Statistical evaluation of tubular joint SCFs from test data and finite element analyses

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Abstract. Stress concentration factors (SCF) derived from both FEA and measurements from laboratory experiments are associated with uncertainties. To understand these uncertainties, SCF values can be determined from a series of tests or possibly also several SCF-values estimated from converged finite element analysis. The challenge is to determine a recommended value for the SCF based on these tests or analysis, ensuring that the resulting SCF along with the SN-curve meets the code required 2.3% exceedance probability. In this paper, structural reliability analysis (SRA) methods are employed to model the combined uncertainty of the parameters representing the SN-curve and the measured SCFs from experimental test joints. Based on this analysis, the SCF that, when paired with the design SN-curve, provides the minimum fatigue life is established. From this analysis, a method to provide characteristic values of the SCF is suggested. Finally, FEA of the same detail, with and without modelling the weld, using both first and second-order elements are performed. From these analyses the SCF is estimated using linear extrapolation and direct extraction. Based on these findings and their comparison to the characteristic SCF, a suggested method to extract SCFs from FEA is provided.

1. Introduction and background

The design of tubular joints against fatigue failure is normally based on the 'hot-spot' stress derivation then using it with the T-curve (DNV RP C203). The T-curve correlates the 'hot spot' stress range at the joint intersection with the number of cycles to develop a through-thickness crack in the stressed member. The design T-curve is assumed to have a normal distribution in logarithmic scale, and can be expressed as:

$$\log N = \log A - m * \log \Delta S$$

where N is the number of cycles, $\log A$ is the curve intercept, and m is the inverse slope of the curve. The value of m = 3 if $N \leq 10^7$ cycles and m = 5 if $N > 10^7$ cycles are applicable for fatigue curve in air environment.

SN-curves are based on linear regression by the least mean square error of selected experimental data points of a range of tubular joint types, loading types and loading ratios. It is well known that there is a significant variation in the resulting fatigue lives and more or less identical tests can be performed gaining different number of cycles to failure. These uncertainties are normally assumed to be related to issues like inherent randomness in material quality, the quality of the welding and associated presence of crack-like defects from the welding, the magnitude of the stress level, stress increase due to local joint geometry and stress increase due to weld geometry. In practical engineering the inherent randomness in material and weld quality is assumed included into SN-curves, determined by several laboratory experiments. Hence, the stress ranges from these experiments are plotted against the number of cycles until occurrence of a through-thickness crack in the specimen and these are plotted in a log-log scale diagram. A regression analysis can then be used to determine the best approximation for a mean value of these data. In addition, the standard deviation (SD) for the $\log A$ parameter is established and as a lower bound the design SN-curve is determined 2 SD below the mean curve, giving a maximum of 2.3% chance of a fatigue crack at the calculated value of cycles.

The data used to establish the design T-curve, along with the mean and design T-curves for in air conditions, are shown in Figure 1, based on HSE (1999).

