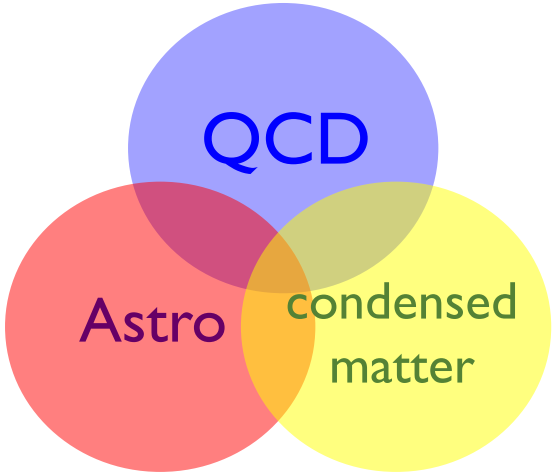
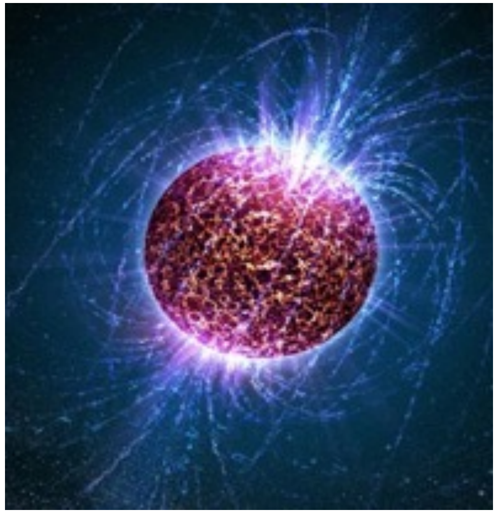
Neutron stars & multi-messenger physics

Toru Kojo

(Tohoku Univ.)



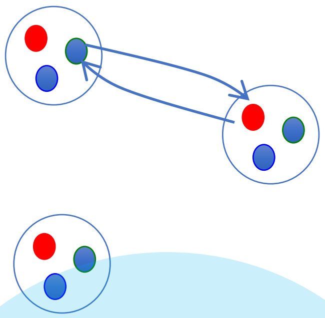


State of matter: overview

($n_0 = 0.16\ \mathrm{fm}^{-3}$)

[Masuda+ '12; TK+ '14]

- few meson exchange
- nucleons only



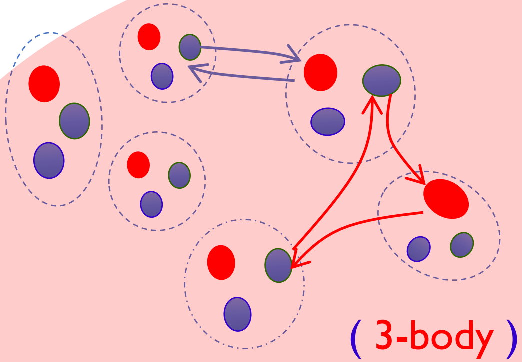
Lect. 1

ab-initio nuclear cal.

laboratory experiments

steady progress

- many-quark exchange
- structural change,...
- hyperons, Δ, ...

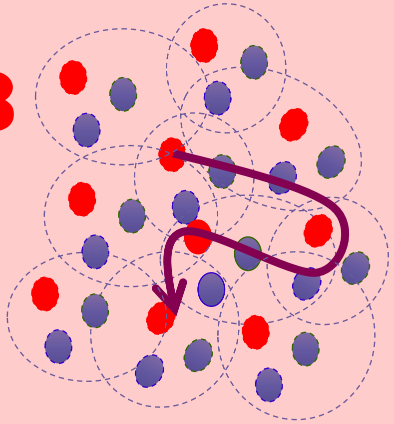


Lect. 3

most difficult

(d.o.f ??)

- Baryons overlap
- Quark Fermi sea



strongly correlated

(d.o.f : quasi-particles??)

not explored well

(pQCD)

[Freedman-McLerran, Kurkela+, Fujimoto+...]

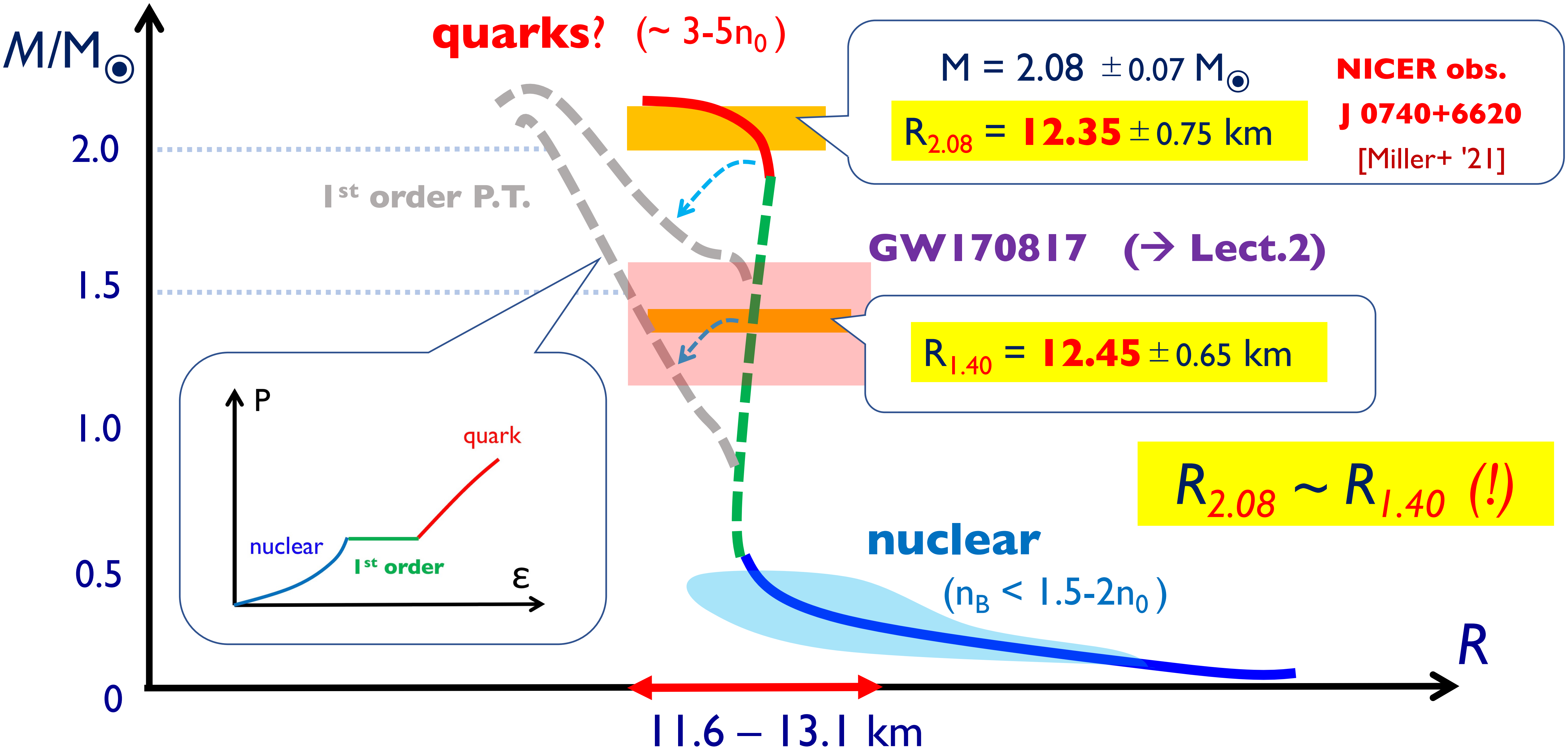
~ 1.4 M☉ **Lect. 1 & 2** ~ 2 M☉

n_B

~ $2n_0$ **Hints from NS** ~ $5n_0$ ~ $40n_0$

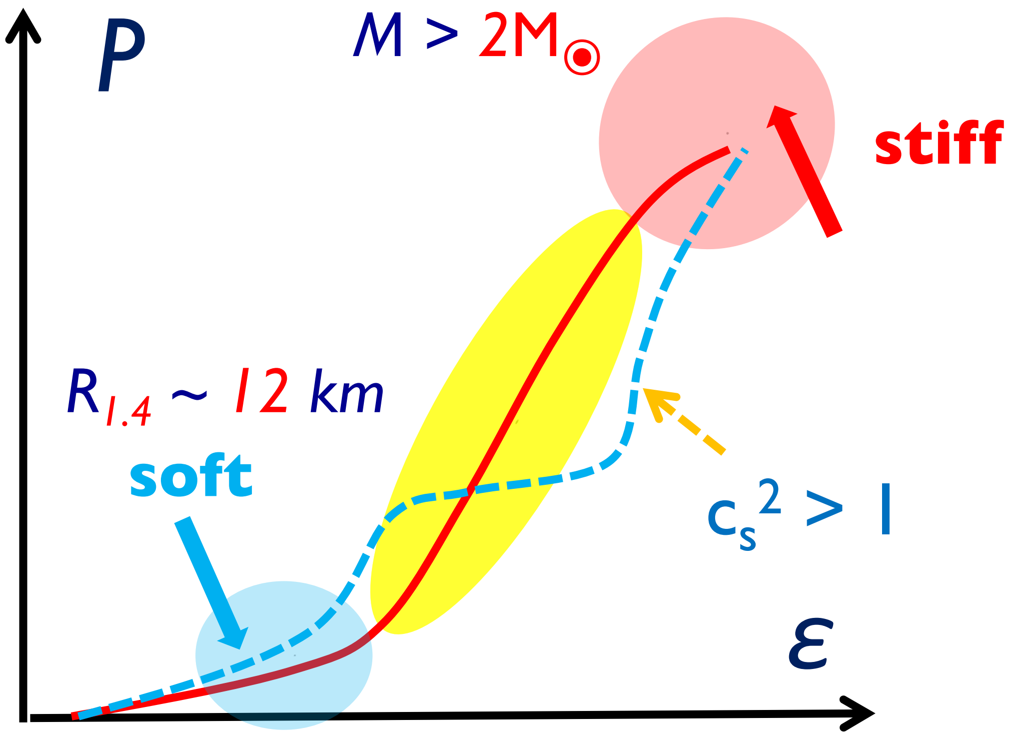
Observations: summary

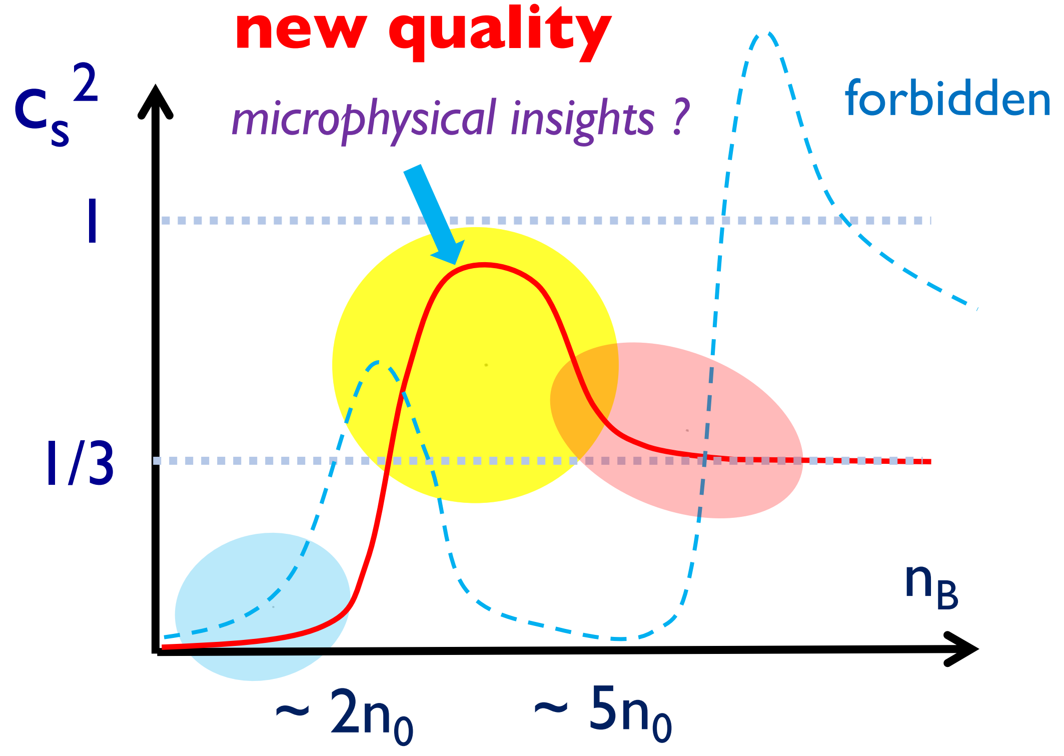
see Lect. 1 & 2



Soft *to* *stiff* *is challenging:* see Lect. I

sound velocity: $c_s^2 = dP/d\varepsilon < 1$ (*causality*) → *nuclear* & *quark* physics constrain each other





baseline: *quark-hadron continuity* **(QHC)**

Plan

Lect 1) Overview

- glancing at NS properties
- M-R relation and EOS
- $R_{1.4}$ & low density EOS

Lect 2) NS-NS mergers

- gravitational waves
- pre-mergers [inspiral & tidal deformation]
- post-mergers [EM-counterparts]

Lect 3) From hadrons to quarks in NS

- quark matter
- 3-window modeling
- stiffening of matter in quark-hadron continuity

Quark matter

from high density down to NS domain

Some notations

We often need to **switch** from nuclear notation to quark's.

Basic relations

number density

$$n_q = n_q^R + n_q^G + n_q^B = N_c n_B$$ (single baryon has N_c-quarks)

note also $n_q^R = n_q^G = n_q^B = n_B$

chemical potential

$$\mu_B n_B = \mu_q n_q$$ (this combo appearing in thermodynamic relations)

➡ $$\mu_B = N_c \mu_q$$

thermodynamic functions

either $P(\mu_q)$ or $\varepsilon(n_B)$ (**all** relations derivable from these)

note: $P(n_B)$ or $P(\varepsilon)$ are less informative

Bag model: simplest quark EOS

massless, N_f -flavors, no-interactions

$$P(\mu_q) = \frac{N_c N_f}{12\pi^2}\mu_q^4 - B \qquad B \sim \Lambda_{\mathrm{QCD}}^4$$

$$\rightarrow \quad n_q = \frac{\partial P}{\partial \mu_q} = \frac{N_c N_f}{3\pi^2}\mu_q^3$$

$$\varepsilon = \mu_q n_q - P$$

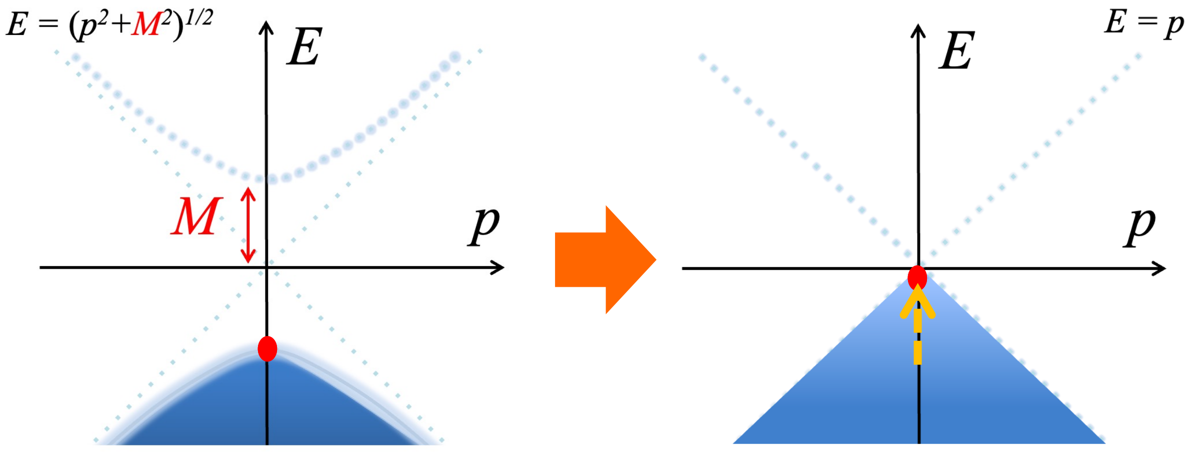
$$\rightarrow \quad \varepsilon = \frac{N_c N_f}{4\pi^2}\mu_q^4 + B = 3P + 4B$$

cost

$$c_s^2 \equiv \frac{\partial P}{\partial \varepsilon} = 1/3$$

independent of B

sym. **broken** | sym. **restored**



more negative E | **less** negative energy

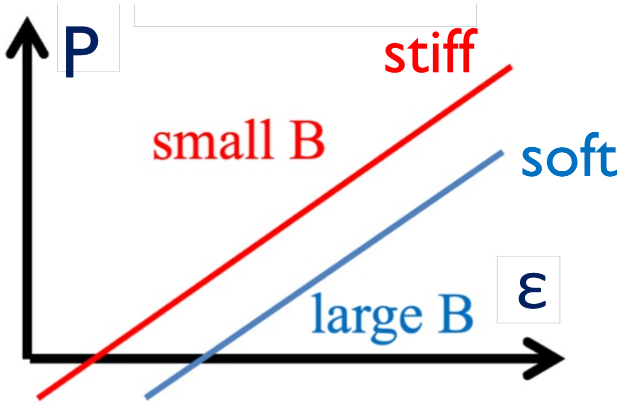
→ energy **cost** !

