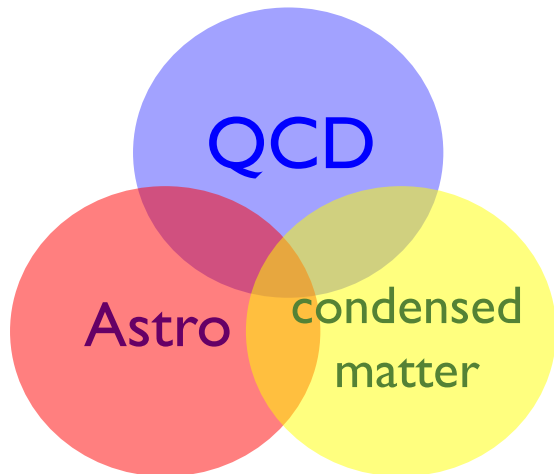


Neutron stars & multi-messenger physics

Toru Kojo

(**Tohoku Univ.**)



Lect 1) Overview

- glancing at NS properties
- M-R relation and EOS
- $R_{1.4}$ & low density EOS

Plan

Lect 2) NS-NS mergers

- gravitational waves
- pre-mergers [inspiral & tidal deformation]
- post-mergers [EM-counterparts]

Lect 3) From hadrons to quarks in NS

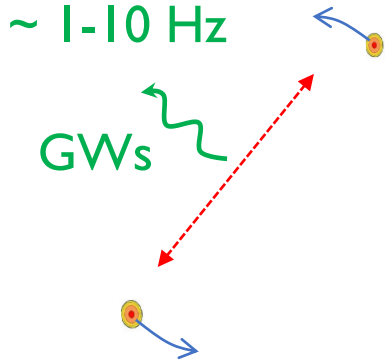
- quark matter
- 3-window modeling
- stiffening of matter in quark-hadron continuity

GWs



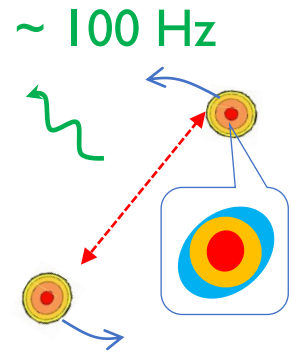
Early inspiral

~ — Myrs-Gyrs



Tidally deformed

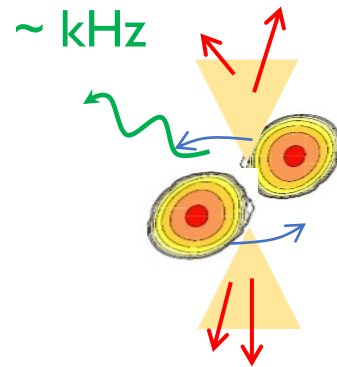
~ — 10 ms



GR + hydro

Merger

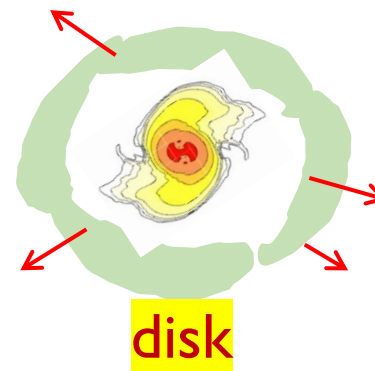
t = 0



GR + magnetohydro
+ neutrino radiation...

HMNS, SMNS

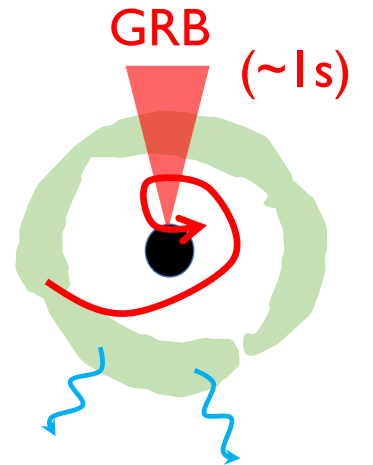
~ 1-10 ms



+ r-process
& radioactive decays

BH

> ~ 1s



kilonova (days-weeks)

References (reviews)

- L. Baiotti and L. Rezzolla, [\[arXiv:1607.03540\]](#)
“Binary neutron-star mergers: a review of Einstein’s richest laboratory”
- M. Shibata and K. Hotokezaka, [\[arXiv:1908.02350\]](#)
“Merger and Mass Ejection of Neutron-Star Binaries”
- D. Radice, S. Bernuzzi, and A. Perego, [\[arXiv:2002.03863\]](#)
“The Dynamics of Binary Neutron Star Mergers and of GW170817”
- K. Chatziioannou, [\[arXiv:2006.03168\]](#)
“Neutron star tidal deformability and equation of state constraints”
- N. Sarin and P. D. Lasky, [\[arXiv:2012.08172\]](#)
“The evolution of binary neutron star post-merger remnants: a review”

Gravitational Waves (GWs)

Einstein eq. $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi GT_{\mu\nu}$

Riemann tensor

Ricci tensor $R_{\alpha\beta} = R^{\rho}{}_{\alpha\rho\beta} = \partial_{\rho}\Gamma^{\rho}{}_{\beta\alpha} - \partial_{\beta}\Gamma^{\rho}{}_{\rho\alpha} + \Gamma^{\rho}{}_{\rho\lambda}\Gamma^{\lambda}{}_{\beta\alpha} - \Gamma^{\rho}{}_{\beta\lambda}\Gamma^{\lambda}{}_{\rho\alpha}$

Ricci scalar $R = g^{\alpha\beta}R_{\alpha\beta}$

Christoffel $\Gamma^{\alpha}{}_{\beta\gamma} = \frac{1}{2}g^{\alpha\sigma} (\partial_{\gamma}g_{\sigma\beta} + \partial_{\beta}g_{\gamma\sigma} - \partial_{\sigma}g_{\beta\gamma})$

highly nonlinear eq. for strong large curvatures

simplified for **weak field regime**: $g_{\mu\nu}(x) = \overset{\text{flat metric}}{\eta_{\mu\nu}} + \overset{\text{small pert.}}{h_{\mu\nu}}(x)$

Gravitational Waves (GWs)

Einstein eq. $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi GT_{\mu\nu}$

1) linearized in metric

2) use the general covariance for coordinates: gauge fixing:
4 + 4 conditions (Lorentz gauge)

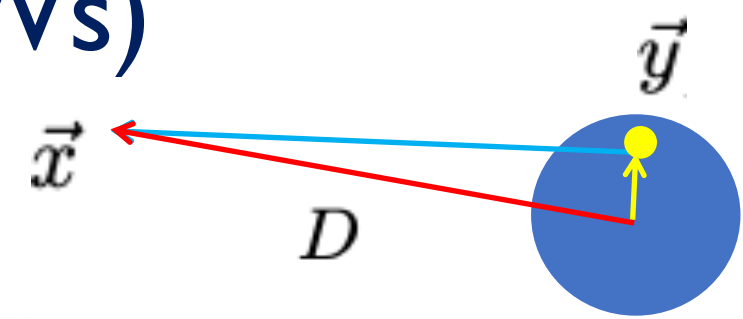
10 d.o.f \rightarrow 2 physical d.o.f in h

Now we define: $\psi_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\eta^{\alpha\beta}h_{\alpha\beta}$

Einstein eq. is reduced to: $\partial^2\psi_{\mu\nu} = -16\pi GT_{\mu\nu}$ wave-eq.

Gravitational Waves (GWs)

$$\partial^2 \psi_{\mu\nu} = -16\pi G T_{\mu\nu}$$



retardation time

D

\vec{y}

\vec{x}

$t_{\text{ret}} \equiv t - D/c$

→
$$\psi_{\mu\nu}(t, \vec{x}) = 4G \int_{\vec{y}} \frac{T_{\mu\nu}(t - |\vec{x} - \vec{y}|, \vec{y})}{|\vec{x} - \vec{y}|} \simeq \frac{4G}{D} \int_{\vec{y}} T_{\mu\nu}(t_{\text{ret}}, \vec{y})$$

formal solutions

$\partial_\mu T^{\mu\nu} = 0$ →

$$T^{0i} = \partial_k (T^{k0} x^i) + x^i \partial_0 T^{00}$$

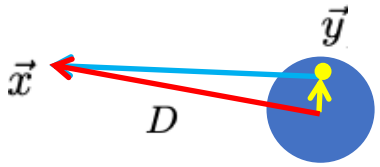
$$T^{ij} = \frac{1}{2} \left[\partial_k (T^{ki} x^j + T^{kj} x^i) + \partial_0 (\underline{T^{0i} x^j} + \underline{T^{0j} x^i}) \right]$$

$$= \partial_k (\dots) + \partial_0^2 (T^{00} x^i x^j)$$

total derivative

2nd time derivative
of energy quadrupole

Gravitational Waves (GWs)



fluctuating quadrupole

$$\psi_{ij}(t, \vec{x}) \simeq \frac{4G}{D} \int_{\vec{y}} T_{ij}(t_{\text{ret}}, \vec{y}) = \frac{4G}{D} \partial_0^2 \int_{\vec{y}} \varepsilon y_i y_j = \frac{4G}{D} \underline{\ddot{I}_{ij}(t_{\text{ret}})}$$