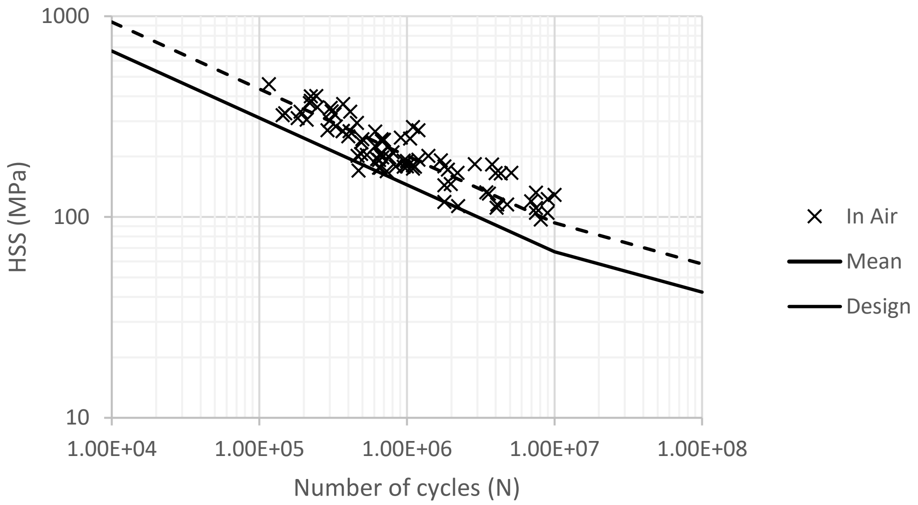

Figure 1. T-curve and data, based on HSE (1999)

The T-curve used in standards for design of offshore structures are reported to have a mean value of the logA of 12.92 and a standard deviation (SD) of the logA value of 0.23. The design S-N T-curves is taken as the mean value of the logA minus two standard deviations of logA.

To create data-points for the SN-curve, careful measurements of the cyclic stress magnitude (S) is needed, along with accurate determination at which number of cycles (N) the specimen is experiencing a through-thickness crack. The cyclic stress magnitude is often determined by the hot-spot stress (HSS) range, which is again often determined by the stress concentration factor (SCF) multiplied by the nominal stress in the brace. The SCF is the ratio between the HSS and the nominal stress in the specimen away from the joint and can be found in design codes, typically providing the Efthymiou stress concentration factors (SCF) for the fatigue life estimation of tubular joints (Efthymiou 1988, ISO 2020). These are often assumed to be slightly biased to the safe side. Hence, many engineers seeking fatigue life optimization are using finite element analysis (FEA) or measurements direct from laboratory tests to obtain unbiased SCFs for tubular joints, both for design optimization and particularly for life extension of existing structures. However, stress concentration factors (SCF) both based on FEA and measurements from laboratory experiments are associated with uncertainties. For laboratory experiments these uncertainties may be a result of the individual geometry of the weld profile, the placement of the strain gauges, the strain measurements and the SCF extrapolation methodology. For an FEA such uncertainties are related to modelling of the weld profile, element type, mesh density, SCF extrapolation methodology and other numerical inaccuracies.

As part of PhD research project (Atteya 2023), the HSSs and their associated SCFs were measured for four double T tubular joints, providing a total of 16 quadrants where the SCFs could be determined. The joint configuration and the arrangement of strain gauges is shown in Figure 2.

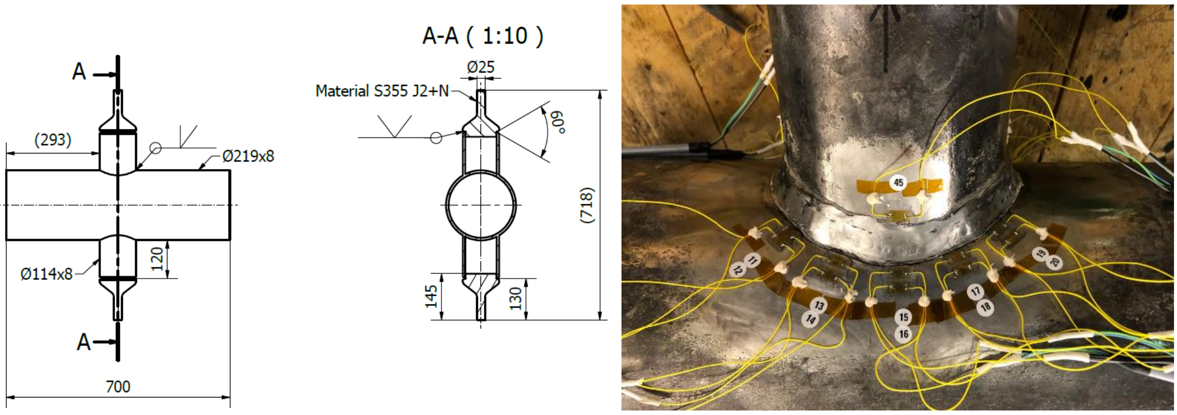

Figure 2. Joint configuration and arrangement of strain gauges

The SCFs were determined by standardised methods and found to be as shown in Table 1, as also reported in Atteya et al (2021). The observed variations in results were assumed to be due to deviations in actual weld profiles, in the precision of actual coordinates and orientation of strain gauges as well as the selected extrapolation method.