$\Delta\varepsilon_{\mathrm{Dirac}} \sim B > 0$

1/3 often called conformal limit : kin E >> interactions

Pure quark stars

$c_s^2 = 1/3$ & no int.



onset of matter: $P(\mu_q = \mu_c) = 0$

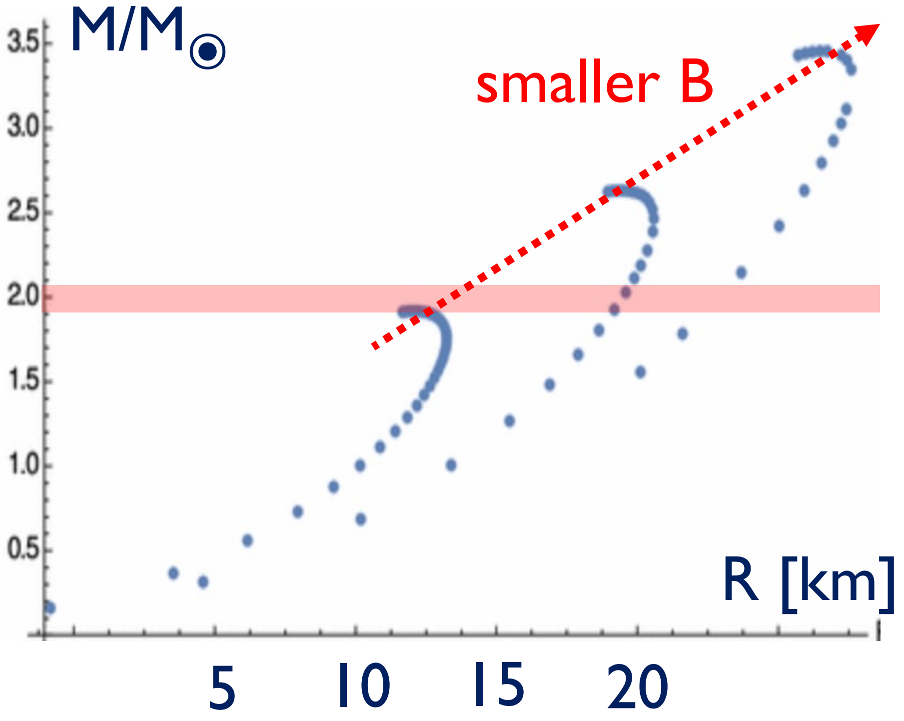
$$n_B(\mu_c) \simeq 1.8 n_0 \times \left(\frac{N_f}{3}\right)^{1/4} \left(\frac{B^{1/4}}{146\,\mathrm{MeV}}\right)^3$$

(P = 0, but $n_B \neq 0$ → self-bound)

For a given B, the EOS leads to

$$M_{\max} \simeq 2M_\odot \times \left(\frac{146\,\mathrm{MeV}}{B^{1/4}}\right)^2$$

$$R|_{M_{\max}} \simeq 10.7\,\mathrm{km} \times \frac{M_{\max}}{2M_\odot}$$

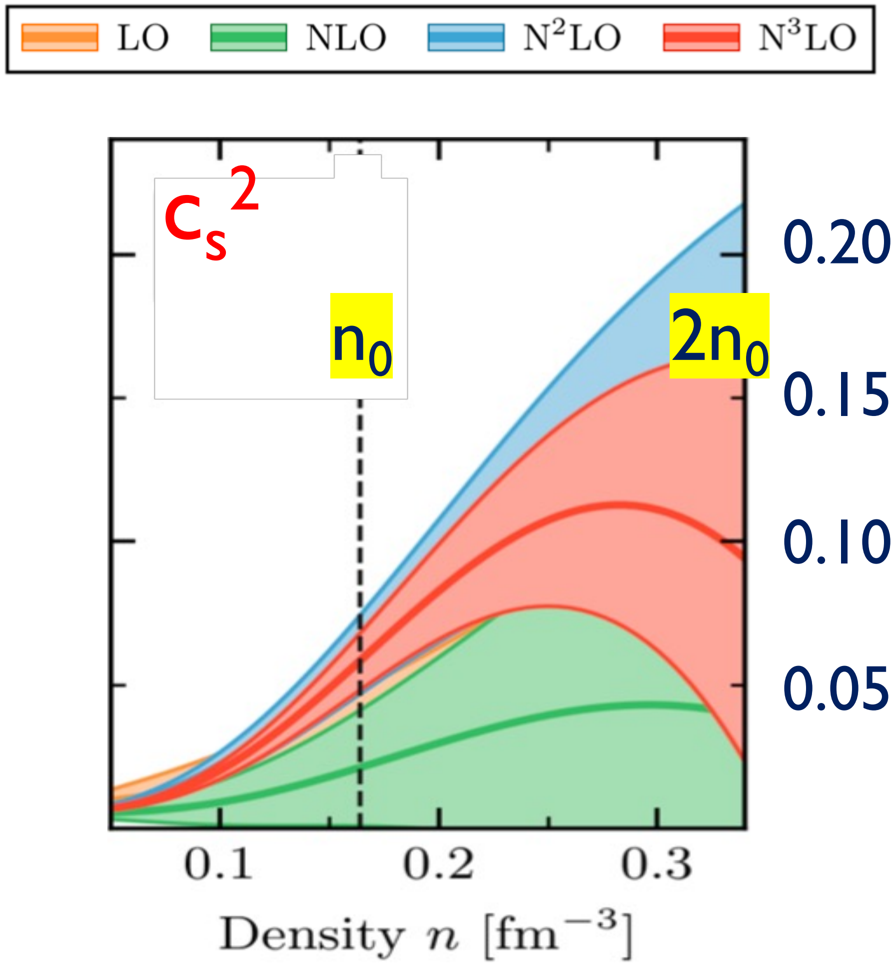


if we ***accept*** quark matter at $n_B < \sim 1.8 n_0$ (small B),

quark stars pass the $2M_\odot$ constraint...

c_s^2 = 1/3 = 0.33… (at 1-3n_0) is large

[e.g., ChEFT, Drischler+ '21]

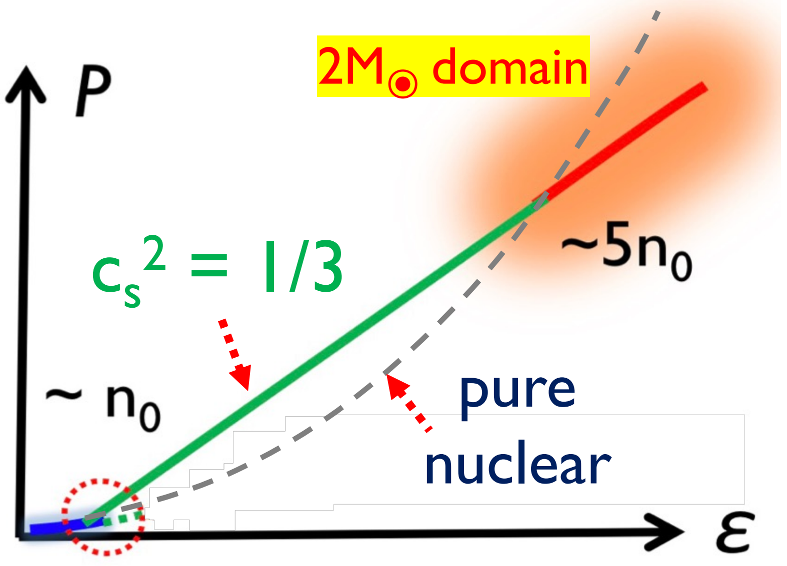


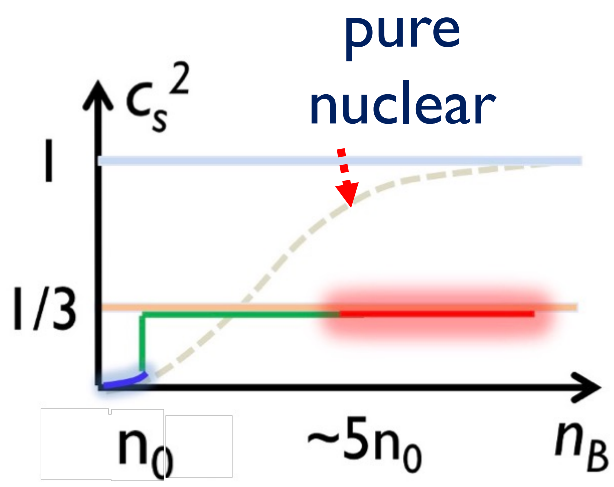
ChEFT (to N³LO)

small..

$c_s^2(n_0)$ ~ 0.05-0.10 $c_s^2(2n_0)$ ~ 0.1-0.2

If we switch to c_s^2 = 1/3 at **low** density…





For systematic analyses, see e.g., Annala+ '20

c_s^2 in purely nucleonic models

mass   kin.   int.

$$\mathcal{P} = n_B^2 \frac{\partial}{\partial n_B}\left(\frac{\varepsilon}{n_B}\right)$$

(the mass term drops !!)

$$\varepsilon(n_B) = \underbrace{m_N n_B}_{\textbf{large (!)}} + a\frac{n_B^{5/3}}{m_N} + b n_B^{\alpha} \quad \Rightarrow \quad P = \underbrace{\frac{2}{3} a \frac{n_B^{5/3}}{m_N}}_{\textbf{small (!)}} + b(\alpha - 1) n_B^{\alpha}$$

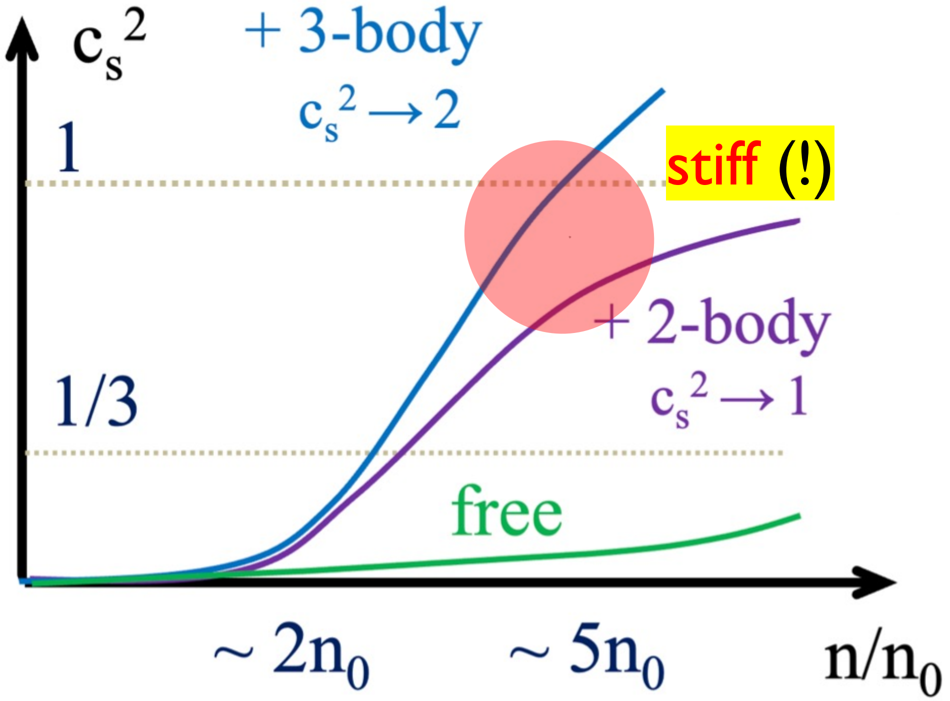
without interactions, $P \sim \varepsilon$ demands $n_B \sim$ **100** n_0 !

with interactions, n_B^{α} term dominates at large n_B

(for $\alpha > 1$)

$$P \sim (\alpha - 1)\varepsilon \ \rightarrow \ c_s^2 \sim (\alpha - 1)$$

[e.g., Sumiyoshi+'21]



Dilemma in purely nucleonic models

low density softness

&

$2M_\odot$ constraint

many-body forces are crucial

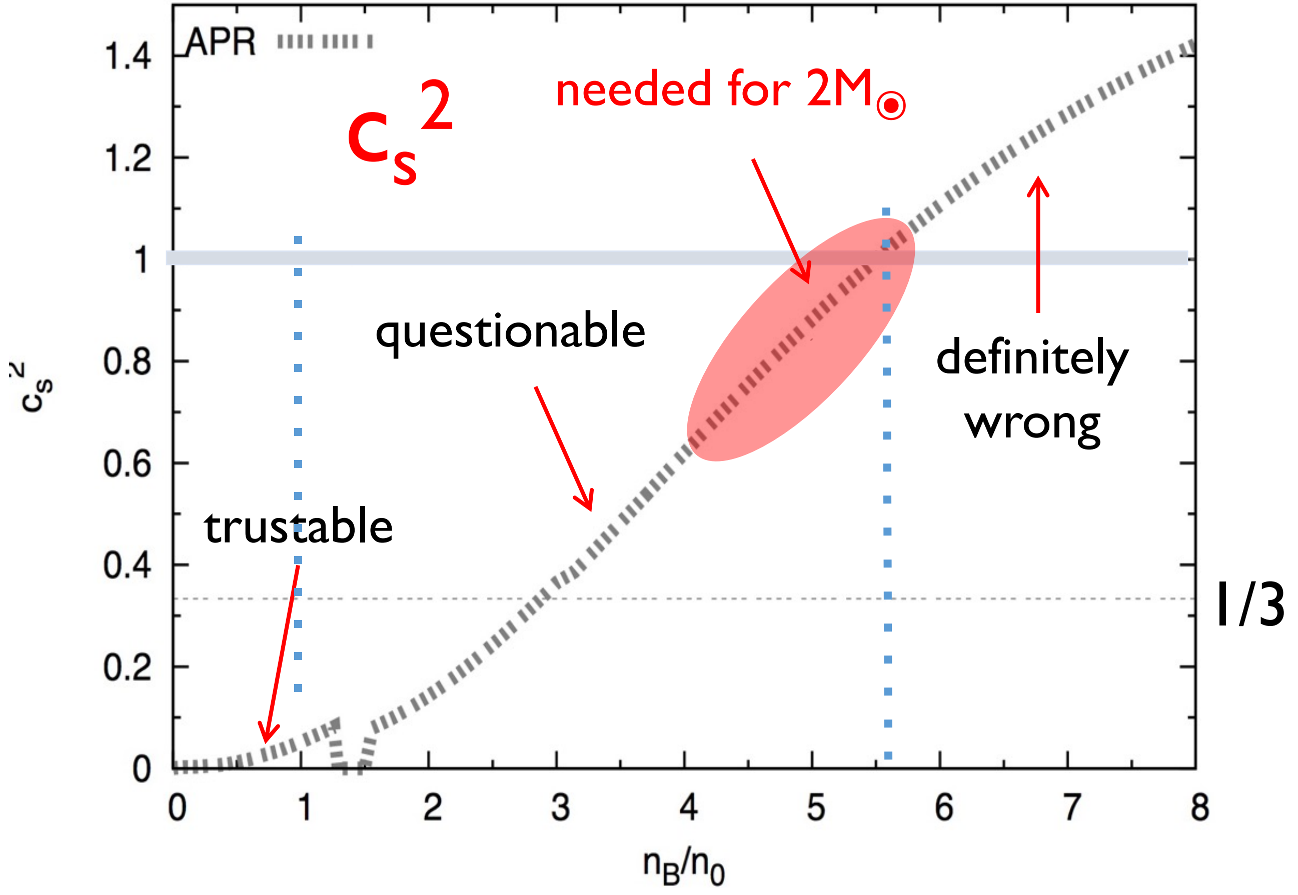
but

the dominance of such forces signals the breakdown of the theory

[2-, 3-, 4-,... –body forces]

this trend is quite common

e.g., APR EOS up to 3NF



Quark matter baseline

$c_s^2 = 1/3$ is the baseline

Now let us first consider corrections assuming weak coupling:

(should be valid at very large density)

$$P(\mu_q) = P_{pQCD}(\mu_q) \underline{- B} + \ldots..$$

normalization

pQCD generally contains corrections of $\sim [\, \alpha_s(\Lambda_{reno}) \ln(E/\Lambda_{reno}) \,]^n$

E is supposed to be $\sim \mu_q$, then natural to choose $\Lambda_{reno} \sim \mu_q$

(note: "*complete*" cal. **should** lead to results independent of Λ_{reno})

Some history of dense pQCD

1977: Freedman-McLerran I, II, III

- **clarify** the **structure** of perturbation theory **at finite density**
- N^2LO EOS for massless quarks
- **plasmon sums** (needed to handle IR divergences)

2010: Kurkela-Romatschke-Vuorinen

- N^2LO EOS with mass corrections
- renormalization scale dependence of α_s → **reliability test**

