Outline:

- Phenomenological prelude
- QCD effective kinetic theory
- Thermalization in simple examples, Bottom-up thermalization

Why kinetic theory:

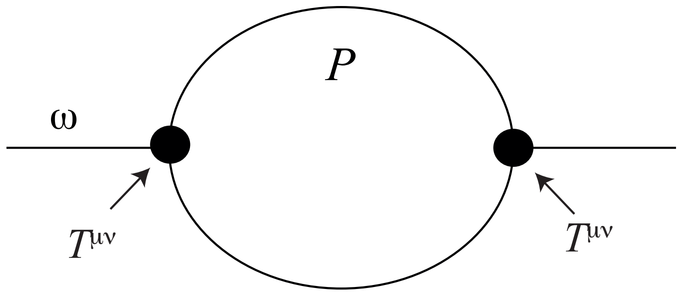
- Start with system in global thermal equilibrium: $T^{\rm eq}_{\mu\nu}$
- Give a kick with coupling to with gravity: $g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}(t)$
- The ripple from the kick described by the retarded $T^{\mu\nu}$ correlator:

$$\delta T^{\mu\nu}(x,t) = \int d^3x' dt G_R^{\mu\nu,\alpha\beta}(\mathbf{x},t;\mathbf{x}',t')h_{\alpha\beta}(\mathbf{x}',t')$$

Why kinetic theory:

$$G_R(\omega = 0, k = 0) \sim \int_P f[1+f]\rho(P)\rho(P)$$

- Free spectral function: $\rho_{\rm free} \sim 2\pi\delta(P^2)$
- Divergent: $G_R \sim \int_p f(1+f)\delta(P^2)\delta(P^2)$



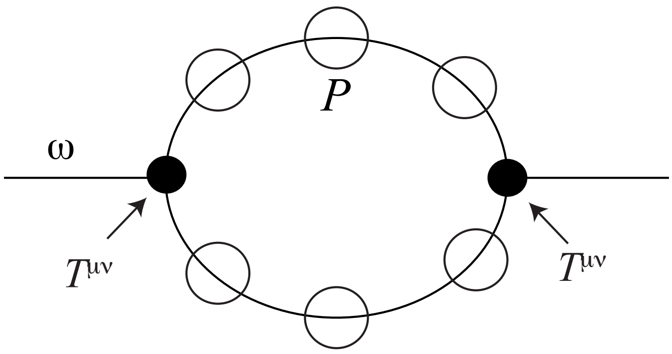
Jeon PRD47 (1993)

Why kinetic theory:

$$G_R(\omega = 0, k = 0) \sim \int_P f[1+f]\rho(P)\rho(P)$$

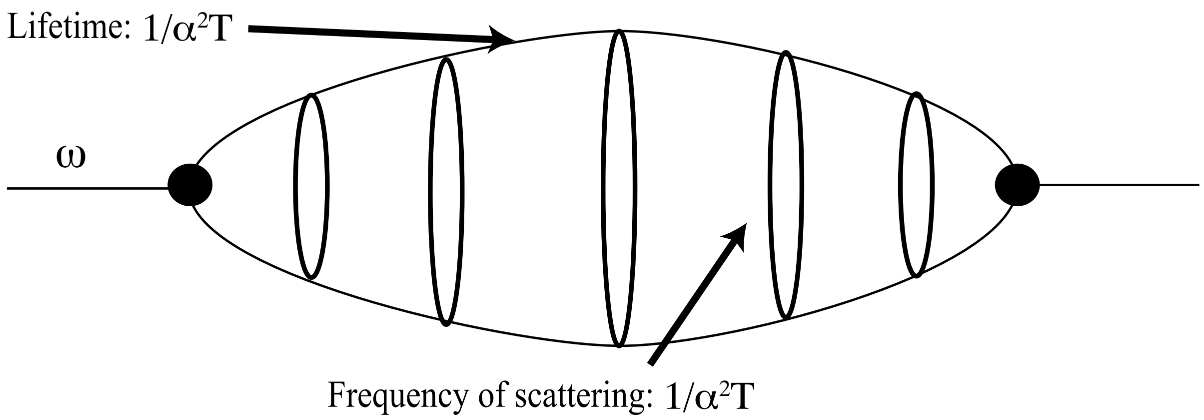
- Resummed spectral function: $\rho_{\rm resum} \sim \frac{p^0\Gamma}{(P^2)^2+\Gamma^2(p^0)^2}$, $\quad \Gamma \sim \frac{1}{\alpha^2 T}$

- Finite but large: $G \sim \int_P f(1+f)\left[\frac{p^0\Gamma}{(P^2)^2+\Gamma^2(p^0)^2}\right]^2 \sim \frac{T^5}{\Gamma}$



Jeon PRD47 (1993)

More resummations needed!