Table 1. SCFs determined by standardised methods for each quadrant of the four test joints.

	Test joints				
	DT1	DT2	DT3	DT4	All
Quadrant 1	18.80	19.09	17.18	21.19	
Quadrant 2	18.22	20.67	22.24	23.30	
Quadrant 3	18.96	20.05	19.96	18.65	
Quadrant 4	17.76	21.60	*no reading*	20.40	
Mean value	18.43	20.36	19.79	20.88	19.87
SD	0.55	1.06	2.54	1.93	1.72
CoV	0.03	0.05	0.13	0.09	8.7%

The distribution of the SCF's at the saddle point from the experimental work shown in Table 1 were plotted in a normal distribution probability paper, as shown in Figure 3, and found to fit reasonably well the normal distribution.

By bootstrapping these data, it was indicated that the 95% confidence interval for the mean value of the SCF was [19.17, 20.63] and for the standard deviation [1.2, 2.0], providing a coefficient of variation (CoV) in the range of 6–10%.

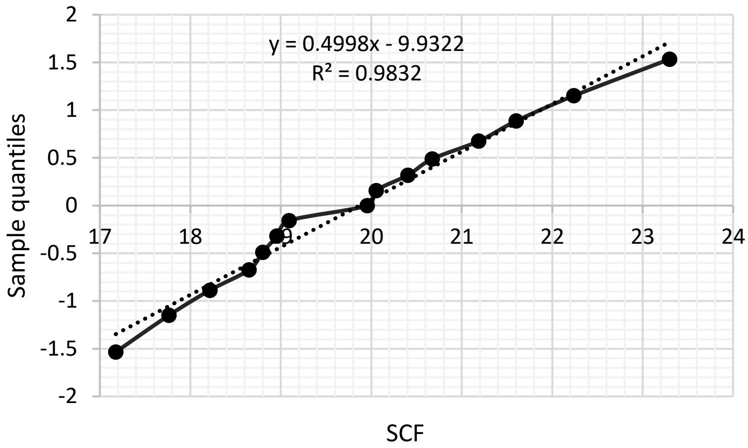

Figure 3. SCF values from experimental work plotted in a normal distribution paper, indicating a mean value of the SCF of 19.9 and a standard deviation of 2.

After these initial evaluations were performed, the precision in the strain gauge locations was studied at eight locations on the saddle of the chord side, using two specimens from a total of four specimens (Atteya et al 2021). The variation between actual and average strain gauge locations serves as useful indicator of the overall accuracy of the SCF for the specimen. The actual strain gauge's locations to the weld toe were measured to an accuracy of ±0.1 mm, then the measured points and their strain readings were used to extrapolate for the SCFs linearly. Figure 4 shows the distribution of strain gauges at the intended "a" and "b" locations and the extrapolation lines to the weld toe to estimate the hot-spot stress and associated SCF value for each pair of points. The results show that the location of the strain gauges at point "a" were scattered without evidence of any systematic error, as indicated by Figure 4. However, for the strain gauges at point "b" there is a scatter around a systematic mean shift of 1.1 mm from the intended "b" location, as indicated by Figure 4.