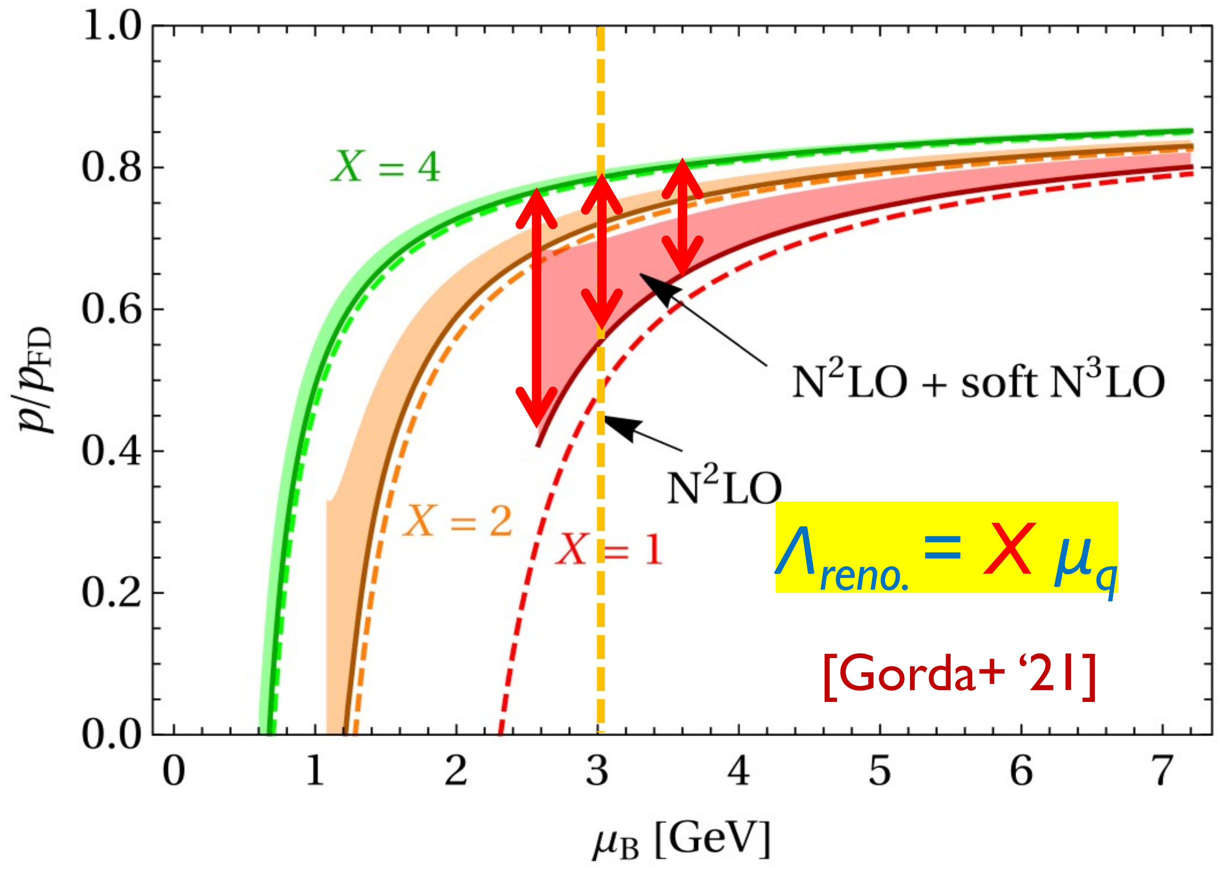
2021: Gorda-Kurkela-Paatelainen-Sappi-Vuorinen

- partial completion of N^3LO EOS; **soft components (HTL)**

[see also Fujimoto+ ’21, Fernandez+ ‘21]

$P_{pQCD}(\mu_B)$ / $P_{SB}(\mu_B)$

X = 1, 2, 4 : renormalization scale dep.

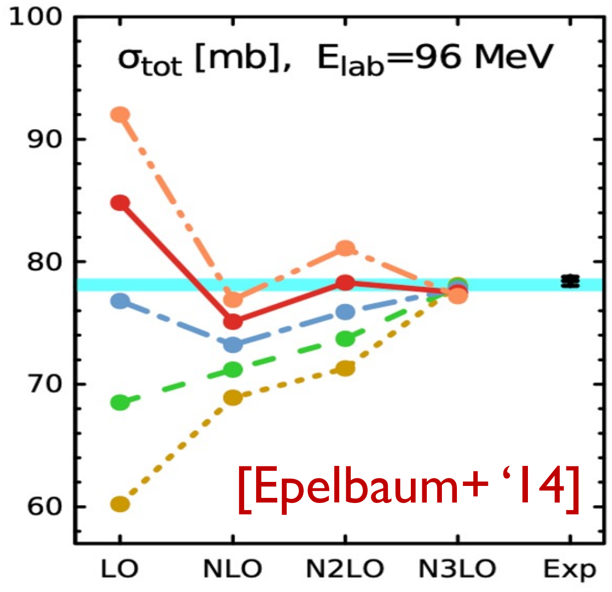


ideally, the result should be indep. of Λ_{reno}

→ measure of **truncation errors**

validity range: $n_B > \sim 40 n_0$

cf) ChEFT with different cutoff scales



higher orders

→ *weaker scale dep.*

c_s^2 in pQCD

soft

$$P_{pQCD}(\mu_q) = c_0\, \mu_q^{4} \left(1 + c_1 \alpha_s + c_2 \alpha_s^2 + c_2' \alpha_s^2 \ln \alpha_s + \dots \right)$$

$(m_D/\mu_q)^2 \sim \alpha_s$

with $\alpha_s(\Lambda_{reno} \sim p_F) \sim 1/\ln(p_F/\Lambda_{QCD})$

1-loop RG: $$p_F \frac{\partial \alpha_s}{\partial p_F} = -\frac{33 - 2N_f}{24\pi} \alpha_s^2$$

$$c_s^2 \rightarrow \frac{1}{3}\left(1 - \frac{15}{16}\left(\frac{\alpha_s}{\pi}\right)^2\right)$$

approaching 1/3 **from below**

(interactions → softening ??)

Note: Λ_{QCD} appears only through logarithms;

but in QCD power corrections often play important roles

(non-perturbative effects) [e.g., Wilson, Shifman+ '70s]

power corrections for stiff/soft EOS ?

what kinds of interactions lead to stiff/soft EOS? cf) [TK-Powell-Song-Baym, '14]

rela. kin. energy, interactions

$$\varepsilon(n) = an^{4/3} + bn^{\alpha}$$

→ ideal gas, interactions

$$P = \frac{\varepsilon}{3} + b\left(\alpha - \frac{4}{3}\right)n^{\alpha}$$

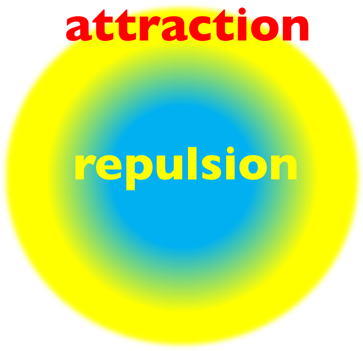
For stiff EoS:

(for large P)

for $\alpha > 4/3$: b > 0 (e.g. bulk repulsion, ~ $+ n_B^{2}/\Lambda_{QCD}^{2}$)

for $\alpha < 4/3$: b < 0 (e.g. surface pairings, ~ $- \Lambda_{QCD}^{2} n_B^{2/3}$)

quark
Fermi sea
(ideal combo)

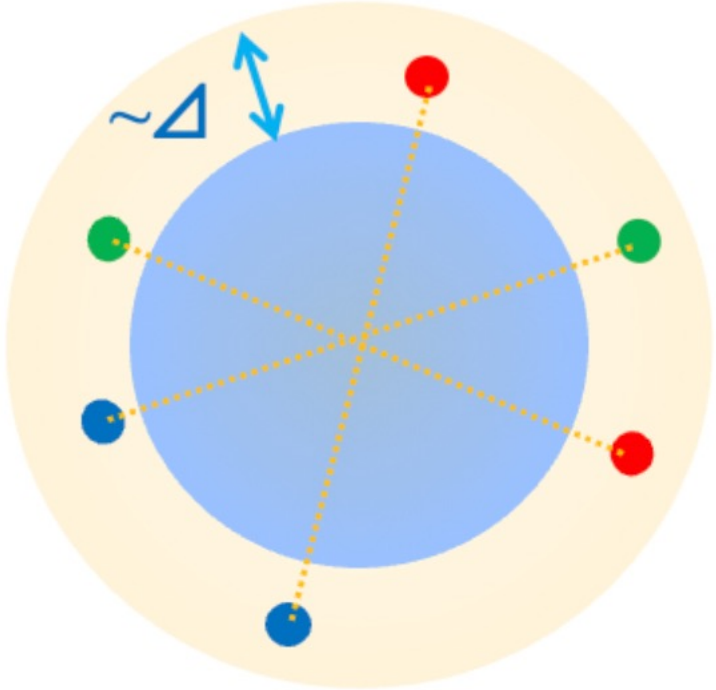


Physics near the Fermi surface (!)
is important

possible physics near the Fermi surface

2-particle correlation

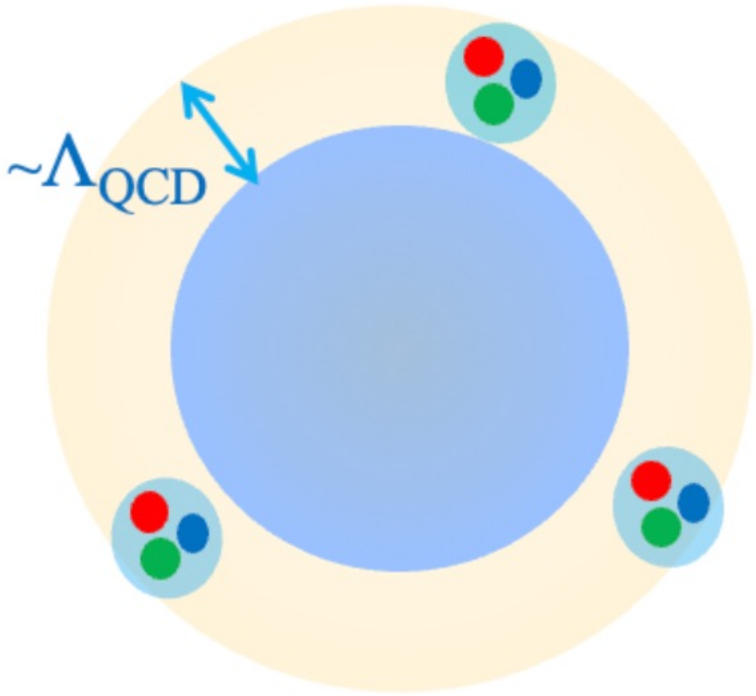
e.g.,
diquark
pairing



color-superconductor (CSC)

[Bailin-Love, Alford, Rajagopal, Wilczek, ...]

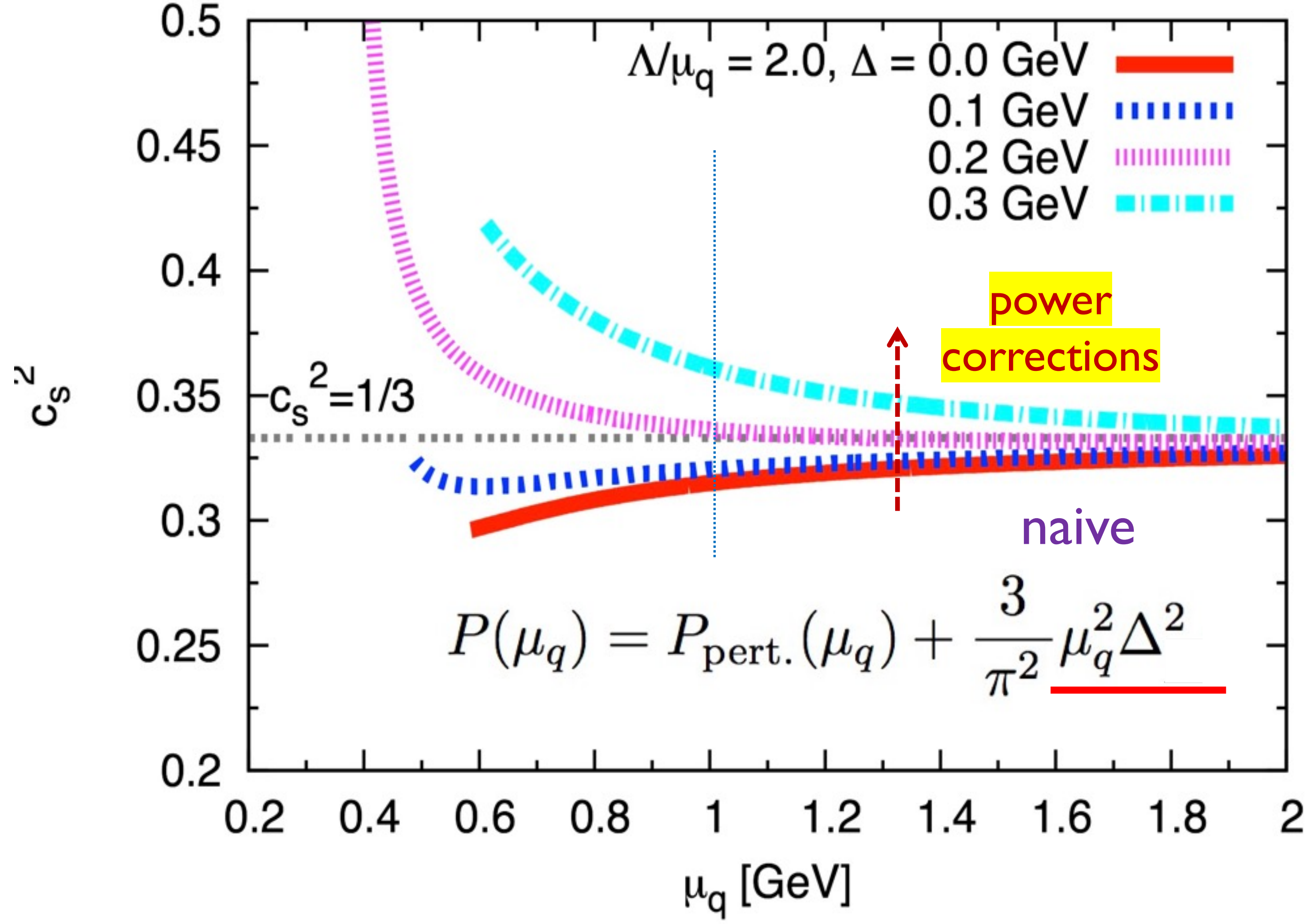
3-particle correlation



quarkyonic matter

[McLerran-Pisarski '07, Hidaka, TK, ...]

c_s^2 vs pQCD + power corrections



e.g. diquark pairing (CFL) terms

For $\Delta \sim$ **0.2** GeV $\sim \Lambda_{QCD}$

$(\Delta/\mu_q)^2 \sim$ **4 %**

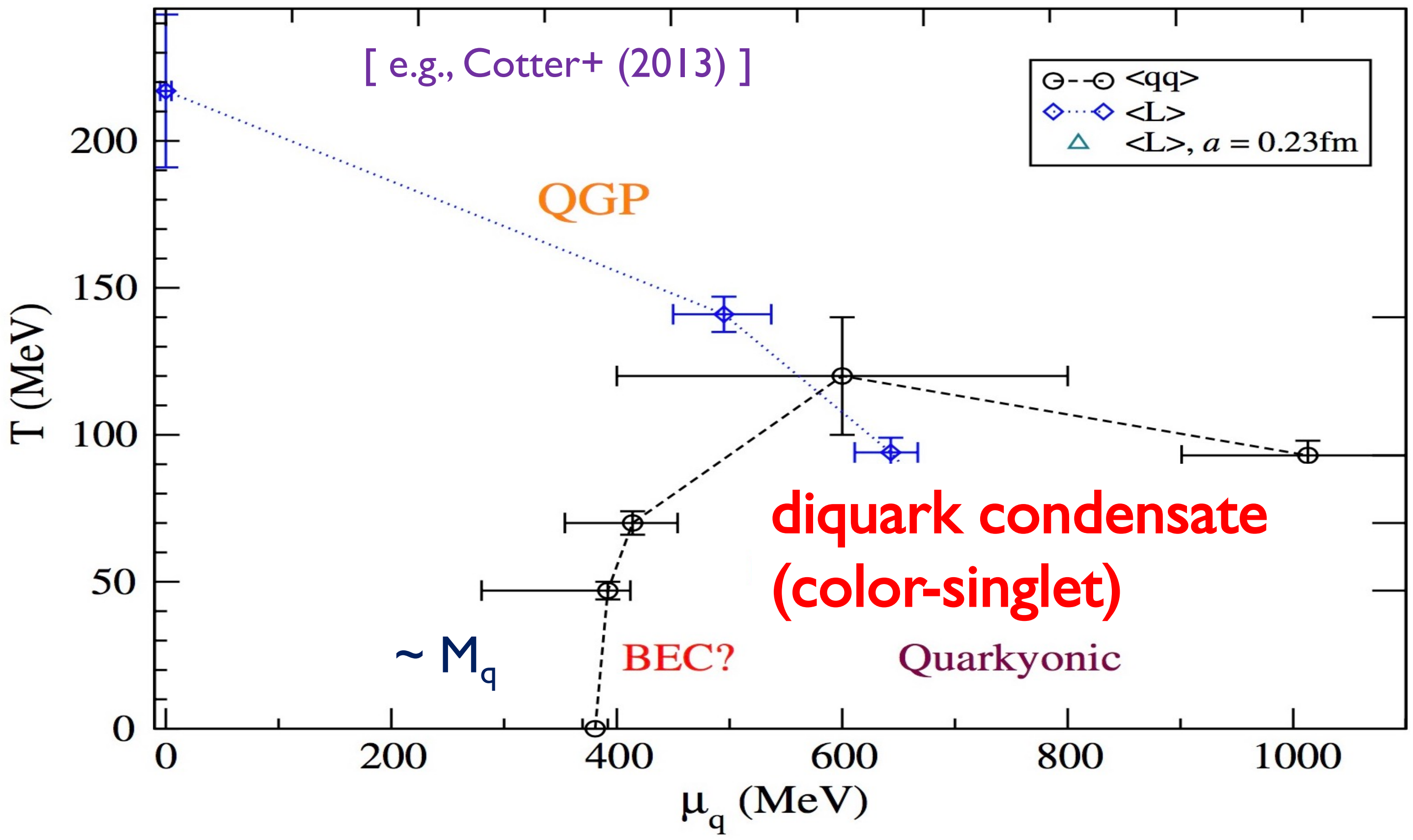
nevertheless,

c_s^2 approach 1/3 from **above**

should be more important toward low density

Non-perturbative effects at very high density?

hints: 2-color QCD → no sign problem in lattice Monte-Carlo



(m_{pi} ~ 700 MeV ~ $2M_q$)

- T_c^{BCS} ~ **100 - 120** MeV

(even at μ_q ~ **1GeV**)

naive BCS estimate

→ **Δ** ~ 1.75 T_c ~ **175 - 210** MeV

~ Λ_{QCD} (!)

