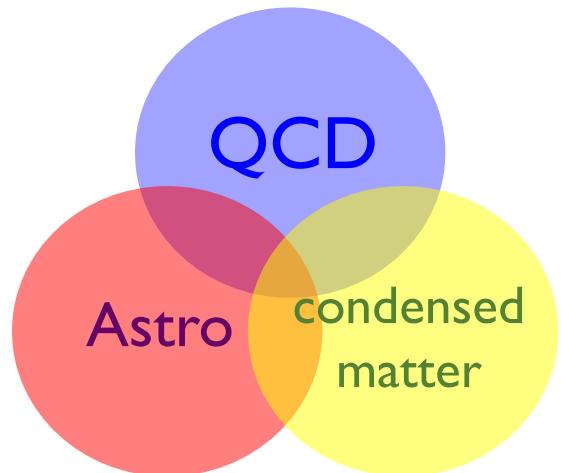
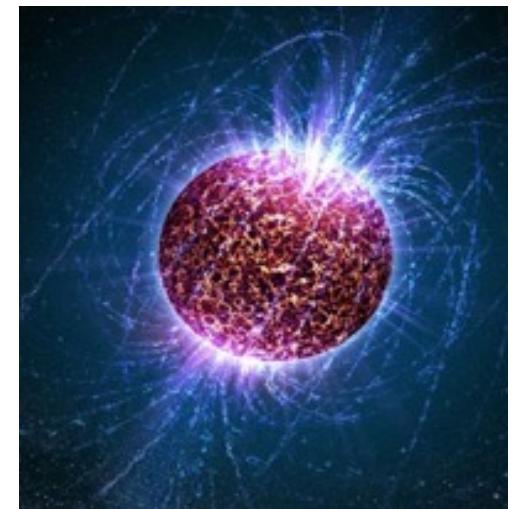


Neutron stars & multi-messenger physics

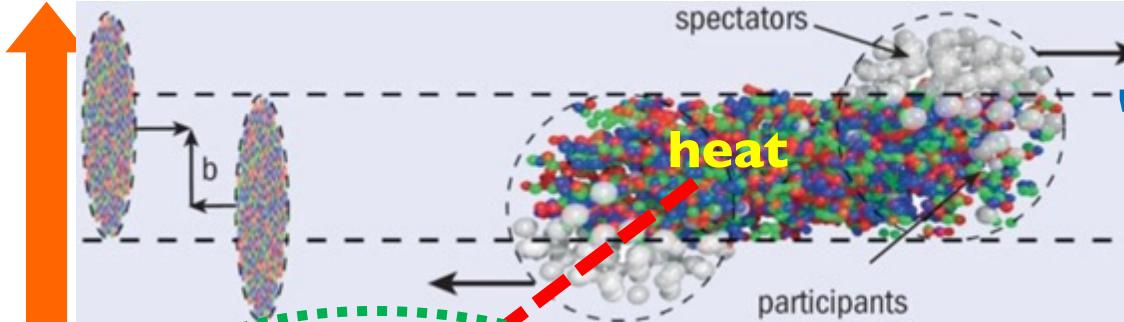


Toru Kojo

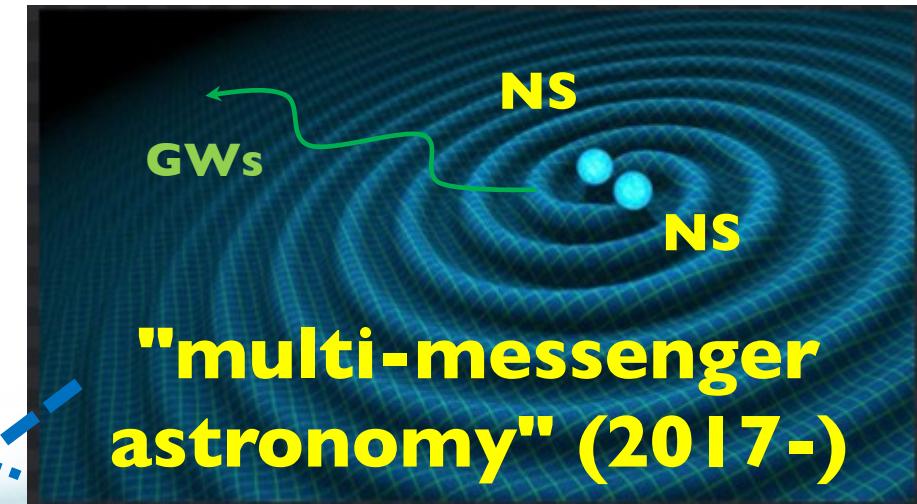
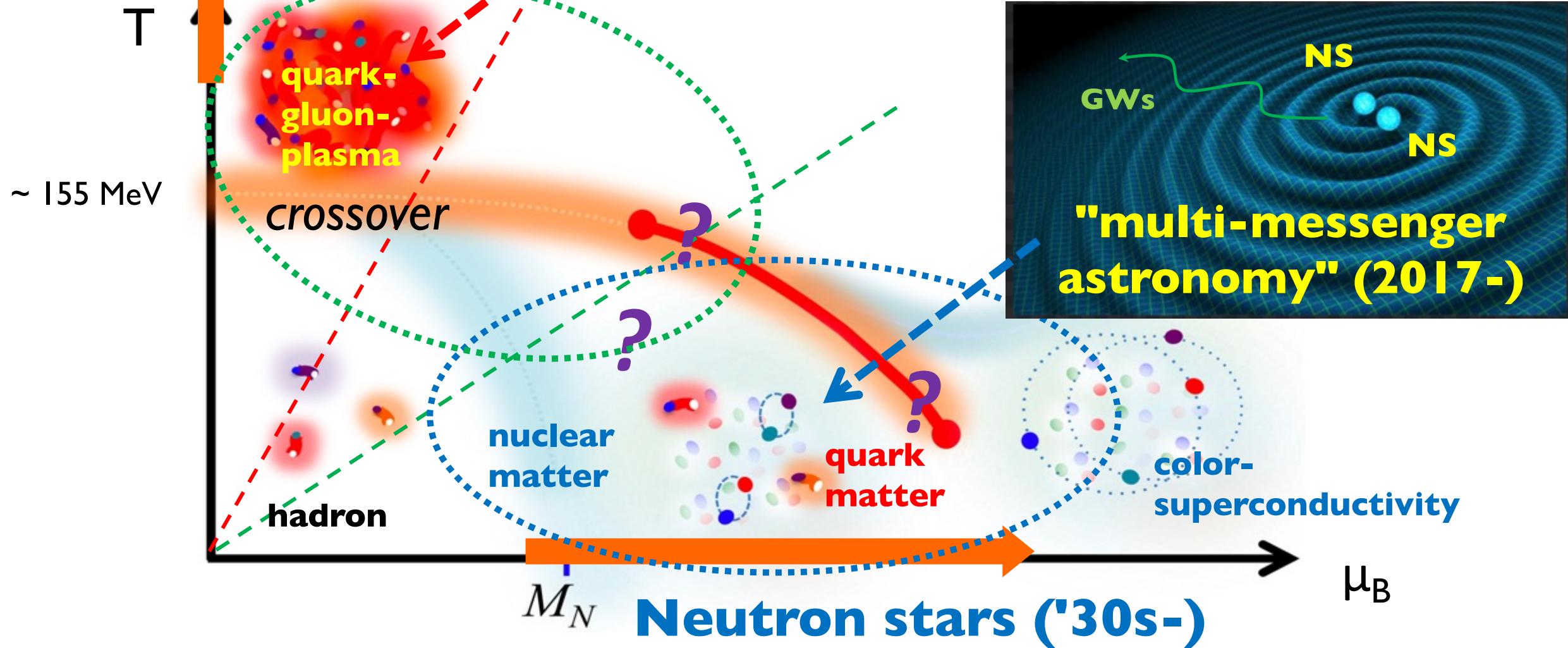
(**Tohoku Univ.**)



**lattice
QCD**



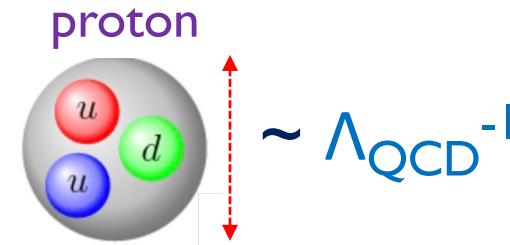
**"heavy-ion collisions"
('80s-)**



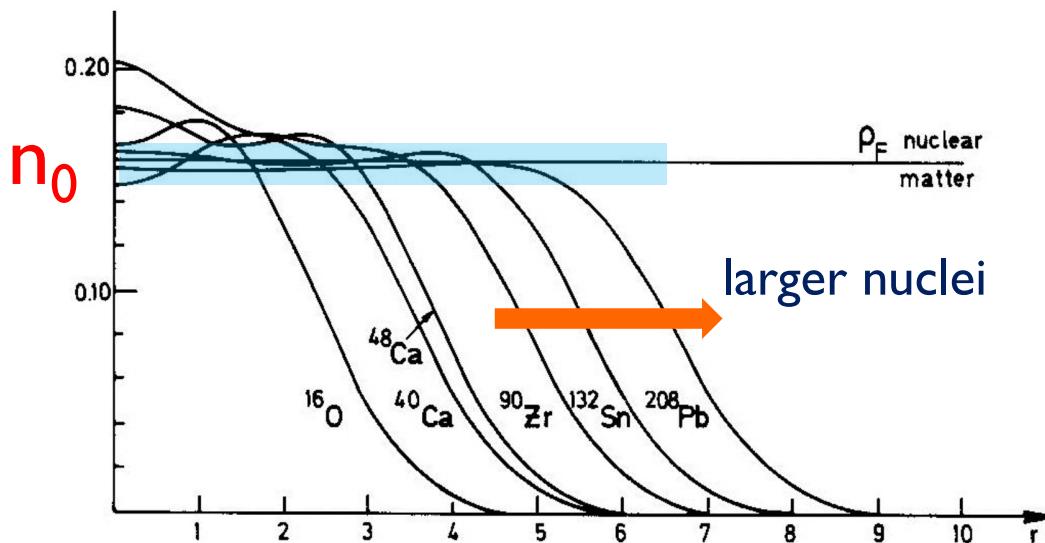
**"multi-messenger
astronomy" (2017-)**

Scales & units to be used

- dynamical scale
- $\Lambda_{\text{QCD}} \sim 200 \text{ MeV} \sim 1 \text{ fm}^{-1}$



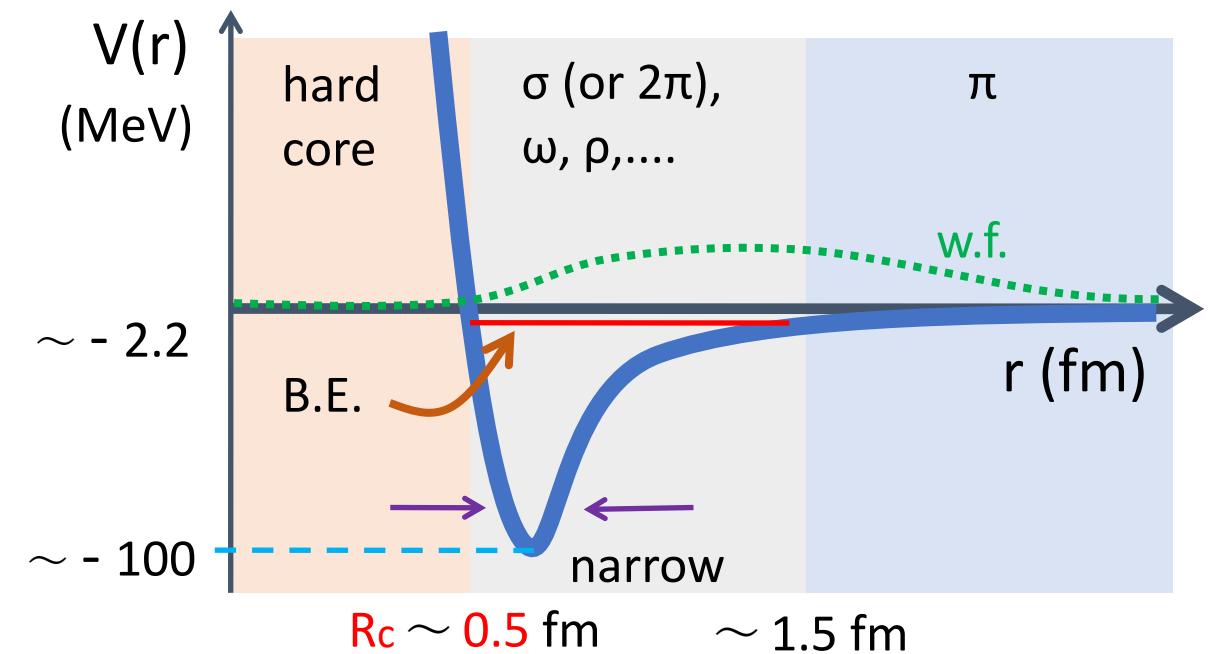
- saturation density: $n_0 = 0.16 \text{ fm}^{-3}$
(~ central density of nuclei)



mass

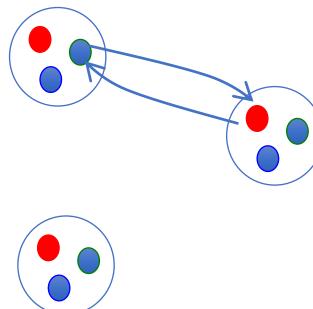
 $M_N \sim N_c \Lambda_{\text{QCD}} \sim 1 \text{ GeV}$

- nuclear binding E : $O(1-10) \text{ MeV}$
(unnaturally small !!)