Both lines long lived $(\alpha^2 T)^{-1}$, of the order of scattering time

Complicated resummation can be dressed in form of an effective kinetic theory:

- Diagrammatic resummation (in $\lambda\phi^4$)

  Jeon PRD52 (1995)

- Interpretation of the diagrammatic resummation in terms of effective kinetic theory

  Jeon, Yaffe PRD53 (1996)

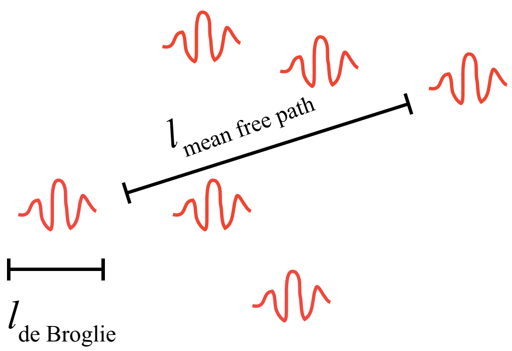
- Generalization to gauge theories through power counting, pQCD effective kinetic theory

  Arnold et al. JHEP 0301 (2003) 030

Scales in weakly coupled QCD

QCD Effective kinetic theory Arnold et al. JHEP 0301 (2003)

- QFT $\rightarrow$ Transport theory $\rightarrow$ fluid dynamics $\rightarrow$ HIC



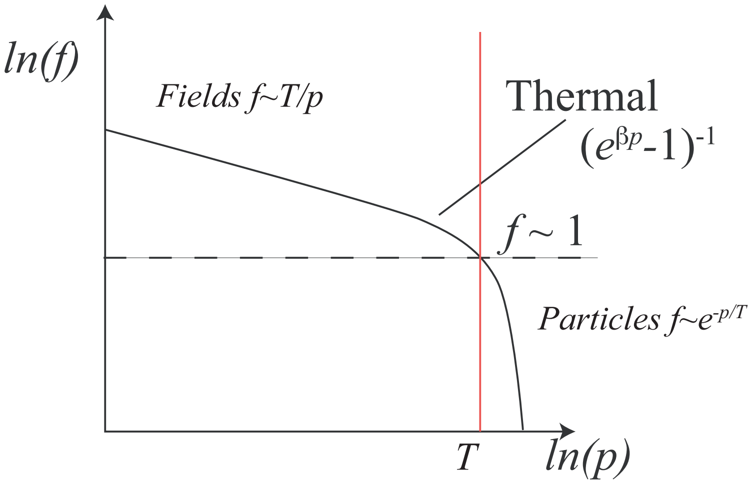
- Far-from-equilibrium physics is non-perturbative: resummation needed Jeon, Yaffe PRD53 (1996)
- Resummation leads to effective kinetic theory of (quasi-)particles

$$(\partial_t + \mathbf{v} \cdot \partial_{\mathbf{x}}) f(t, \mathbf{x}, \mathbf{p}) = C[f]$$

- Philosophy: Find all necessary processes needed for leading-order QCD description
- Kinetic evolution classical, quantum hidden in collision kernel $C[f]$

Scales in thermal QCD

Free theory in thermal equilibrium T:



- For bosons the distribution function $n_B(p) = \frac{1}{e^{\beta p}-1}$
  - for $p \gg T$, classical particles