μ_q ~ **1** GeV or n_B ~ **40** n_0 → chances for power corrections

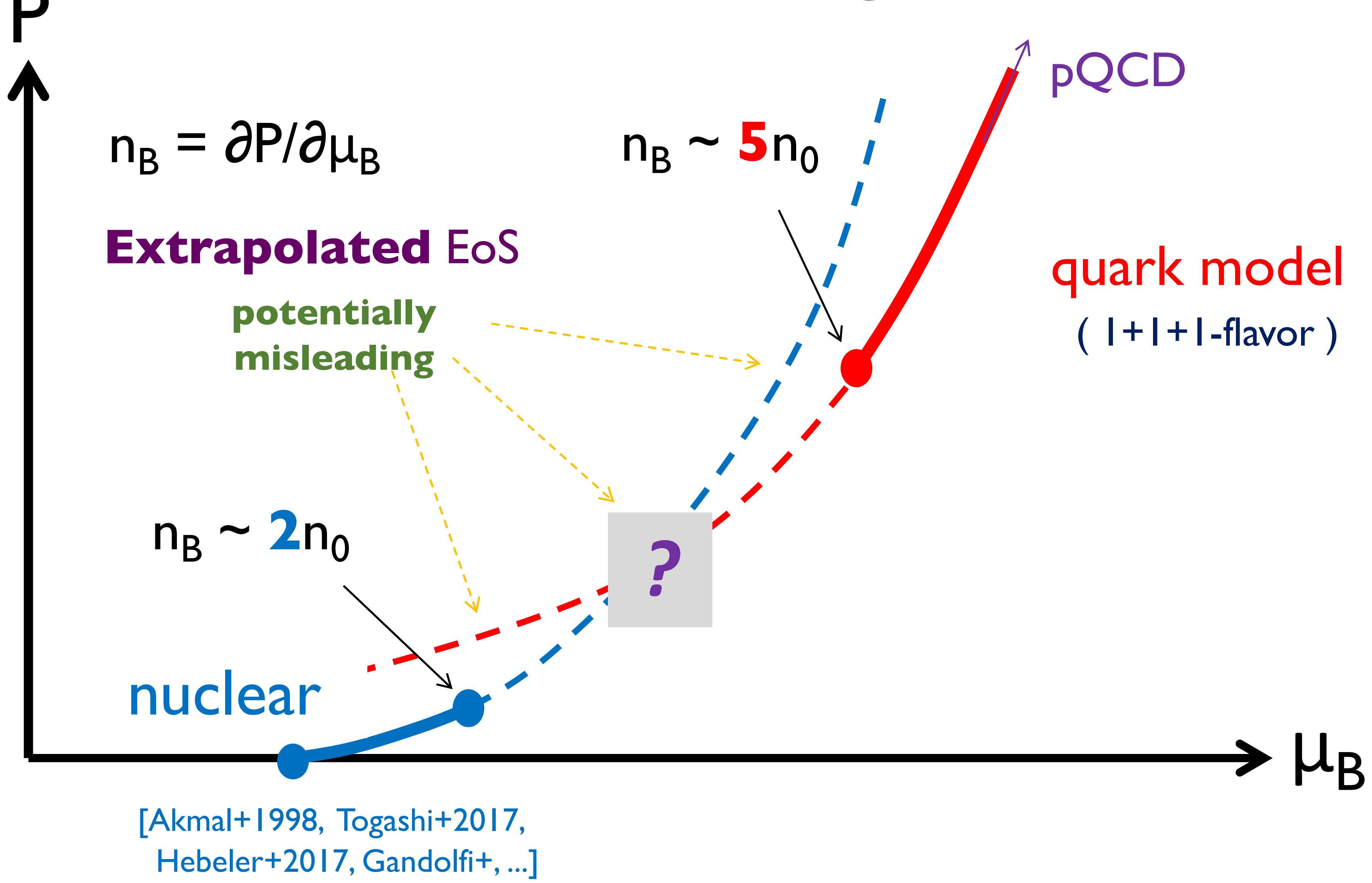
Three window modeling

[see also Fraga+ ‘14, Gorda ’22,.... for QCD → EOS constraints]

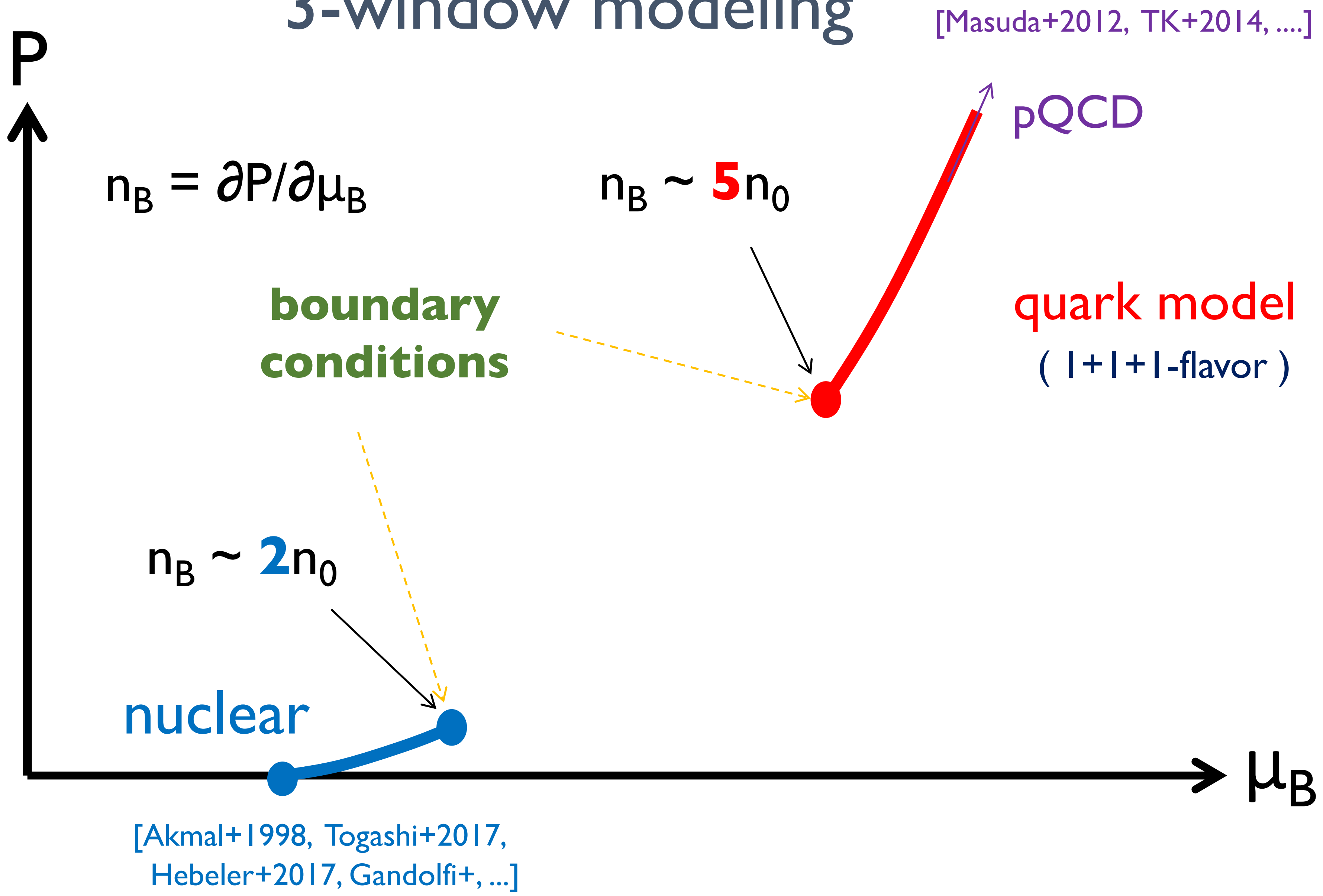
[TK+ ’14, Baym+ ’18,.... for EOS constraints → insights]

3-window modeling

[Masuda+2012, TK+2014, ...]

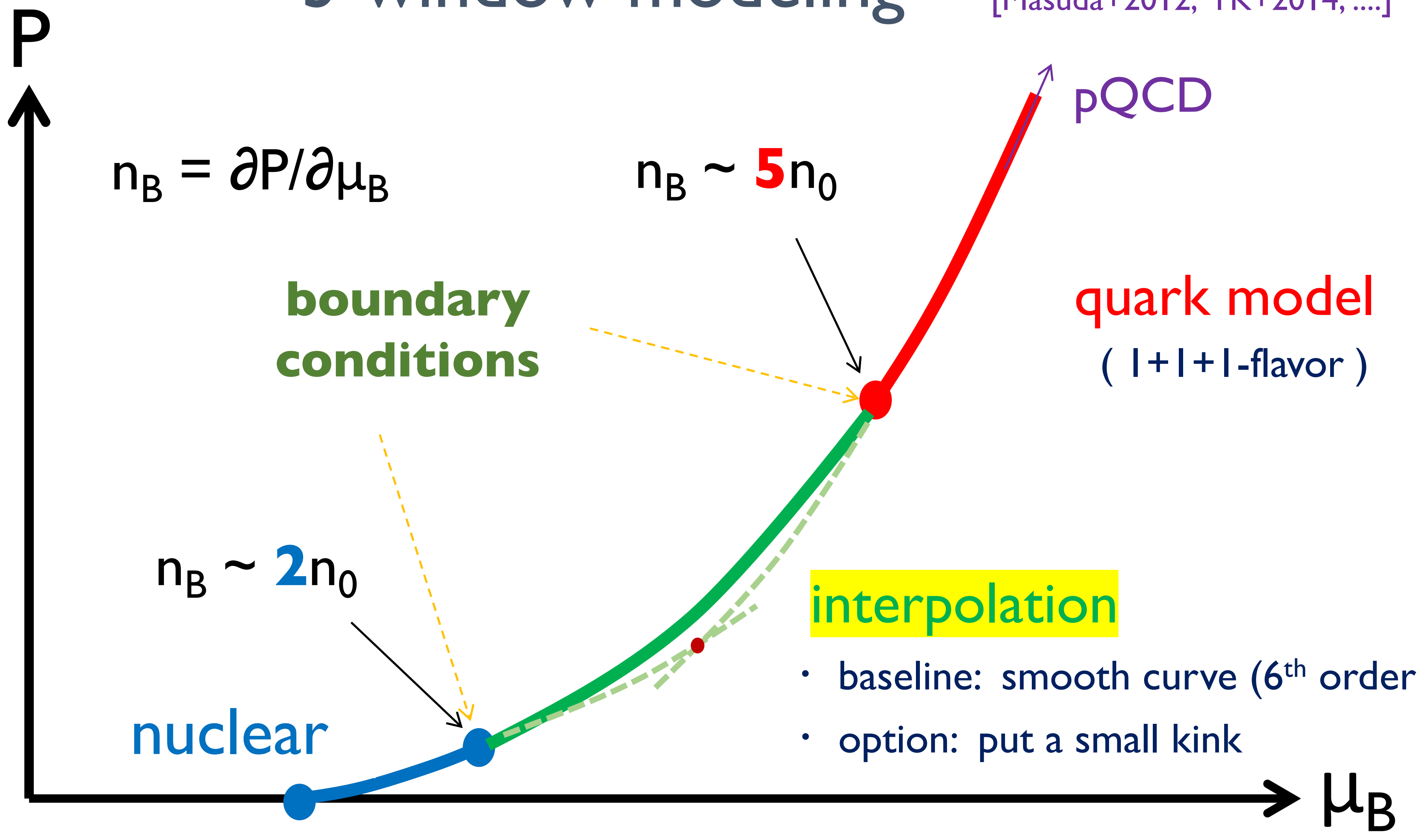


3-window modeling



3-window modeling

[Masuda+2012, TK+2014, ...]

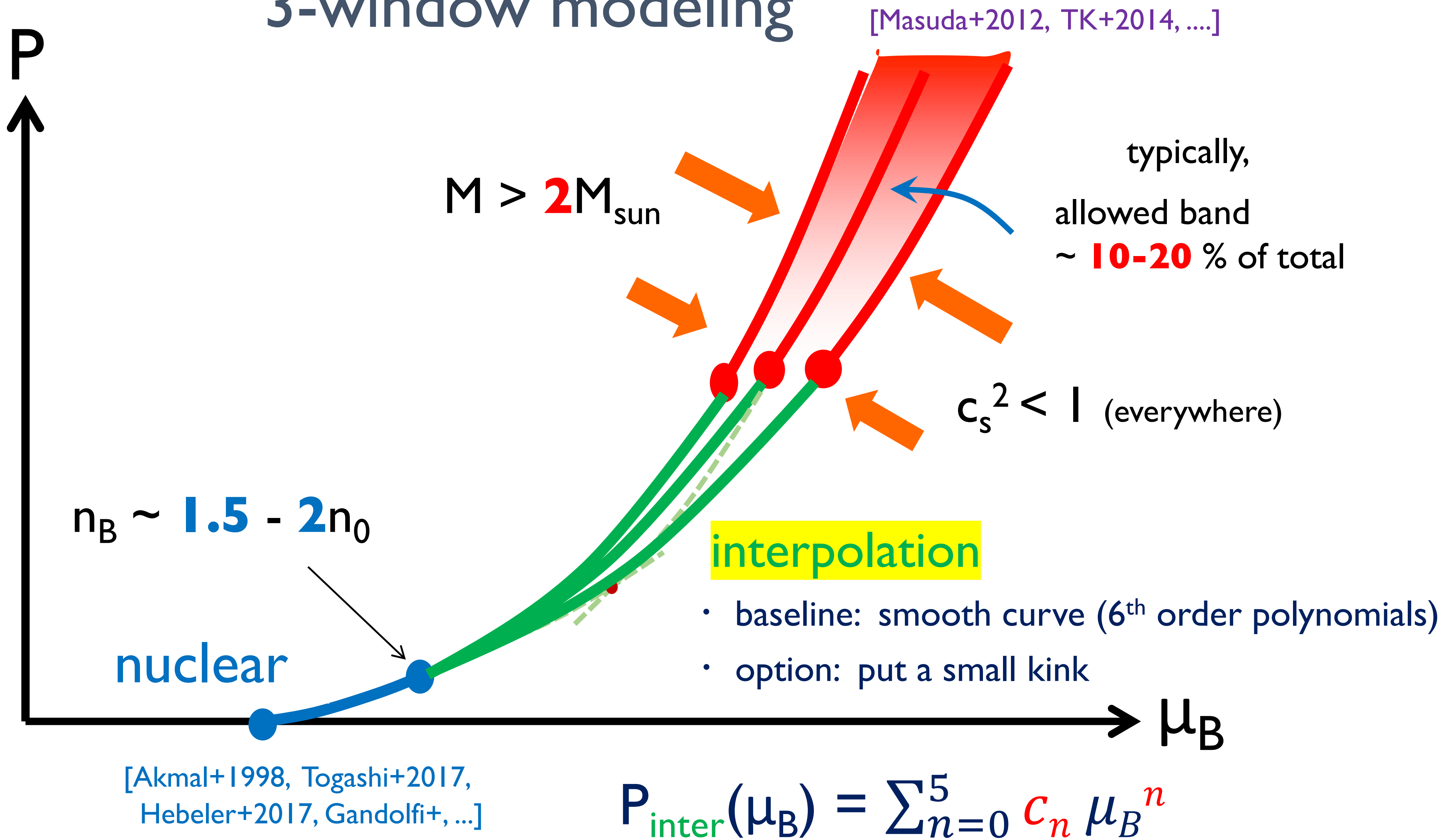


- baseline: smooth curve (6^{th} order polynomials)
- option: put a small kink

[Akmal+1998, Togashi+2017, Hebeler+2017, Gandolfi+, ...]

$$P_{inter}(\mu_B) = \sum_{n=0}^{5} c_n \, \mu_B{}^n$$

3-window modeling



- baseline: smooth curve (6th order polynomials)
- option: put a small kink

$$P_{inter}(\mu_B) = \sum_{n=0}^{5} c_n \mu_B^n$$

A quark model for n_B > ~ **5**n_0 (~ 1fm^{-3})

A guide : *Quark-Hadron Continuity* : eff. Hamiltonian continuously evolves from hadron physics

"3-window" [Manohar-Georgi 1983, Weinberg 2010,...]

Q < ~0.2 GeV	0.2 GeV < Q < 1-2 GeV	~2 GeV < Q
very long-range (> 1fm)	constituent quarks + OGE (quasi-particles)	short range
confinement	**chiral** SB & color-**mag**. int. & **baryon-baryon**. int.	pQCD

A template) chiral color-mag. nB-nB int.

$$\mathcal{H} = \mathcal{H}_{\rm NJL} - \underline{H} \sum_A (q\Gamma_A q)(\bar{q}\Gamma_A \bar{q}) + \underline{g_V} \, (\bar{q}\gamma_0 q)^2$$

solve within **MF**
+ color- & charge- neutrality
+ β-equilibrium

[Masuda+2015, TK+2014, Blaschke+....]