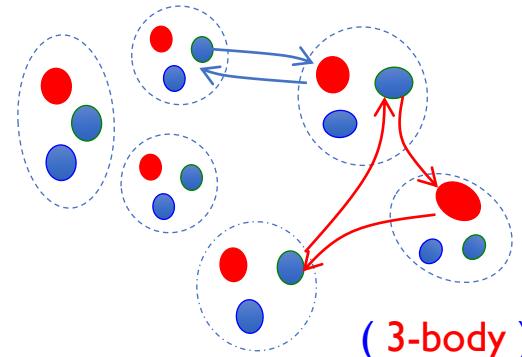
State of matter: overview

- few meson exchange
- nucleons only



ab-initio nuclear cal.
laboratory experiments
steady progress

- many-quark exchange
- structural change,...
- hyperons, Δ , ...



most difficult
(d.o.f ??)

$$\sim 1.4 M_{\odot}$$

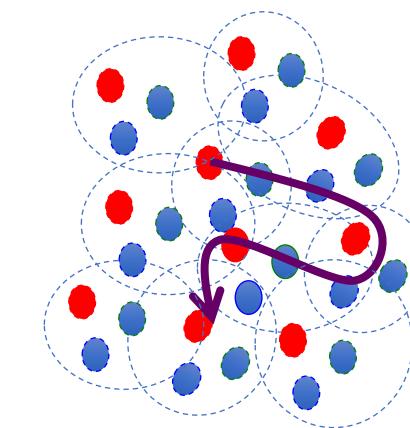
$$\sim 2n_0$$

Hints from NS

$$(n_0 = 0.16 \text{ fm}^{-3})$$

[Masuda+ '12; TK+ '14]

- Baryons overlap
- Quark Fermi sea



(pQCD)

[Freedman-McLerran,
Kurkela+, Fujimoto+...]

not explored well

$$\sim 2 M_{\odot}$$

$$\sim 5n_0$$

$$n_B$$

$$\sim 40n_0$$

Lect 1) Overview: NS phenomenology & QCD

- glancing at NS properties
- M-R relation and EOS
- $R_{1.4}$ & low density EOS

Plan

Lect 2) NS-NS mergers

- gravitational waves
- pre-mergers [inspiral & tidal deformation]
- post-mergers [EM-counterparts]

Lect 3) From hadrons to quarks in NS

- quark matter
- 3-window modeling
- stiffening of matter in quark-hadron continuity

Glancing at neutron star properties

Neutron stars (NSs)

rot. period : $P \sim 1 \text{ ms} - 1 \text{ s}$ ($\sim 10^{-9} - 10^{-6} P_{\odot}$)

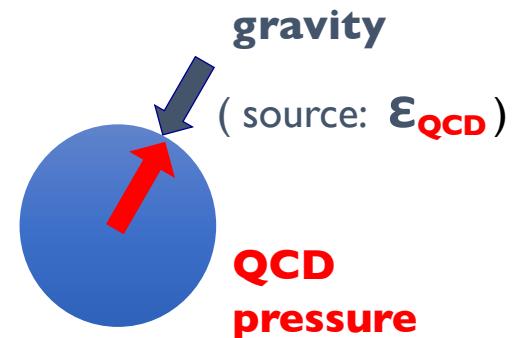
Mass : $M \sim 1-2 M_{\odot}$

Radius : $R \sim 11-13 \text{ km}$ ($\sim 10^{-6} R_{\odot}$)

n_p/n_B : $Y_p \sim 0.05$ (neutron rich)

Temp. : $T \sim \text{KeV}$ ($\ll p_F$ of nucleons)

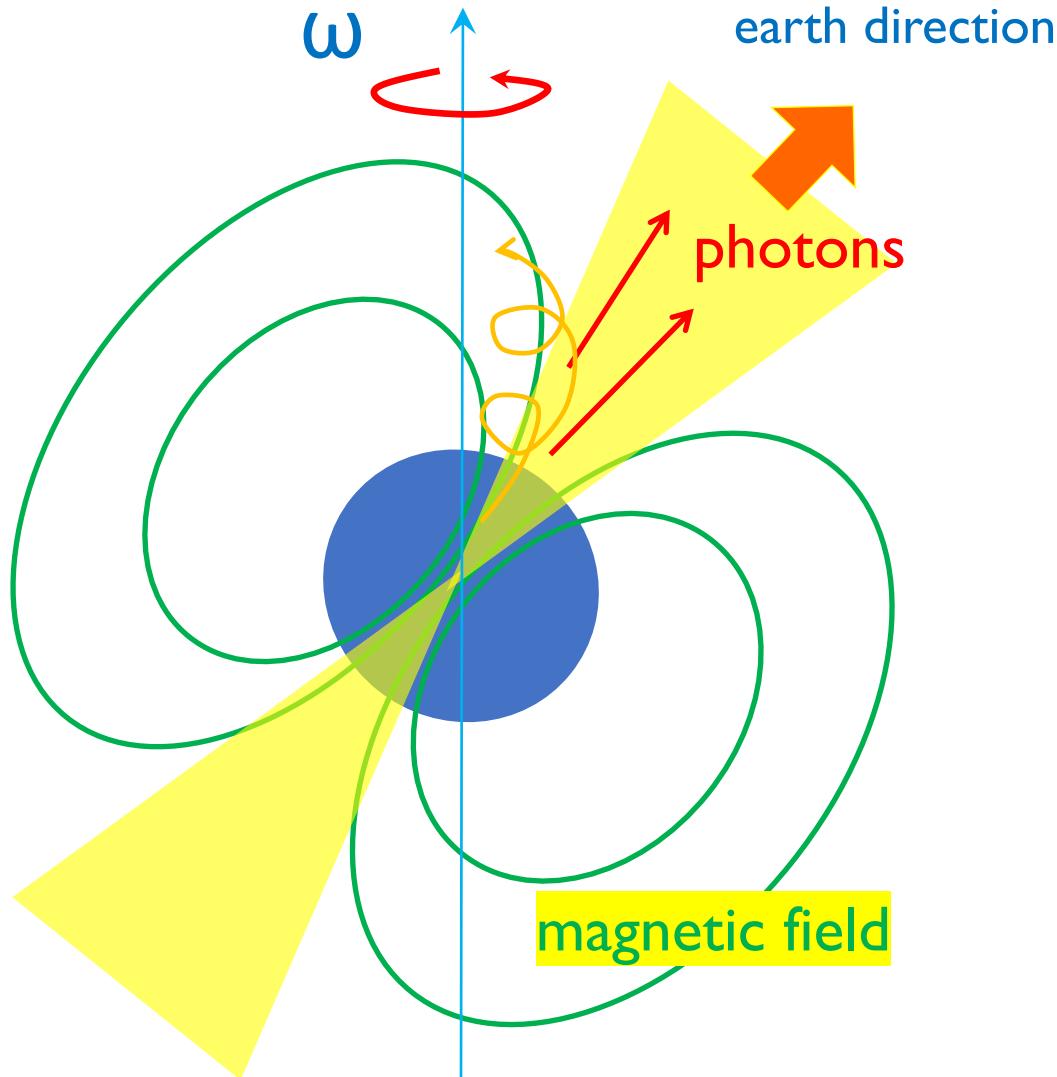
mag. field : $B \sim 10^6 - 10^{15} \text{ Gauss}$ ($\sim 10^5 - 10^{11} B_{\odot}$)



how can we infer
these extreme properties ?

gravitationally bound

fast rotating pulsars & NS



the **existence** of pulsars with
pulse period (for earth) $< 1\text{ s}$

(very quickly rotating, large ω)

with constraints

$$v_{\text{surf}} = \omega R < c \quad (\text{light velocity})$$

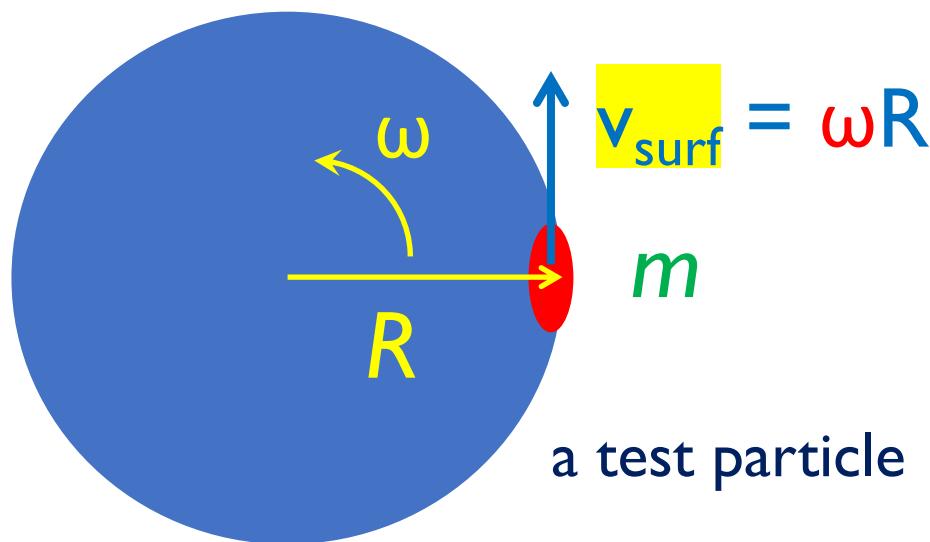
$$\rightarrow R \sim 10\text{-}100 \text{ km (!)}$$

compact objects !!

fast rotating pulsars & NS

pulse periods are known to be extremely stable & precise

how can such compact objects be stable ?



$$GMm/R^2 = mR\omega^2 \quad \text{mechanical balance}$$

$$\rightarrow \omega = [GM/R^3]^{1/2} \quad \text{Kepler frequency}$$

$$\text{large } \omega \rightarrow M_{\text{pulsar}} \sim M_{\text{sun}}$$

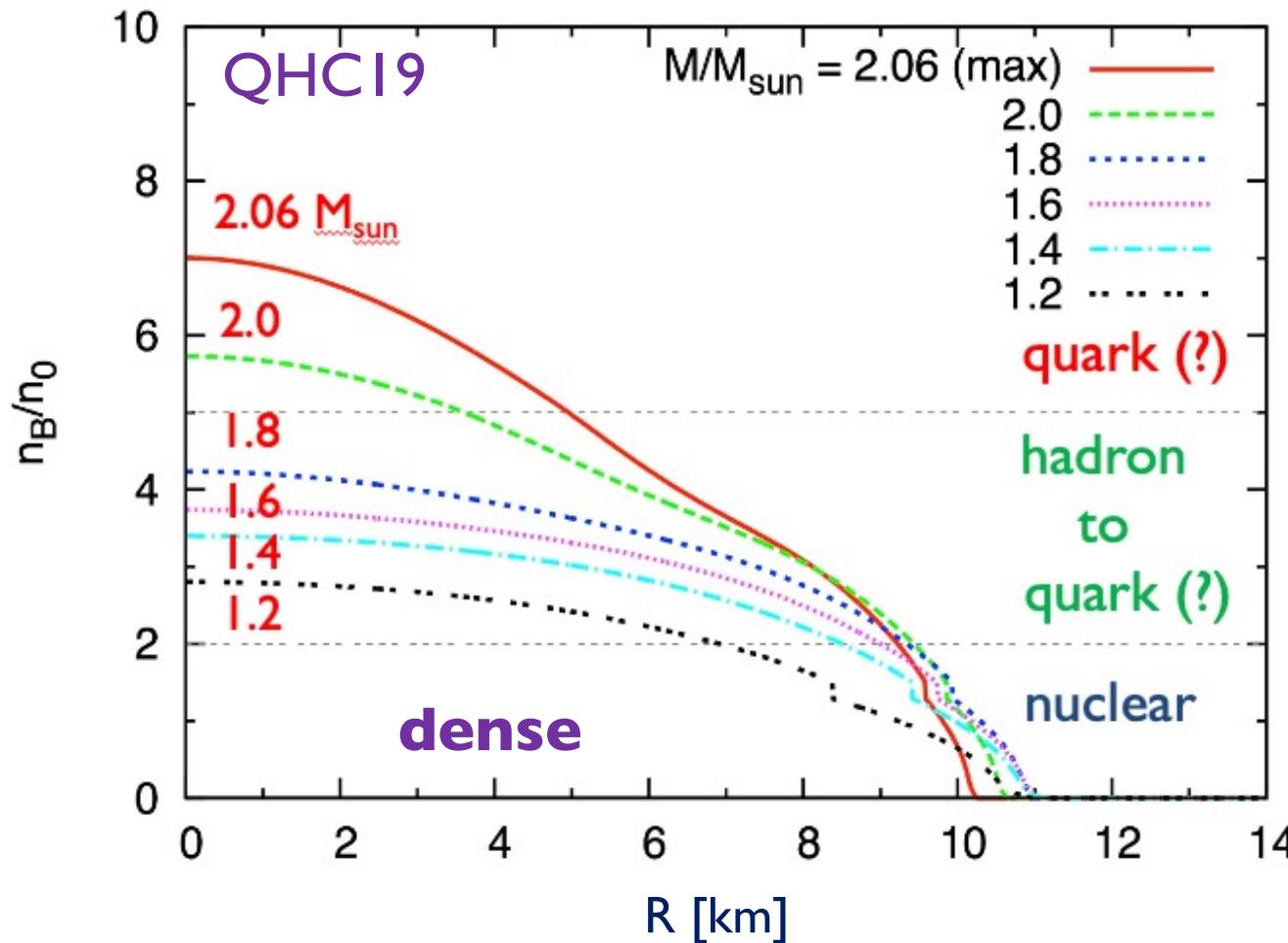
(unless R is extremely small $< \sim 10$ km)

massive & compact \rightarrow hardly affected by other objects

How dense?

very dense

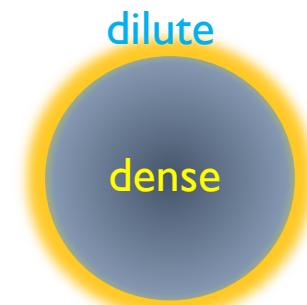
For $M \sim M_{\text{sun}}$ & $R \sim 10 \text{ km}$ $\rightarrow n_B \sim M/(4\pi R^3 \times m_N) \sim O(0.1-1) \text{ fm}^{-3}$



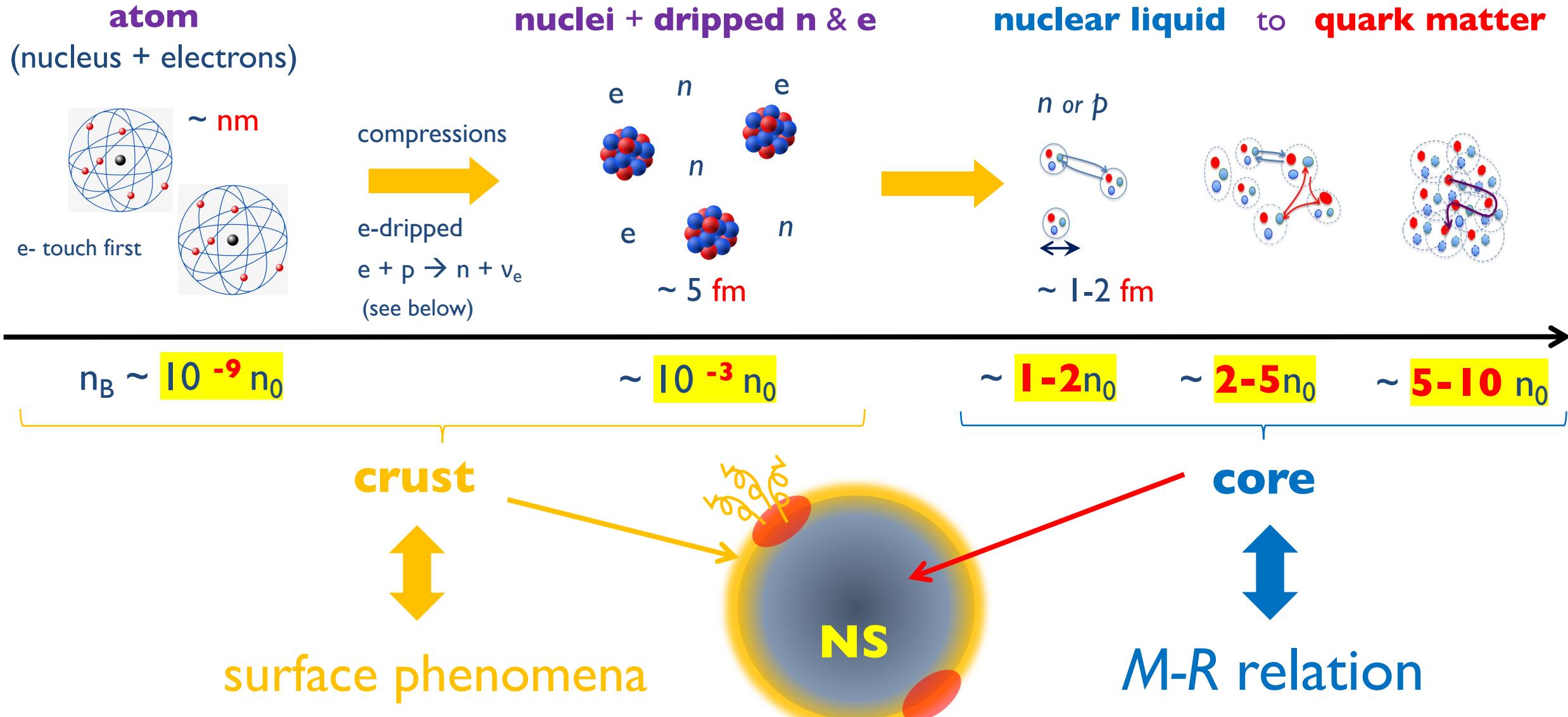
density distribution
from the surface to the center

for $M \sim M_{\text{sun}}$

the **majority** of matter
is **very dense**, $n_B > n_0$



gravitational compression & matter content



why “neutron” stars ?

assuming ideal gases of rela. electrons + non-rela. nucleons

$$\text{charge neutrality} \rightarrow n_p (= Y_p n_B) = n_e \quad \text{or} \quad P_F^p = P_F^e$$

$$\beta\text{-equilibrium} \rightarrow m_N + (P_F^n)^2 / 2M_N = m_N + (P_F^p)^2 / 2M_N + P_F^e + \dots$$