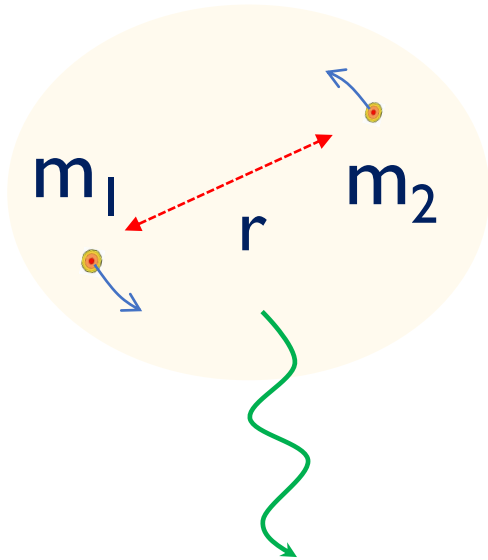
e.g.) **binary**

$$I_{xx} = \mu r^2 \cos^2 \omega t$$

$$\omega = [GM_{\text{tot}}/r^3]^{1/2} \quad [\text{Kepler}]$$

$$I_{yy} = \mu r^2 \sin^2 \omega t$$

$$\mu = m_1 m_2 / M$$



eliminate r

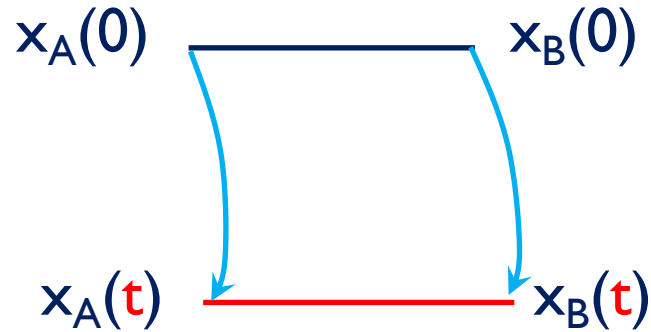
$$h_{\text{max}} = \frac{4G}{D} \mu r^2 \omega^2 = \underline{\underline{\frac{4G^{5/3}}{D} \mu M^{2/3} \omega^{2/3}}}}$$

$$\mu M^{2/3} \equiv \mathcal{M}_{\text{chirp}}^{5/3} \quad \text{“chirp mass” (observable)}$$

GWs

the signals are large for **large** $\mathcal{M}_{\text{chirp}}$, ω , and **small** D

Geodesic deviation



grav. pot.

$$\ddot{x}_A^i = -\partial^i \Phi(x_A)$$

$$\ddot{x}_B^i = -\partial^i \Phi(x_B)$$

Newtonian

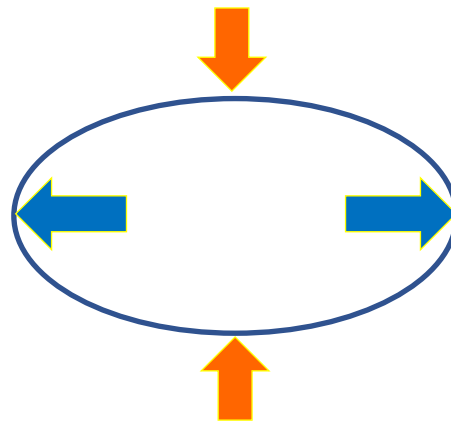
$$\delta \ddot{x}^i = -(\partial^i \partial_j \Phi_A) \delta x^j$$

GR version:

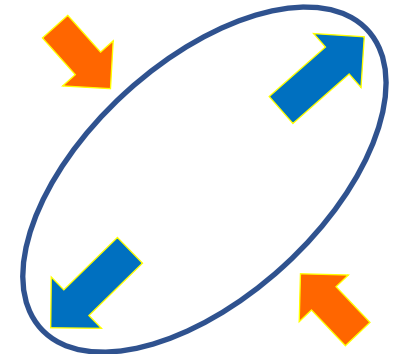
$$\delta \ddot{x}^i = R^i{}_{00j} \delta x^j = R_{i00j} \delta x^j = \frac{\ddot{h}_{ij}}{2} \delta x^j$$

For GWs:

+ mode



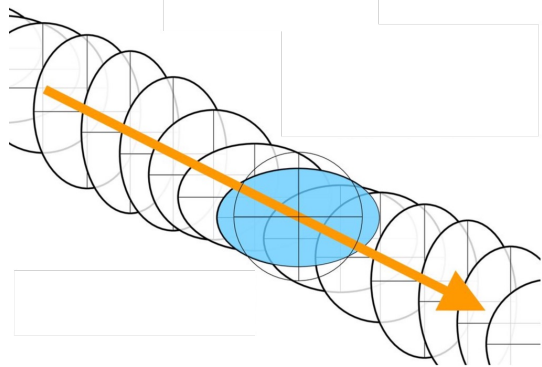
X mode



GW170817

Detecting GWs from NS-NS mergers

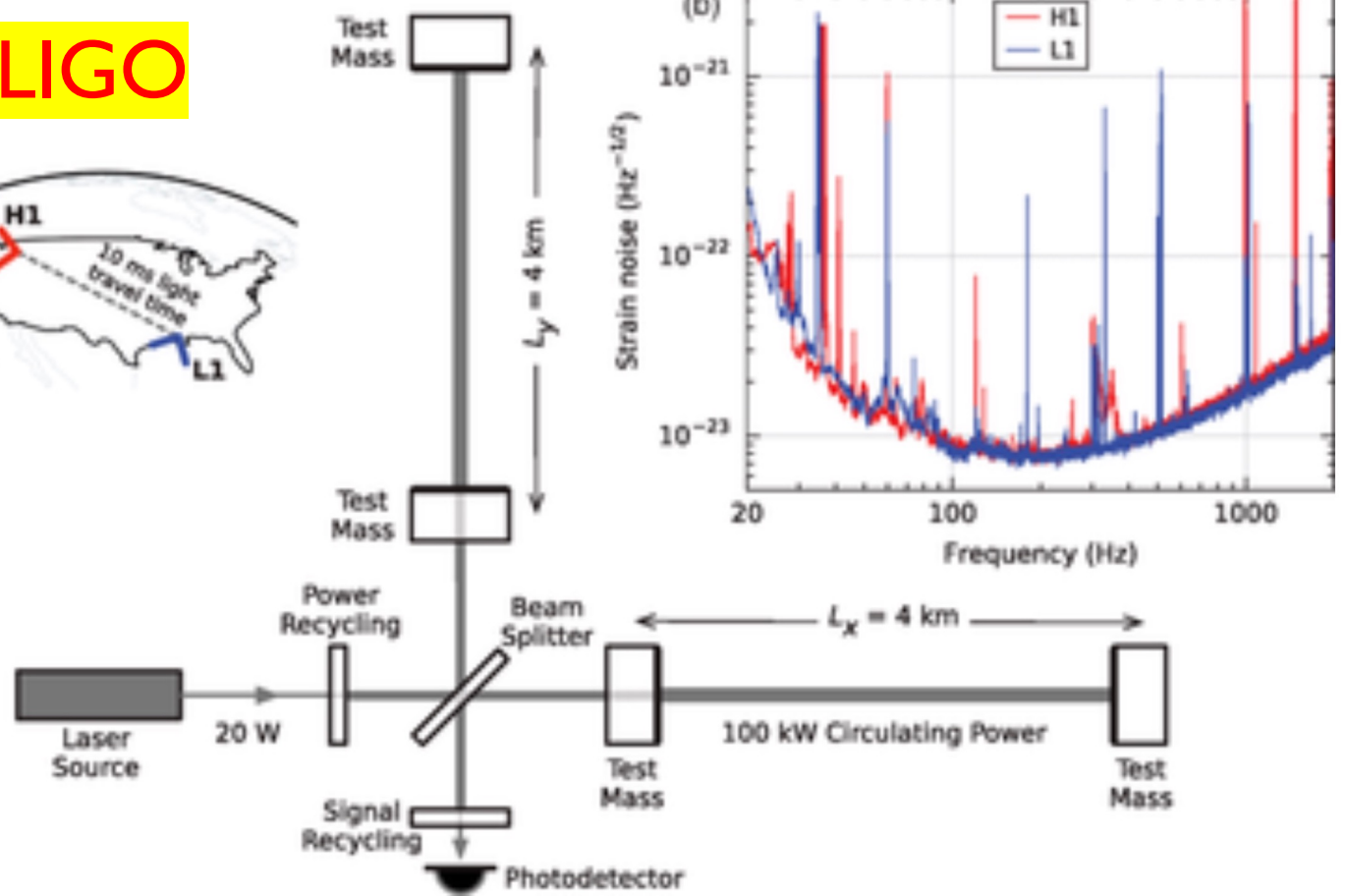
distortion of space-time



$$g_{\mu\nu} \simeq \eta_{\mu\nu}^{\text{flat}} + \underline{h_{\mu\nu}}$$

$$\Delta L/L \sim 10^{-23} !!$$

LIGO

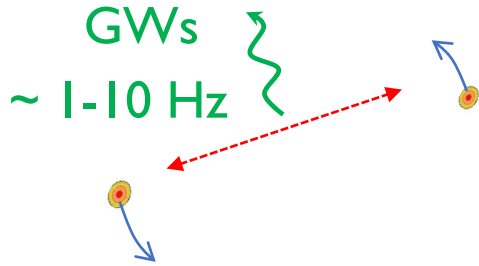


magnitude $h \simeq \underline{10^{-23}} \left(\frac{M_{\text{NS}}}{M_{\odot}} \right)^{5/3} \left(\frac{f}{200 \text{ Hz}} \right)^{2/3} \left(\frac{100 \text{ Mpc}}{R} \right)$
 distance to the NS-NS

How long do we have to wait?