(gv, H): both inspired from color-mag. interactions

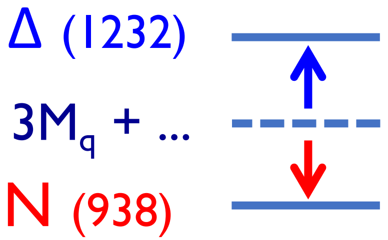
[e.g., Oka-Yazaki '80]

Color-magnetic interaction play **many** roles

1) **Coupling** ∝ **velocity** ~ **p/E**

become important in relativistic regime & high density

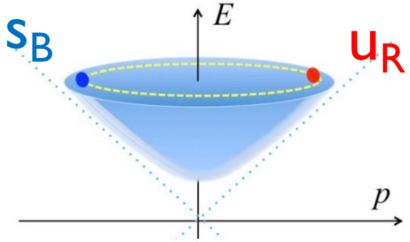
cf)



2) **Pairing** : strongly channel dependent

hadron mass ordering: N-Δ, etc. [DeRujula+ (1975), Isgur-Karl (1978), ...]

color-super-conductivity [Alford, Wilczek, Rajagopal, Schafer,... 1998-]



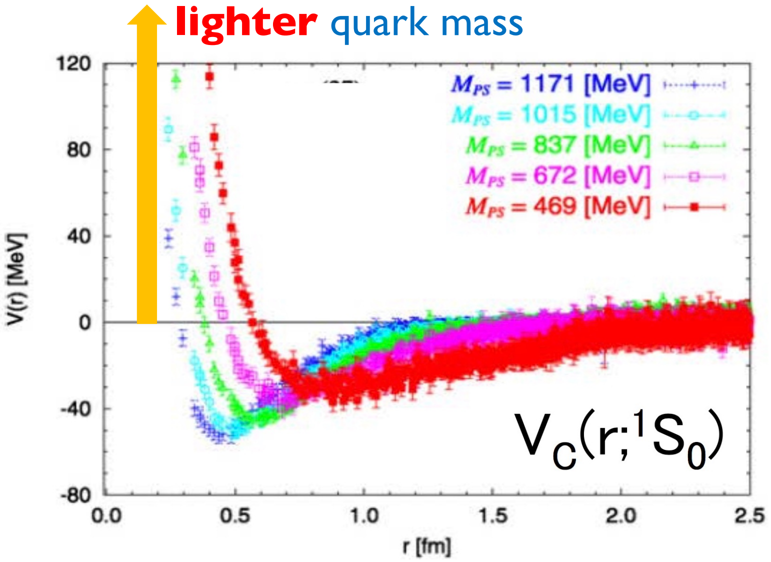
3) **Baryon-Baryon int.** : **short-range** correlation

(**Pauli** + **color-mag.**) [Oka-Yazaki (1980),...]

channel dep. → non-universal hard core (some are attractive!)

mass dep. → stronger hard core in relativistic quarks

→ **consistent with the lattice QCD** [HAL-collaboration]

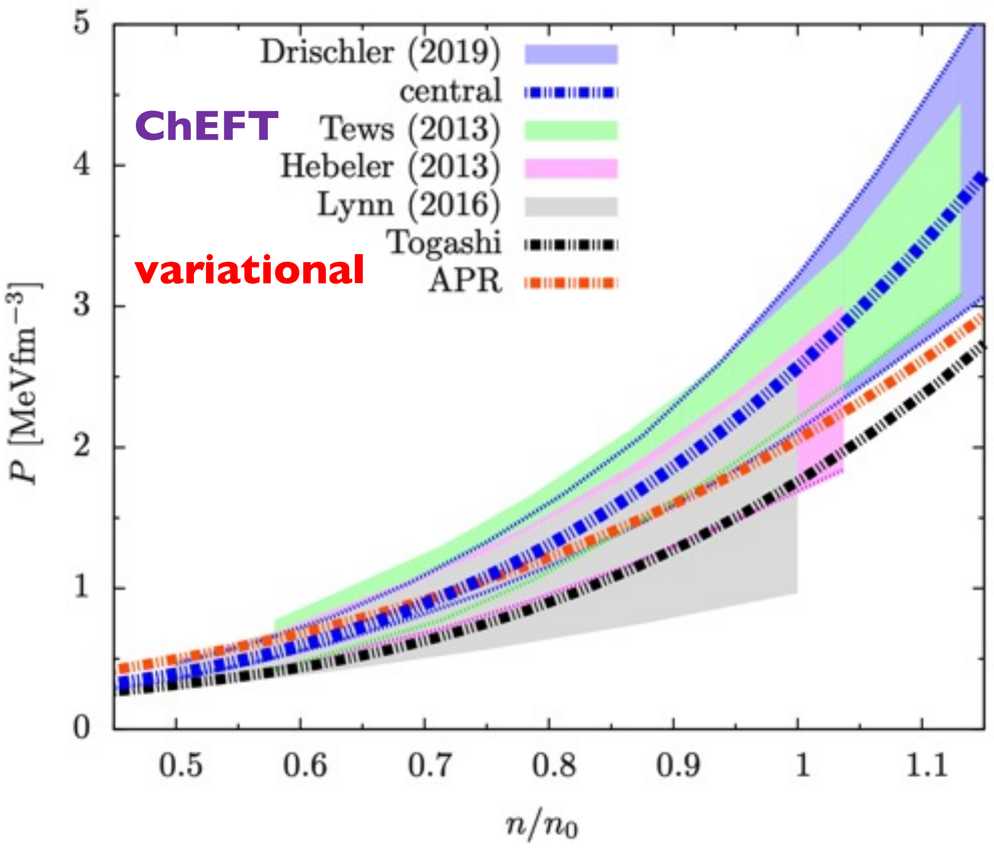


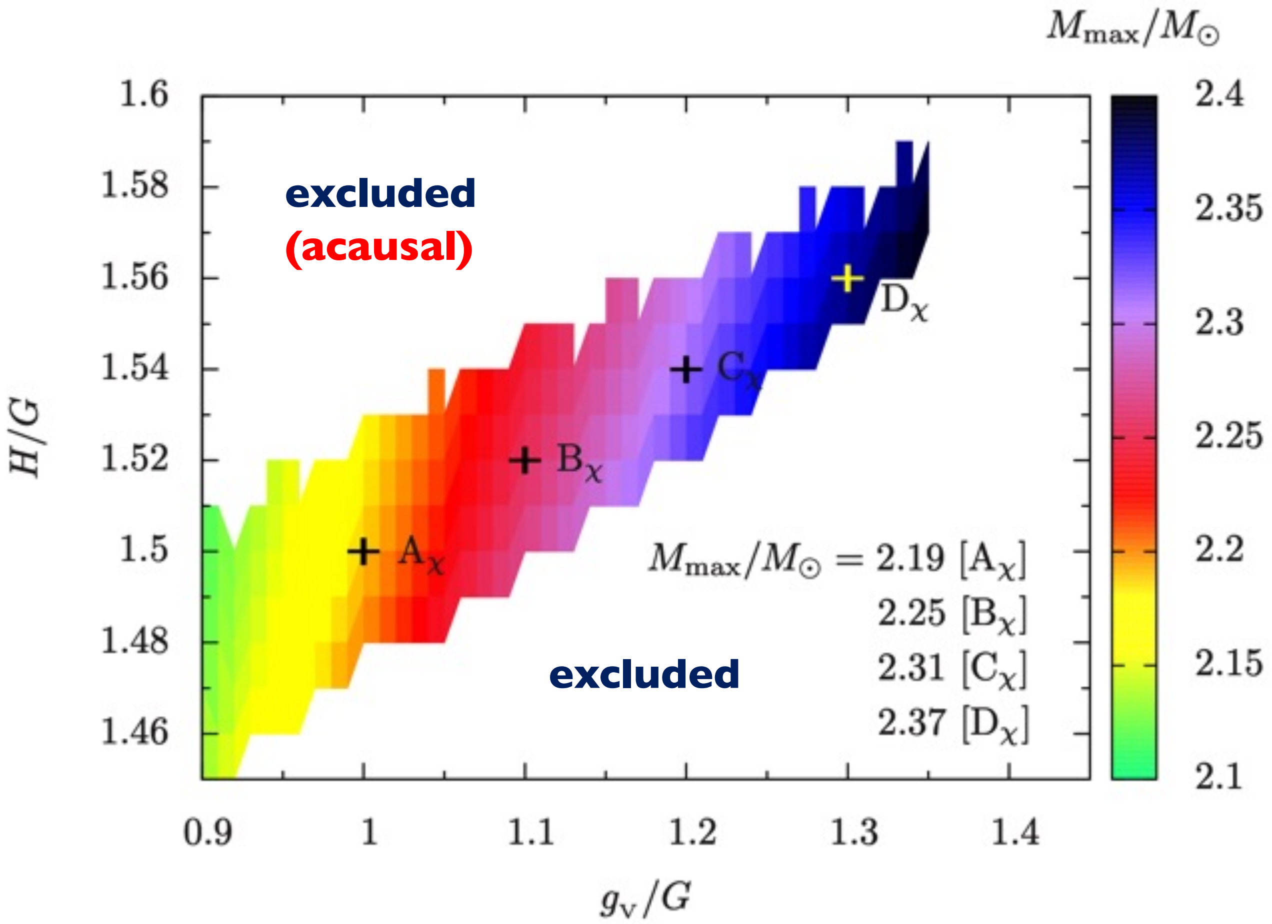
An exercise: survey for (g_V, H)@3.5-5n0 [Baym+ '19, TK '21]

Step1) Prepare **realistic** nuclear EoS up to **1.5-2**n_0

[e.g. Akmal+1998, **Togashi+2017**, **ChEFT**, ...] → 30-40% uncertainties in P @ ~ n_0

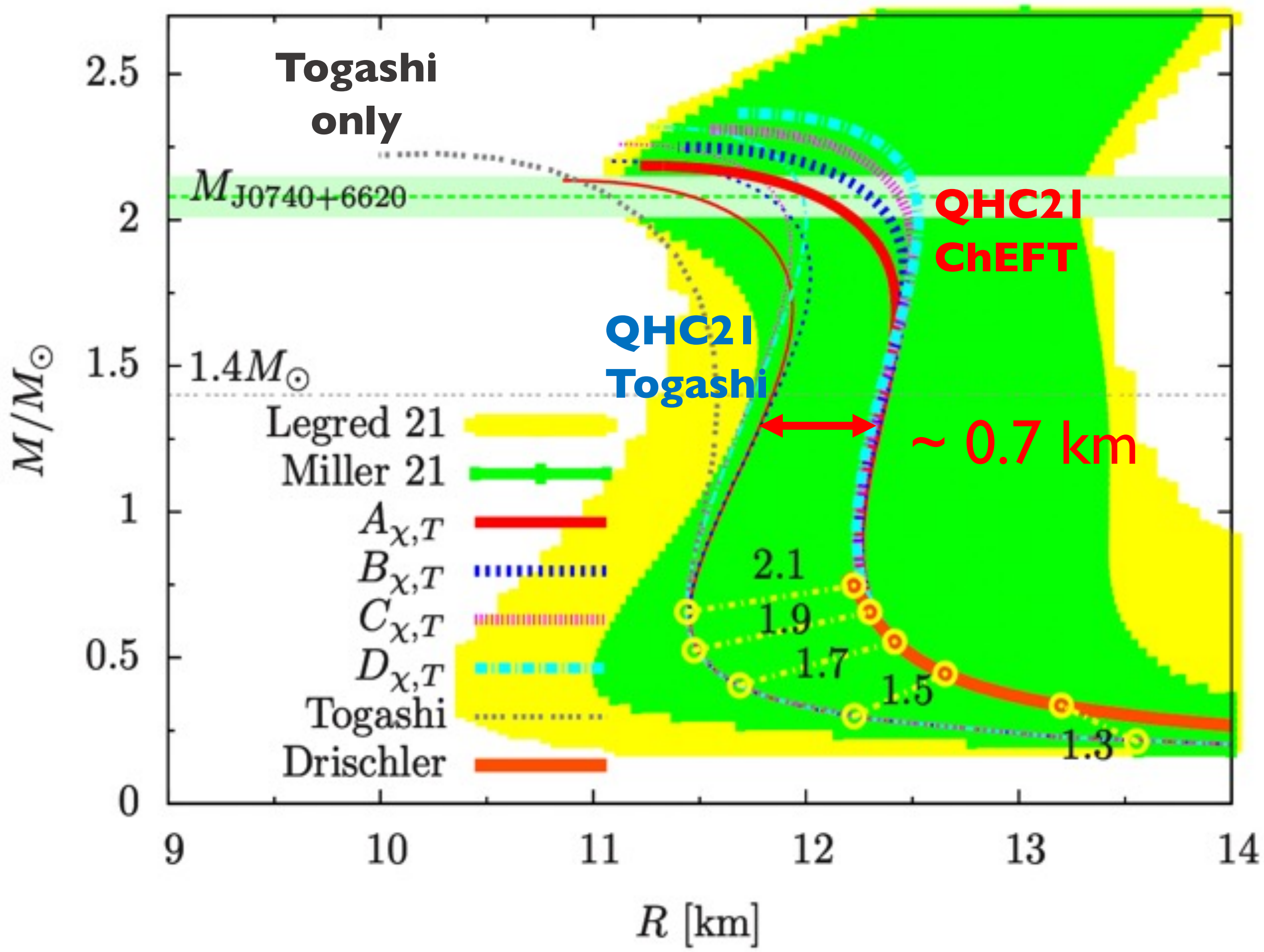
Step2) Survey the range of (g_V, H) consistent with **causality & stability**

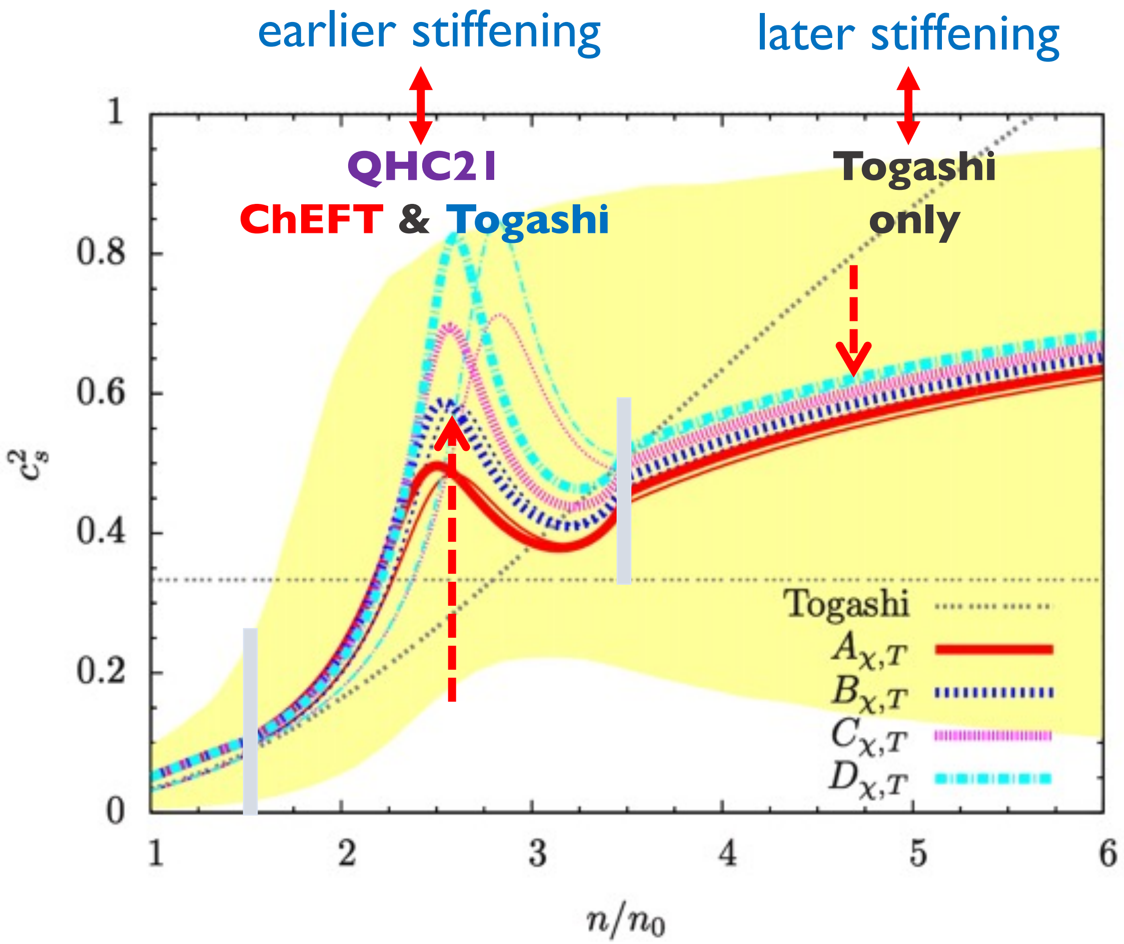




An exercise: survey for (g_V, H) @3.5-5n0

[Baym+ '19, TK '21]





- nuclear uncertainties → $\Delta R_{1.4}$ ~ 0.7 km, but the peak in c_s^2 robust
- QHC type models → earlier stiffening than in pure hadronic models

Trends found in this exercise (for quark matter part)

for quark EoS consistent with all constraints

- bottom line: $(g_V, H)_{@3.5\text{-}5n0} \sim (G_s)_{@vac}$

  interactions remain non-perturbative (!)