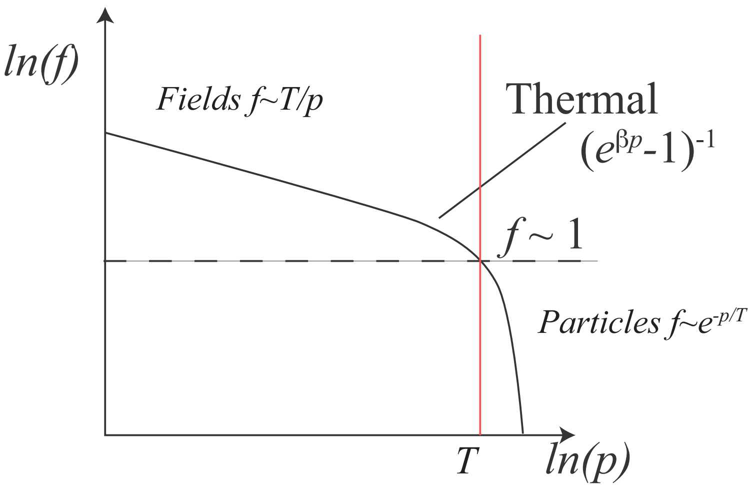
$$n_B(p) \approx e^{-\beta p}$$

  - for $p \ll T$, classical fields

$$n_B(p) \sim T/p$$

Scales in thermal QCD

Free theory in thermal equilibrium T:



- Most quantities dominated by particles at the scale T:
  - Energy density

$$\epsilon \sim \int \frac{d^3p}{(2\pi)^3} f p \sim T^4$$

  - Number density

$$n \sim \int \frac{d^3p}{(2\pi)^3} f \sim T^3$$

  - Interparticle distance

$$\Delta x \sim n^{1/3} \sim T$$

- mean free path $\lambda_{\mathrm{mfp}} \sim \frac{1}{\alpha^2 T}$

Wave packets overlap, but collisions can be treated separately

Interaction scales:

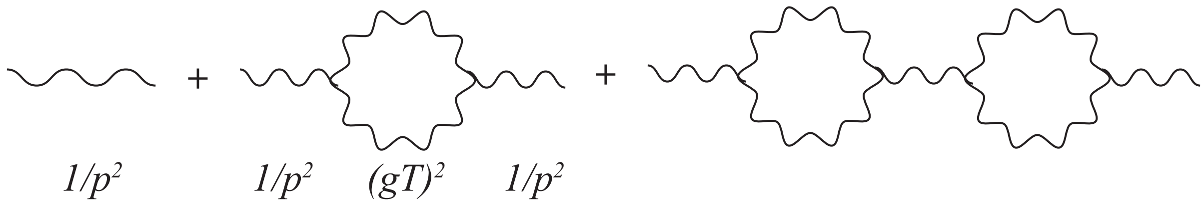
Interactions rise from $D_\mu = \partial_\mu + igA_\mu$

In medium, there are always statistical field fluctuations:

$$\underbrace{\langle A(\mathbf{x},t)A(\mathbf{x},t)\rangle}_{\int_p a^\dagger a} \sim \int_p \frac{1}{p}\left(\frac{1}{2} + f(p)\right) \sim T^2$$

For typical modes: $p \sim T, \quad p + gA \sim T + gT$

- Interactions with medium lead to small modifications of dispersion



- modes with $p \sim T$, get a small thermal mass $p^2 + m^2$ from interaction with the medium $m^2 \sim g^2 \int_p \frac{f(p)}{p}$

Interaction scales:

For *soft modes:* $p \sim gT$, $p + gA \sim gT + gT$

- Non-perturbative interaction with typical modes at scale T
- Interactions among soft modes still perturbative

$$\langle A(\mathbf{x},t)A(\mathbf{x},t)\rangle_{gT} \sim \int_p^{gT} \frac{f_p}{p} \sim g^3T^3 \frac{T}{g^2T^2} \sim gT^2 \tag{2}$$

so that $\underbrace{p}_{gT} + ig \underbrace{A_{soft}}_{g^{3/2}T}$

- but the expansion parameters if now only g

- Soft modes are classical fields $f \gg 1$, can use classical methods.
- The wavelength of the typical modes is $\frac{1}{p} \gg \frac{1}{T}$, the hard modes appear as classical particles to soft modes
  - At time scales of interest $1/gT$, the hard particles don't interact $\lambda_{mfp} \sim \frac{1}{g^4 T}$.
  - In linear level, QCD is like QED and the interaction between the soft and the hard modes is given by the linear response