$$t_{\text{merge}} - t \approx \underline{7 \times 10^6 \text{ yr}} \left(\frac{T}{\underline{1 \text{ hr}}} \right)^{8/3} \left(\frac{M_c}{1.21 M_\odot} \right)^{-5/3} \approx 3 \times 10^{10} \text{ yr} \left(\frac{T}{1 \text{ day}} \right)^{8/3} \left(\frac{M_c}{1.21 M_\odot} \right)^{-5/3}$$

orbit



NS-NS binaries
in **our galaxy**

$T_{\text{orbit}} \sim$ **hr -day**

will **NOT** be merged
in next **$\sim 10^6$ yrs**

Name	M_{tot} [M_\odot]	M_A [M_\odot]	M_B [M_\odot]	q	T_{orb} [days] [light s]	R	e_{orb}	D [kpc]	f_s [Hz]	B_{surf} [G]
J0453+1559 [8]	2.734	1.559	1.174	0.75	4.1	14	0.11	1.8	22	9.3E+09
J0737-3039 [9]	2.587	1.338	1.249	0.93	0.10	1.4	0.088	1.1	44	6.4E+09
J1518+4904 [10]	2.718	<1.766	>0.951	>0.54	8.6	20	0.25	0.7	24	9.6E+08
B1534+12 [11]	2.678	1.333	1.345	0.99	0.42	3.7	0.27	1.0	26	9.6E+09
J1753-2240 [12]	–	–	–	–	14	18	0.30	3.5	10	9.7E+09
J1756-2251 [13]	2.577	1.341	1.23	0.92	0.32	2.8	0.18	0.73	35	5.4E+09
J1807-2500B [14]	2.571	1.366	1.21	0.89	1.0	29	0.75	–	239	$\leq 9.8E+08$
J1811-1736 [15]	2.571	<1.478	>1.002	>0.68	19	35	0.83	5.9	9.6	9.8E+09
J1829+2456 [16]	2.59	<1.298	>1.273	>0.98	1.2	7.2	0.14	0.74	24	1.5E+09
J1906+0746 [17]	2.613	1.291	1.322	0.98	0.17	1.4	0.085	7.4	6.9	1.7E+12
J1913+1102 [18]	2.875	<1.84	>1.04	>0.56	0.21	1.8	0.090	13	1.1	2.1E+09
B1913+16 [19]	2.828	1.449	1.389	0.96	0.32	2.3	0.62	7.1	17	2.3E+10
J1930-1852 [20]	2.59	<1.199	>1.363	>0.88	45	87	0.40	2.3	5.4	6.0E+10
B2127+11C [21]	2.713	1.358	1.354	1.0	0.34	2.5	0.68	13	33	1.2E+10

To detect rare events

1 pc = 3.26 ly

- our **galaxy** (milky-way) ~ 31-55 **kpc**
- to **the edge of universe** ~ 14 **Gpc**