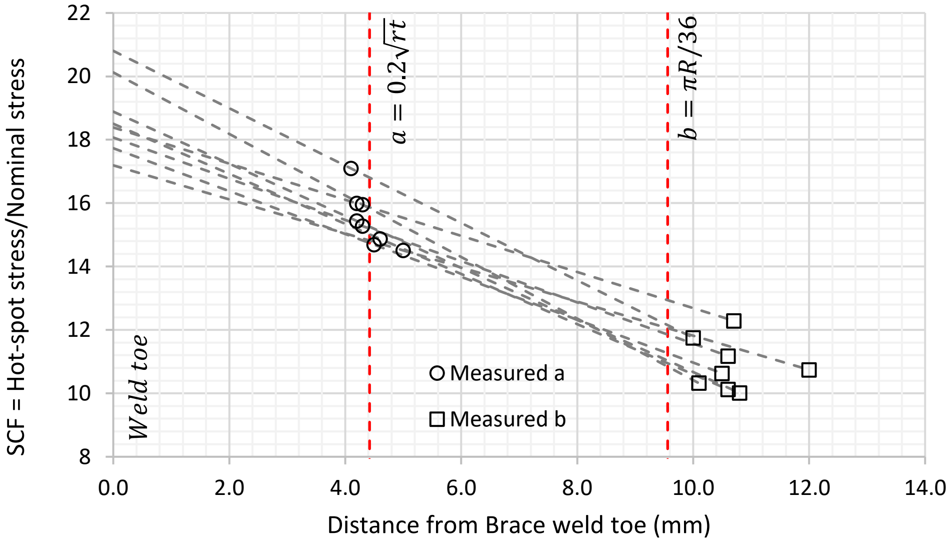

Figure 4. Experimentally measured SCFs sensitivity

The systematic shift in mean value can be explained by the length of the strain gauge carriers. The carrier's length was 5 mm while the distance between point "a" and "b" was only 5.1 mm, which made it challenging to manually position the strain gauges spot on. After strain gauges installation, the average distance between “a” and “b” was approximately 6.3 mm.

The actual extrapolated SCFs for the four quadrants on the two test joints, prior to correction and after correction, are shown in **Table** 2.

Table 2. SCFs at test joints after correcting for the location of the strain gauge positions.

	Test joints – uncorrected and corrected (*)					
	DT1	DT2	All	DT1*	DT2*	All *
Quadrant 1	18.80	19.09		18.07	18.39	
Quadrant 2	18.22	20.67		17.73	20.12	
Quadrant 3	18.96	20.05		18.50	18.89	
Quadrant 4	17.76	21.60		17.19	20.80	
Mean value	18.43	20.36	19.4	17.87	19.55	18.71
SD	0.55	1.06	1.29	0.55	1.11	1.21
CoV	3%	5%	6.6%	3%	5.7%	6.5%

For the SCFs corrected for the strain gauge location, the average from these eight points is 18.71 with a standard deviation of 1.21, providing a CoV of 6.5%. This should be compared to the uncorrected average for the same two specimens, providing a mean SCF of 19.39, a standard deviation of 1.29 and a CoV of 6.6%. As reported in Atteya et al (2021), the difference between the mean value between these two samples can be assumed to be due to the systematic inaccuracy in the location of strain gauge b, while the variation in the results from the eight points is a combination of other inherent uncertainties (such as weld geometry, material properties and measurements). This implies that it could be argued that the shift in strain gauge location introduces a bias of 3.7% to the safe side, which can also easily be seen to be reasonable by a geometric evaluation of the mean values of locations and stresses at “a” and “b”.

$$\frac{S_a - \frac{S_b - S_a}{x_b - x_a} \cdot x_a}{S_a - \frac{S_b - S_a}{x_{b\_corrected} - x_a} \cdot x_a} = \frac{16 - \frac{11 - 16}{9.56 - 4.42} \cdot 4.42}{16 - \frac{11 - 16}{10.66 - 4.42} \cdot 4.42} = 1.04$$

where $S_a - \frac{S_b - S_a}{x_b - x_a} \cdot x_a$ is the extrapolation formula for the stress at the hot-spot, S_a and S_b the stress reading at locations a and b respectively (average values of S_a and S_b is used), x_a and x_b is the intended locations of strain gauge a and b respectively and $x_{b\_corrected}$ is the corrected mean location of strain gauge b.