$$J^{\mu}_{a}(\omega, k) = G^{\mu\nu}_{J,ab}(\omega, k) A^{b}_{\nu}(\omega, k)$$

+ a delta function in color

Interactions scales

For *ultrasoft modes*: $p \sim g^2 T$

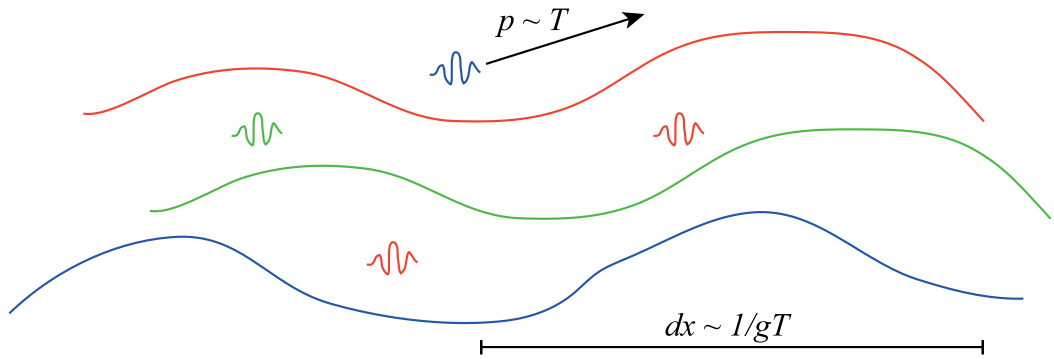
Interaction even among ultrasoft modes is nonperturbative

$$\langle A(\mathbf{x},t)A(\mathbf{x},t)\rangle_{g^2T} \sim \int_p^{g^2T} \frac{f_p}{p} \sim g^6 T^3 \frac{T}{g^4 T^2} \sim g^2 T^2$$

so that $\underbrace{p}_{g^2T} + ig \underbrace{A_{soft}}_{g^2T}$

No expansion parameter, but classical fields

Scales in thermal equilibrium at weak coupling

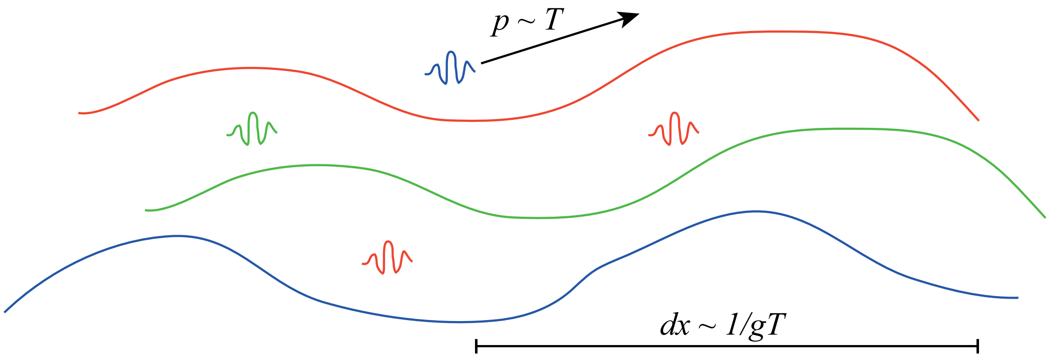


Degrees of freedom at weak coupling $g \ll 1$:

- Hard *particle* modes $p \sim T$: kinetic theory
- Soft (bosonic) *field* modes $p \sim m \sim gT$: classical field theory

$$n_B(p) = \frac{1}{e^{\omega/T} - 1} \sim \frac{T}{\omega} \sim \frac{1}{g}$$

Scales in thermal equilibrium at weak coupling



- Soft fields evolve according to classical nonabelian field equations

$$D_\mu F^{\mu\nu}(t, \mathbf{x}) = J^\mu(t, \mathbf{x})$$

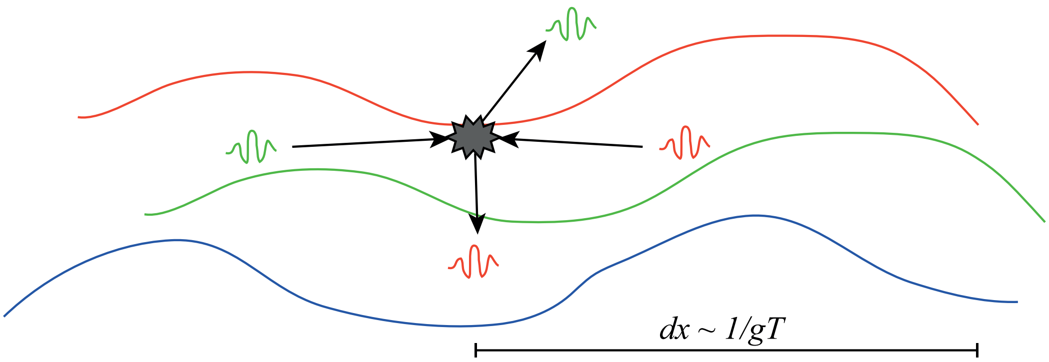
- Hard modes see soft fields as classical fields: Hard loop theory

$$J^{\mu,a}_{ind.}(t) = \delta^{ab} \int dt' G^{HL}_{\mu\nu}(t, t')\, \delta A^\nu_b(t'), \quad G^{HL}_{\mu\nu} \sim g^2 T^2$$