($\mu_n = \mu_p + \mu_e$)

small ($= P_F^p$)

large scale

$$\xrightarrow{\text{m}_N \gg P_F^p} (P_F^n)^2 = 2M_N P_F^p + \dots \quad (\ P_F^n \gg P_F^p) \quad \text{neutron rich !}$$

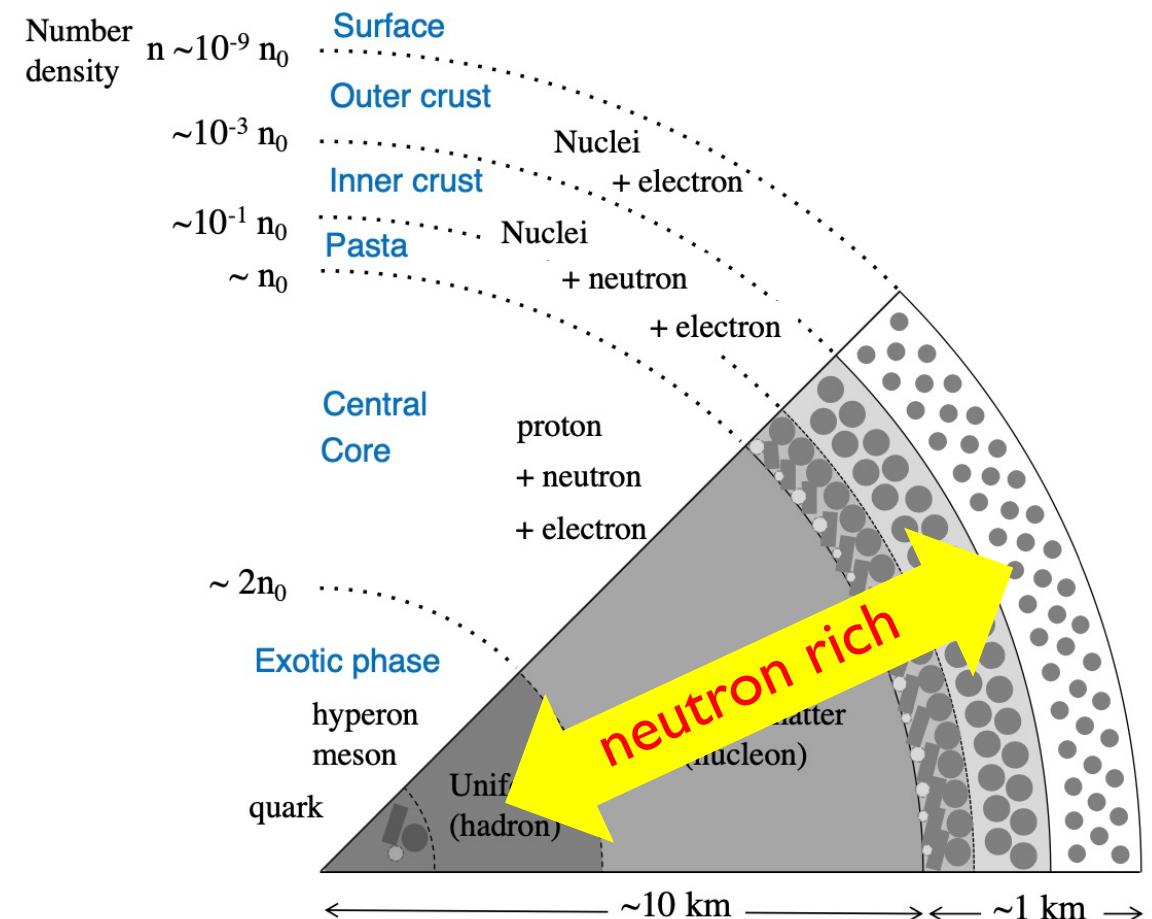
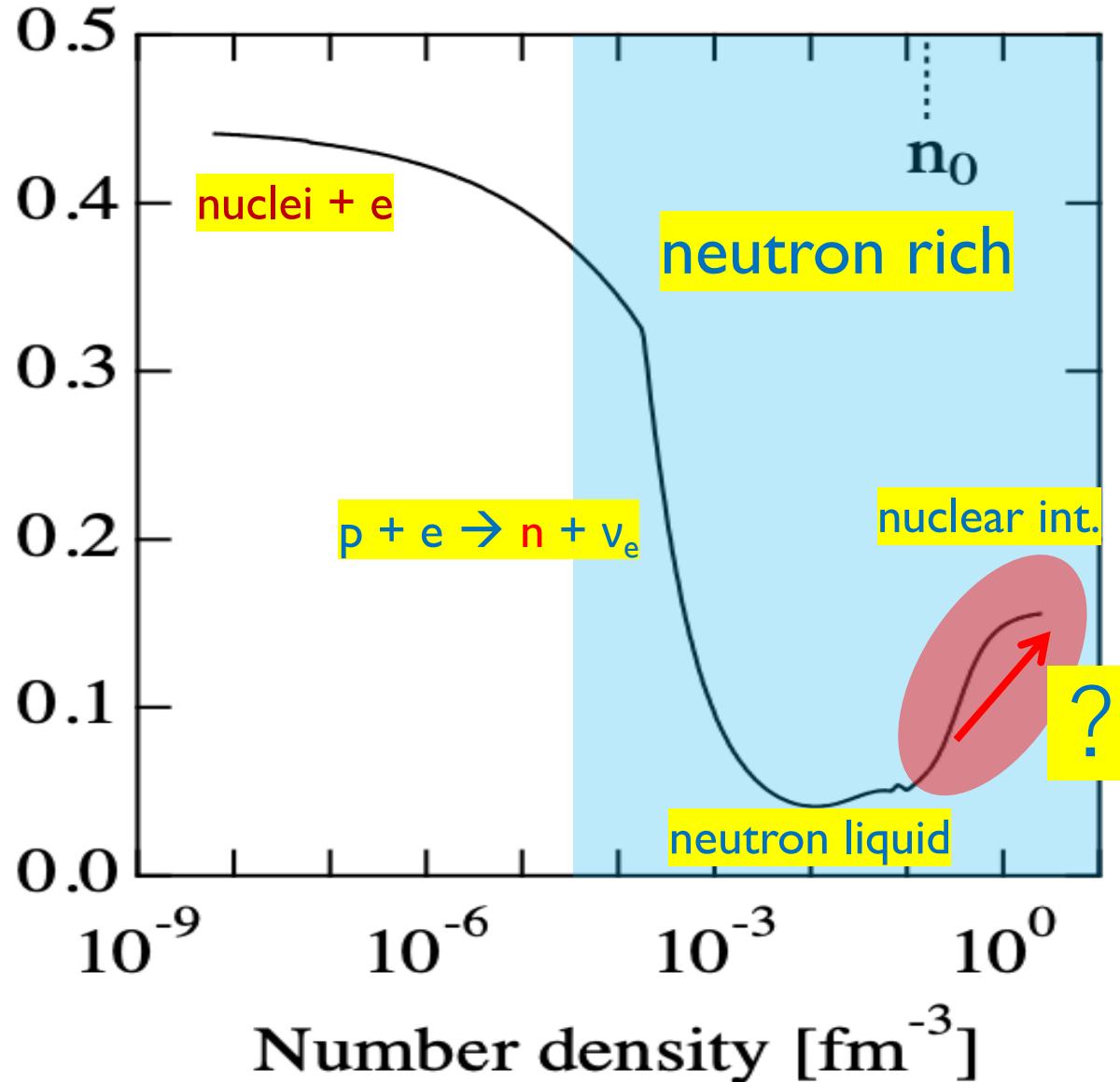
in intuitive terms,

electron kin. energy cost is sensitive to Y_p , while nucleon kin E cost is NOT (tempered by $1/M_N$)

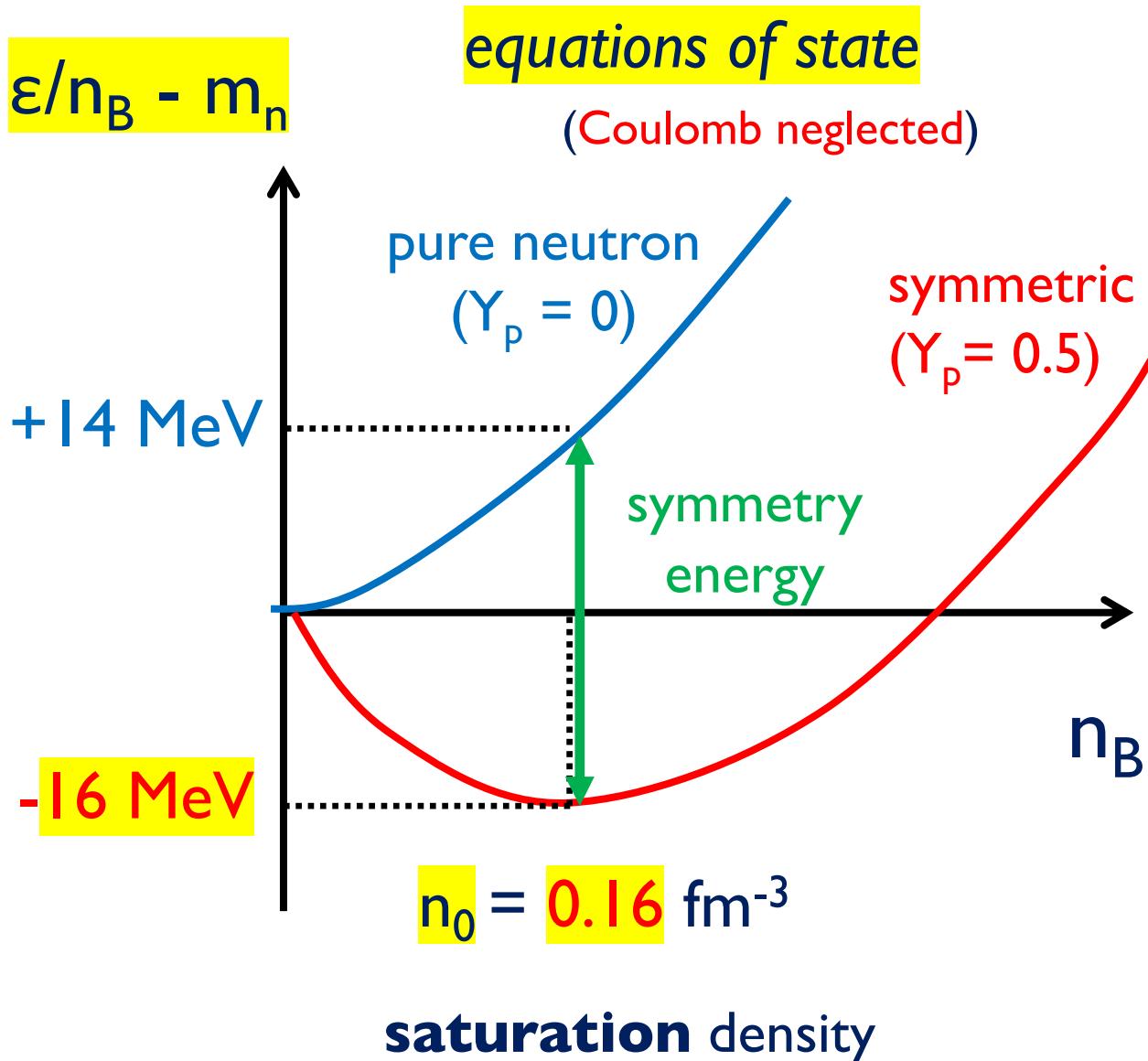
 the priority is to reduce electron density as much as possible → small Y_p

the trend continues until nucleon interactions become important

how wide neutron rich domain?



Y_p in nuclear sector becomes important at $\sim n_0$



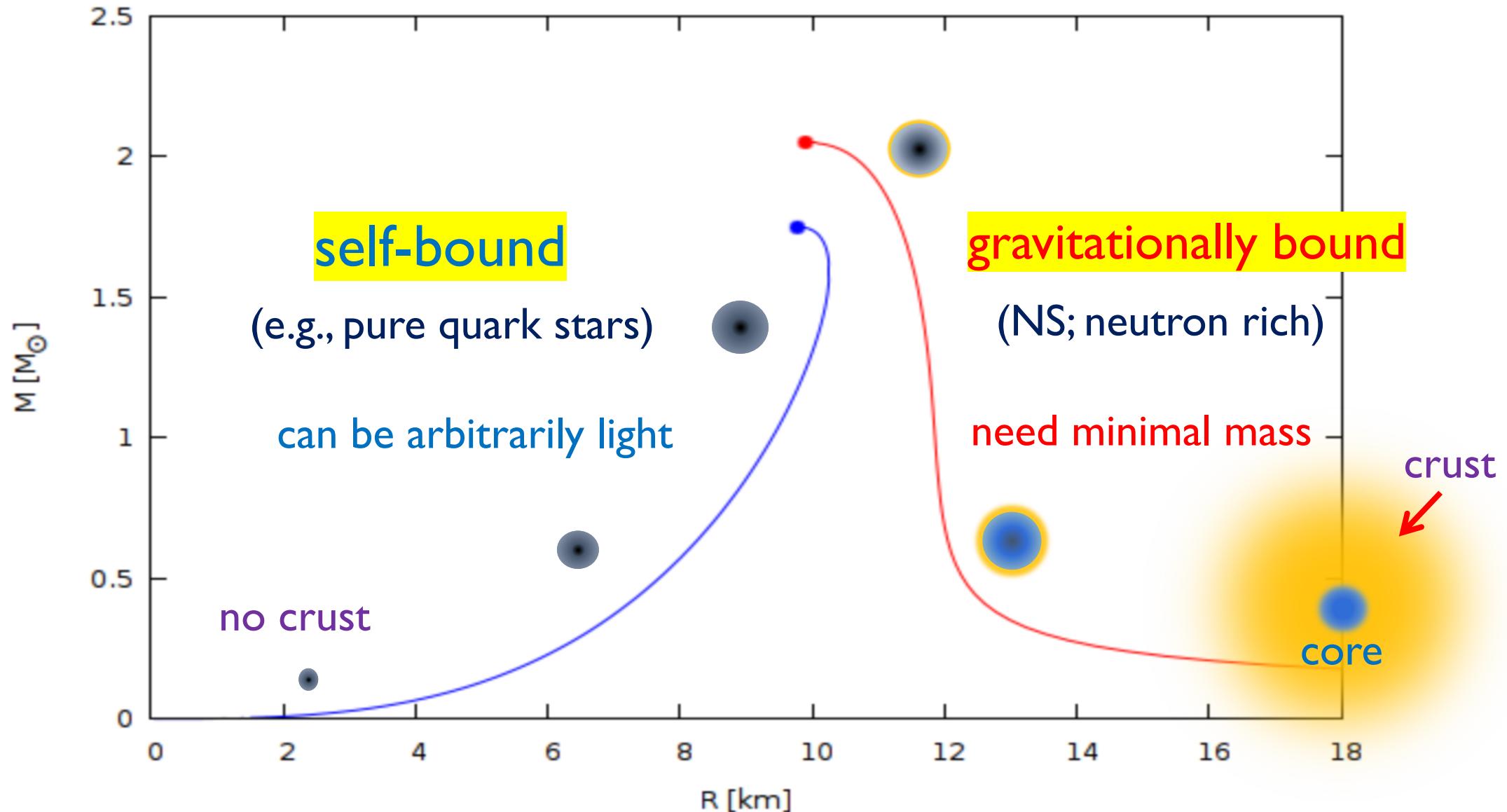
- *nuclear liquid* at $n_B > \sim 0.5n_0$
(clusters are all dissolved)
 - SNM is **self-bound**