- Slow chiral restoration

  at $5n_0$: $M_u \sim M_d \sim$ 50 MeV >> ~ 5 MeV, $M_s \sim$ 300 MeV >> ~100 MeV

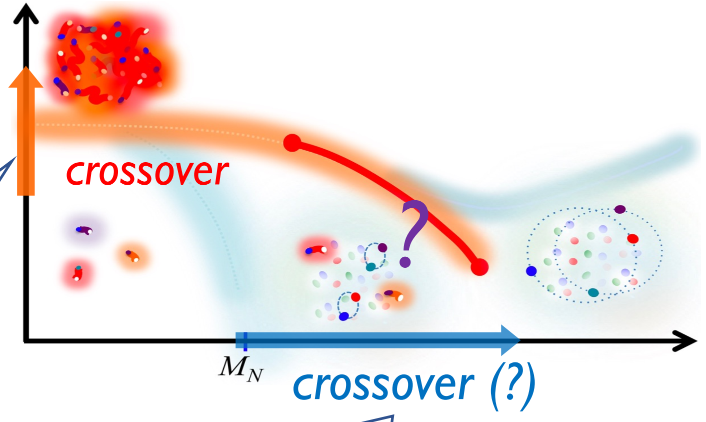
- Pairing effects important

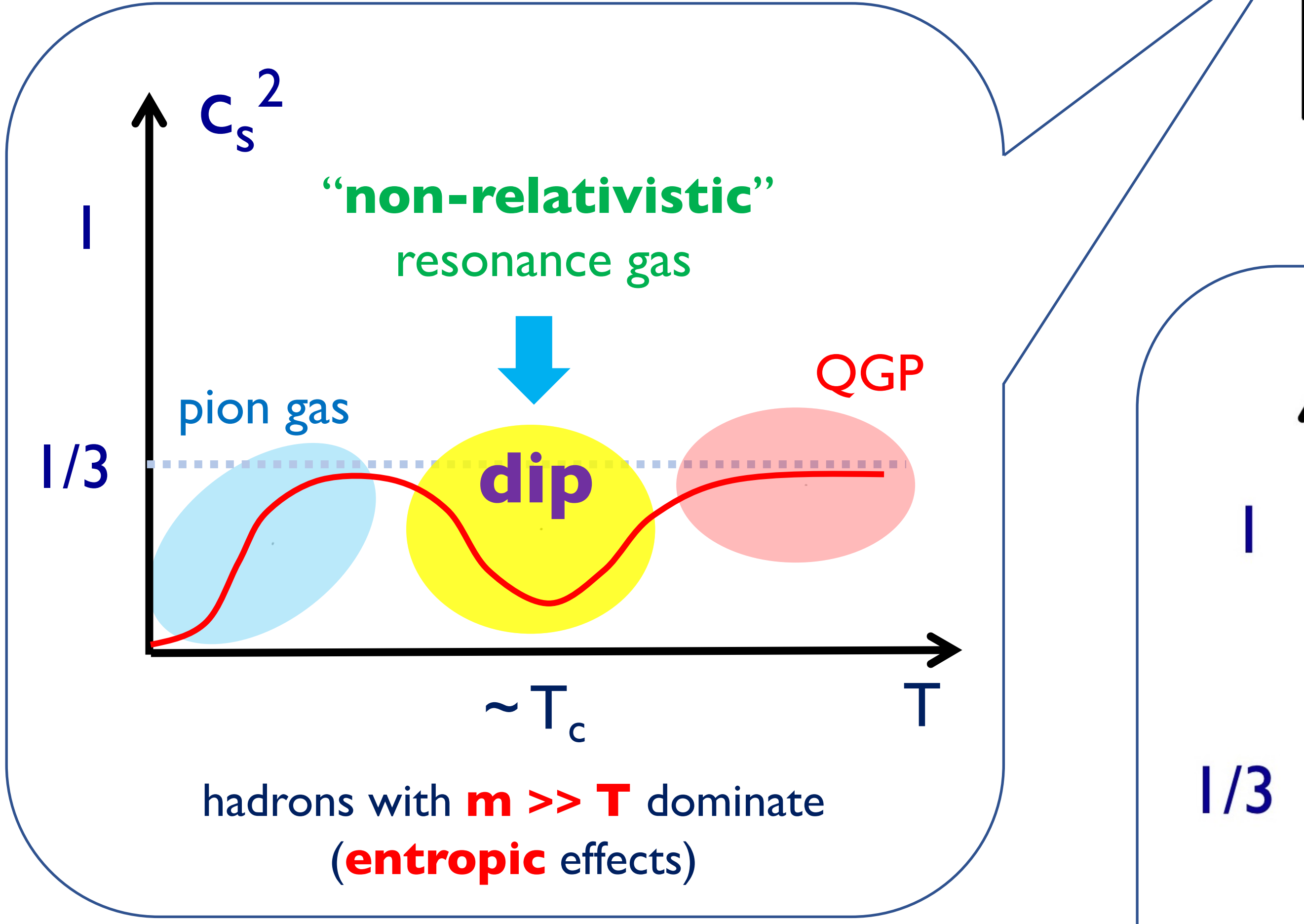
  at $5n_0$: $\Delta_{CFL} \sim$ **200 - 250** MeV (!) ~ Λ_{QCD}

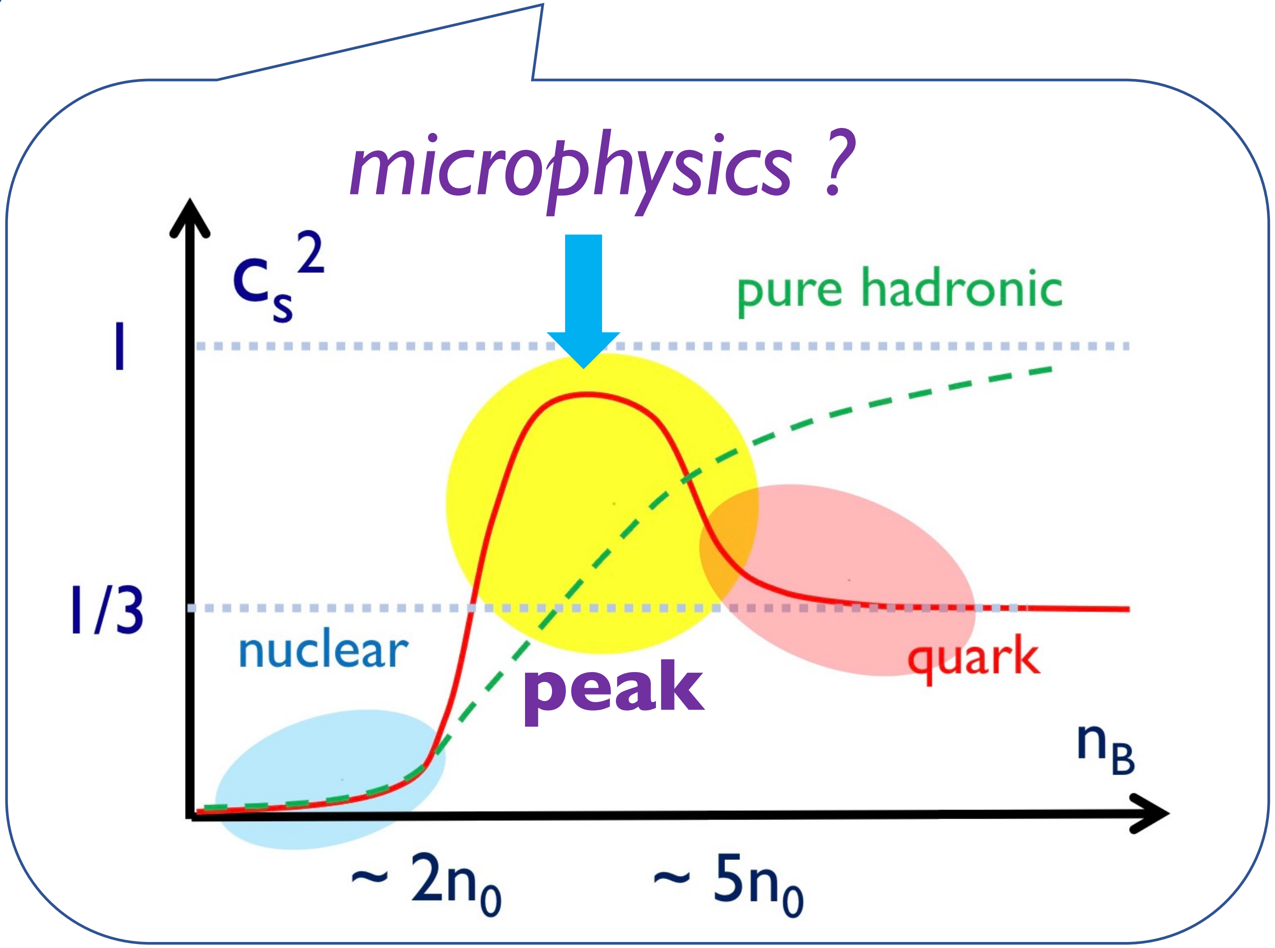
- For allowed range of (g_v, H), $M_{max} \sim 2.4\ M_{sun}$ (with ChEFT B.C. at $1.5n_0$)

Stiffening of matter in quark-hadron continuity

Crossovers & $c_s^2 = dP/d\varepsilon$







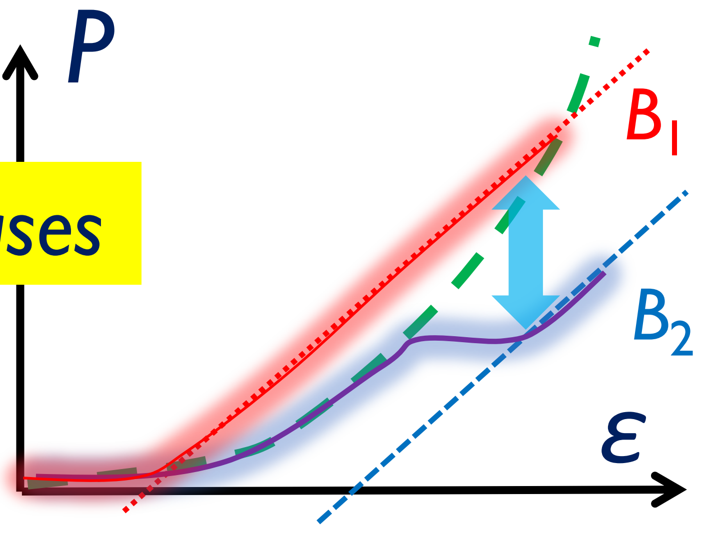
Direct descriptions for 2-5n_0 ?

confusing point:

- *Switching from baryonic to quark bases*

→ *a source of confusions in hybrid models*

(e.g. normalization of energy)



Strategy

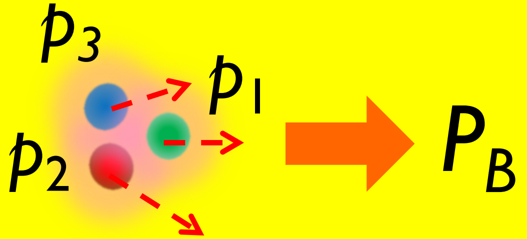
Follow quark states from nuclear to quark matter

(within a *single* model, e.g., percolation model, Fukushima-TK-Weise '20)

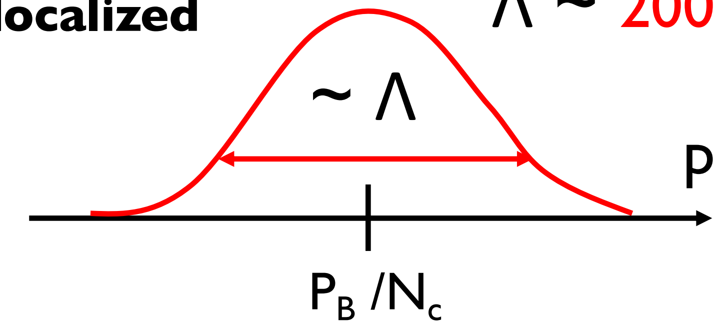
Quarks in a baryon

N_c (=3): number of colors

probability density: $$Q_{\rm in}(\boldsymbol{p}, \boldsymbol{P}_B) = \mathcal{N} e^{-\frac{1}{\Lambda^2}\left(\boldsymbol{p}-\frac{\boldsymbol{P}_B}{N_c}\right)^2}$$



localized



$\Lambda \sim 200$ MeV

mean: $$\langle \boldsymbol{P}_B \rangle = N_c \int_{\boldsymbol{p}} \boldsymbol{p} Q_{\rm in}(\boldsymbol{p}, \boldsymbol{P}_B)$$

variance: $$\left\langle \left(\boldsymbol{p} - \frac{\boldsymbol{P}_B}{N_c}\right)^2 \right\rangle \sim \Lambda^2$$

energetic !

$$\langle E_q(\boldsymbol{p}) \rangle_{\boldsymbol{P}_B} = \mathcal{N} \int_{\boldsymbol{p}} E_q(\boldsymbol{p}) e^{-\frac{1}{\Lambda^2}\left(\boldsymbol{p}-\frac{\boldsymbol{P}_B}{N_c}\right)^2} \simeq \langle E_q(\boldsymbol{p}) \rangle_{\boldsymbol{P}_B=0} + \frac{1}{6}\left\langle \frac{\partial^2 E_q}{\partial p_i \partial p_i} \right\rangle_{\boldsymbol{P}_B=0} \left(\frac{\boldsymbol{P}_B}{N_c}\right)^2 + \cdots$$

average energy (quark)

x N_c x N_c

$\sim N_c (M_q + \Lambda)$ $\gg$ $\sim P_B^2 / (N_c E_q)$

baryon mass baryon kin. energy

A new **unified** model for **QHC**

cf) [TK '21, TK-Suenaga '21]

occupation **probability** of **quark** state with p

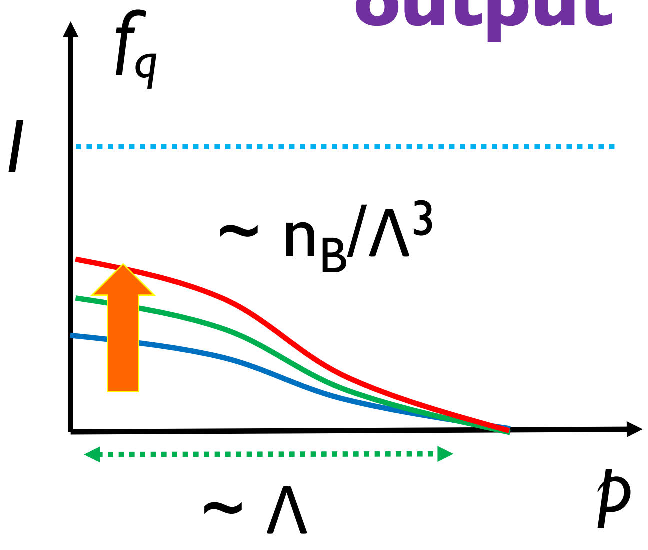
occupation **probability** of **baryon** state with P_B

quark mom. distribution **in a baryon**

$$f_q(\underline{p}; n_B) = \int_{\underline{\boldsymbol{P}_B}} \mathcal{B}(\underline{P_B}; n_B) Q_{\rm in}(\underline{\boldsymbol{p}}, \underline{\boldsymbol{P}_B})$$

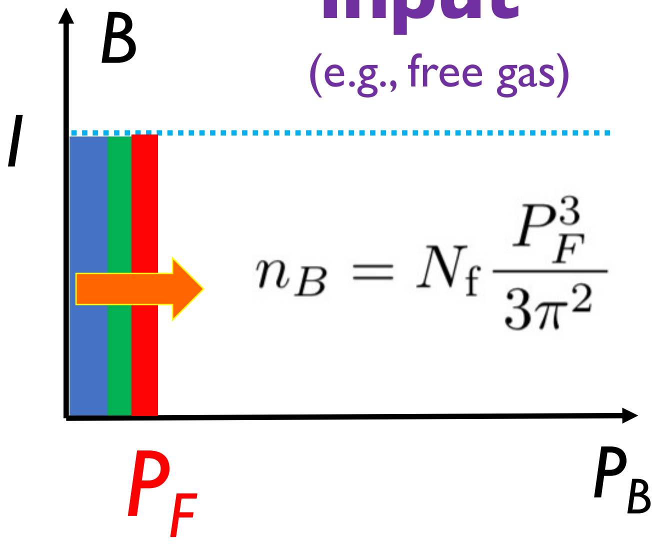
e.g.) in **ideal** baryonic matter

output



input

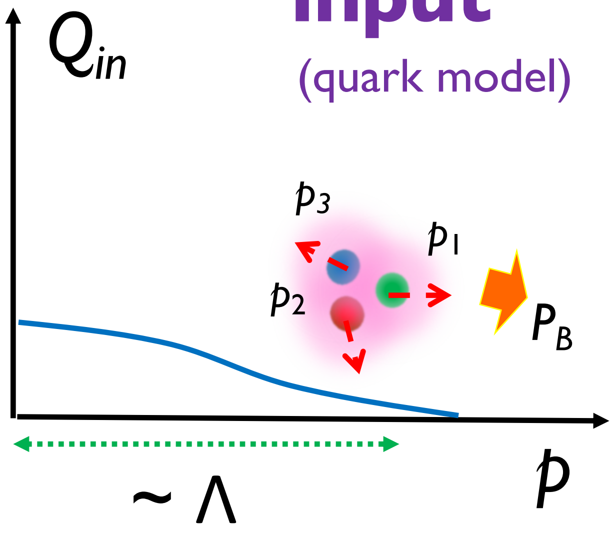
(e.g., free gas)