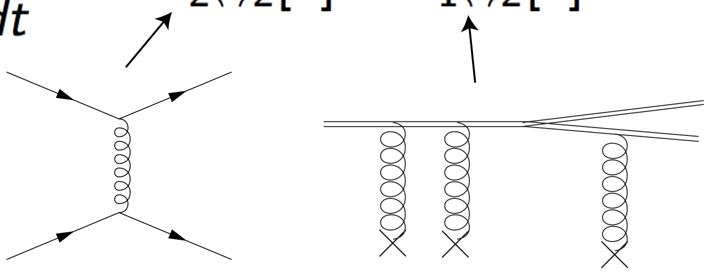
Blaizot, Iancu, Phys.Rept. 359 (2002) 355-528

Kinetic theory of hard modes

Arnold, Moore, Yaffe JHEP 0301 (2003) 030



$$\frac{df}{dt} = -C_{2\leftrightarrow 2}[f] - C_{1\leftrightarrow 2}[f]$$



- pQCD kinetic theory for the hard modes

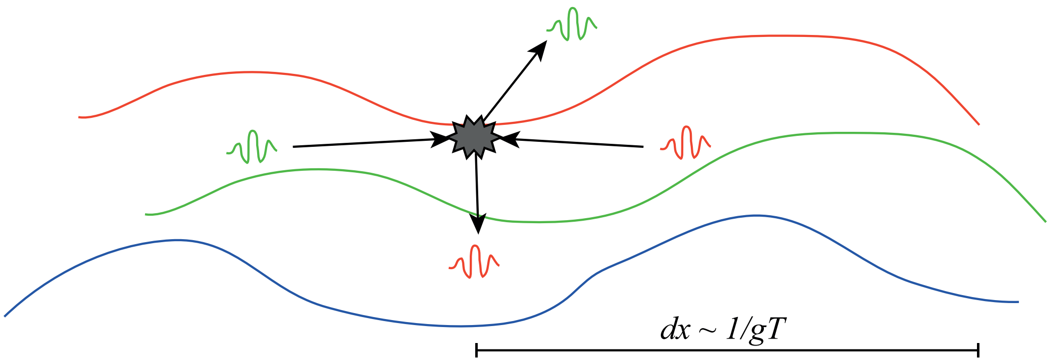
Kinetic theory of hard modes:

Contribution from different scales:

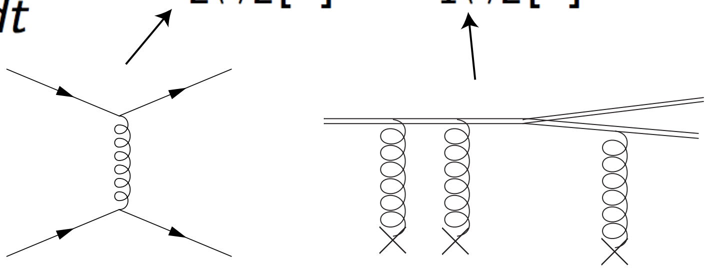
- In principle, all the aspects are present in any perturbative calculation
- In practice, to low orders in g for some quantities, one may get away just from looking the hard (or soft, ultrasoft) sectors