The data in **Table** 2 can be plotted in a normal probability paper as shown in Figure 5, still to some extent indicating that the data follows a normal distribution. The shift in the mean value is obvious and the slope is showing signs of being similar.

For the entire sample, the average SCF was found to be 19.87, with a standard deviation of 1.72 and CoV of 8.7%. To adjust for the strain gauge location, assuming the same trend for the two remaining specimens as for the two tested, the mean value of the SCF is set to 19.87/1.037 = 19.16, while there were not found any obvious reason to reduce the CoV from 8.7% (Atteya et al 2021). It is also assumed that the data still can be modelled by a normal distribution function.

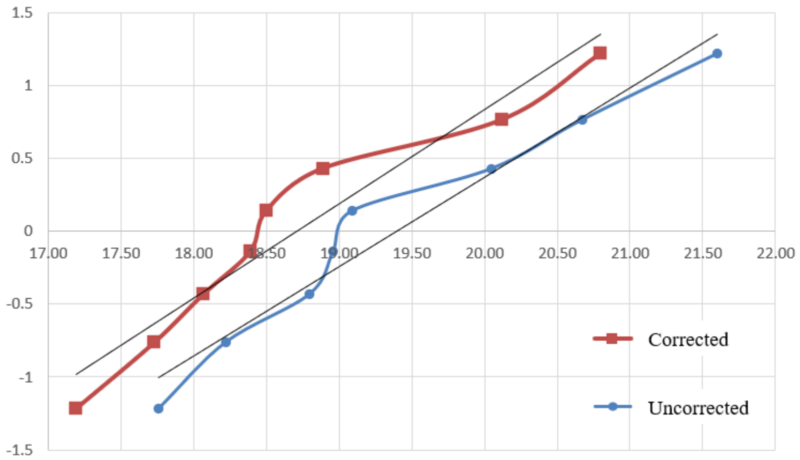

Figure 5. Corrected and uncorrected SCF values from the two experimental test joints plotted in a normal distribution paper, indicating a change in the mean value for the SCF from 19.4 to 18.7.

In Atteya et al (2021), the SCFs were also determined by different finite element analysis (FEA). The variation of the SCFs for models with different weld profiles and different extrapolation methods is shown in Table 3. The “smallest” accepted weld profile was the shortest acceptable weld leg length as per fabrication specifications and the “largest” was the most extended acceptable weld leg length as per fabrication specifications (Atteya et al 2021).

Table 3. SCFs for different weld profiles based on unaveraged principal stresses with 1st and 2nd order brick elements.

Weld profile	Direct Extraction from $0.1\sqrt{rt}$ Location		Linear Extrapolation	
	1st Order	2nd Order	1st Order	2nd Order
No Weld	-	-	20.2	22.2
Idealised smallest accepted weld profile	20.1	19.8	18.5	20.4
Idealised largest accepted weld profile	18.3	20.3	19.1	21.0

The mesh discretization with convergence study and the use of extrapolation method is reported in Atteya et al (2021). With the mean SCF determined above, only two of the models are underpredicting the SCF. However, the further studies will determine if the mean value of SCF are the correct SCF to be used in design fatigue analysis.

In this work we will investigate the combined uncertainty of the inherent randomness in material and weld quality and the stress concentration factor (SCF). One method of investigating uncertainties is the use of structural reliability analysis (SRA), for example by the use of the so-called first-order reliability method (FORM). By this method it is possible to model the uncertain parameters by their assumed distribution functions and determine a so-called design point providing a given probability of limit state exceedance (where the limit state in this case is the SN-curve being the assumed formula for when a through-thickness crack is expected). Standard practice is to use the mean value of the measured SCFs and this study is essentially to check if the mean value is the correct choice.

