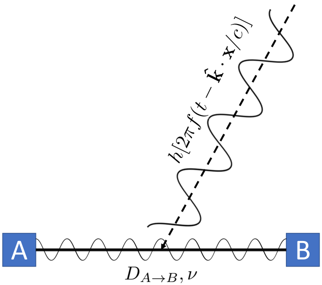
Golat & CC 2022





Phase vs Frequency Measurements

LISA is a phasemeter. Measures the perturbation to the *distance* between two stations (TDI - Time-Delay Interferometry).



Phase change:

$$\delta D_{A\to B} \sim c \int_{t_A}^{t_B} h \, dt \sim c \, h/f$$

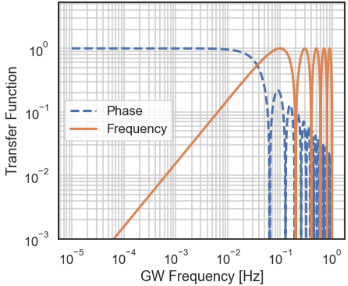
Frequency change (Doppler):

$$\frac{dD_{A\to B}}{dt} \sim \Delta\nu/\nu \sim h$$

LISA uses TDI because it cannot compare frequencies between stations - local oscillator ("clock") is not stable enough leading to overwhelming laser frequency noise.

GW Observation with space clocks?

- Lab-based optical lattice atomic clocks routinely reach 10^{-19} relative frequency stability.
- This raw sensitivity is sufficient to measure astrophysical GWs if it can be integrated on to the required frequencies.
- Measuring the Doppler shift directly may have significant advantages for the same technology and scale of e.g. LISA.



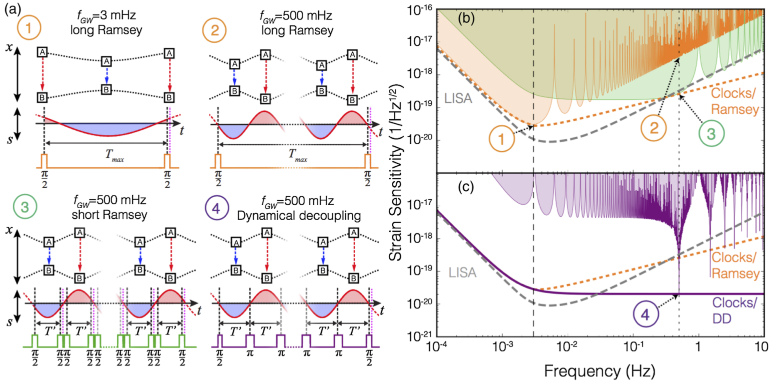
Phase:
$$d_p \sim s_p + n_p$$

Frequency:
$$d_\nu \sim s_\nu + n_\nu$$

Strain:
$$h \sim d_\nu - n_\nu \sim (d_p - n_p)f$$

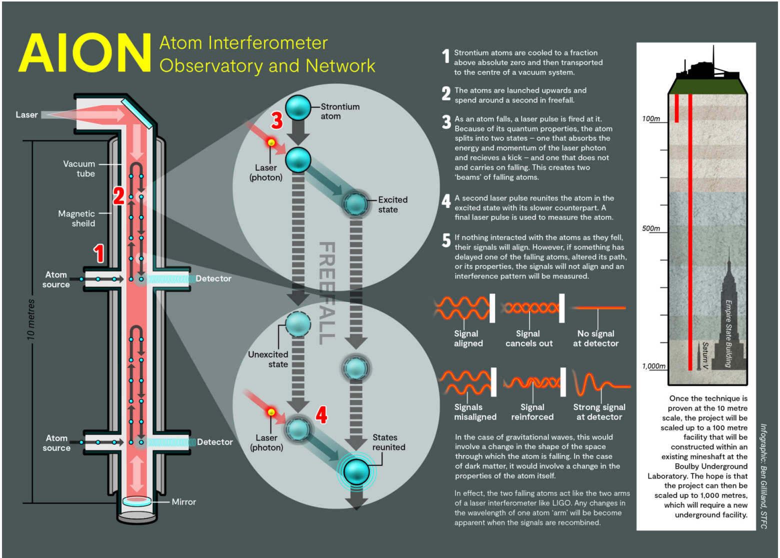
GW Observations with space clocks

Kolkowitz et al., PRD 94, 124043 (2016)



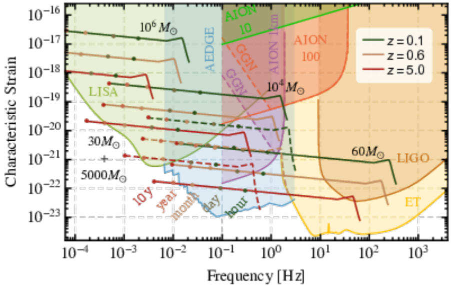
GW Observations with cold atoms

O. Buchmueller, Cold atom group, Imperial College



Fundamental physics with cold atoms in space?

Badurina et al. 2020



- Atom interferometry: *MAGIS* (US), *AION* (UK/EU?), AEDGE (SPACE?).
- Phase or frequency measurements?
- "Tunable" target frequency range.
- Anisotropies: higher angular resolution *cf.* LISA.
- Other tests of GR (scalar and vector modes of time dependent metric perturbations).

Summary

- Great prospects for characterisation of SWGBs over big range in frequency.
- LISA: significant real-world challenges - separation of stochastic components/residuals will be difficult. Exploit both frequency and angular structure.
- Angular resolution will improve with addition of baselines to ground-based network (but still ~ 10 degrees at current frequencies).
- Long-baseline in space (~ 1 degree?)
- Cold atoms?