• detector horizon

• **aLIGO**

Livingston ~ 218 **Mpc**

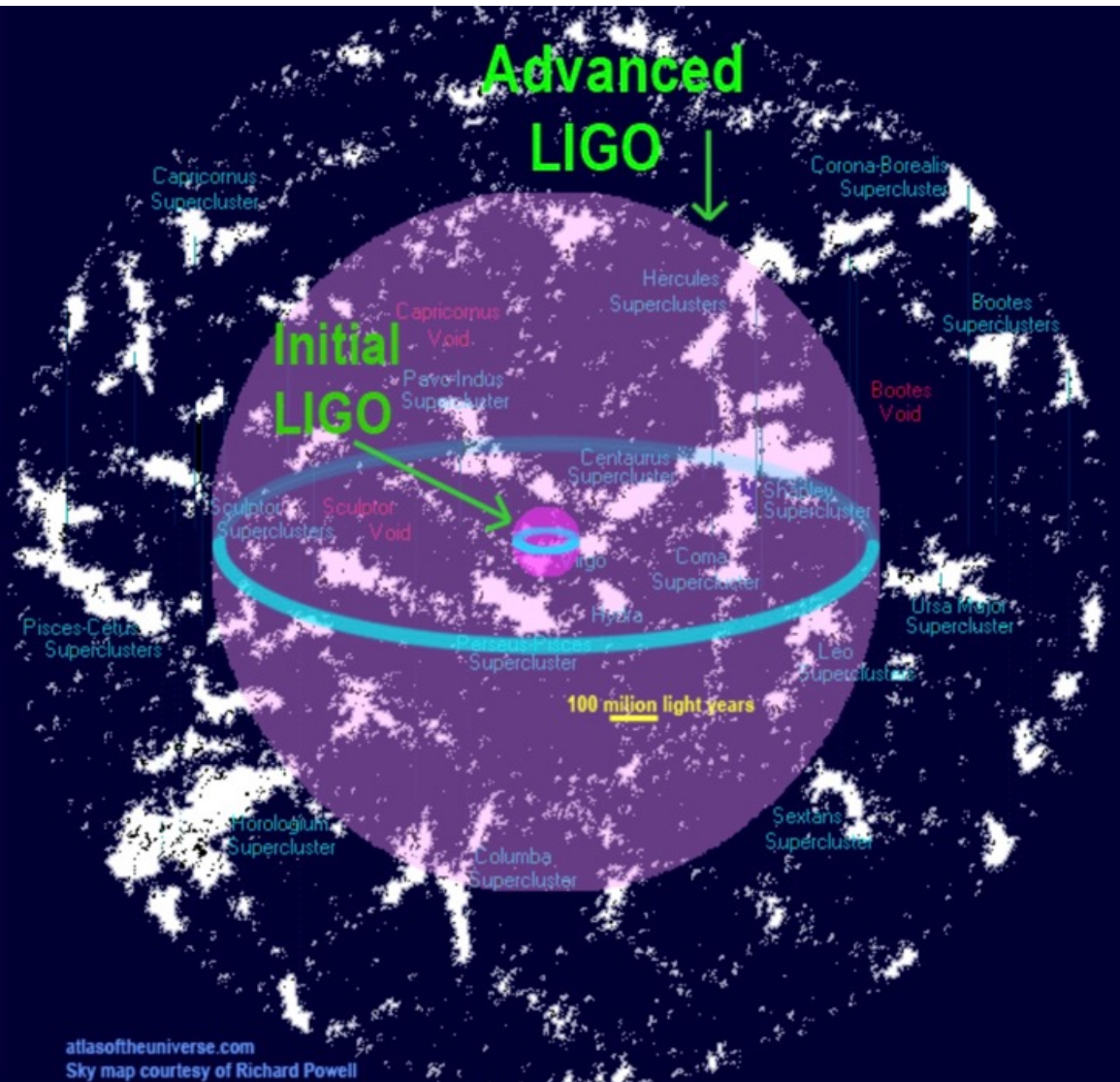
Hanford ~ 107 **Mpc**

• **Virgo** ~ 58 **Mpc**

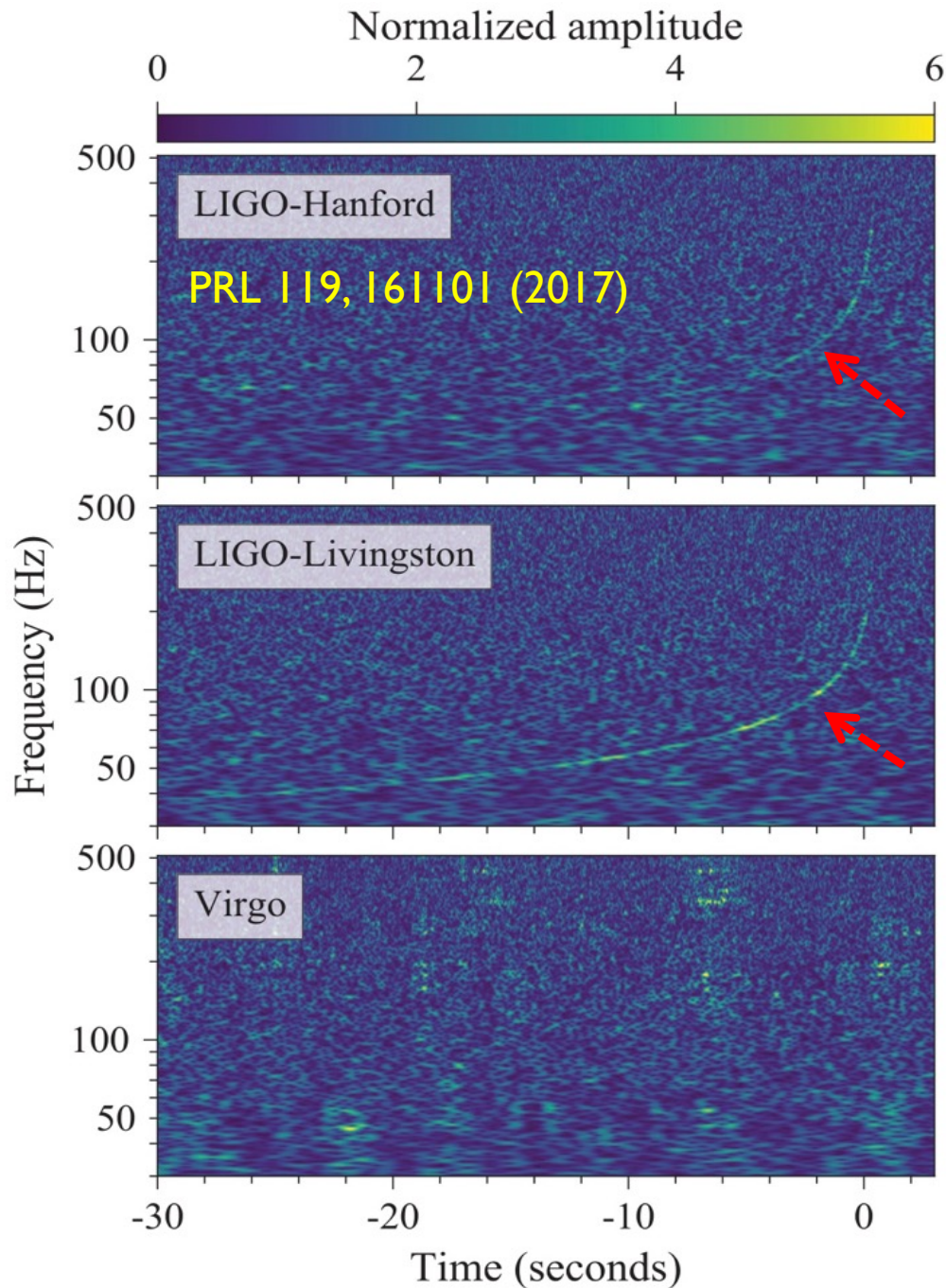
→ expected detection rate

0.1 – 100 events/year

- **GW170817** happened at 40_{-14}^{+8} **Mpc**



On Aug. 17, 2017



- aLIGO: signal-to-noise = **32.4** !
- Virgo did not find it
 - GWs from the blind spot of Virgo
 - strongly **constrain the location**
 - trigger follow-up EM studies
- clear signal 20 Hz - 1 kHz
 - inspiral** – **tidal deformed** phases
- NOT measured for $> \sim 1$ kHz
 - larger noise at higher frequency
- **EM signals** from objects just after merger
 - but neutrino signals too small

Pre merger

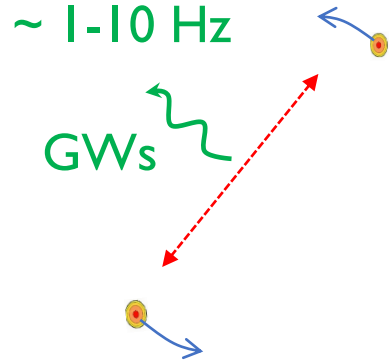
inspiral & tidally deformed phases

GWs



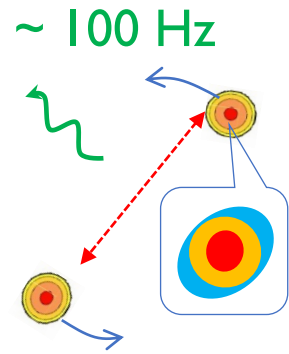
Early inspiral

~ - Myrs-Gyrs



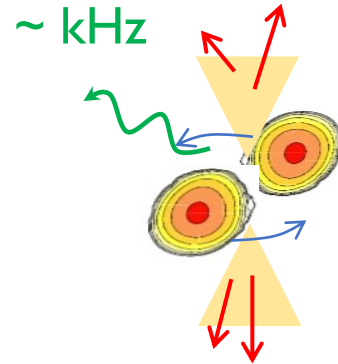
Tidally deformed

~ - 10 ms



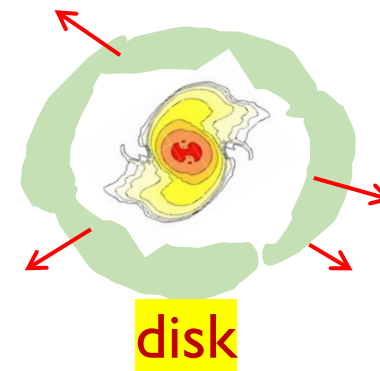
Merger

t = 0



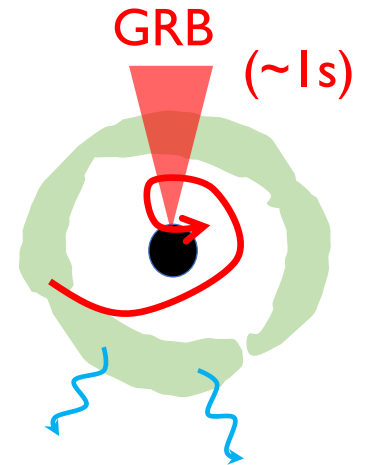
HMNS, SMNS

~ 1-10 ms



BH

> ~ 1s



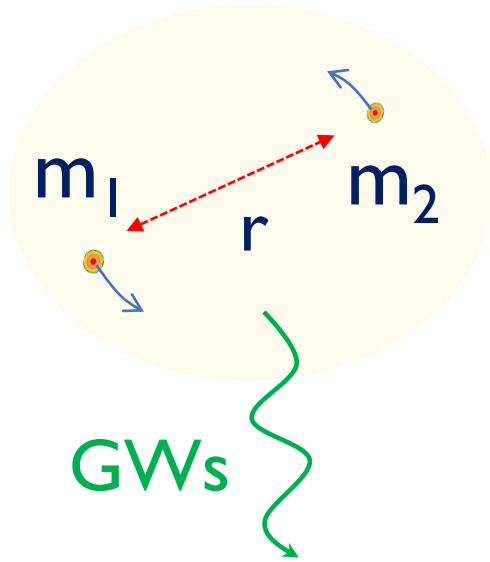
kilonova (days-weeks)

GR + hydro

GR + magnetohydro
+ neutrino radiation...

+ r-process
& radioactive decays

Inspiral phase



chirp mass $\mathcal{M} := (m_1 m_2)^{3/5} / (m_1 + m_2)^{1/5}$
 $\approx 1.188^{+0.004}_{-0.002} M_{\odot}$ (well constrained)

mass ratio: $q = m_2/m_1 = 0.7 - 1.0$ (not well constrained)

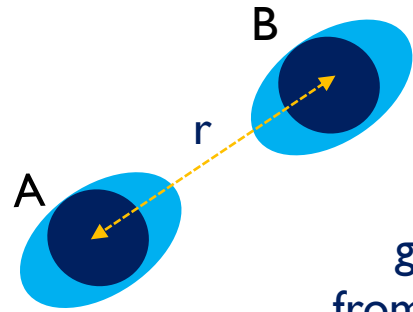
Nevertheless, $m_{\text{tot}} = m_1 + m_2$ is well constrained:
 (insensitive to details of q)

$$m_{\text{tot}} = 2.74^{+0.04}_{-0.01} M_{\odot}$$

$$m_{1,2} = 1.3 - 1.5 M_{\odot}$$

(typical)

Tidally deformed phase



quadrupole moment

polarizability

$$Q_{ij} = -\lambda(M) E_{ij}$$

external field

$$E_{ij} = -\frac{\partial^2 V}{\partial x_i \partial x_j}$$

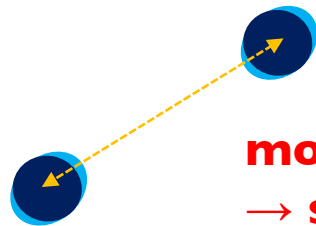
grav. pot. from the star A

$$V_A(r) \simeq -\frac{GM_A}{r} - \frac{GQ_{AB}}{r^3}$$

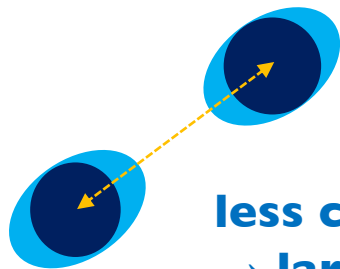
deformation of A by B

additional **attraction**

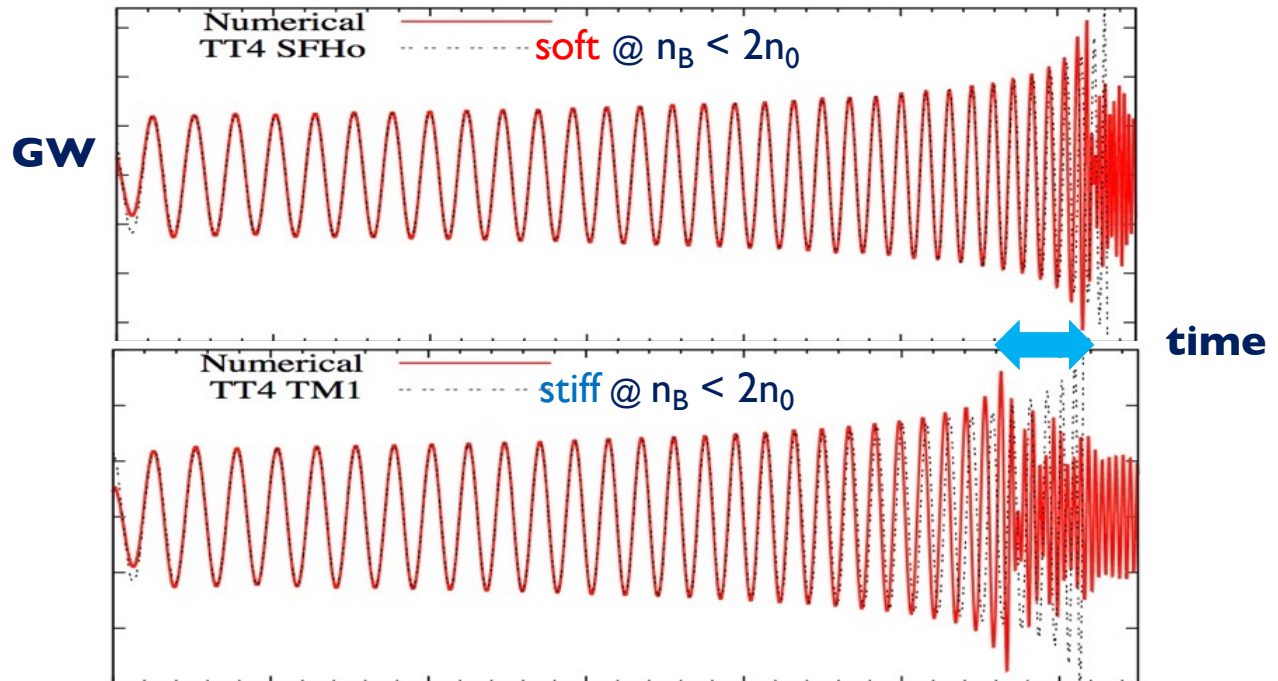
→ **NSs approach faster**



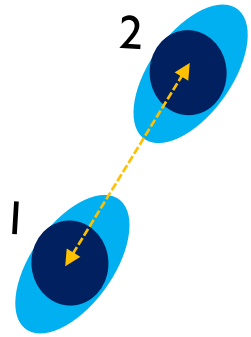
more compact
→ **smaller Q**



less compact
→ **larger Q**



Dimensionless tidal deformability $\rightarrow R_{NS}$



more common to use

$$\Lambda(M) = 32 \frac{\lambda G}{R^5} \leftarrow \mathbf{R^{-5}} (!)$$

What GW analyses measure: combination of Λ for star 1 & 2 :