Further, the same joint has been numerically modelled by several methods using finite element analysis (FEA), as previously presented by Atteya et al (2021). These simulated SCFs will be compared with what is found to be the “recommended” SCF to determine some initial recommendation for determining SCFs from FEA.

2. Probabilistic modelling of uncertainty in experimental SCFs

To study the combined uncertainty of the inherent randomness in material properties and weld quality and the stress concentration factor (SCF), the problem is investigated by a first order reliability method (FORM) analysis (Melchers 1999). A simple limit state is established for the problem illustrating the probability of the number of cycles determined from the SN-curve based on $logA$ and SCF as stochastic variables exceeding the number of cycles determined from characteristic values of $logA$ and SCF. The purpose is to determine the characteristic value of the SCF providing a limit state exceedance probability of 2.3% (as per the definition of the fatigue limit state).

The limit state function is described as:

$$g(logA, SCF) = [logA_{ch} - m \cdot \log(SCF_{ch} \cdot \Delta\sigma_{nom})] - [logA - m \cdot \log(SCF \cdot \Delta\sigma_{nom})]$$

where the stochastic variables are modelled as shown in Table 4.

Table 4: Description of stochastic variables

Variable	Modelling	parameter 1	parameter 2
$logA_{ch}$	Fixed	12.46	
m	Fixed	3	
SCF_{ch}	Fixed	Determined by the analysis	
$\Delta\sigma_{nom}$	Fixed	10 MPa	
$logA$	Normal distributed	Mean=12.92	SD=0.23
SCF	Normal distributed	Mean=19.16	SD=1.66

Based on these parameters, a FORM analysis was performed for the given limit state, providing a 2.3% exceedance probability ($\beta = 2$) of the limit state function for a SCF_{ch} of 19.9. The design points in the FORM analysis are $logA_{DP} = 12.5$ and $SCF_{DP} = 20.45$. The design points can be argued to be the values that should be used in a design calculation. However, since $logA$ is given in design codes as 12.46 the SCF should be adjusted to take this $logA$ value into account.

$$SCF_{adj} = \frac{10^{\frac{\left(12.46 - \left(logA_{DP} - m \cdot log(SCF_{DP} \cdot \Delta\sigma_{nom})\right)\right)}{m}}}{\Delta\sigma_{nom}} = 19.9$$

The limit state function with a “characteristic” SCF of 19.9 is illustrated in Figure 6, with a circle of radius 2 illustrating the exceedance probability 2.3% to indicate the design point for the limit state. The sensitivity to changes in the two stochastic parameters is shown in Figure 7 and clearly indicate that the uncertainty in $logA$ by far supersedes the uncertainty in the SCF.

An alternative to the above FORM analysis would be an inverse first-order reliability method (IFORM) analysis (Winterstein et al 1993). By an IFORM analysis any combinations of $logA$ and SCF that gives 2.3% probability of exceedance (probability of fatigue crack according to the SN-curve) can be found approximately. In these analyses both $logA$ and SCF are modelled as normal distributed in accordance with the data given above. However, as the limit state includes a term log (SCF), this term needs to be modelled as a log-normal distribution. Hence, the $logA$ and SCF variables are transformed into a U-space by the following transformation:

$$logA = SD_{logA} \cdot U_{logA} - \mu_{logA}, SCF = 10^{[SD_{\log\_SCF} \cdot U_{SCF} + \mu_{\log\_SCF}]}$$

By defining the parameters in the U-space in such a manner that they define a circle with a radius of $\beta = 2$ (indicating a 2.3% exceedance probability), the contour line of allowable combinations of $logA$ and SCF are found.