Kinetic theory of hard modes

Arnold, Moore, Yaffe JHEP 0301 (2003) 030



$$\frac{df}{dt} = -C_{2\leftrightarrow 2}[f] - C_{1\leftrightarrow 2}[f]$$

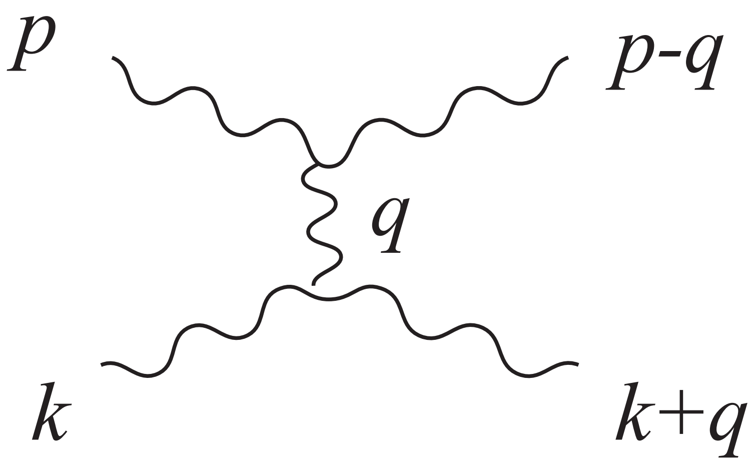


- pQCD kinetic theory for the hard modes

Naive guess:

Naive guess: sum over elementary (2↔ 2) tree-level processes

$$C_a^{2\leftrightarrow 2}[f](p) = \frac{1}{2}\frac{1}{2p}\frac{1}{\nu_a}\int_{\mathbf{k}\mathbf{p}'\mathbf{k}'} |M^{ab}_{cd}|^2 (2\pi)^4 \delta^{(4)}(P + K - P' - K')$$
$$\times \left\{ f_p f_k (1 \pm f_p')(1 \pm f_{k'}) - f_p' f_k' (1 \pm f_p)(1 \pm f_k) \right\}$$



Naive guess:

Naive guess: sum over elementary ($2 \leftrightarrow 2$) tree-level processes

$$C_a^{2\leftrightarrow 2}[f](p) = \frac{1}{2}\frac{1}{2p}\frac{1}{\nu_a}\int_{\mathbf{k}\mathbf{p}'\mathbf{k}'} |M_{cd}^{ab}|^2 (2\pi)^4 \delta^{(4)}(P + K - P' - K')$$
$$\times \left\{ f_p f_k (1 \pm f_p')(1 \pm f_{k'}) - f_p' f_k' (1 \pm f_p)(1 \pm f_k) \right\}$$

- Sum over all processes and kinematics
- Matrix elements (in non-relativistic normalization) for the different processes

$$M_{gg}^{gg} = \frac{\alpha^2 \left(3 - \frac{su}{t^2} - \frac{st}{u^2} - \frac{tu}{s^2}\right)}{(2p^0)(2k^0)(2p'^0)(2k'^0)}$$

- Energy and momentum conserving δ-function
- Gain and loss terms

Naive guess:

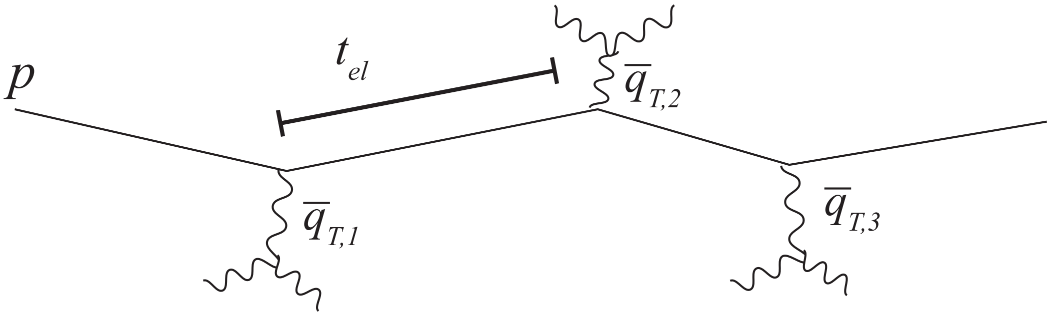
Naive guess: sum over elementary (2$\leftrightarrow$ 2) tree-level processes

$$C_a^{2\leftrightarrow 2}[f](p) = \frac{1}{2}\frac{1}{2p}\frac{1}{\nu_a}\int_{\mathbf{k}\mathbf{p}'\mathbf{k}'} |M_{cd}^{ab}|^2 (2\pi)^4 \delta^{(4)}(P + K - P' - K')$$
$$\times \left\{ f_p f_k (1 \pm f_p')(1 \pm f_{k'}) - f_p' f_k' (1 \pm f_p)(1 \pm f_k) \right\}$$

This is fine :