$$P = \mu_B n_B - \varepsilon = n_B^2 \partial(\varepsilon/n_B)/\partial n_B$$

$$\rightarrow P(n_0) = 0$$

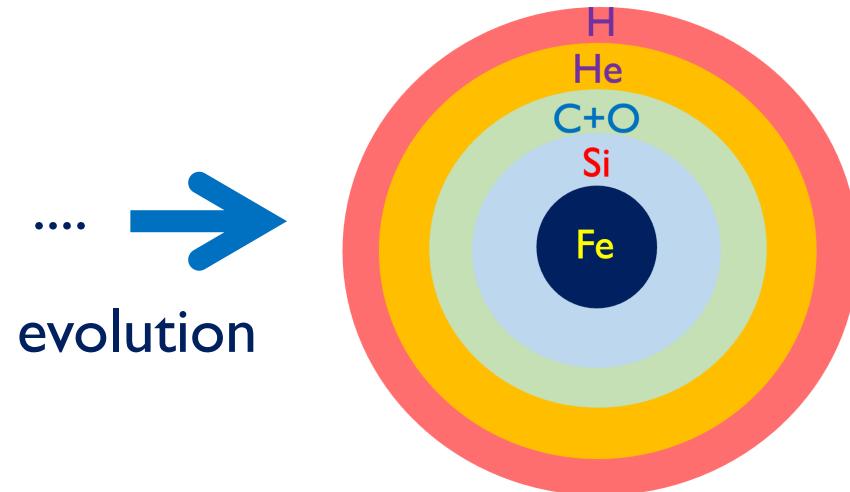
(finite density **without** pressure!)
 - PNM is **NOT self-bound**
(finite density needs **external** pressure!)
- the external force → **gravity**

self-bound vs gravitationally bound stars

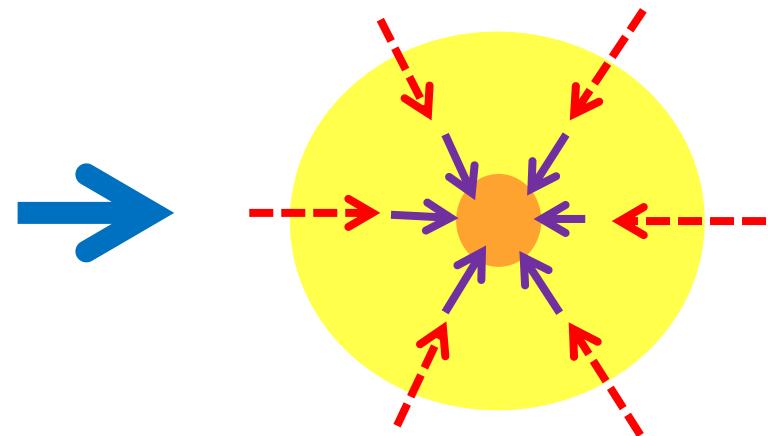


Core collapse supernovae: birth of NS

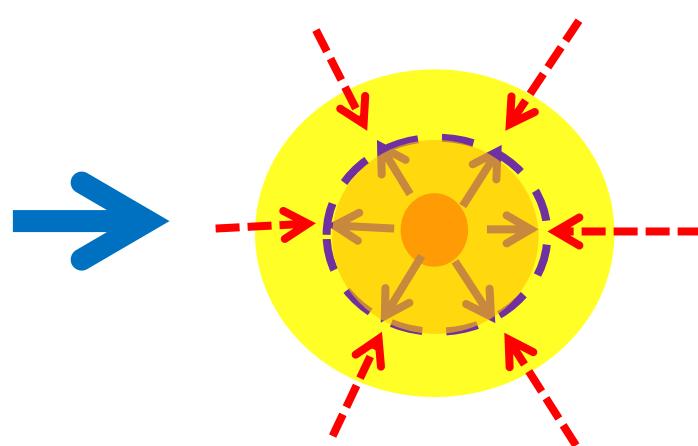
exhaust nuclear fuel at core



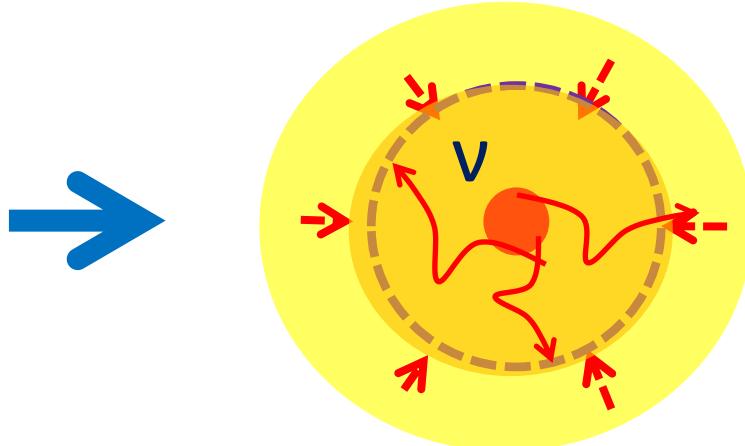
core collapse



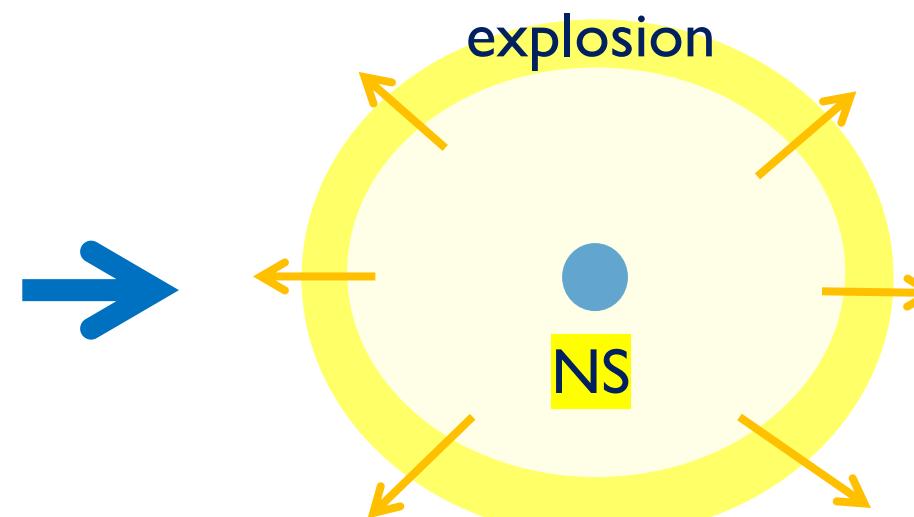
core bounce



neutrino reheating



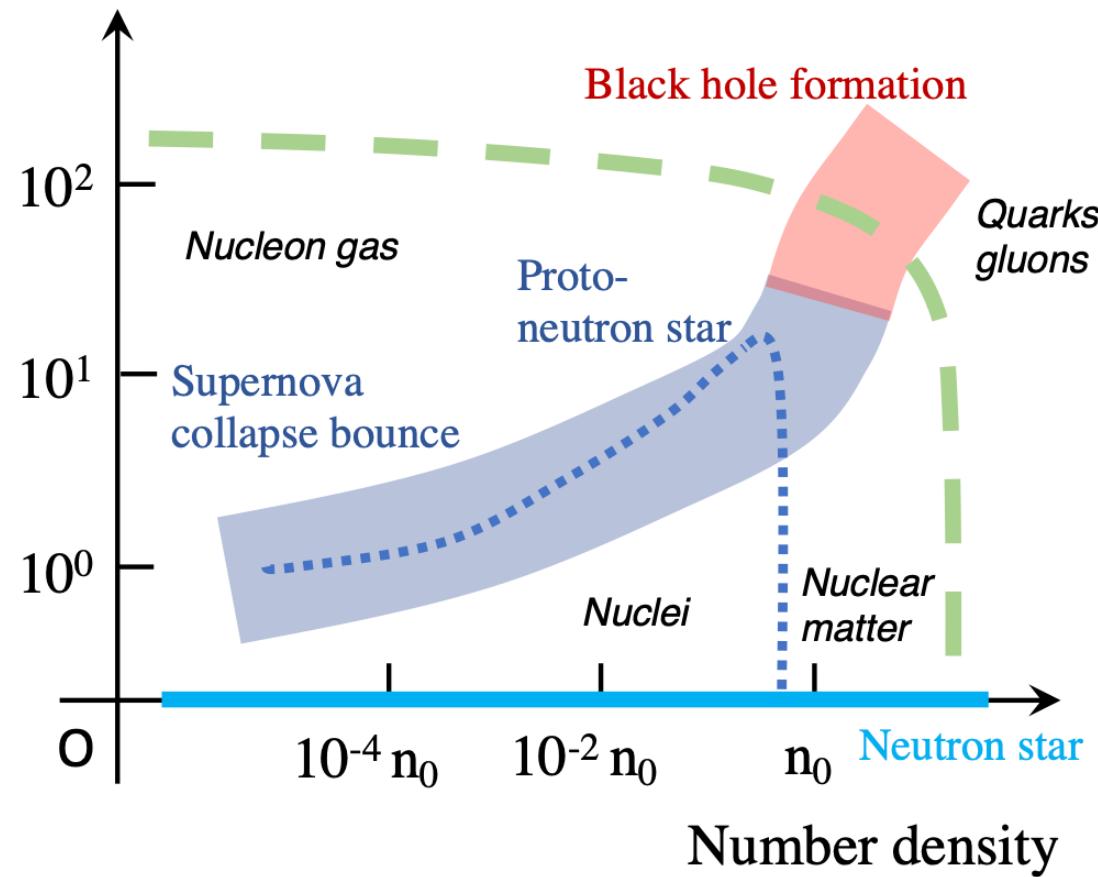
explosion



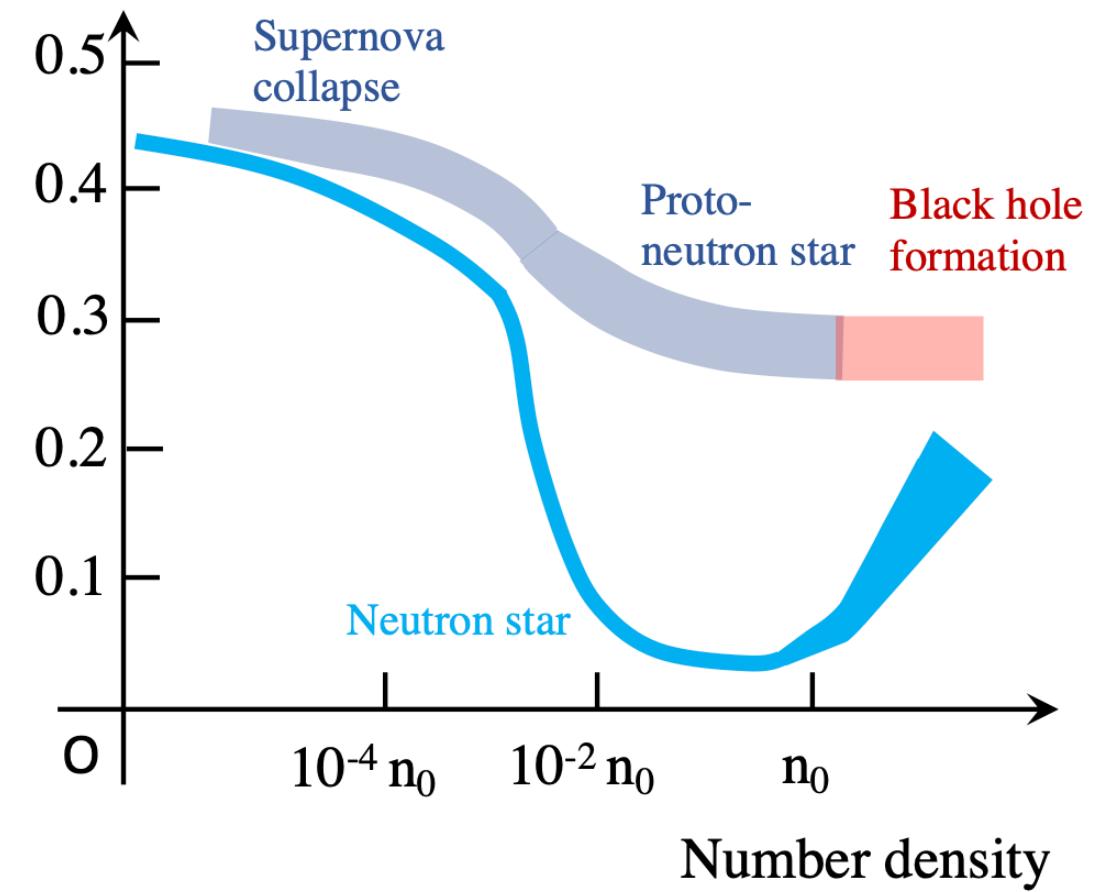
BH

Supernova matter & proto-NS

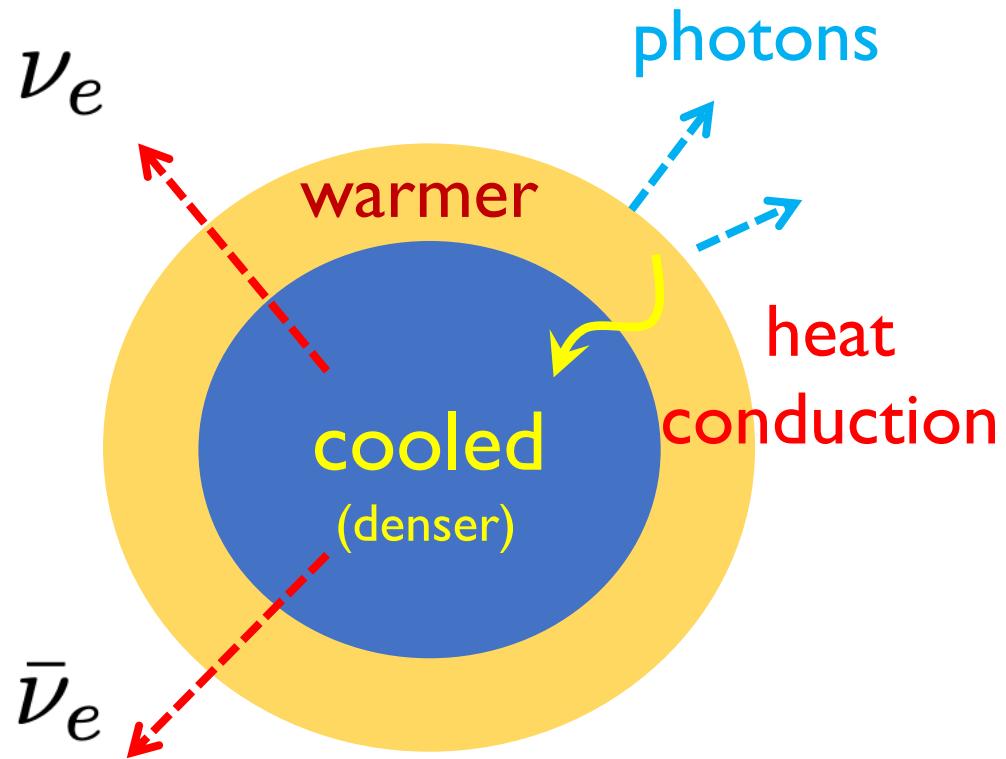
Temperature [MeV]



Proton fraction



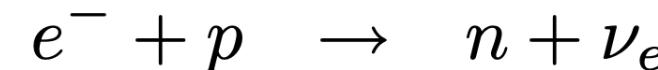
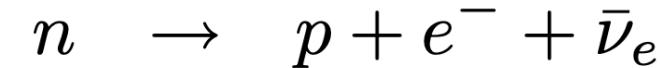
Temperature in NSs



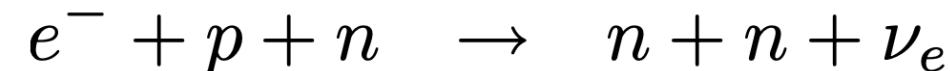
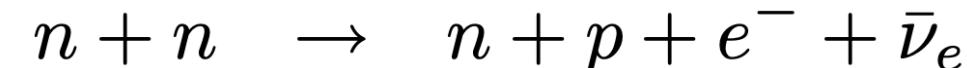
in the core:

- thermal nucleons lose energy via

direct URCA (Fast):

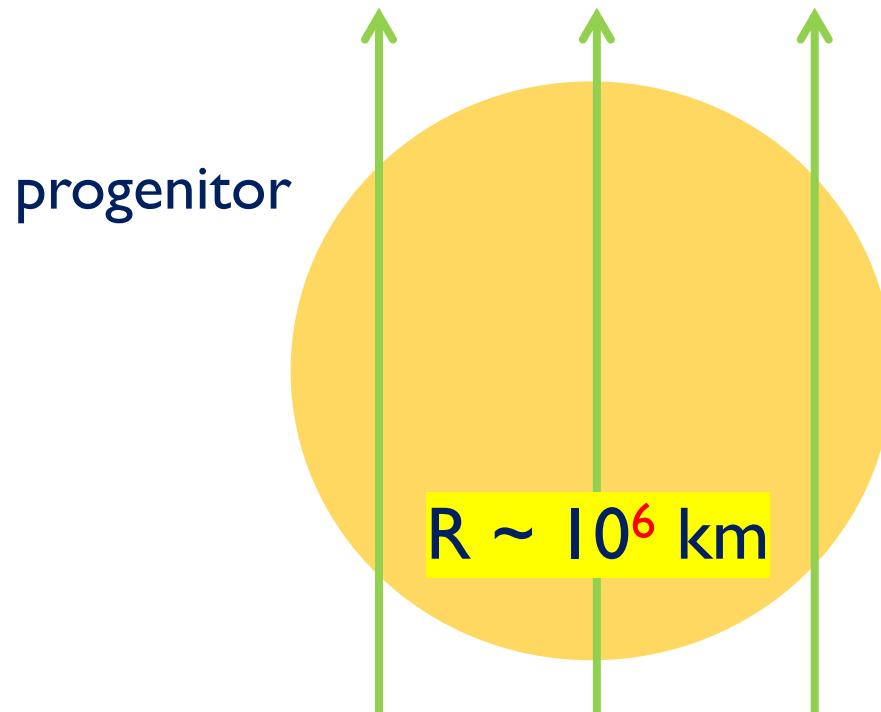


modified URCA (Slow):



~10-100s after the birth, the T_{surf} drops to ~ keV

magnetic fields & angular momentum



typical initial
conditions

$$P \sim 10^7 \text{ s}$$

core collapse
supernovae



$$BR^2 = \text{const.}$$

$$I\omega \sim R^2\omega = \text{const.}$$

squeezing fluxes



$$B \sim 10^{12} \text{ G}$$

size

consistent
with obs.

$$P \sim 10^{-3} \text{ s}$$

Neutron stars (NSs)

rot. period : $P \sim 1 \text{ ms} - 1 \text{ s}$ ($\sim 10^{-9} - 10^{-6} P_{\odot}$)



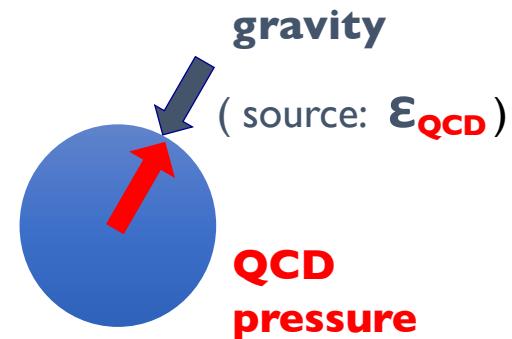
Mass : $M \sim 1-2 M_{\odot}$

Radius : $R \sim 11-13 \text{ km}$ ($\sim 10^{-6} R_{\odot}$)

n_p/n_B : $Y_p \sim 0.05$ (neutron rich)

Temp. : $T \sim \text{KeV}$ ($<< p_F$ of nucleons)

mag. field : $B \sim 10^6 - 10^{15} \text{ Gauss}$ ($\sim 10^5 - 10^{11} B_{\odot}$)



now we have (very) rough ideas on NS

gravitationally bound

How many NSs?

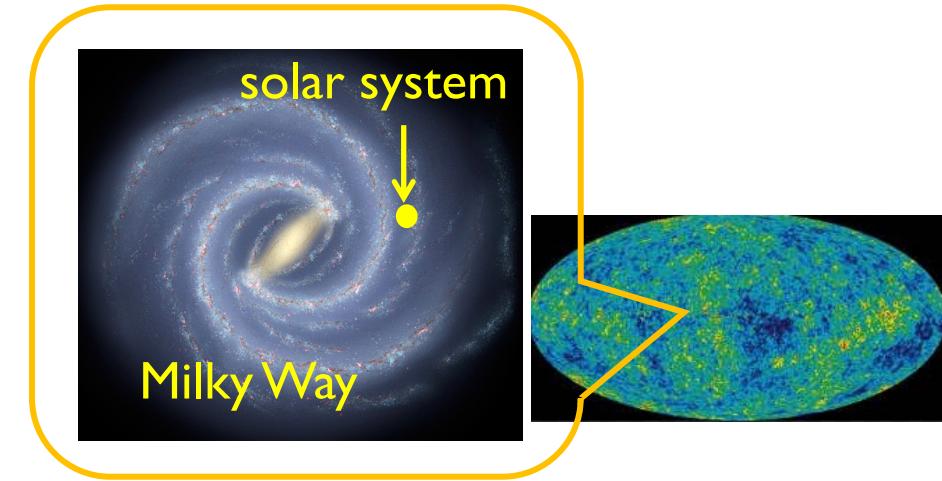
1 parsec (**pc**) = 3.26.. light year

earth to sun: $\sim 10^{-8}$ pc $\sim 10^8$ km

diameter of **Milky Way galaxy**: $\sim 10^4$ pc

range of **GW detection (aLIGO)**: $\sim 10^8$ pc

diameter of **visible universe (CMB)**: $\sim 10^{10}$ pc



In our Milky Way galaxy:

- NS formation (SNe) \rightarrow a few events per century
- age of the galaxy $\rightarrow \sim 10^9$ years

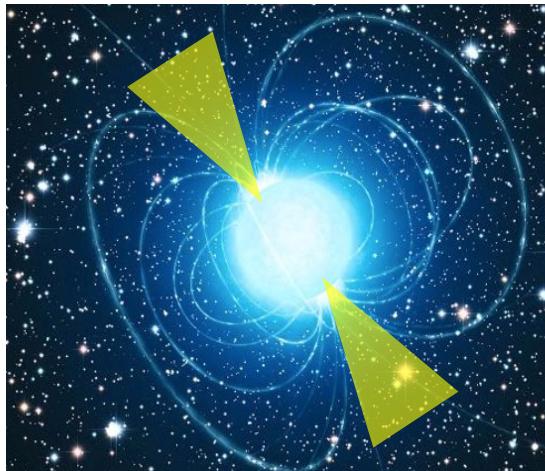


the num. of NSs $\sim 10^8 - 10^9$; only $\sim 2 \times 10^3$ were observed

Types of NSs?

observed $\sim 2 \times 10^3$ pulsars

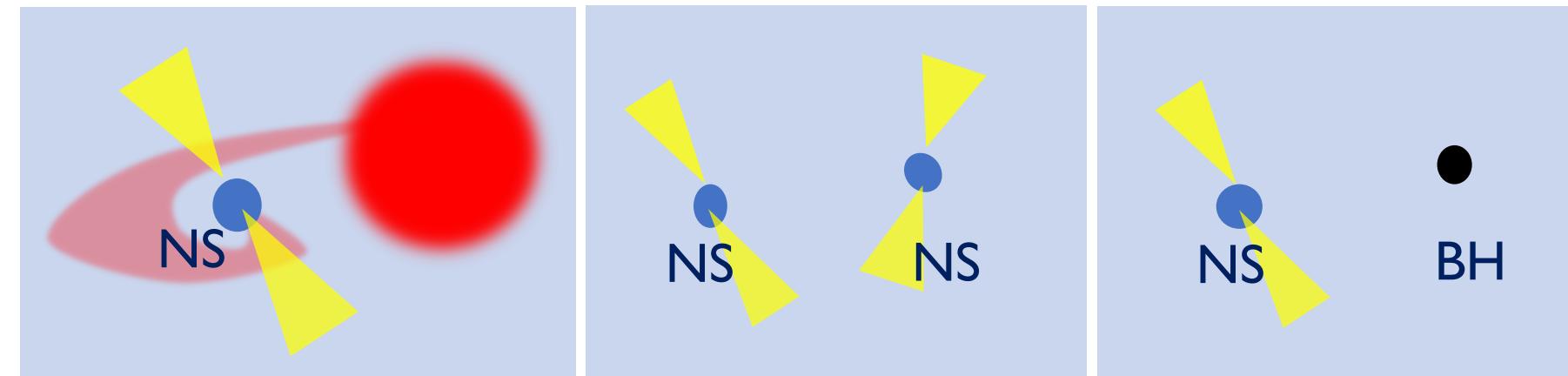
isolated radio pulsars



> 1500 NS

(including ~ 400 millisec.
pulsars)

pulsars in binaries



~ 100 NSs (pulsars) : member of binaries

... + many invisible NSs (**old**, age $> \sim 10^8$ yrs)

which have exhausted the rot. energy to power radio pulses

EOS & NS structure

NS: brief history --- where are we now?

1934) Zwicky & Baade (prediction) [1931: neutron discovered (Chadwick)]

"supernova: ordinary star → NS"

1967) Bell & Hewish: discovery of a pulsar (NS)

"existence of NS"

1974) Taylor & Hulse: a binary pulsar (double NSs)

"indirect confirmation of gravitation waves (GWs)"

2010) Demorest+: discovery of $2M_{\odot}$ -NS

"new guide on high density equations of state"

2017) LIGO-Virgo: GWs from NS mergers (+ electromagnetic counterparts)

"dawn of multi-messenger (GW, EM, neutrino) astronomy"

The **first** (established) $\sim 2M_{\odot}$ NS : **PSR J1614-2230**

NS – WD (white dwarf) binary

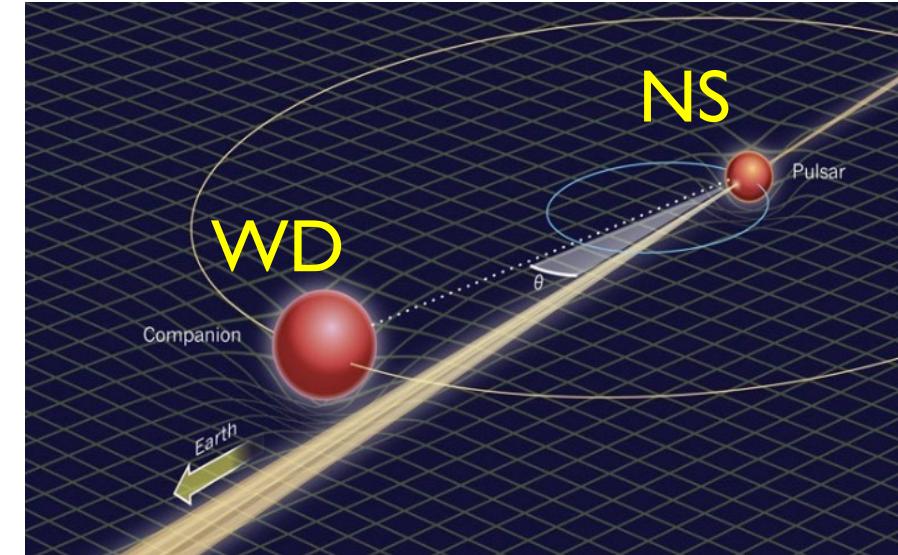
[v1) Demorest+ (2010); v2) Fonseca+ (2016)]

spin period = 2.15 ms; orbital period = 8.7 days

inclination angle = **89.17** \pm 0.02 ; edge on

$$M_{\text{NS}} = 1.928 \pm 0.017 M_{\odot}$$

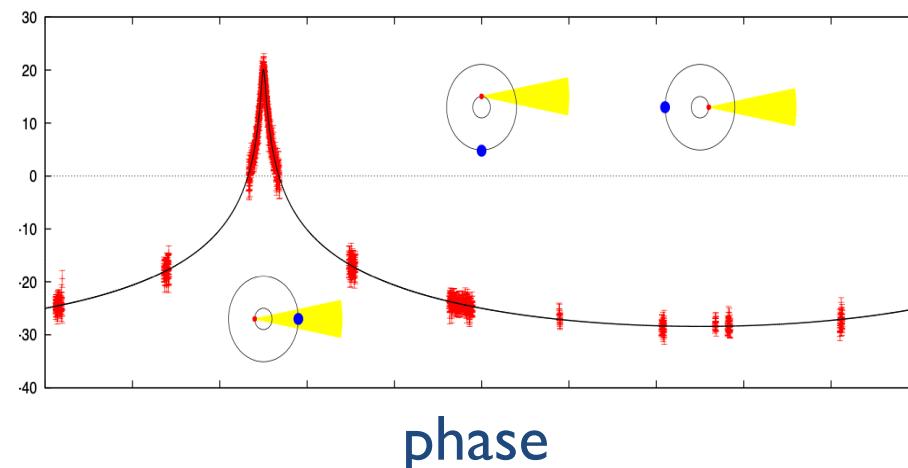
$$M_{\text{WD}} = 0.500 \pm 0.006 M_{\odot}$$



- 1) **binary orbit** \rightarrow relations btw M_{NS} & M_{WD}
- 2) **WD in the direction of pulses (!)**

Shapiro delay of light

1 & 2 \rightarrow separate determination of M_{NS} & M_{WD}



The **heaviest** NS known: **PSR J0740+6620**

NS – WD (white dwarf) binary

[v1) Cromartie+ (2019); v2) Fonseca+ (2021)]

spin period = 2.89 ms; orbital period = 4.77 days

inclination angle = **87.56** \pm 0.17 ; edge on

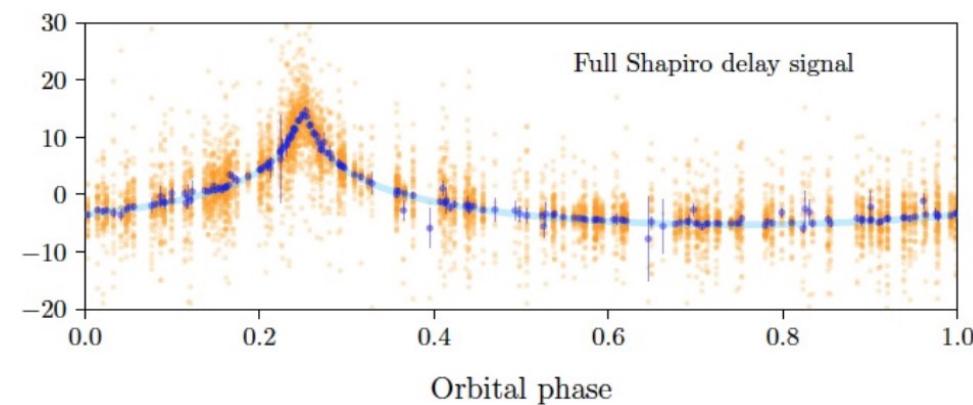
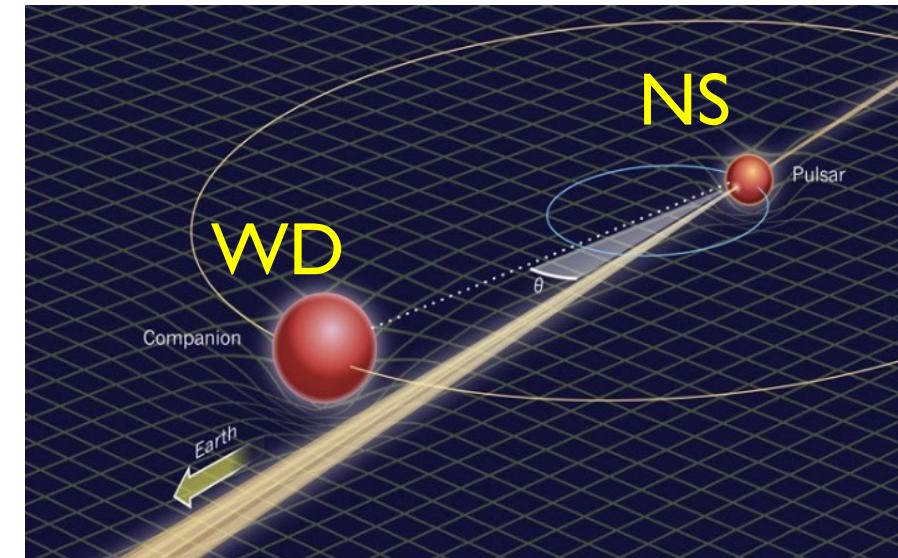
$$M_{\text{NS}} = \mathbf{2.08} \pm 0.07 M_{\odot}$$

$$M_{\text{WD}} = 0.253 \pm 0.005 M_{\odot}$$

distance from the earth : 1.14 kpc

Also, a target of NICER

M & R (see below)



Mass-Radius (M-R) relations

Einstein eq. QCD EoS

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

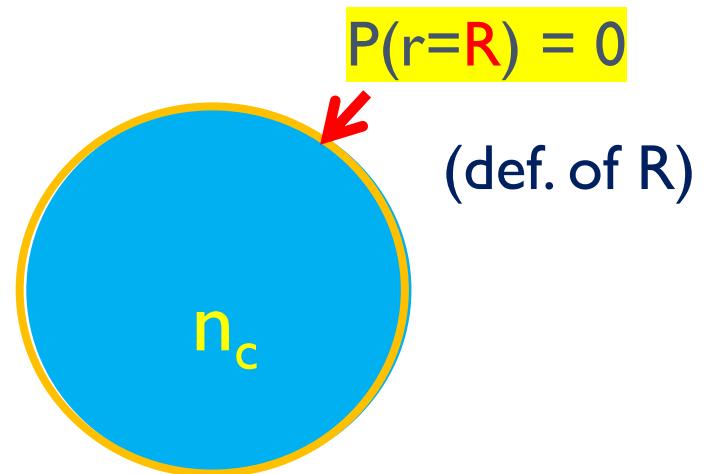


static
& spherical sym.