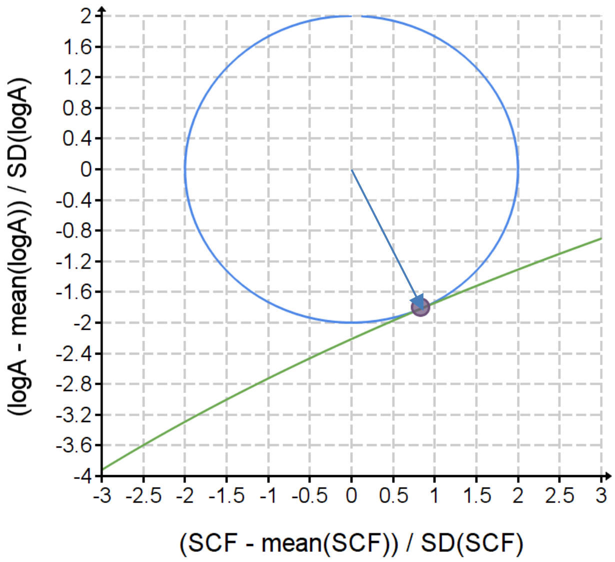

Figure 6. Graphical representation of the limit state function with a characteristic SCF of 19.9

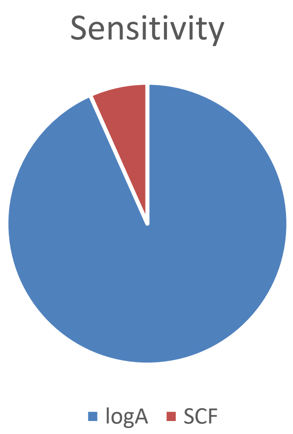

Figure 7. Sensitivity of the two stochastic parameters in the limit state

At the minimum value of $logA$, the mean value of SCF is found and, hence, it could be argued that a characteristic value of $logA$ should be combined with the mean value of SCF. However, if the number of cycles according to the SN-curve is calculated for all allowable combinations of $logA$ and SCF, the set of parameters ensuring the lowest fatigue life with exceedance probability of 2.3% can be found. This minimum $logN$ is found to be 5.565 for the variable pair $logA = 12.51$ and $SCF = 20.63$. This SCF is then adjusted to account for the fact that the $logA$ in codes are 12.46:

$$SCF_{ch} = \frac{10^{\left(\frac{12.46-5.565}{3}\right)}}{\Delta\sigma_{nom}} = 19.9$$

The "contour line" for values of logA and SCF that fulfils the required exceedance probability of 2.3% are shown in Figure 8, with marking of the design point pair of variables pair $logA = 12.51$ and $SCF = 20.63$ (black circle) in addition to the adjusted pair of variables pair $logA = 12.46$ and $SCF = 19.9$ (red square, slightly off the contour line indicating 2.3% exceedance probability). In addition, the closest value on the contour line with $logA = 12.46$ and $SCF = 19.3$ is indicated (black cross).

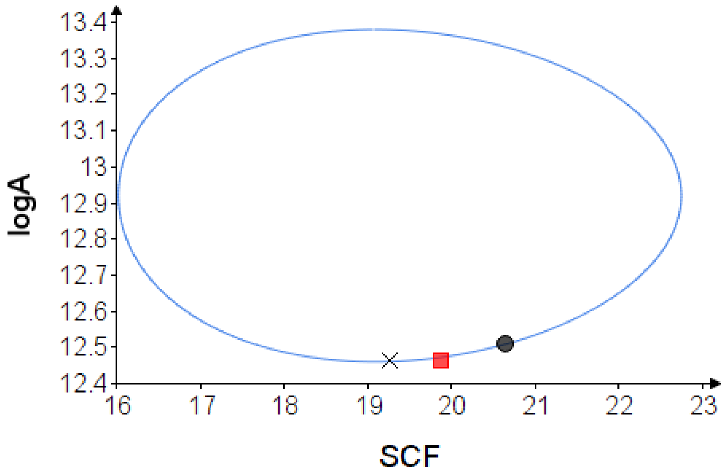

Figure 8. Contour line for logA and SCF providing a 2.3% exceedance of SN-fatigue limit state, indicating the SCF at the logA and the minimum logN and the adjusted SCF to account for standardised logA.