- if all the lines are unaffected by the medium
  Screening of the internal and external lines
- No other processes induced my the medium
  splitting
- All processes happen fast enough that they can be separated
  LPM-suppression

Momentum diffusion coefficient $\hat{q}$



- Random walk in mom. space

$$\mathbf{Q}_\perp = \mathbf{q}_\perp^1 + \mathbf{q}_\perp^2 + \dots$$

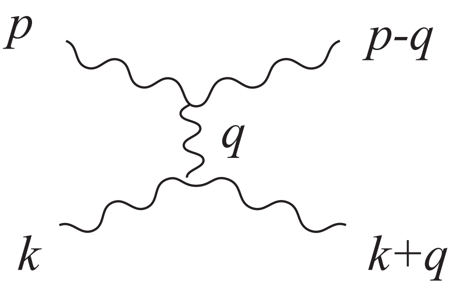
- Incoherent angles add in quadrature:

$$|Q_\perp| \propto \sqrt{\text{number of collisions}} \propto \sqrt{t}$$

- Momentum diffusion coefficient

$$\frac{|Q_\perp|^2}{t} \equiv \hat{q}_\perp \sim \frac{\Delta q_\perp^2}{t_{\text{el}}(q_\perp)}$$

Naive computation of $\hat{q}$



$$\frac{d\Gamma_{el}}{d^2q_\perp} \sim \int dq_z \int d^3k f_k(1 \pm f_{k+q}) \frac{d\sigma}{d^3q} \quad (3)$$

$$\hat{q} \sim \int d^2q_\perp \frac{d\Gamma_{el}}{d^2q_\perp} q_\perp^2 \quad (4)$$

$\Gamma_{\rm el} \equiv \frac{1}{t_{el}}$

For soft scattering $q \ll k, p$

$$\hat{q} \sim \underbrace{\int d^3k f_k(1 + f_k)}_{\sim T^3} \times \int d^2q_\perp \frac{d\sigma}{d^2q_\perp} q_\perp^2 \quad (5)$$

The soft scattering is dominated by t-channel coulomb scattering:

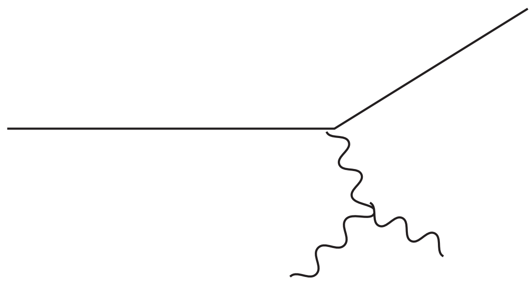
$$\frac{d\sigma}{d^2q_\perp} \sim \frac{\alpha^2}{(q_\perp^2)^2}$$

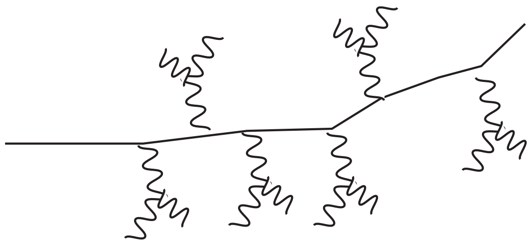
The momentum broadening coefficient has a IR log divergence $\int \frac{dq_\perp}{q_\perp}$!

UV taken care by relaxing the soft assumption

Soft log-digergence

$$\hat{q} \sim \alpha^2 T^3 \int d^2 q_\perp \frac{1}{(q_\perp^2)^2} q_\perp^2$$





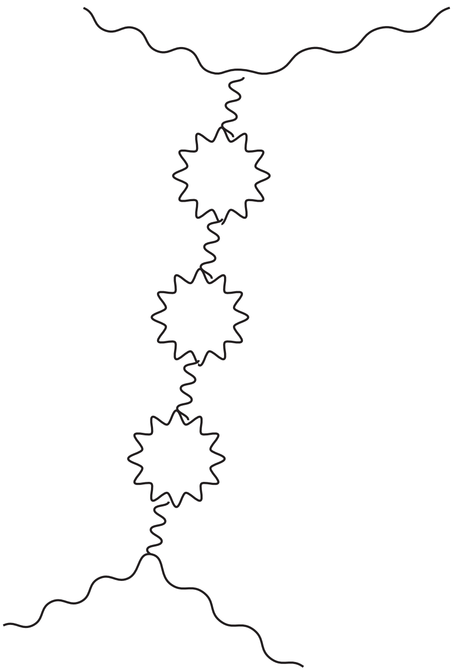
$$q_\perp \sim T \qquad\qquad q_\perp \sim gT$$