$r \rightarrow$ integrated



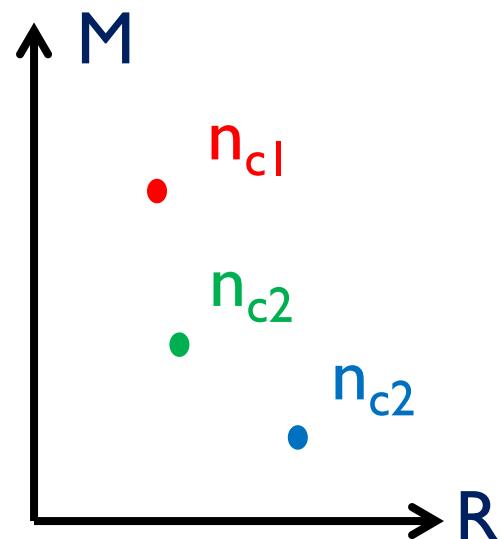
$n_c = n_B(r = 0)$



Tolman-Oppenheimer-Volkoff (TOV) eq.

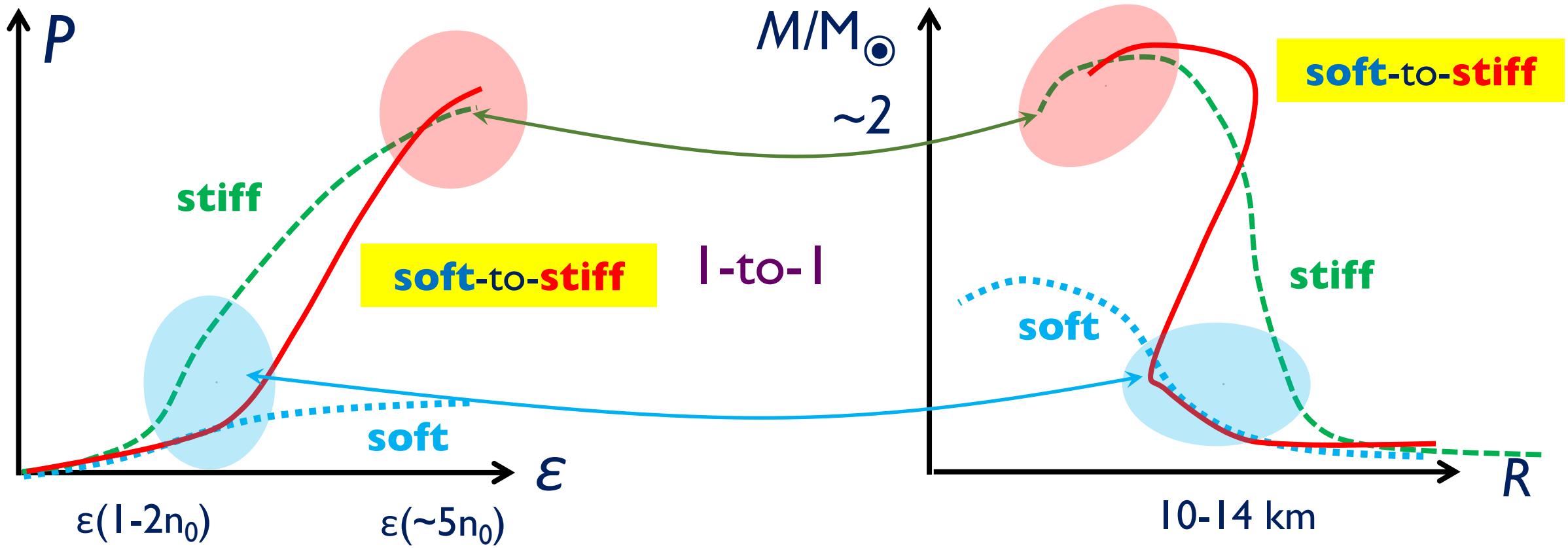
$$\left[\begin{array}{l} \frac{dP(r)}{dr} = -\frac{GM(r)\varepsilon(r)}{r^2} \left(1 + \frac{P}{\varepsilon}\right) \left(1 + \frac{4\pi r^3 P}{M(r)}\right) \left(1 - \frac{2GM(r)}{r}\right)^{-1} \\ \frac{dM(r)}{dr} = 4\pi r^2 \varepsilon(r) \end{array} \right] \text{GR effects } (> 1)$$

$M(n_c)$
 $R(n_c)$
at *given* n_c



EoS & Neutron Star M-R relation

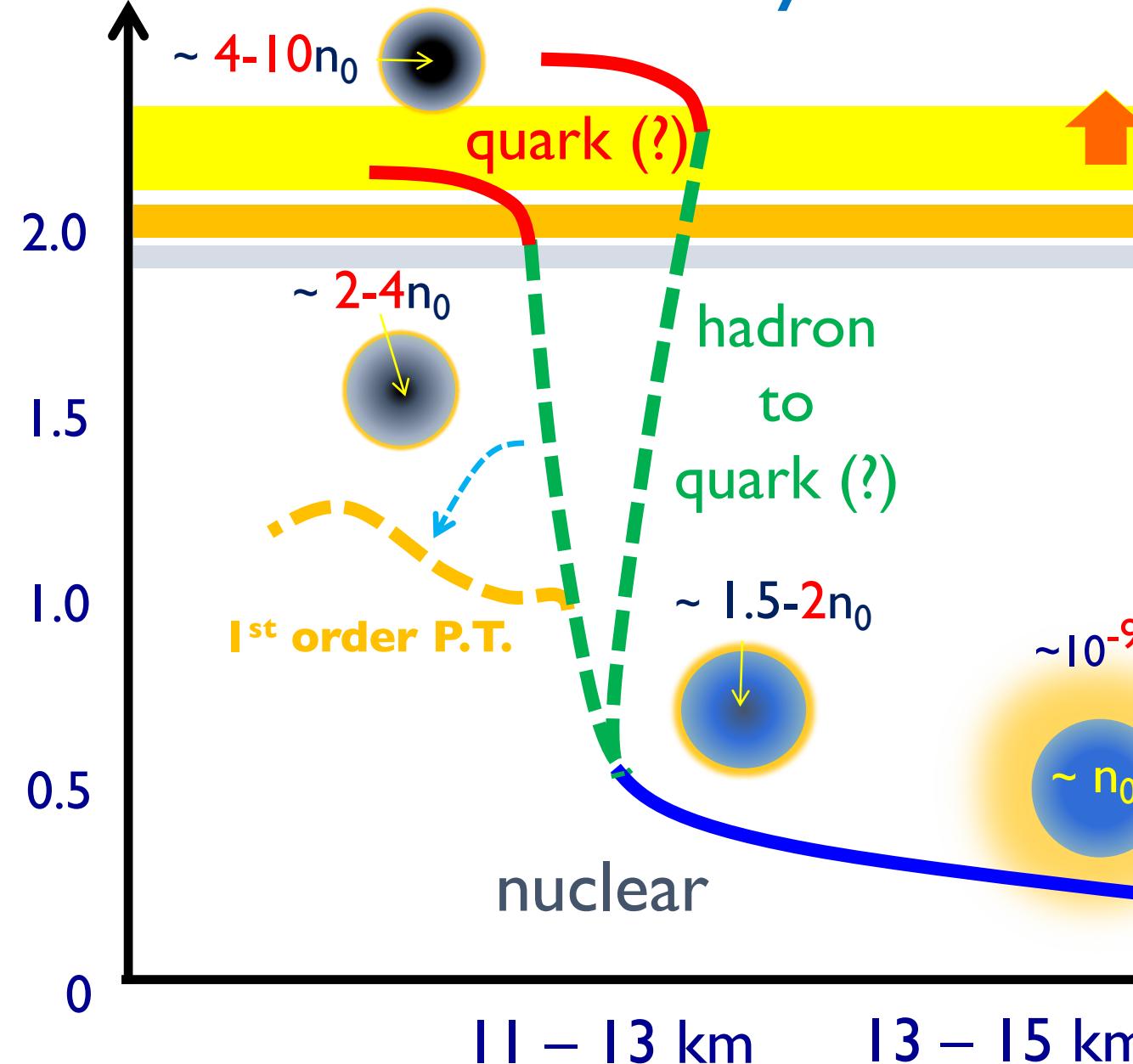
Ref) Lattimer & Prakash (2001)



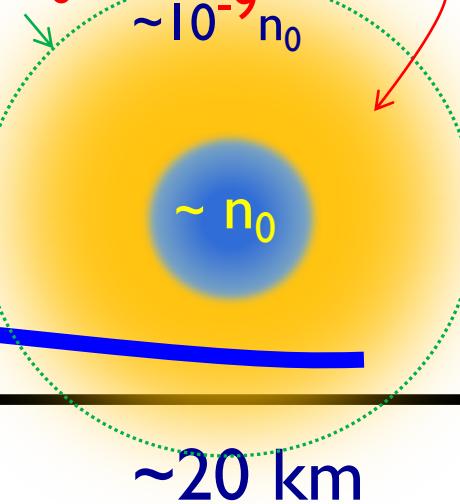
M/M_{\odot}

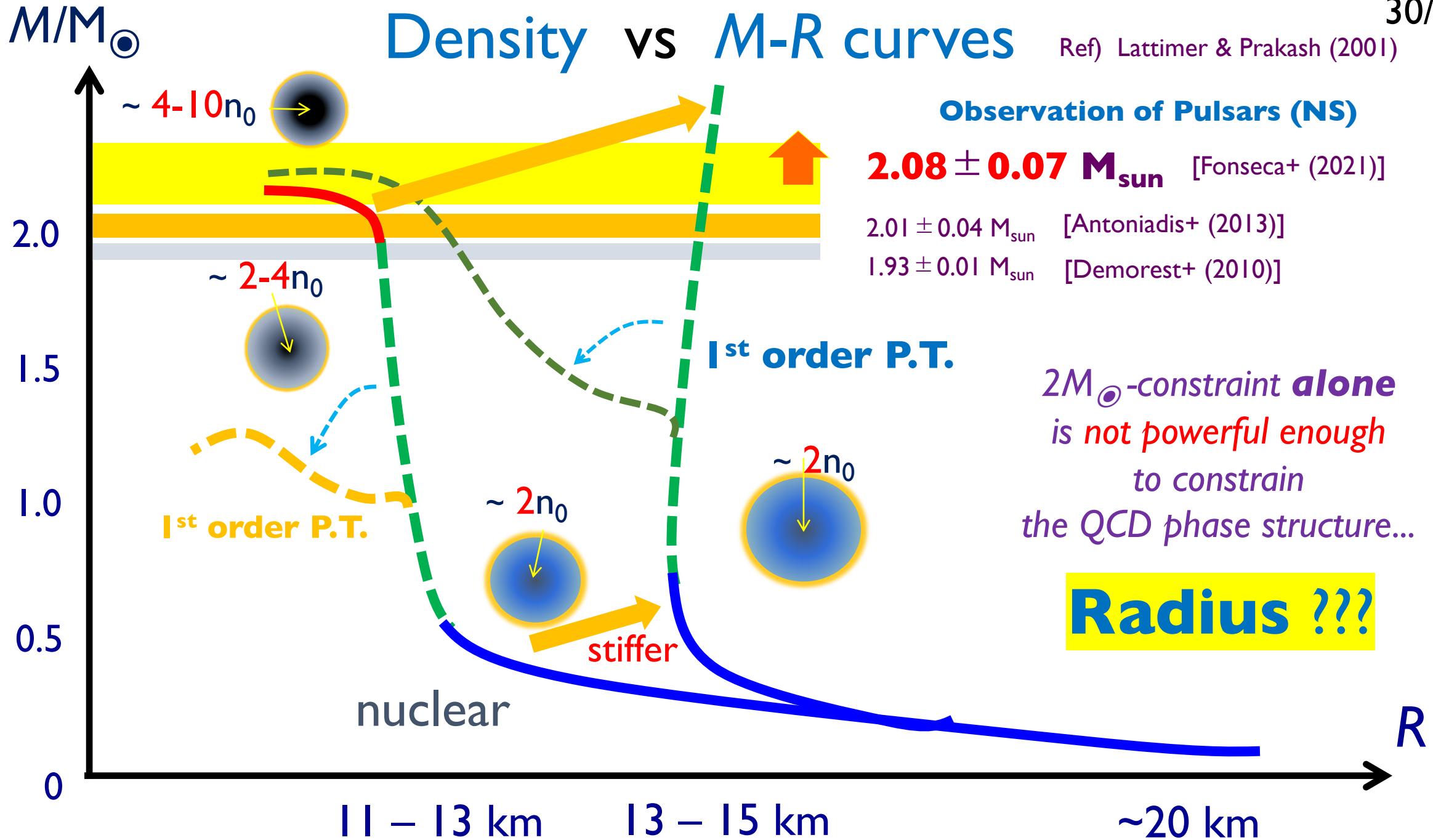
Density vs M-R curves

Ref) Lattimer & Prakash (2001)