$$\tilde{\Lambda} = \frac{16 (M_1 + 12M_2)M_1^4 \Lambda_1 + (M_2 + 12M_1)M_2^4 \Lambda_2}{(M_1 + M_2)^5} \quad \text{2-parameters: } M_1 \text{ \& } M_2$$

(measured)

For GW170817 :

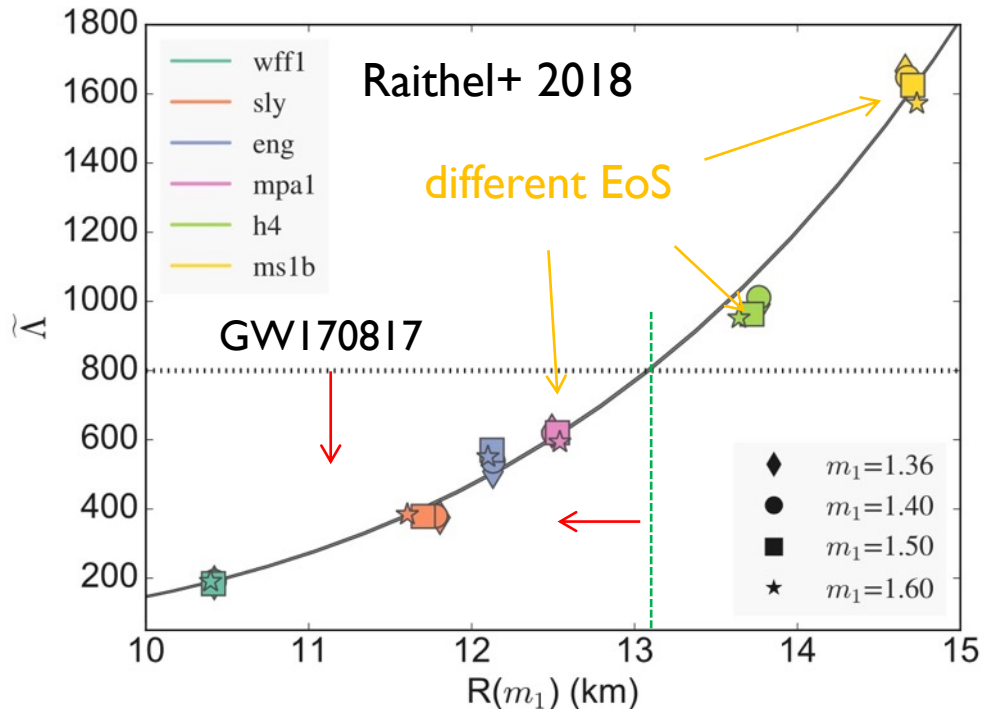
chirp mass ($1.188 M_{\text{sun}}$) (determined)

$$\mathcal{M}_c = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} = m_1 \frac{q^{3/5}}{(1 + q)^{1/5}}$$

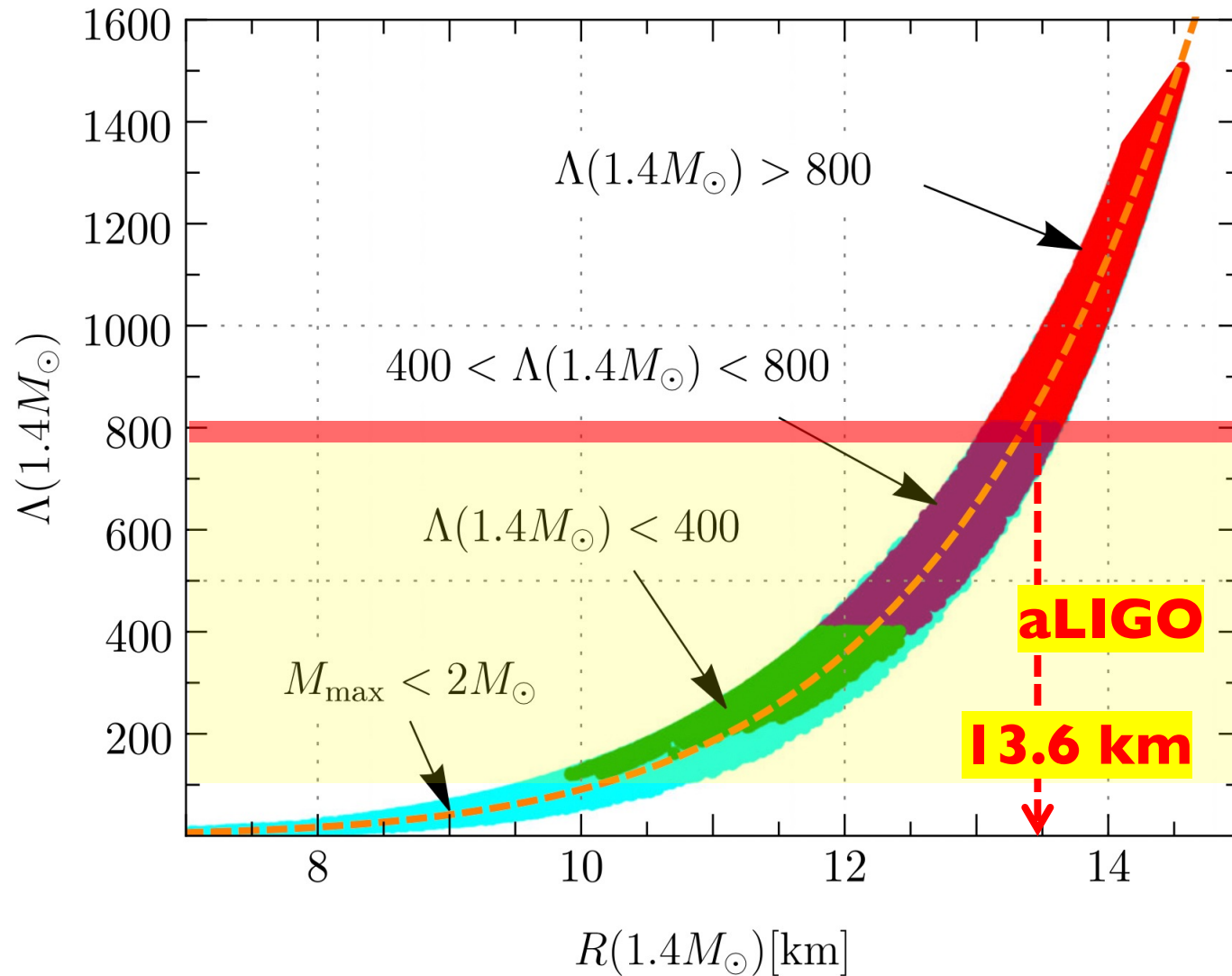
mass ratio $q = M_2/M_1$ (undetermined)



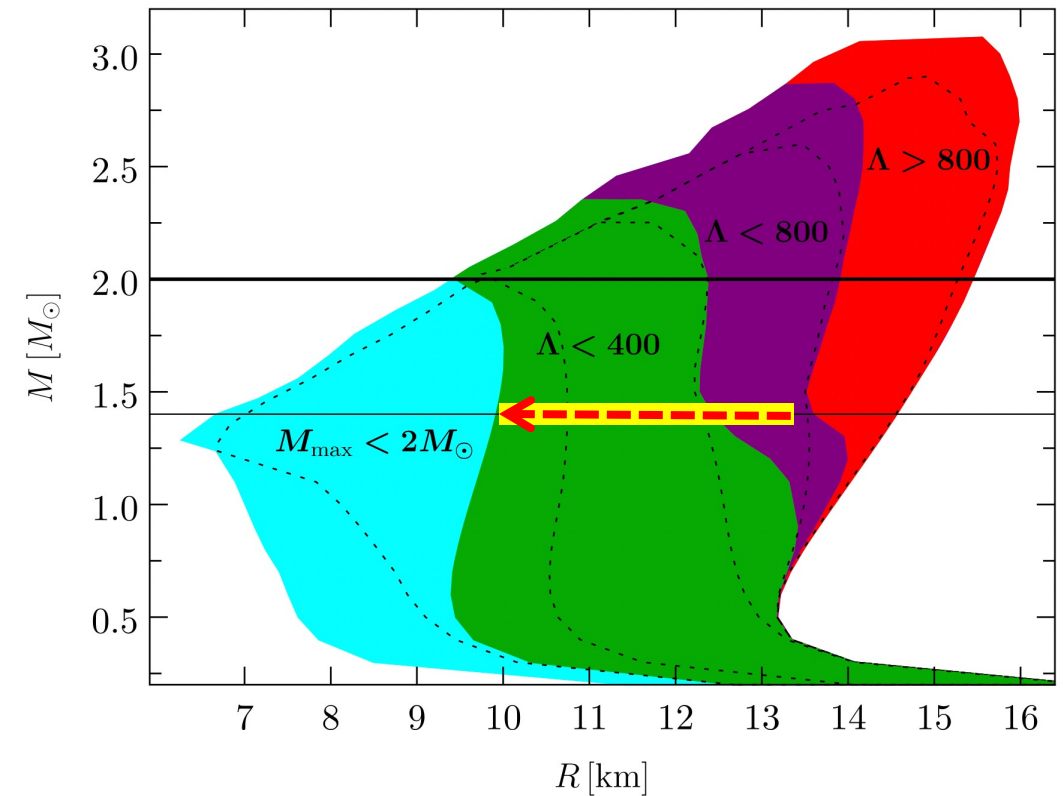
- different q degenerate !
- $R < \sim 13$ km