⊗

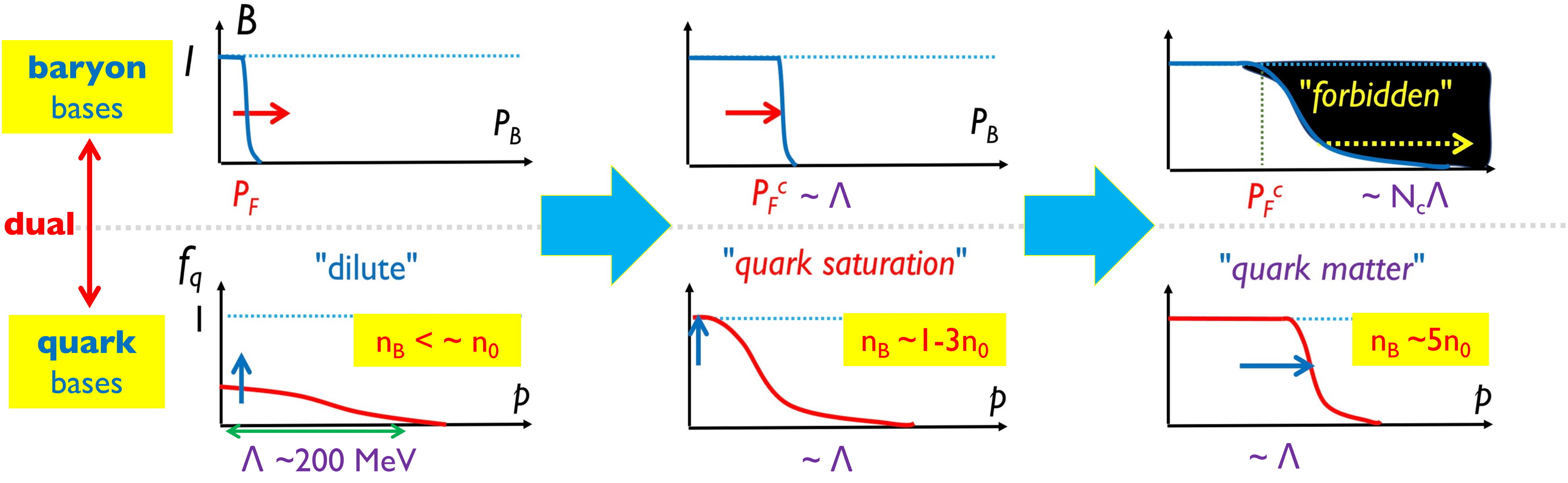
input

(quark model)



Evolution of occ. probabilities

$$f_q(p; n_B) = \int_{\boldsymbol{P}_B} \mathcal{B}(P_B; n_B) Q_{\rm in}(\boldsymbol{p}, \boldsymbol{P}_B)$$



"**quark saturation**" constraint

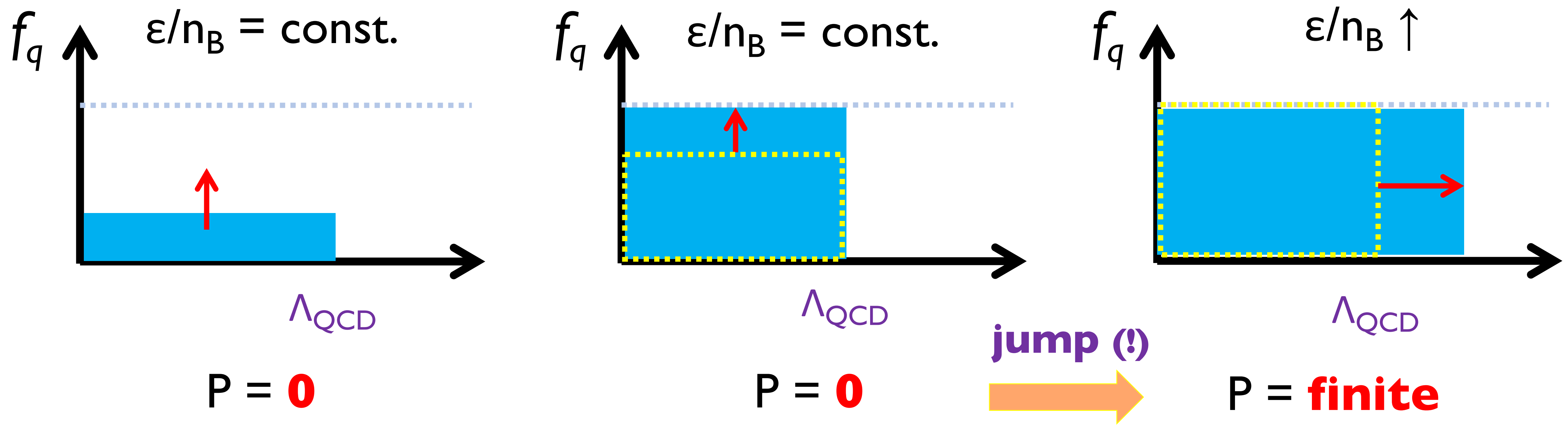
→ **relativistic baryons at low density, n_B ~ 1-3n_0 !**

cf) McLerran-Reddy model (2018) of quarkyonic matter

Jump in pressure : schematic picture

$$\mathcal{P} = n_B^2 \frac{\partial}{\partial n_B}\left(\frac{\varepsilon}{n_B}\right)$$

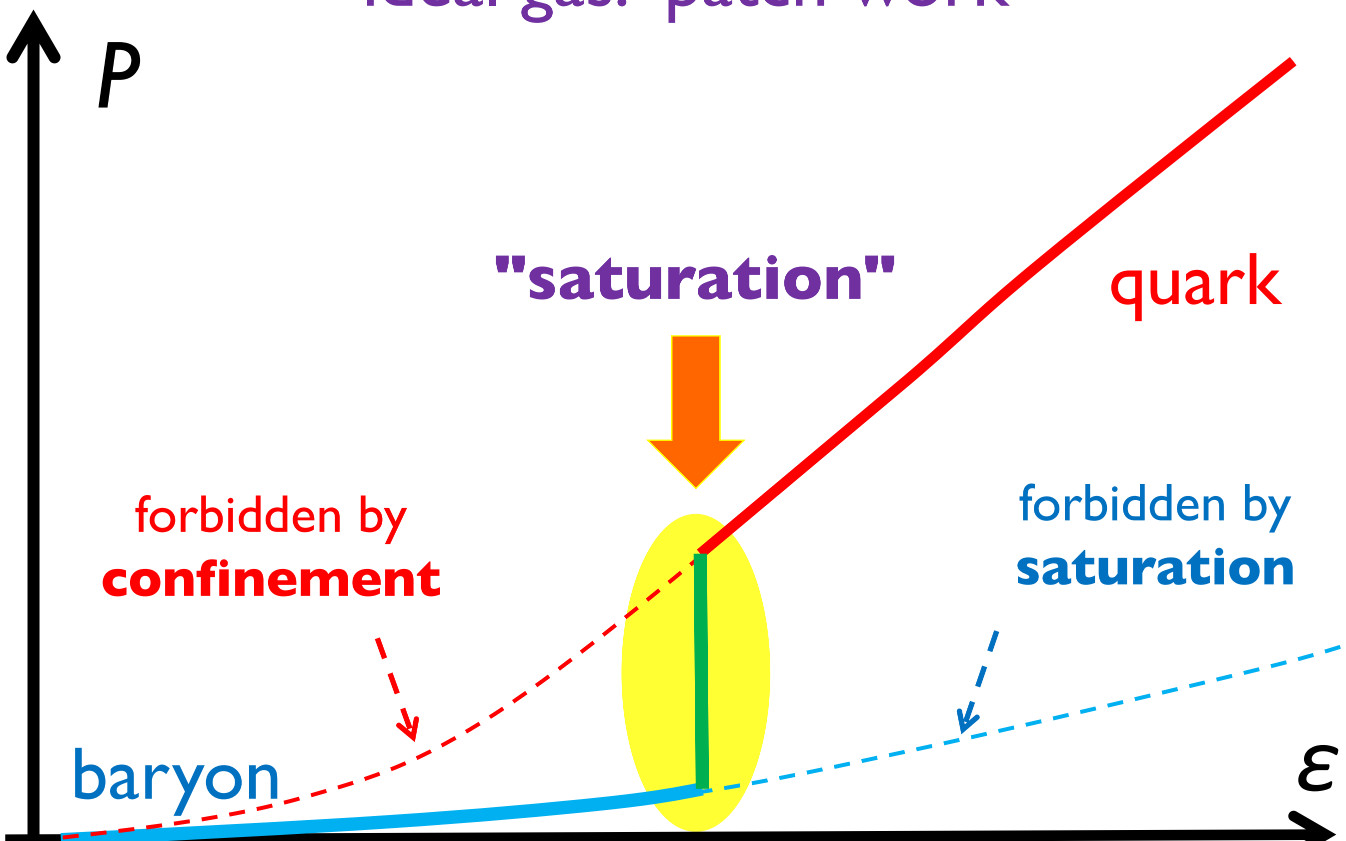
energy per particle



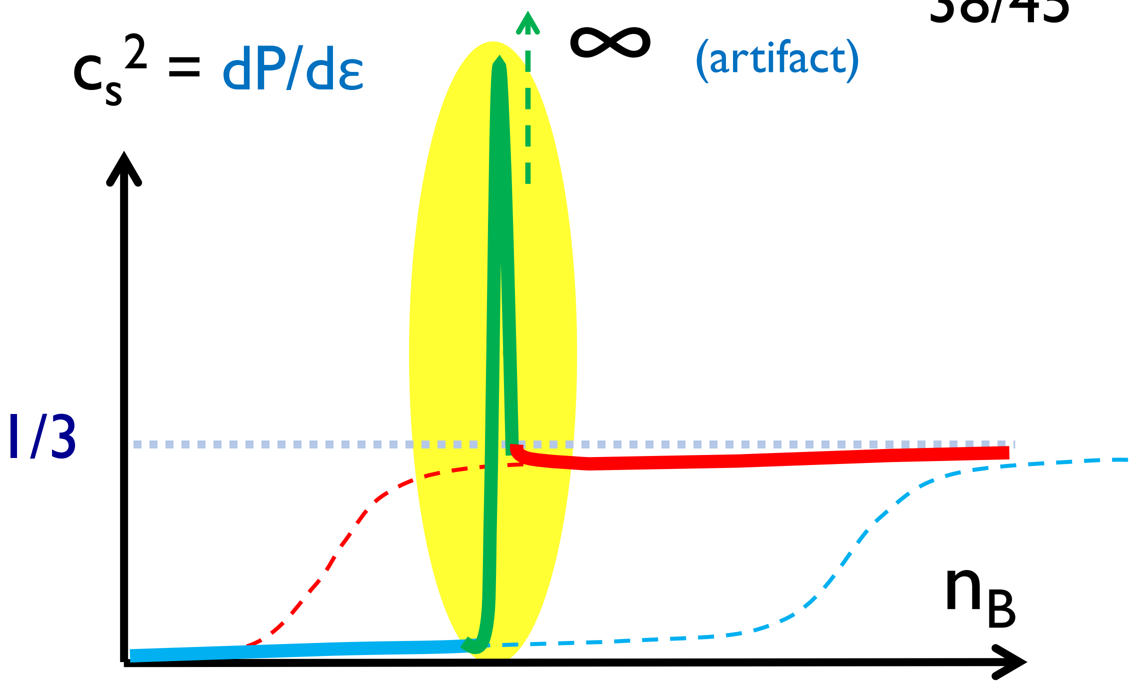
ε, n_B are continuous (f_q continuous)