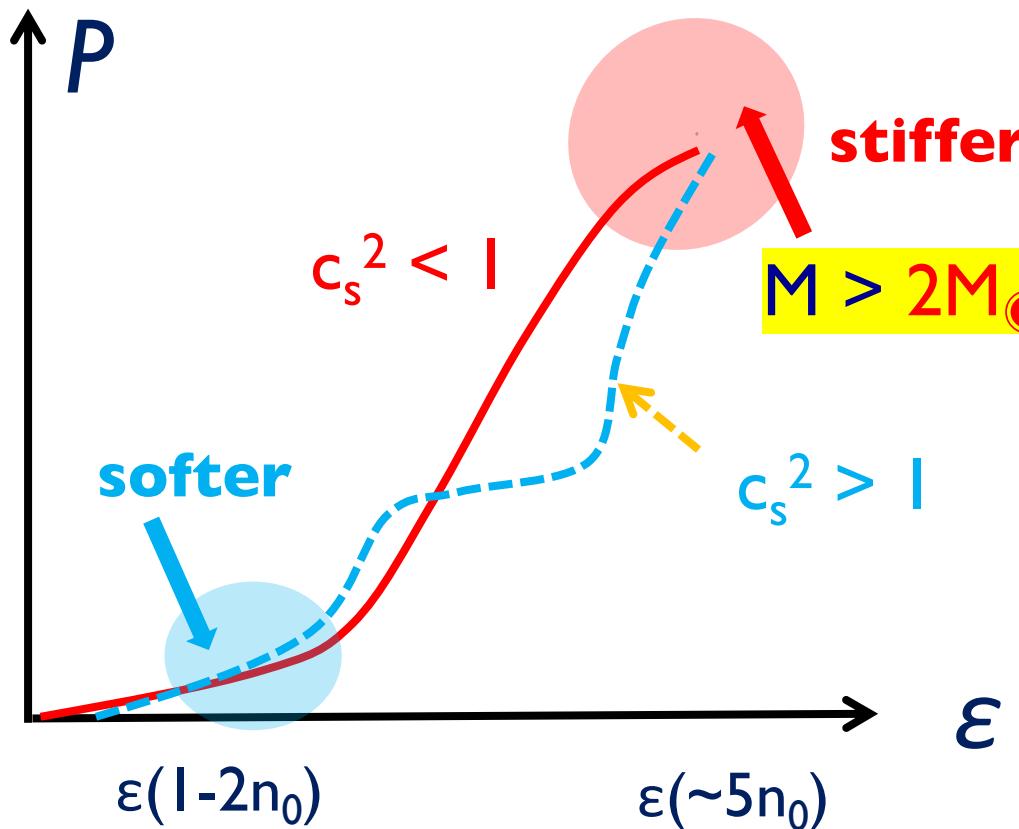
Observation of Pulsars (NS)

 $2.08 \pm 0.07 M_{\text{sun}}$ [Fonseca+ (2021)] $2.01 \pm 0.04 M_{\text{sun}}$ [Antoniadis+ (2013)] $1.93 \pm 0.01 M_{\text{sun}}$ [Demorest+ (2010)]crust \rightarrow loosely bound by gravity $P=0$ 



Correlating **low** ($<\sim 2n_0$) & **high** ($>\sim 5n_0$) density EOS

speed of sound: $c_s^2 = dP/d\varepsilon < 1$ (causality)



The constraints are **tighter**
if

- **high** density EOS is **stiffer**
- **low** density EOS is **softer**

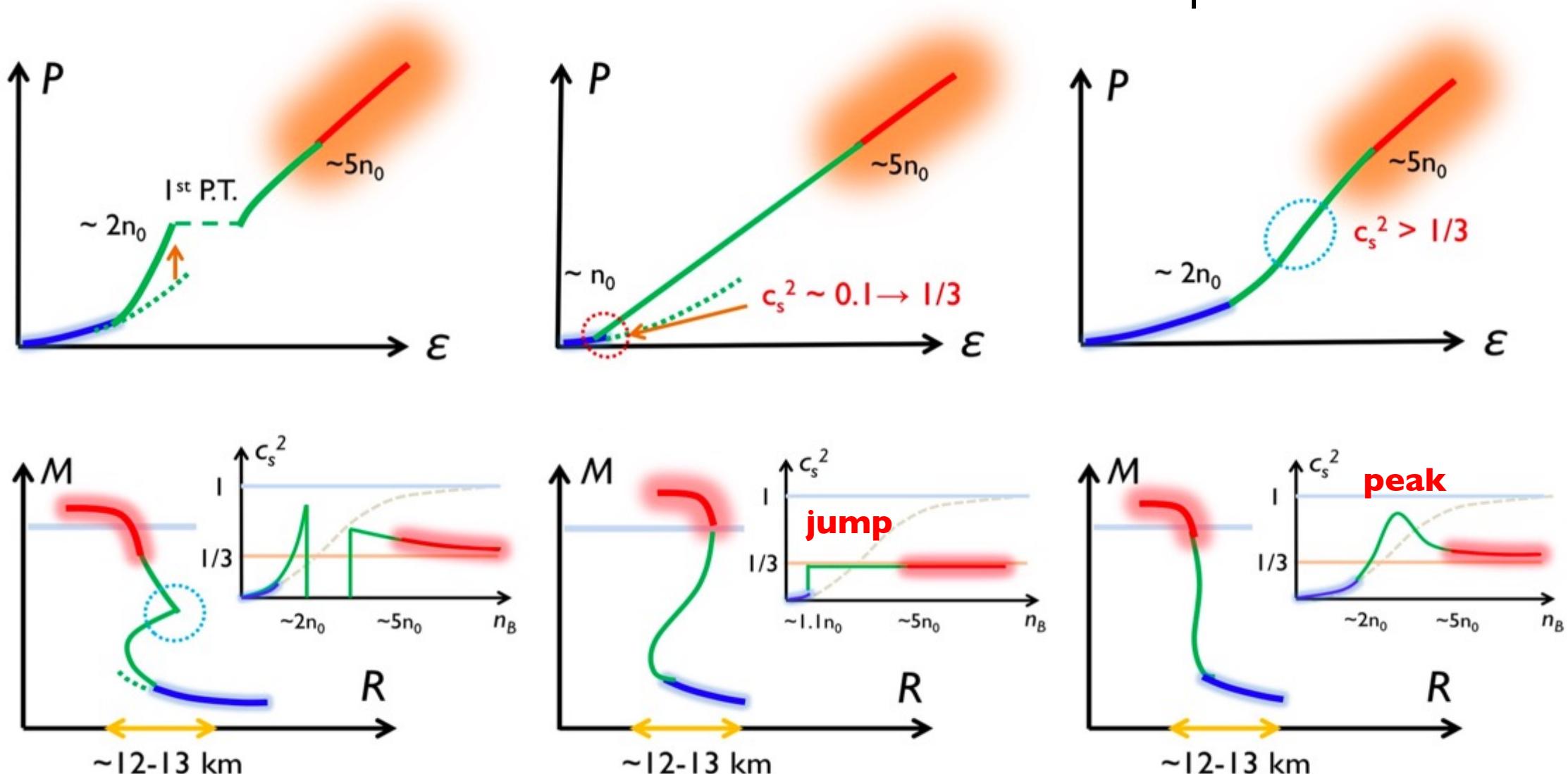
“**soft-to-stiff**” combination
is **theoretically challenging**

(but *likely* from *observations & nuclear physics*)

Several possible scenarios

[e.g. TK '21, mini-review]

→ topics in **Lect. 3**



Key questions to be addressed

- **low density:** $(n_B < \sim 2n_0)$
 - **observational** constraints for $R_{1.4}$?
 - **domain of applicability?** ($1.1-2.0n_0$?)
 - **precision** of low density **calculations?**
- **high density:** $(n_B > \sim 5n_0)$
 - **mechanisms** for **stiff EOS**?
 - effective **d.o.f?** Hadrons or quarks or..?
 - **when** matter becomes **weakly coupled?**
- **Inbetween:**
 - hadron-to-quark **phase transitions?**
 - **mechanism** for stiffening? **peaks in c_s^2 ?**

see
below

Lect.3

$R_{1.4}$ & low density EOS ($<\sim 2n_0$)

Soft vs **stiff** low density EOS ($n_B < \sim 2n_0$)

sources of info

NS observations :

$R_{1.4} \rightarrow$ EOS at $1\text{-}3n_0$
[Y_p for β -equilibrium]

Nuclear physics :

EOS at $< \sim 2n_0$

[theory for all Y_p , exp. for $Y_p \sim 0.5$]

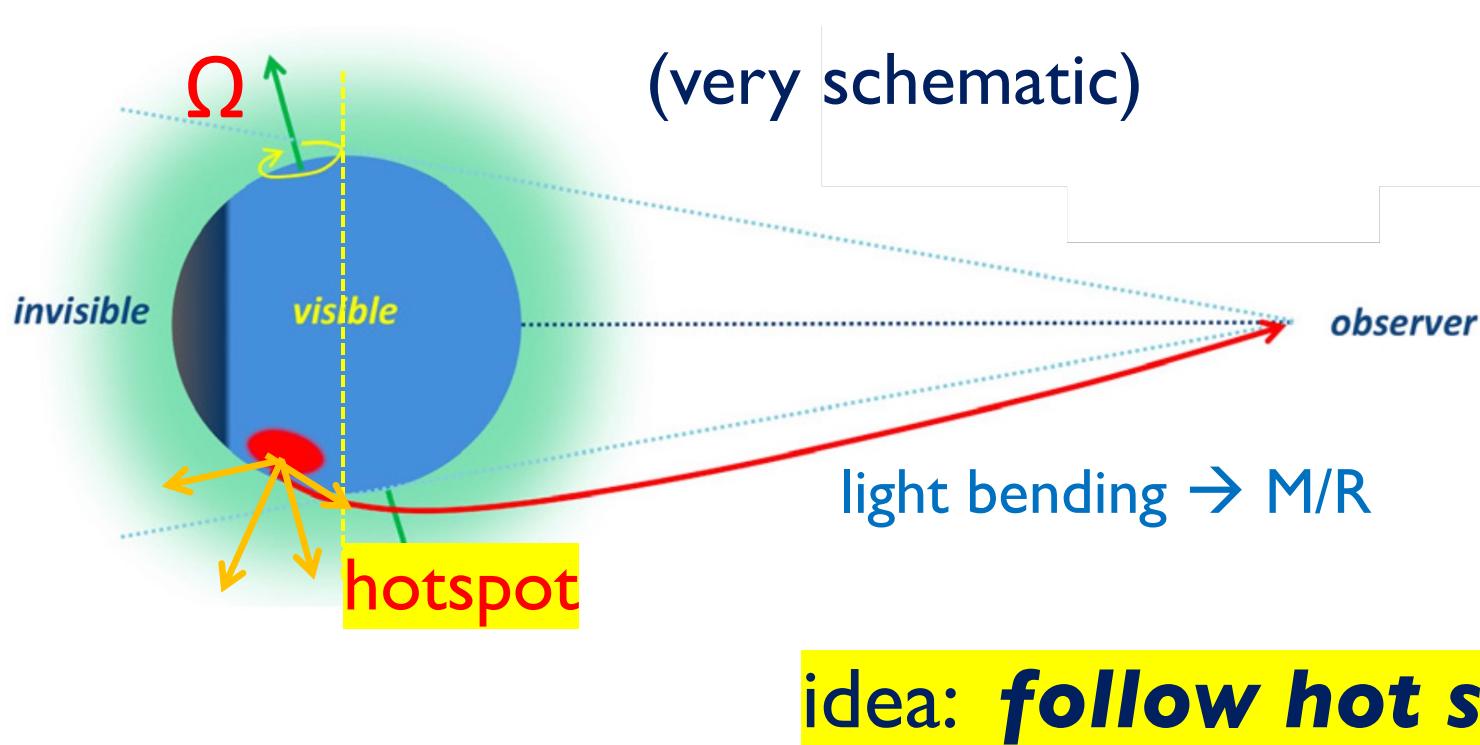
Heavy ion experiments :

“EOS” for $2\text{-}5(?)n_0$

[but $Y_p \sim 0.5$ & $T \sim 20\text{-}100$ MeV,
considerable extrapolations needed]

NICER

(Neutron star Interior Composition Explorer, 2017-)



period

Doppler shifted spectra

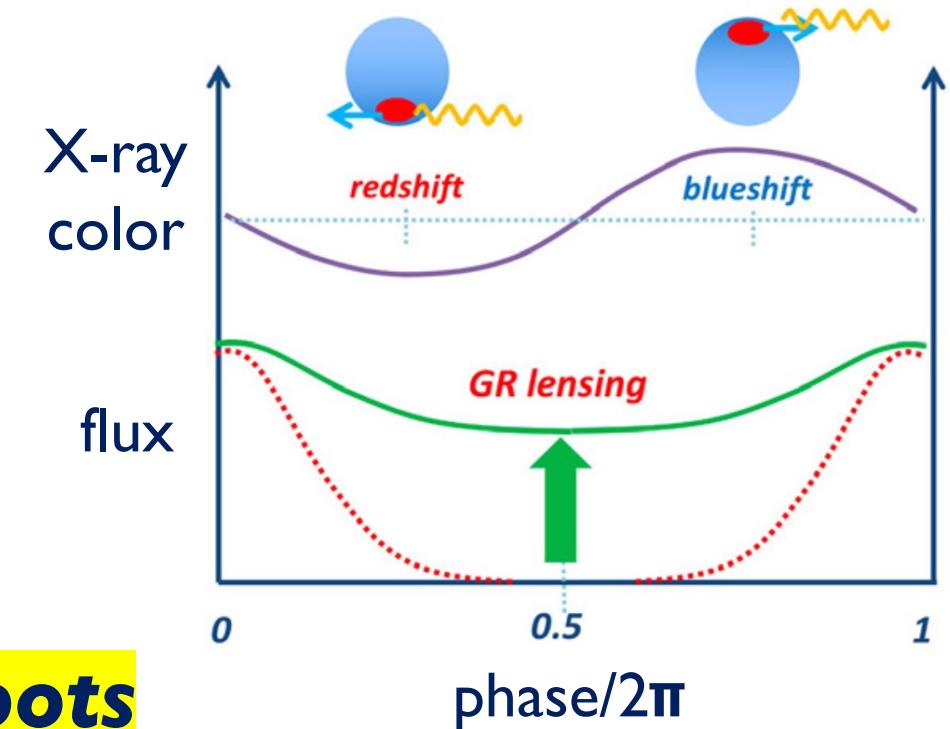
GR lensing (in principle)



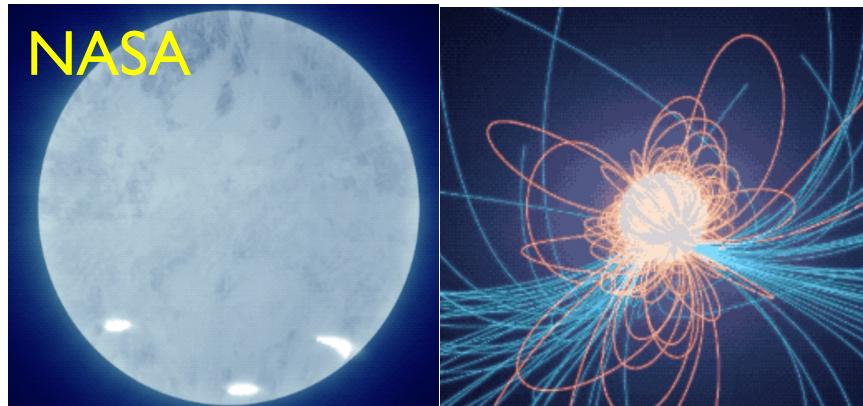
Ω (pulse period)

$R\Omega$ (surface velocity)

M/R (red shift)



NICER analyses for R (&M)



...reality is more complicated

hot spots → not simple dipole types
shapes? numbers of spots?
[examined by two teams]

PSR J0030+0451

$$M = 1.44^{+0.15}_{-0.14} M_{\odot}$$

$$R = 13.02^{+1.24}_{-1.06} \text{ km}$$

Miller+ '19

$$M = 1.34^{+0.15}_{-0.16} M_{\odot}$$

$$R = 12.71^{+1.14}_{-1.19} \text{ km}$$

Riley+ '19

PSR J0740+6620

$$M = 2.08^{+0.07}_{-0.07} M_{\odot}$$

NICER + XMM Newton

$$R = 13.7^{+2.6}_{-1.5} \text{ km}$$

Miller+ '21

$$R = 12.39^{+1.30}_{-0.98} \text{ km}$$

Riley+ '21

Nuclear calculations (microscopic)

I) Prepare **NN + 3N** forces + ...

- a) meson exchange models (traditional) [e.g., Illinois, Bonn, Argonne, Nijmegen,...]
- b) Chiral EFT (ChEFT) for π & N [e.g., Weinberg, Epelbaum, Meissner, Schwenk,...]
- c) Lattice QCD for NN, NY, YY int. [e.g., HAL collaboration,...]