$$t_{el} \sim \frac{1}{g^4 T} \qquad\qquad t_{el} \sim \frac{1}{g^2 T}$$

$$\hat{q} \sim g^4 T^3 \qquad\qquad \hat{q} \sim g^4 T^3$$

- IR divergence is cured by the physics of screening

Effective matrix element



- Interaction of the soft mode non-perturbative with hard modes needs to be resummed

$$\frac{g_{\mu\nu}}{Q^2} \Rightarrow \left[Q^2 + G_J(\omega, k)\right]^{-1}_{\mu\nu}$$

- Infrared divergence then cancelled by the screening scale

$$\int_m \frac{d^2 q_\perp}{(q_\perp^2 + m^2)^2} q_\perp^2 \sim \log\left(\frac{T}{m}\right)$$

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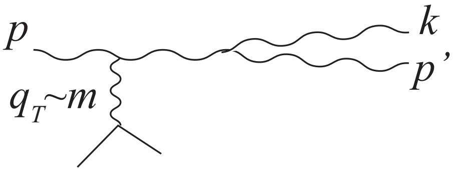
Induced splitting/merging

Screening makes the total scattering rate finite but it is still very fast:

technically speaking, has a log-divergence...

$$\Gamma_{el} \sim \frac{1}{t_{el}} \sim \alpha^2 T^3 \int \frac{d^2 q_\perp}{(q_\perp^2 + m^2)^2} \sim \alpha^2 \frac{T^3}{m^2} \sim \alpha T$$

Large angle scattering rate $\alpha^2 T_*$



- Massless onshell particles don't have phase space to decay
- But even a very soft scattering enough to induce a splitting

Induced splitting/merging rate as big as large angle scattering rate:

The rate at which a given particle emits at given k

$$\frac{d\Gamma^{BH}_{\text{split}}}{d\log k} \sim \alpha \Gamma_{el} \sim \alpha^2 T$$

BH = Bethe-Heitler

Induced splitting/merging

- For a leading order description, need to include an effective "1 ↔ 2" collision kernel

$$C^{\text{“}1 \leftrightarrow 2\text{”}}[f] \sim \int dk \frac{d\Gamma^{\mathrm{BH}}_{split}}{dk} \Big\{ f(p)[1+f(p-k)][1+f(k)] - f(p-k)f(k)[1+f(p)] \Big\},$$

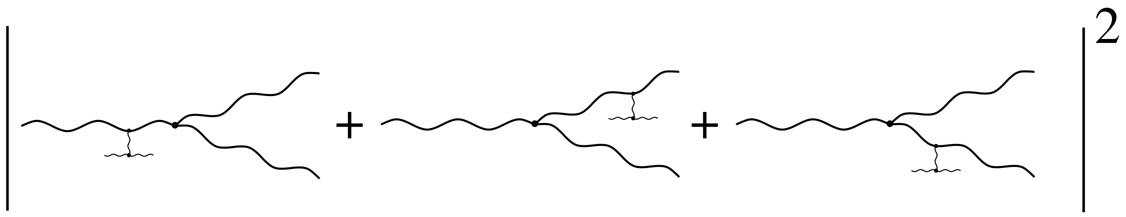
  - The soft scattering that induced the splitting is hidden in Γ^{BH}_{split}

Induced splitting/merging

The effective splitting element: $\gamma = p^2 \frac{d\Gamma^{BH}_{split}}{dp'}$

$$\gamma_{BH}(p; p', p-p') = p^2 \frac{\mathcal{Q}(m_\infty^2/m_D^2)}{4(2\pi)^4} N_c g^2 T_* \frac{p^2}{p'}$$

$$\mathcal{Q}(m_\infty^2/m_D^2) \equiv 8 \int_{p_\perp, q_\perp} \left[\frac{1}{\mathbf{q}_\perp^2} - \frac{1}{\mathbf{q}_\perp^2 + m_D^2}\right] \left(\frac{\mathbf{p}_\perp}{m_\infty^2 + \mathbf{p}_\perp^2} - \frac{\mathbf{p}_\perp - \mathbf{q}_\perp}{m_\infty^2 + (\mathbf{p}_\perp - \mathbf{q}_\perp)^2}\right)^2,$$



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