Constraints on $R_{1.4}$ [e.g., Annala+ '18]



with ChEFT constraint up to $1.1n_0$
 & pQCD constraint down to $40n_0$



Post merger

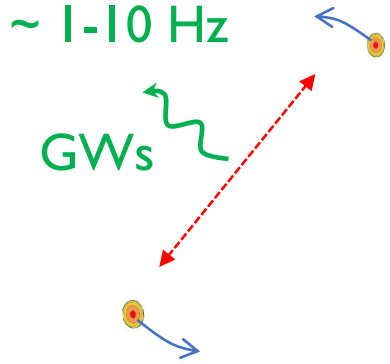
short-lived vs long-lived NS remnants

GWs



Early inspiral

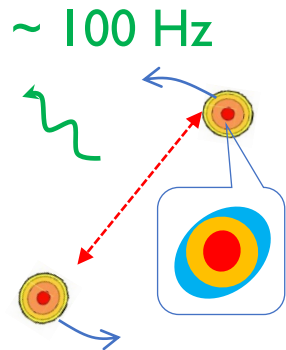
~ - Myrs-Gyrs



GR + hydro

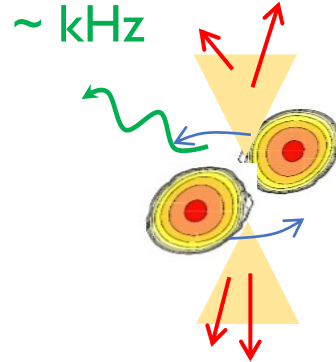
Tidally deformed

~ - 10 ms



Merger

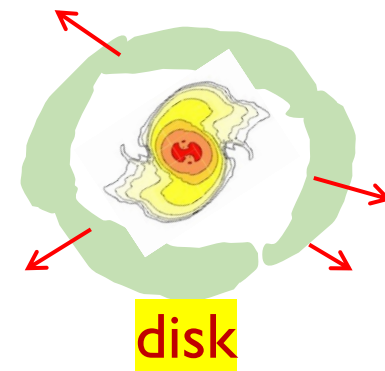
t = 0



GR + magnetohydro
+ neutrino radiation...

HMNS, SMNS

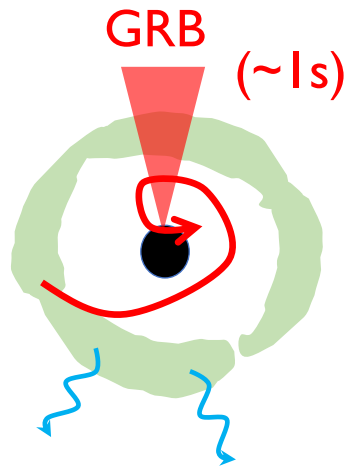
~ 1-10 ms



+ r-process
& radioactive decays

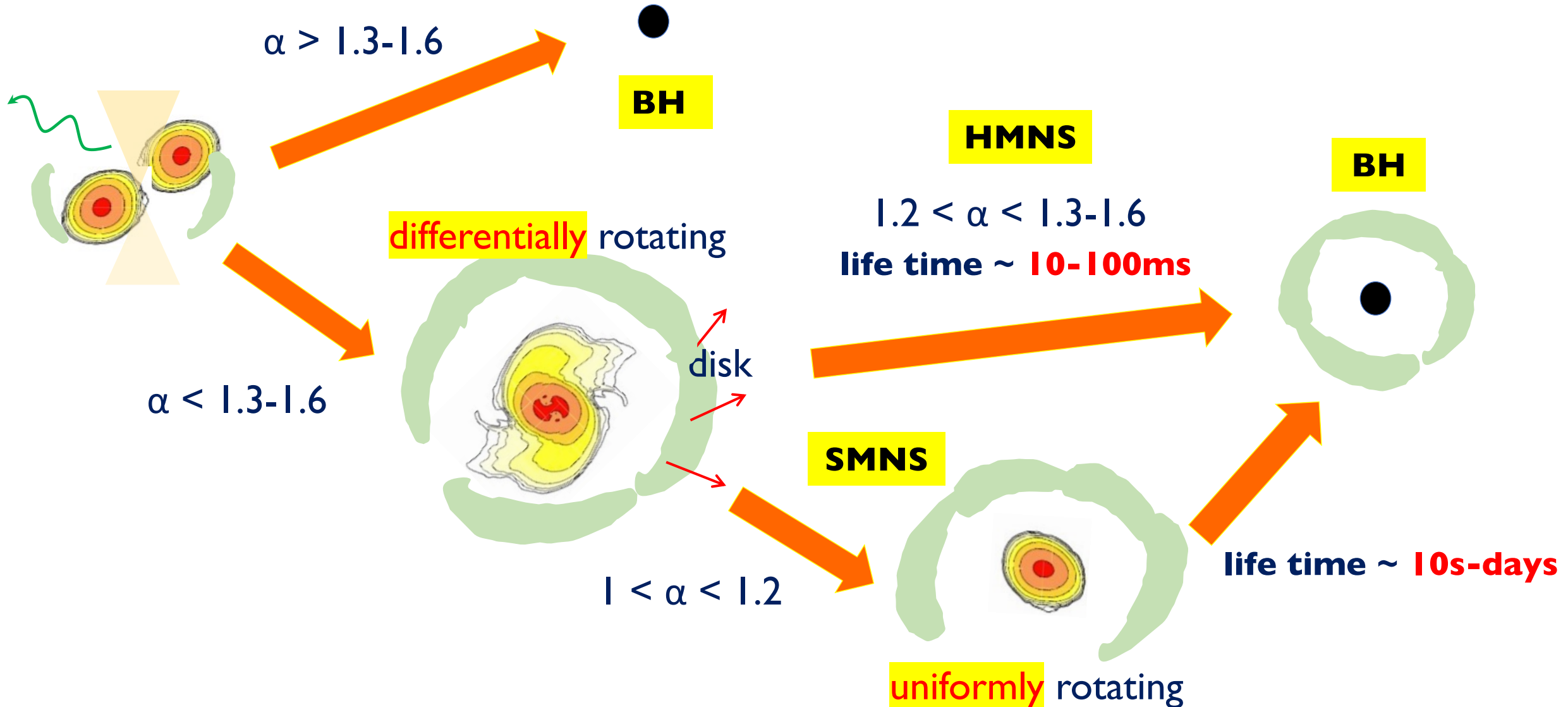
BH

> ~ 1s



kilonova (days-weeks)

Fate of post-merger depends on $\alpha = M/M_{\text{TOV}}$



Supports for threshold masses

$$M_{\text{thre}} = M_{\text{TOV}}^{\text{max}} + \Delta M_{\text{rot, rigid}} + \Delta M_{\text{rot, diff}} + \Delta M_{\text{therm}}$$

$\Delta M_{\text{rot, rigid}}$: additional support from rigid rotation, $\sim 0.2 M_{\text{TOV}}^{\text{max}}$

→ for long time:

$\Delta M_{\text{rot, diff}}$: additional support from differential rotation, $\sim 0.1-0.4 M_{\text{TOV}}^{\text{max}}$

→ only for short time: viscosity & magnetic field braking

ΔM_{therm} : additional thermal support

→ only for short time: neutrino emissions remove thermal support

➔ need sophisticated “numerical GR + magnetohydro + microphysics”

Life time \rightarrow constraints on M_{TOV}

If the merger in **GW170817** ($M_{\text{tot}} = 2.73-2.78M_{\text{sun}}$)

1) promptly collapsed to BH **[unlikely, see below]**

$$M_{\text{tot}}/M_{\text{TOV}} > 1.3-1.6 \rightarrow M_{\text{TOV}} < 1.71-2.14$$

2) is **HMNS** (life time $\sim 10-100$ ms \rightarrow diff. rot. stops)