2) Use the microscopic forces in **many-body methods**

- a) variational [e.g., Pandharipande, Takano, Togashi, ...] (w. soft nucleons $p^2/m_N < m_\pi$)
- b) Quantum Monte-Carlo [e.g., Carlson, Gandolfi,...]
- c) Hartree-Fock + many-body perturbation [e.g., Schwenk, Drischler, ...]

Advantage: can check how *systematic uncertainties* propagate;
 → the methods can predict *the domains of applicability*.

Domain of nuclear methods: rough estimate

$$\varepsilon(n_B)$$

2-body int. $\sim n_B^2$ (contact) & $n_B^{4/3}$ (long-range)

3-body int. $\sim n_B^3$ (contact) & $n_B^{5/3}$ (long-range)

Interactions **dominate** over kin. E (NR) $\sim n_B^{5/3} / m_N$

e.g.1) Akmal-Pandharipande-Ravenhall EoS (**APR**)

PNM	2 –body int.		3 –body int.	
	$\langle v_{ij}^\pi \rangle$	$\langle v_{ij}^R \rangle$	$\langle V_{ijk}^{2\pi} \rangle$	$\langle V_{ijk}^R \rangle$
n_B				
n_0	-4.1	-29.9	1.2	4.5
$2n_0$	-25.1	-36.4	-17.4	30.6
$3n_0$	-35.7	-44.7	-34.1	78.0
$4n_0$	-52.2	-41.1	-76.9	160.3

grow rapidly!

e.g.2) parameterized **pure neutron** matter EoS

[Gandolfi+ '09]

fit to Quantum Monte-Carlo

~kin. + 2-body

~3-body

$$\varepsilon = n_0 \left[(12 \pm 1 \text{ MeV}) \left(\frac{n_B}{n_0} \right)^{1.45 \pm 0.05} + (4 \pm 2 \text{ MeV}) \left(\frac{n_B}{n_0} \right)^{3.3 \pm 0.3} \right]$$

$n_B \sim 2n_0$ is the upper bound

Chiral EFT

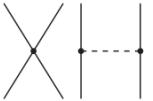
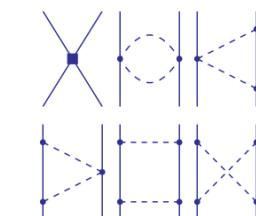
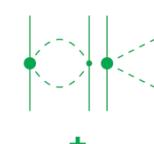
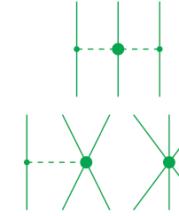
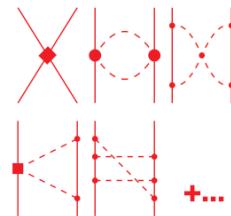
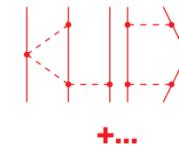
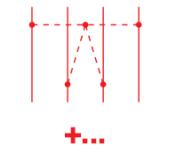
[Weinberg 1990-]

chiral symmetry

- 1) pions must appear together with derivatives or explicit sym. breaking terms
 - 2) Short distance nucleon contact terms
 - 3) other heavy d.o.fs : “integrated out”
- systematic organization of terms in Lagrangian (!)

advantages especially in organizing many-body forces

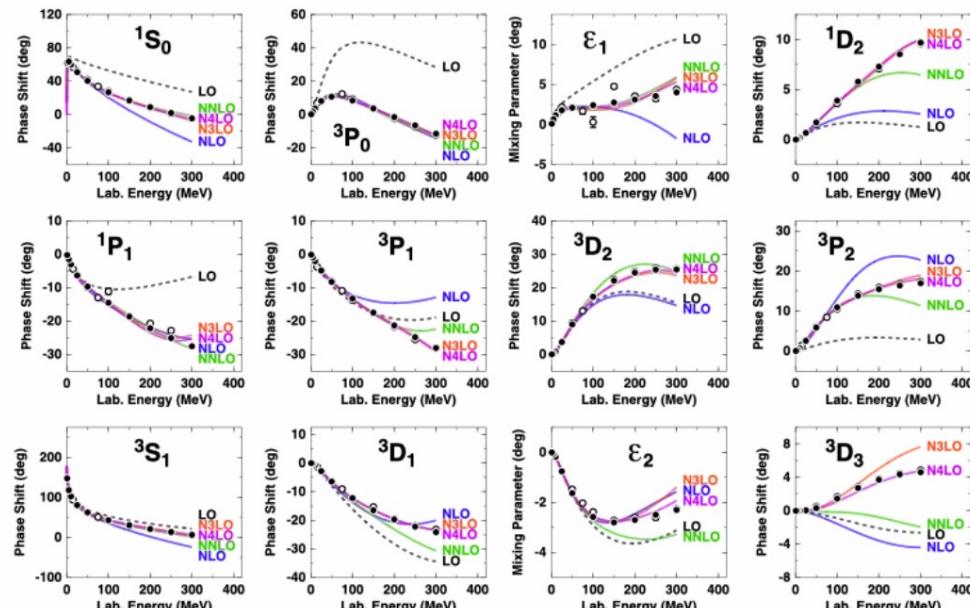
expansions of Q/Λ_χ , m_π/Λ_χ

	2N Force	3N Force	4N Force
LO $(Q/\Lambda_\chi)^0$			
NLO $(Q/\Lambda_\chi)^2$			
NNLO $(Q/\Lambda_\chi)^3$			
N^3LO $(Q/\Lambda_\chi)^4$			

Low energy constants

2N forces (well determined)

Neutron-proton scattering phase shifts



excellent fits (~6000 data)

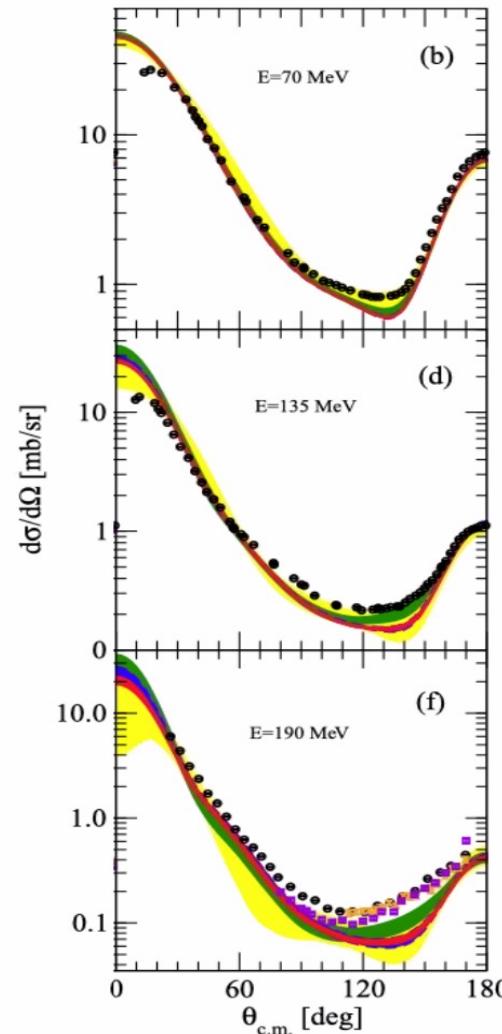
~40 LECs (N^4LO , 2NF)

3N forces (progressing)

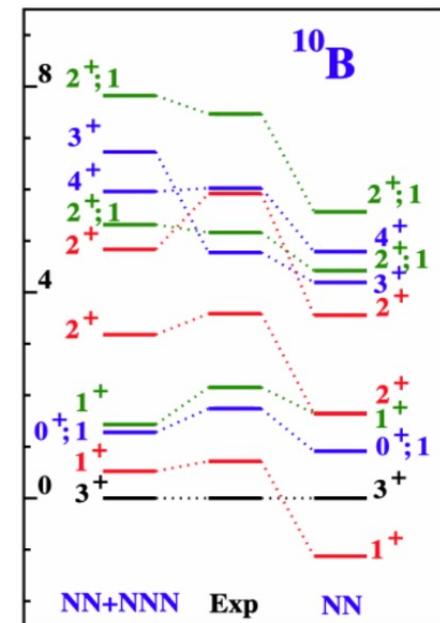
d(np) – p scatt. [Sekiguchi+ '17]

2NF to N^4LO
miss some contributions

spectra of light nuclei

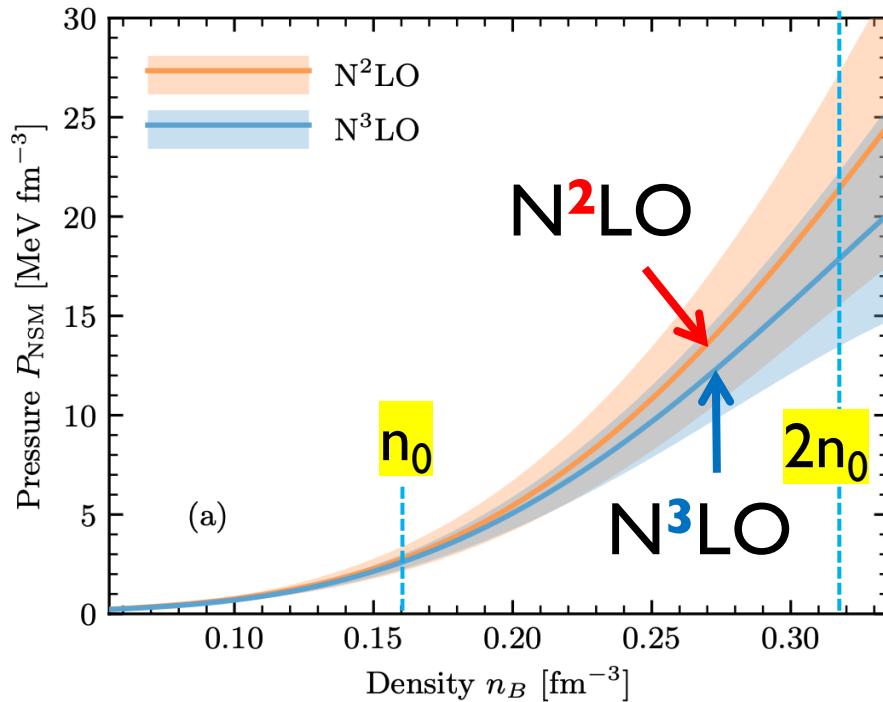


significant
improvement
with 3NF

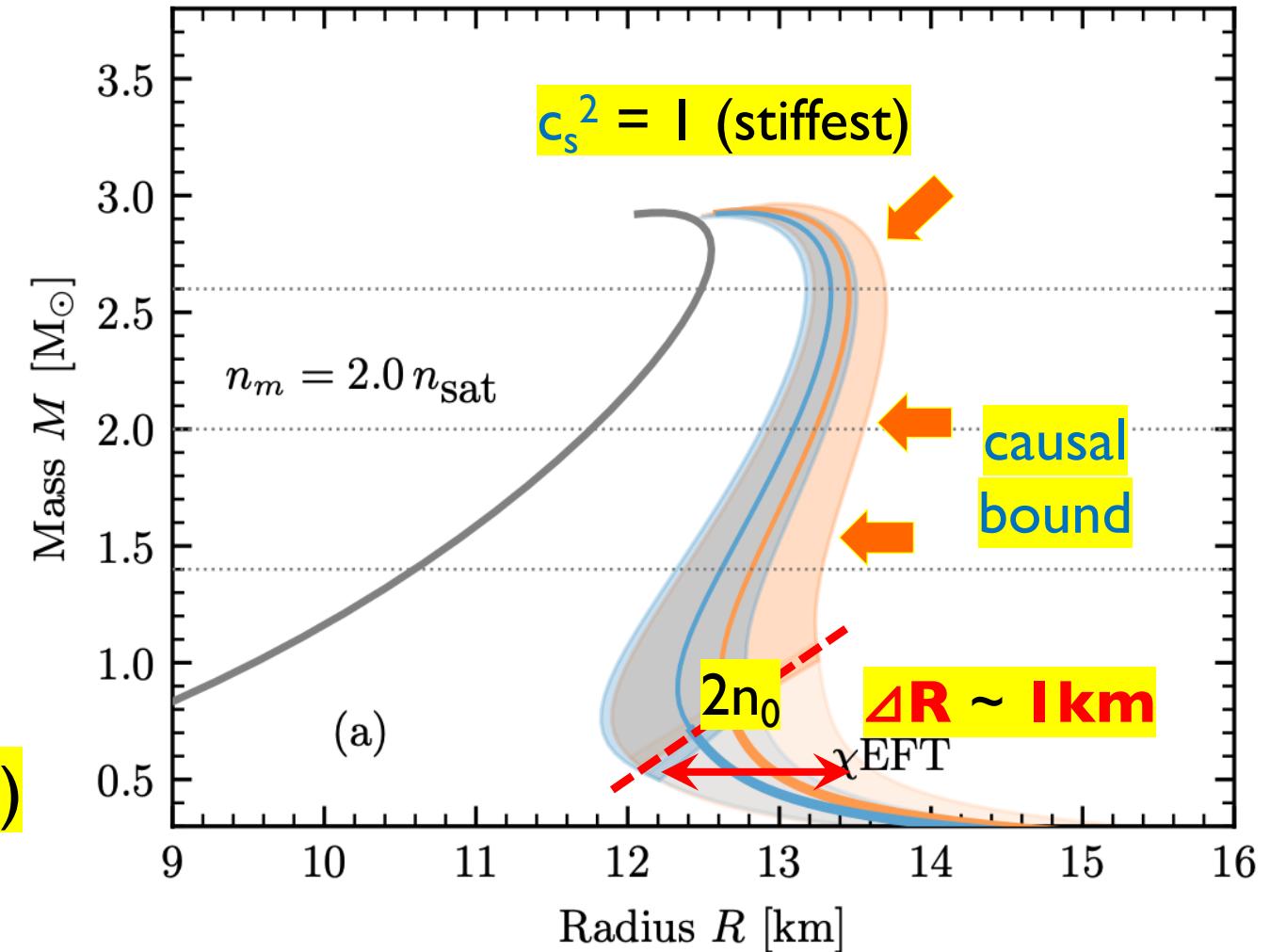


EOS with N³LO ChEFT band

[Drischler+ '21]



EOS: $P = P|_{x=2n_0} + c_s^2 (\varepsilon - \varepsilon|_{x=2n_0})$



if we trust ChEFT to $2n_0$, the causality $\rightarrow R_{1.4} < \sim 12.9 \text{ km}$

NICER + XMM + GW + nuclear physics (+ “ $c_s^2 < 1$ ”)

(see Lect.2)

NICER + XMM Newton

$$R_{2.08} = 13.7^{+2.6}_{-1.5} \text{ km}$$

Miller+ '21

+ GW + nuclear ($0.5n_0$) [Miller+ '21]



$$R_{2.08} = 12.35^{+0.75}_{-0.75} \text{ km}$$

$$R_{1.4} = 12.45^{+0.65}_{-0.65} \text{ km}$$

reduction of errors (!)

NICER + XMM Newton

$$R_{2.08} = 12.39^{+1.30}_{-0.98} \text{ km}$$

Riley+ '21

+ GW + ChEFT ($0.5-1.1n_0$) [Raaijmakers+ '21]



$$R_{1.4} = 12.33^{+0.76}_{-0.81} \text{ km}$$

reduction of errors (!)

$$R_{2.08} \sim R_{1.4} (!)$$

Summary of Lecture I

- glancing at NS properties; M, R, Y_p, B, T, \dots
- EOS and M-R relations
- EOS at low density; NICER $R_{1.4}$ and ChEFT

$$R_{\textcolor{red}{2.08}} \sim R_{\textcolor{red}{1.4}} (!) \quad \text{hints for soft-to-stiff EOS}$$

Lect. 2 : Gravitational waves & NS-NS mergers

Lect. 3 : From hadrons to quarks in NS