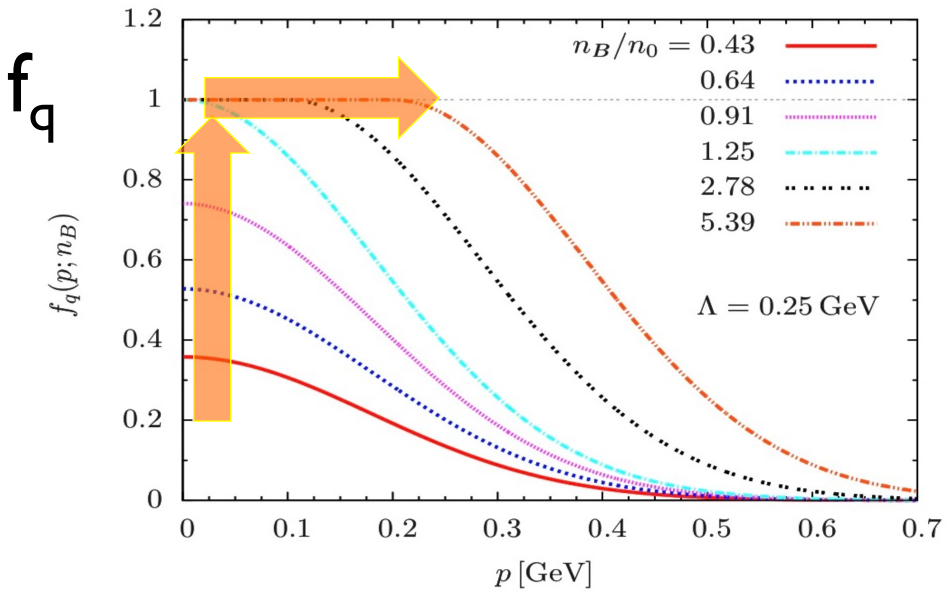
Peak in sound velocity

ideal gas: patch work

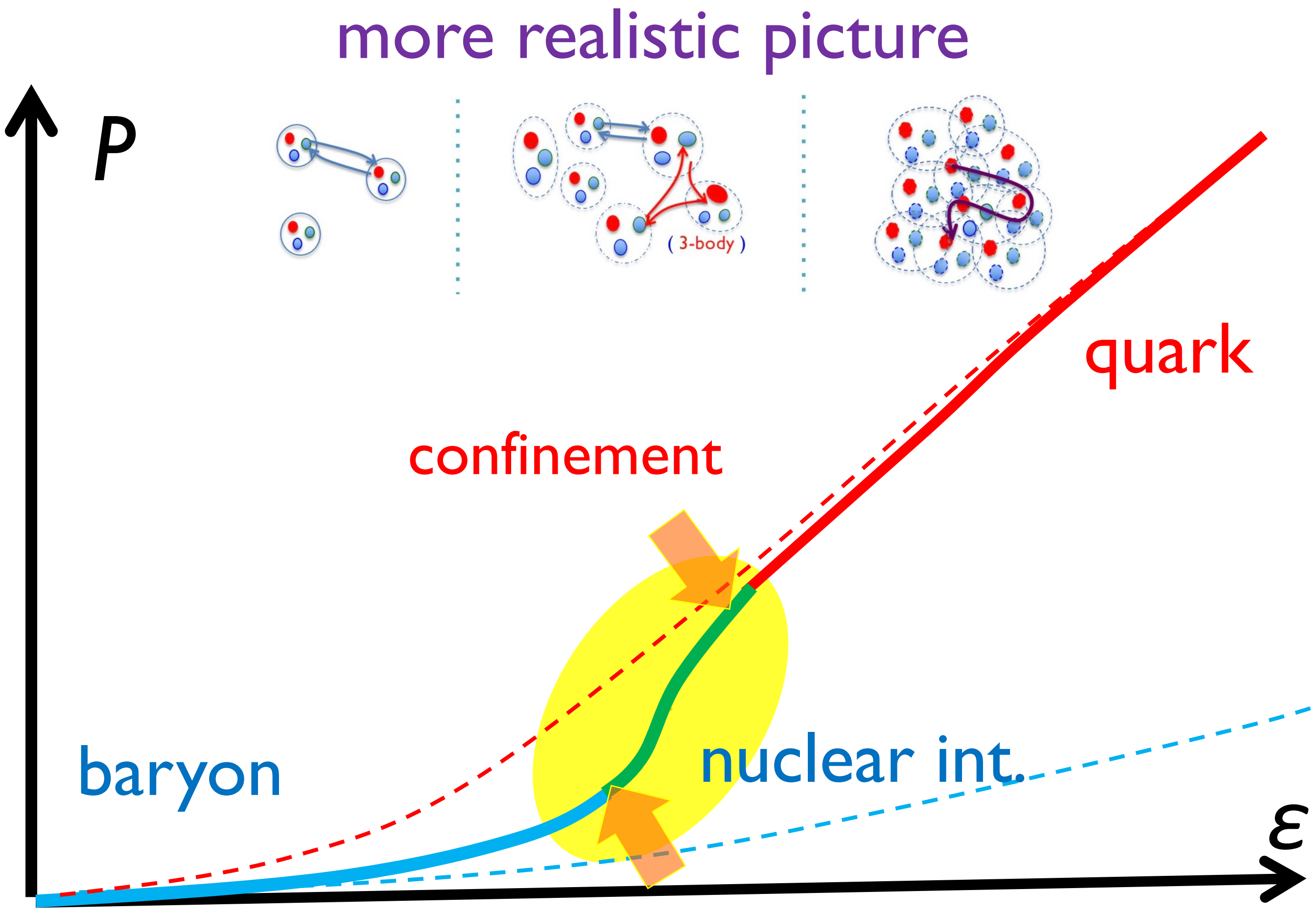


“inevitable” stiffening

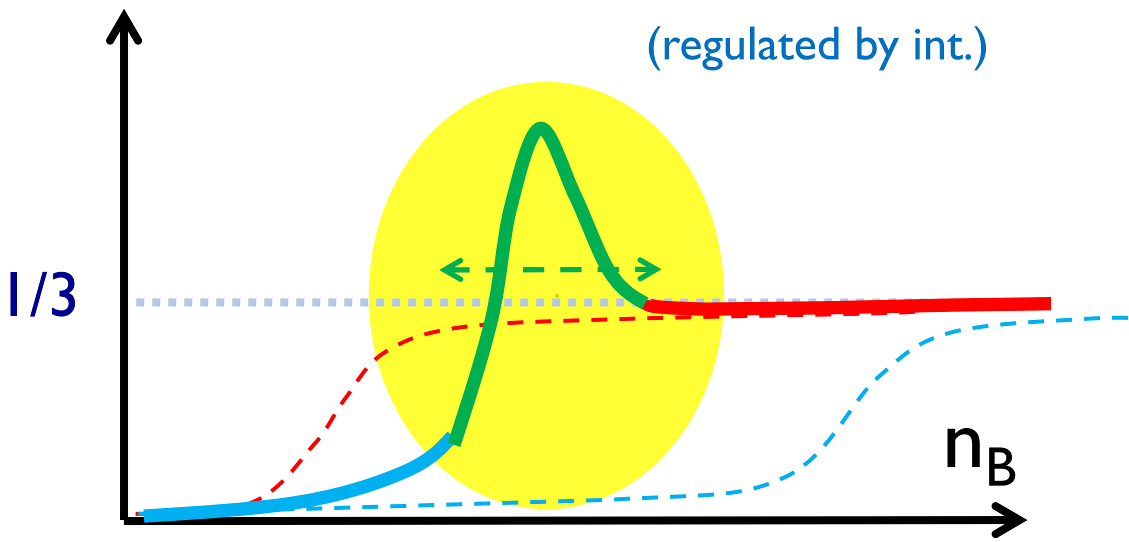


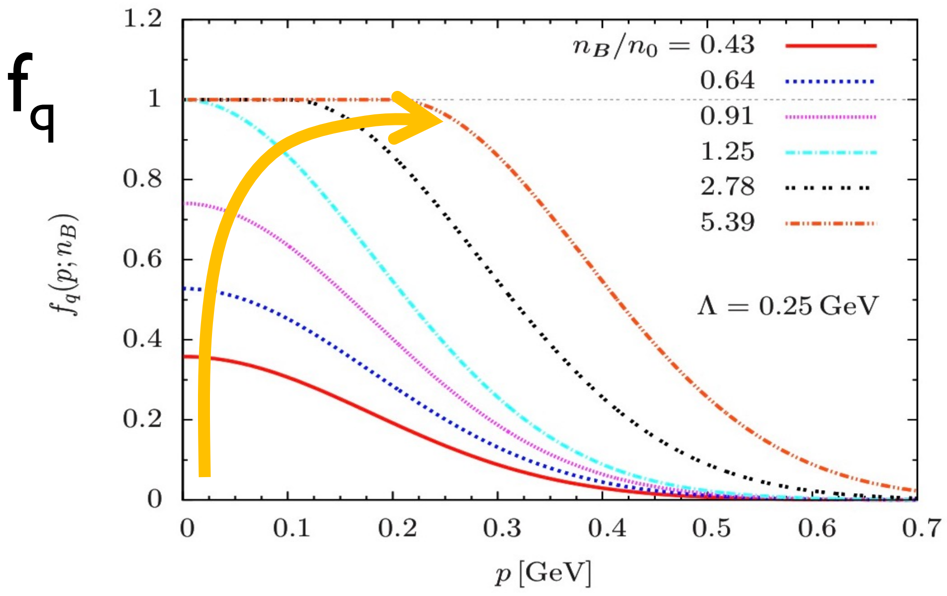


Peak in sound velocity

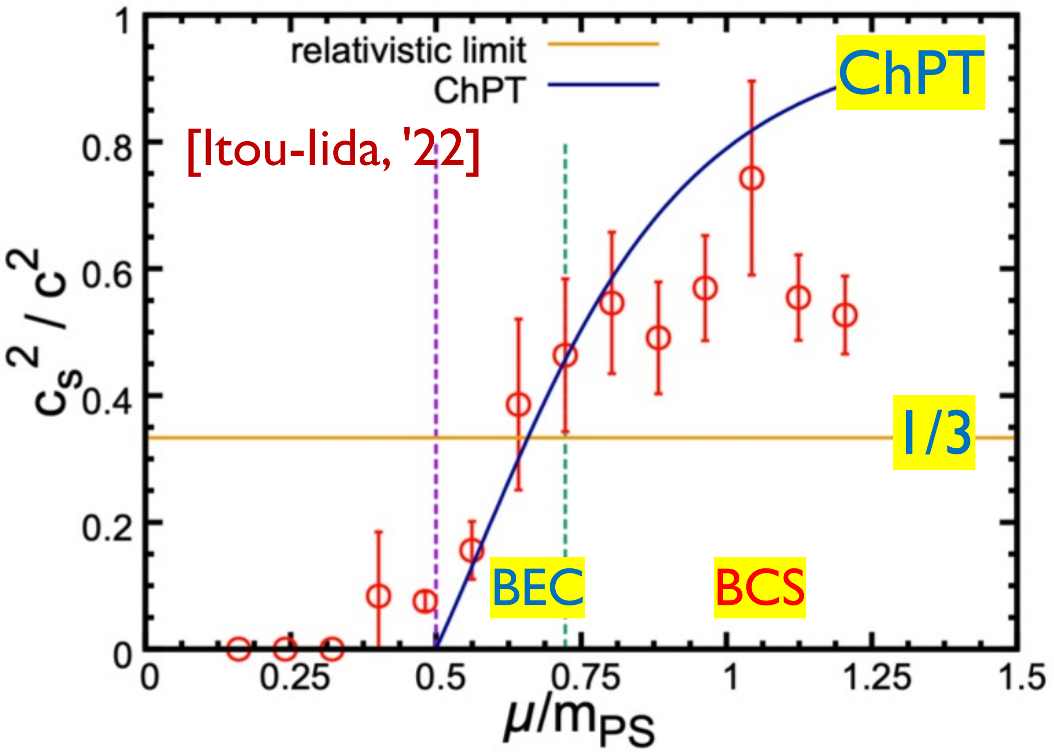


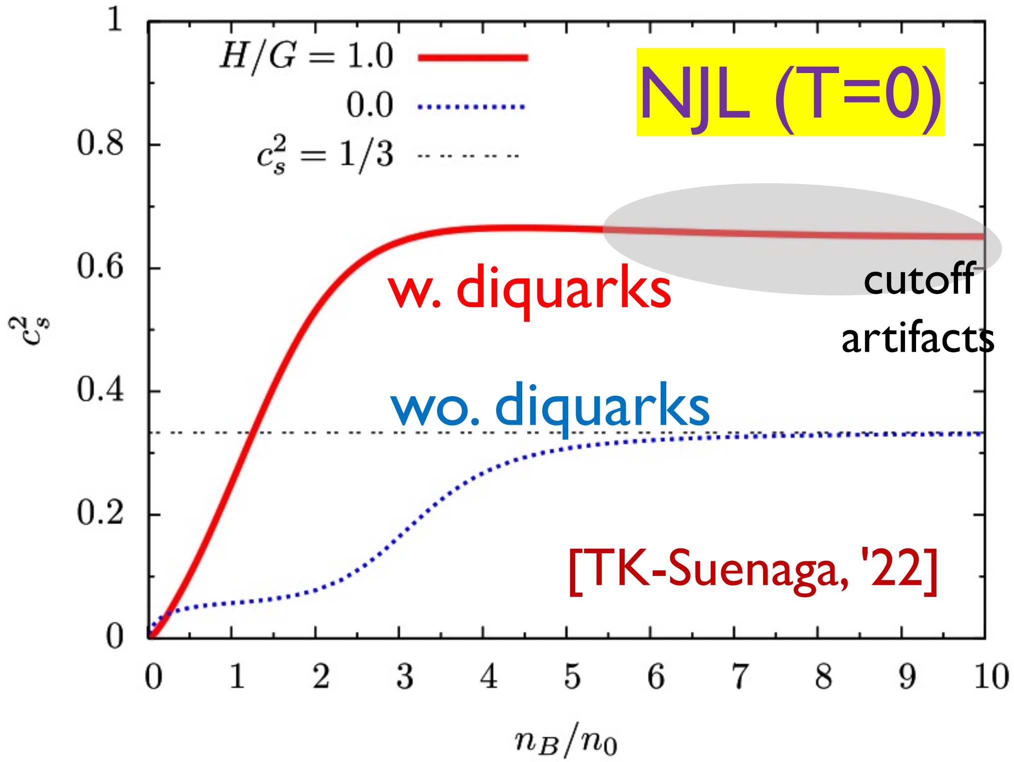
$c_s^2 = dP/d\varepsilon$

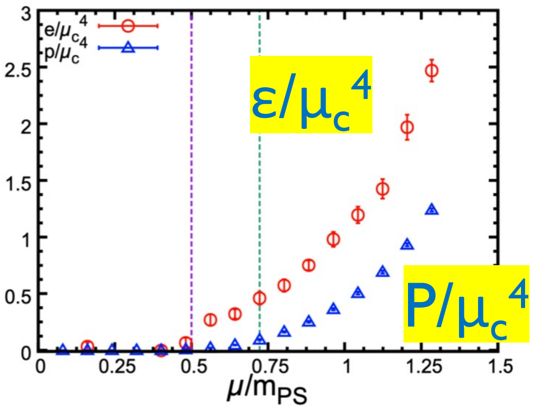




Peak in c_s^2 on the lattice (2-color) T ~ 80 MeV







diquark **bosons** subject to quark Pauli blocking

c_s^2-peak around $\sim 2n_0 = 0.32$ fm^{-3}

(as T↓, the peak should be more pronounced)

Quantum numbers ?

quark quantum numbers; N_c, N_f, 2-spins (for a given spatial w.f.)

how many baryon species are needed to saturate quark states?

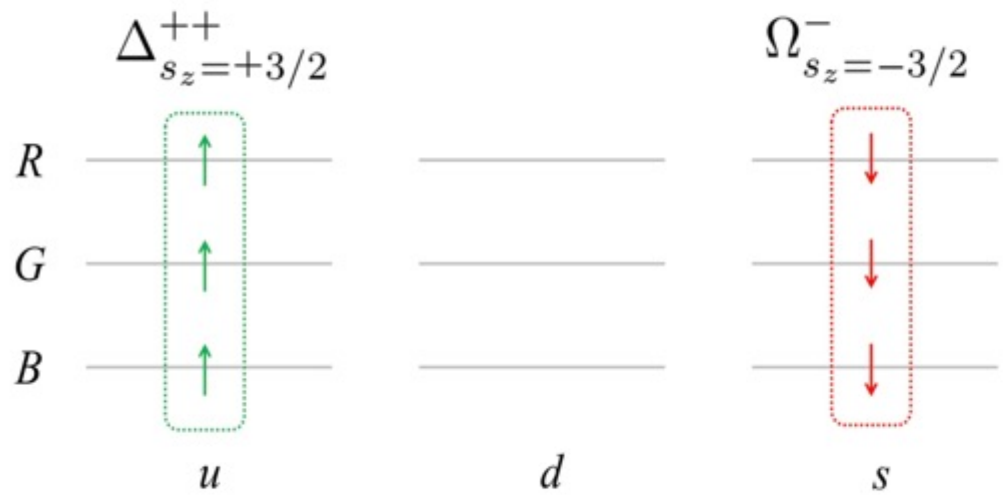
→ need only **$2N_f$ = 6** species for $N_f = 3$

(full members of singlet, octet, decuplet are NOT necessary)

convenient **color-flavor-spin** bases

[neglect N-⊿ splitting etc. for simplicity]

$$\Delta^{++}_{s_z=\pm 3/2} = [u_R\!\uparrow u_G\!\uparrow u_B\!\uparrow], \quad [u_R\!\downarrow u_G\!\downarrow u_B\!\downarrow],$$
$$\Delta^{-}_{s_z=\pm 3/2} = [d_R\!\uparrow d_G\!\uparrow d_B\!\uparrow], \quad [d_R\!\downarrow d_G\!\downarrow d_B\!\downarrow],$$
$$\Omega^{-}_{s_z=\pm 3/2} = [s_R\!\uparrow s_G\!\uparrow s_B\!\uparrow], \quad [s_R\!\downarrow s_G\!\downarrow s_B\!\downarrow],$$



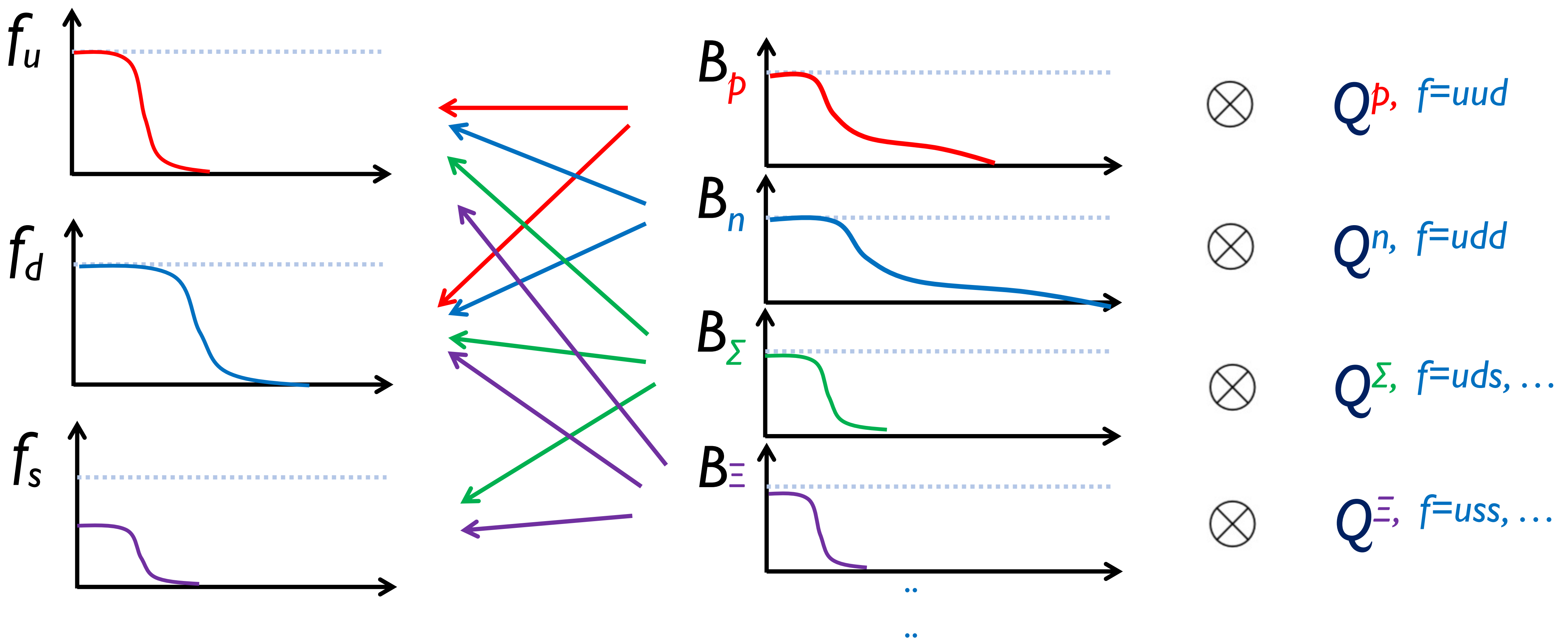
A model for crossover ; flavor-dep.

$$f_{\boldsymbol{q}}(\boldsymbol{k};n_B) = \sum_I \int_{\boldsymbol{K}_B} \mathcal{B}_I(\boldsymbol{K}_B;n_B) Q^{I\boldsymbol{q}}_{\text{in}}(\boldsymbol{k},\boldsymbol{K}_B)$$

q (color, flavor, spin)

I : baryon species (p, n, ⊿, Σ, Ξ, Λ….)

quark dist. in a baryon I (p, n, ⊿, Σ, Λ….)



A model for crossover ; flavor-dep.

$$f_{\boldsymbol{q}}(\boldsymbol{k}; n_B) = \sum_I \int_{\boldsymbol{K}_B} \mathcal{B}_I(\boldsymbol{K}_B; n_B) Q_{\rm in}^{I\boldsymbol{q}}(\boldsymbol{k}, \boldsymbol{K}_B)$$

Issues to be addressed (work in progress):

1) $f_{u,d,s} < 1$ → **constraints** on $(B_p, B_n, B_\Sigma, B_\Xi, B_\Lambda, B_\Delta, \ldots)$

[baryons are NOT independent]

2) the onset of *quark saturation* -> sensitive to baryon size

[size in medium ???]

3) Hamiltonian ?? minimization problems to be formulated

Summary of Lecture 3

- Quark matter from high density approach
- 3-window modeling → insights for 2-5n_0 & 5-40n_0
- stiffening of matter in quark-hadron continuity

c_s^2 peak is associated with the quark substructure of hadrons

c_s^2 peak → signature of quark matter formation

it may appear at density not far from n_0

attempts to find signatures in NS-NS merger simulations

[Huang+ '22, Fujimoto+ '22, ...]

Summary of Lecture 1-3

1) $R_{1.4} \sim R_{2.08} \sim 12.5$ km → hints for soft-to-stiff EOS
2) GW170817; EM signals → $M <\sim 2.3\ M_{\odot}$
3) c_s^2 peak → quark matter formation

(need further check, stay tuned)

Future

- more data from astrophysics will come
- more theoretical constraints on $<\sim 2n_0$ from ChEFT,...
- more theoretical constraints for $5\text{-}40n_0$ based on QCD
- quark descriptions for hadron physics → more insights for $2\text{-}5n_0$