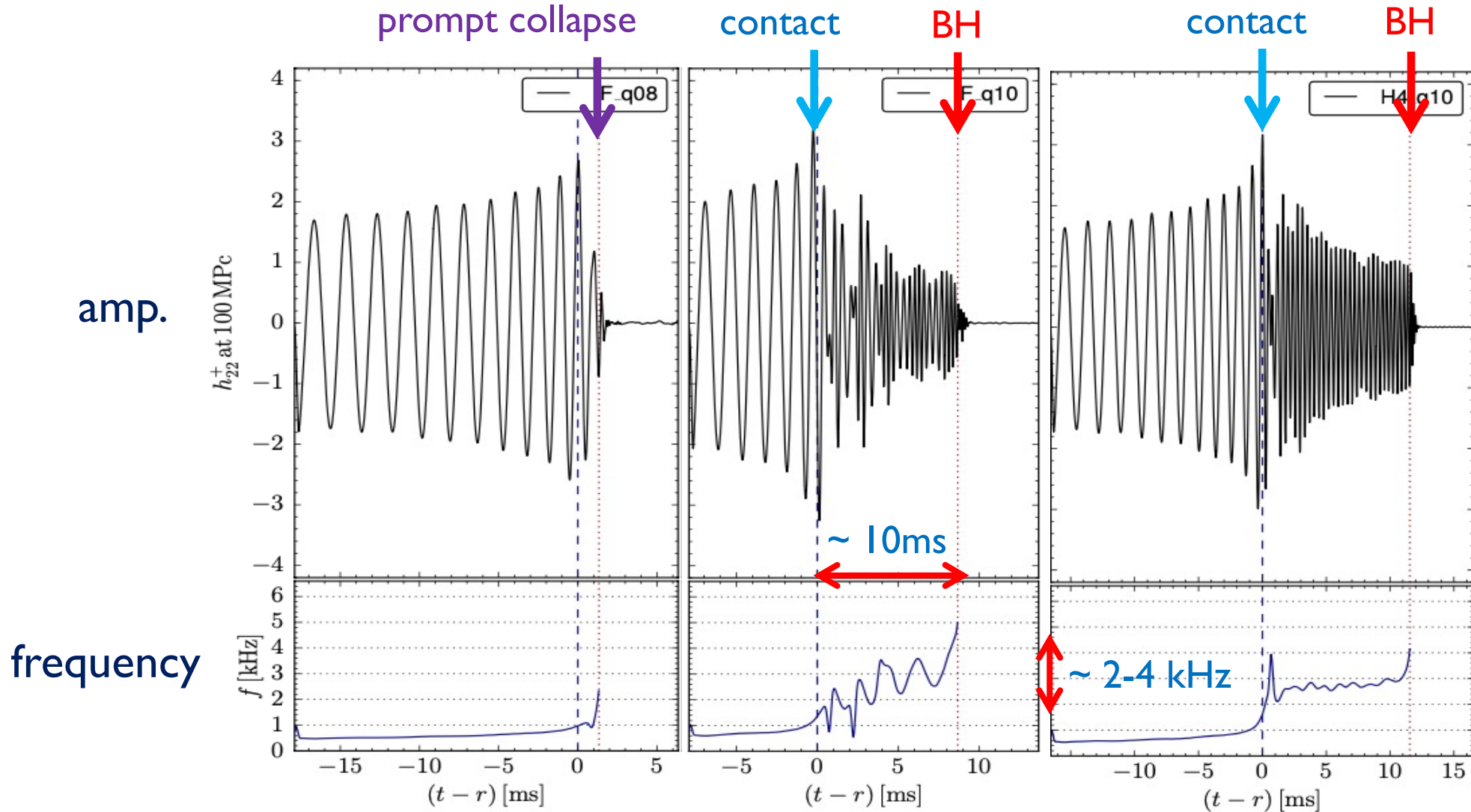
$$1.2 < M_{\text{tot}}/M_{\text{TOV}} < 1.3-1.6 \rightarrow 1.71-2.14 < M_{\text{TOV}} < 2.28-2.32$$

3) is **SMNS** (life time ~ 10 s - days \rightarrow loss of angular mom.)

$$M_{\text{tot}}/M_{\text{TOV}} < 1.2 \rightarrow M_{\text{TOV}} > 2.28-2.32 (!?)$$

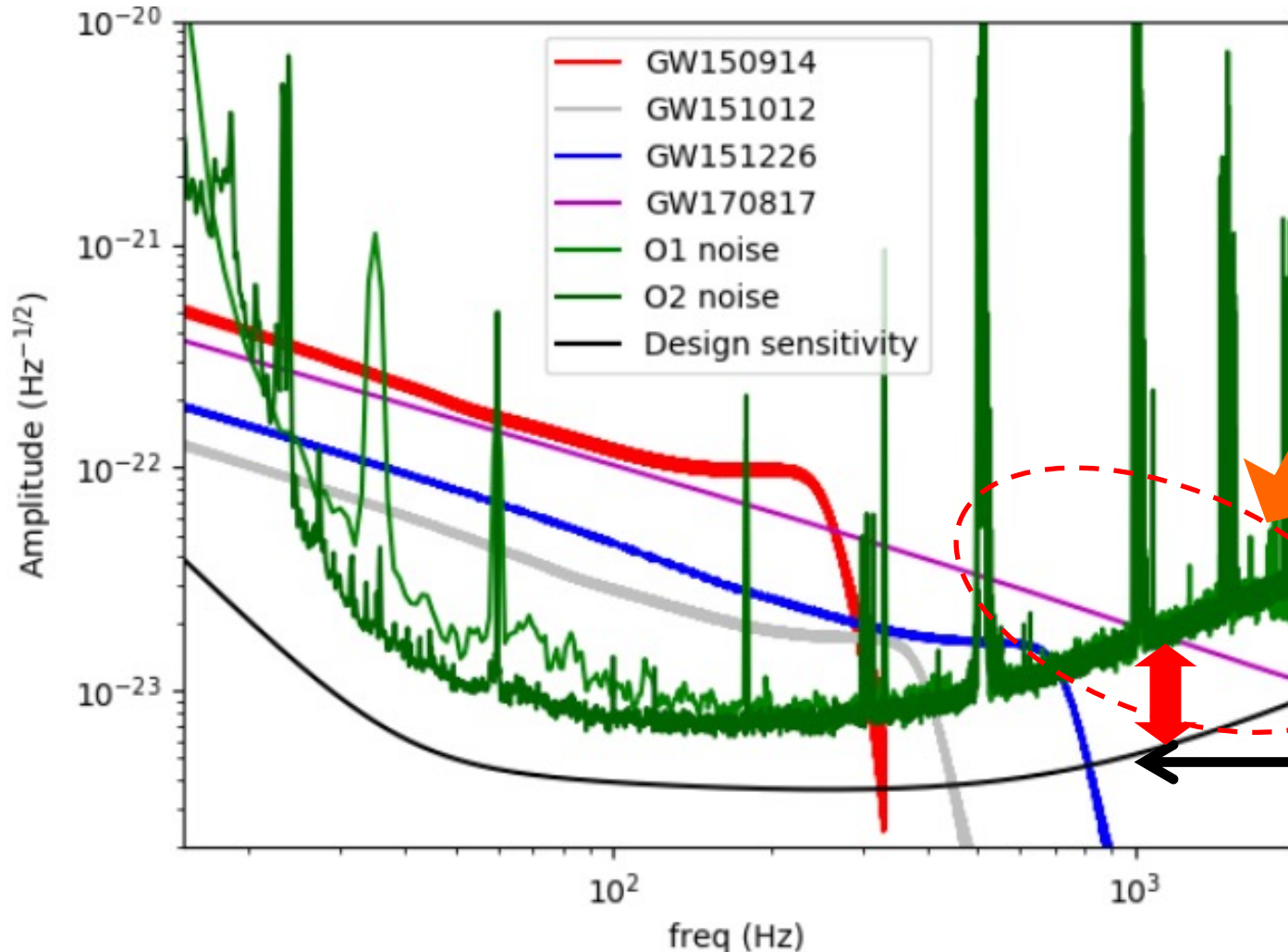
GWs from HMNS: life time

[Fig. from Kawamura+ '16]



GWs from **post-mergers** ??

ideal to determine the **life time**, but **not measured yet (!)**



GWs from **post-mergers**
with **> ~ 1 kHz**
are difficult to measure

- smaller signals for f with **less cycles**
(as in usual Fourier analyses)
- noises in detectors are large

design sensitivity

Alternative signals for post-mergers

electromagnetic (EM) counterparts

- Gamma Ray Burst (GRB170817A)
- kilonova light curves (AT 2017gfo)
- heavy r-process nuclei (solar-abundance)

key quantities: ejecta mass M_{ej} & proton fraction Y_p

(sensitive to short- vs long-lived NS)