The argument for using the adjusted value of 19.9 is, hence, to ensure that the minimum value of $logN$ is obtained, even if this set of parameters provides an exceedance probability that is slightly less than 2.3%. The alternative pair of variables that are on the contour line ($logA = 12.46$ and $SCF = 19.3$) in contrast does not provide a $logN = 5.565$. Hence, the pair of values recommended here are a substitute set of parameters for the design values providing the same $logN$ with the boundary condition given by the standardised $logA$.

The recommended SCF of 19.9 could possibly be estimated by two simplified methods:

1) the mean value of 95% confidence level from the bootstrap, which adjusted by the bias of 3.7% also provides a value of 19.9,
2) the mean value of the measured SCFs plus 0.443 times the standard deviation of the SCFs, as presented in Table 4, which result in a SCF of 19.9.

However, this example is insufficient to indicate whether any of these ways to determine the recommended SCF can be used as a rule. Nevertheless, these approaches appear to be reasonable estimates until more data becomes available.

3. Implication of the probabilistic analysis on the choice of FEA and model

The findings of the probabilistic analysis can be used to evaluate the FEA models and extrapolation methods presented in Atteya et al (2021). As shown in **Table** 3, the variation in the SCF results is depending both on the element type used (first-order or second-order), the modelling of the weld and the extrapolation method. In Table 5, which is similar to **Table** 3, the FEA models and extraction methods that produced SCFs lower than what has been found to be the recommended SCF of 19.9 is greyed out.

An analysis neglecting the weld profile combined with linear extrapolation will provide SCFs on the safe side, but it is worth noticing that second-order elements are overestimating the stress by approximately 10%. The direct extraction method is in most cases providing acceptable results, except from first-order elements with the idealized largest accepted weld profile. The linear extrapolation method is providing acceptable results only for second-order elements.

Table 5. SCFs for different weld profiles based on unaveraged principal stresses.

Weld profile	Direct Extraction from $0.1\sqrt{rt}$ Location		Linear Extrapolation	
	1st Order	2nd Order	1st Order	2nd Order
No Weld	-	-	20.2	22.2
Idealised smallest accepted weld profile	20.1	19.8	18.5	20.4
Idealised largest accepted weld profile	18.3	20.3	19.1	21.0
Efthymiou	22.4			

There seems to be some system for the linear extrapolation cases where second-order elements are providing higher SCFs than first-order elements. In addition, in both cases the idealised largest accepted weld profile is providing larger SCFs than the idealised smallest accepted weld profile. The results from the direct extraction method are less systemised.

4. Conclusion

In this paper, a method to determine the SCF that, with the SN-curve given in standards, are expected to provide the code-recommended 2.3% fatigue limit state exceedance. The recommended SCF seems to fit with the upper mean value with 90% confidence interval. In addition, it is shown that $\mu_{SCF} + 0.443 \cdot SD_{SCF}$, where μ_{SCF} is the mean value of the measured SCFs and SD_{SCF} is their standard deviation, may also be used.

Based on the experimental SCF measurements from this study (corrected for strain gauge location), it seems like the SCF predicted by this method will underestimate the experienced SCF in 5 of 15 cases. Hence, this may be a middle route that can be taken in some cases. However, due considerations to the savings in design cost versus the possible significant increase in operational costs, should significant repairs be needed, should be made. A life cycle-cost benefit analysis may not necessarily favour an "optimised" design.

It is important to note that there is no definitive way to prove that measurements and related uncertainties discussed here are representative of similar work conducted in other laboratories. Nevertheless, it is argued that the issues discussed in this paper hold broader significance in a general sense.

A general recommendation regarding FEA to determine SCFs is difficult to give based on this one example, but an indication that an FE model without the weld included in the model seems to be a reasonable choice and is likely the easiest to model. Alternatively, reliable SCF estimates can also be found by using send-order elements in combination with linear extrapolation.

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