Y_p again (see also lect. 1, page 12)

cold NS (wo neutrinos) $(p_F^n)^2 = 2M_N p_F^p + \dots$ (large scale) $(p_F^n \gg p_F^p)$ **neutron rich !**

we used:

charge neutrality $\rightarrow n_p (= Y_p n_B) = n_e$ or $p_F^p = p_F^e$

β -equilibrium $\rightarrow m_N + (p_F^n)^2 / 2M_N = m_N + (p_F^p)^2 / 2M_N + p_F^p + \dots$
small $(= p_F^e)$

with *neutrinos equilibrated*: $\mu_n + \mu_\nu = \mu_p + \mu_e$

β -equilibrium $\rightarrow (p_F^n)^2 / 2M_N + p_F^\nu = (p_F^p)^2 / 2M_N + p_F^p + \dots$

large scale

$(p_F^n)^2 = 2M_N (p_F^p - p_F^\nu) + \dots$ **p_F^p needs not be small**

Ejecta mass & proton fraction Y_p

1) **dynamical** ejecta

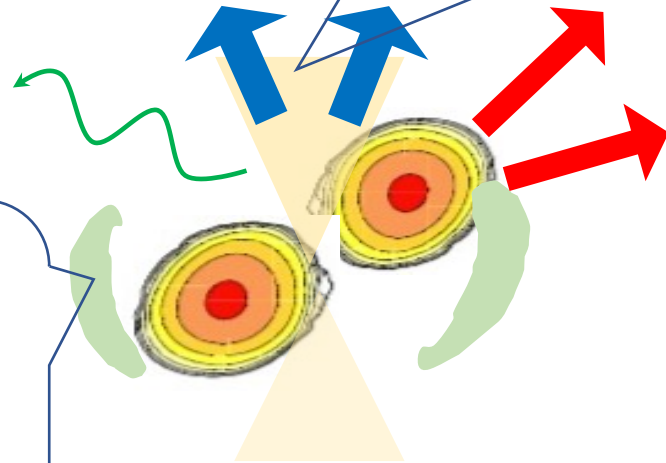
$t \sim 1-10$ ms

shock heated ejecta $Y_p \uparrow$
 ($T > 10$ MeV, positron capture by n)

tidal ejecta

$Y_p \sim 0.05-$

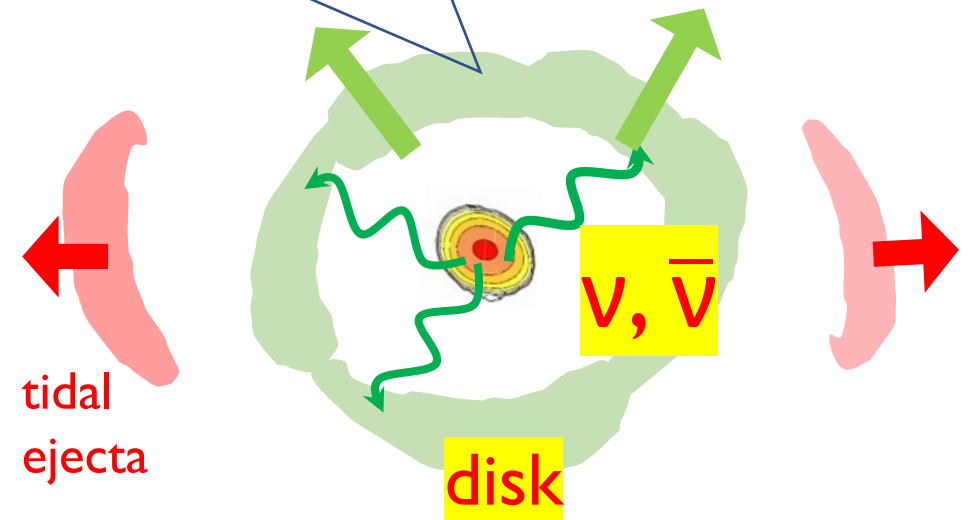
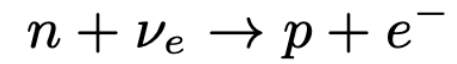
(as in NS)



2) **post merger** ejecta

$t > \sim 100$ ms

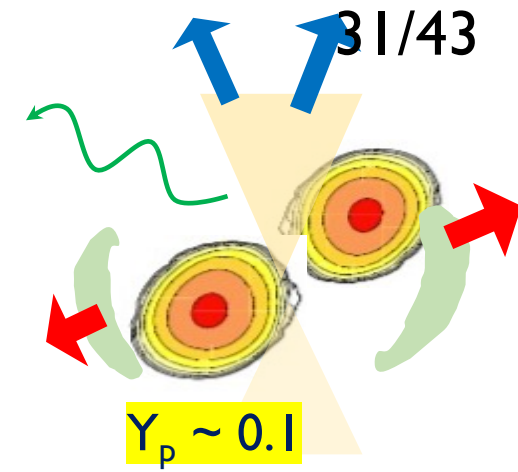
post-merger ejecta $Y_p \uparrow$ in disk



1) **Dynamical** ejecta (~ 10 ms)

many GR simulations \rightarrow mechanical, well-understood

key factors: **binary parameters** (m_1, m_2) & **EOS**



1) $M_{ej} = 10^{-4} - 10^{-2} M_{sun}$ \rightarrow compactness of NSs
mass ratio: $q = m_1/m_2$

[Hotokezaka, Sekiguchi,
Foucart, Radice,...]

2) average velocity = $0.15-0.25c$ [Hotokezaka, ...]

[Wanajo, Sekiguchi, Roberts,...]

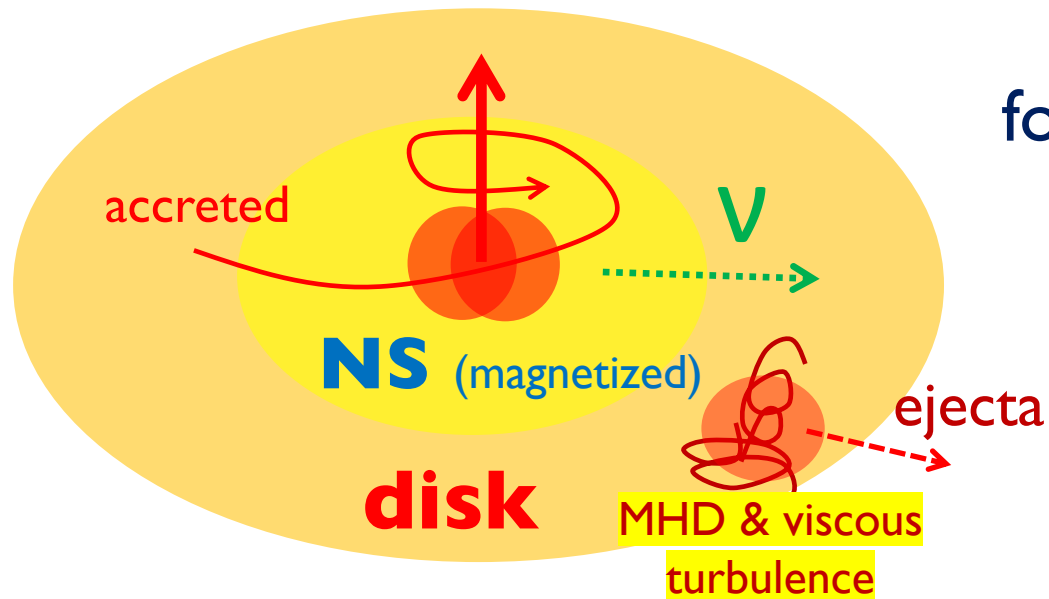
3) $Y_p = 0.05 - 0.4$ \rightarrow polar direction (shock heated) $\rightarrow Y_p > \sim 0.2, T > 10$ MeV
equatorial direction (tidal + ...) $\rightarrow Y_p \sim 0.05-$

important for **r-process nuclei** & **kilonova light curves** (see below)

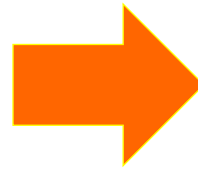
2) Post-merger ejecta ($> \sim 100$ ms)

complicated, long time duration \rightarrow less-understood (difficult to simulate)

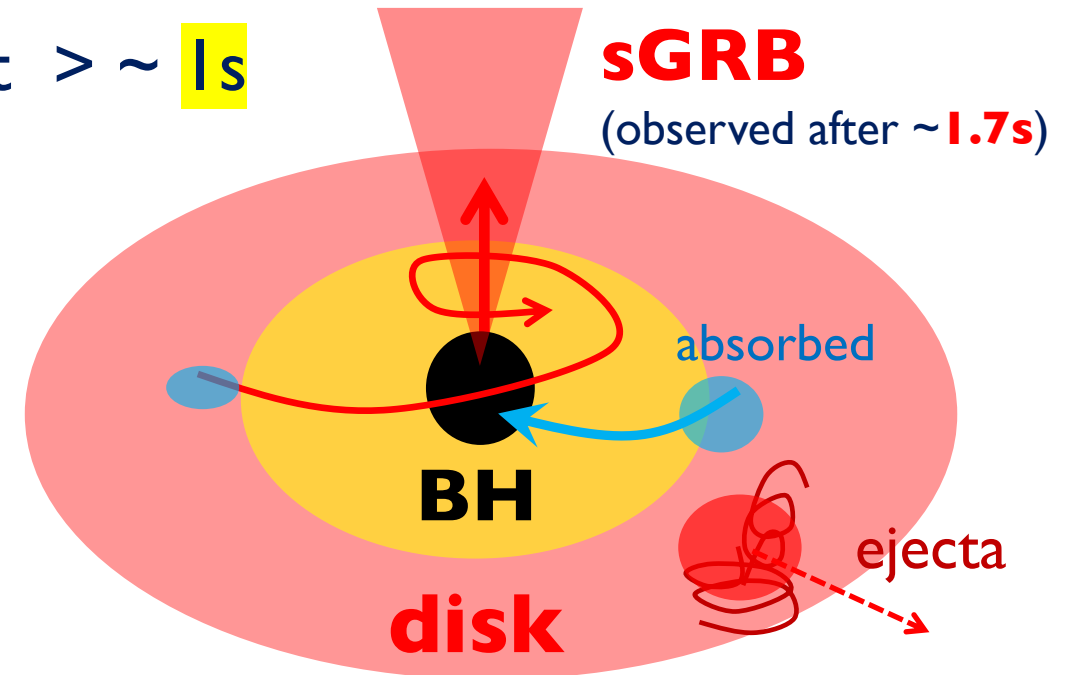
$t < \sim 10$ ms



for HMNS



$t > \sim 1$ s



MHD instabilities $\rightarrow eB \sim 10^{14-16}$ G

neutrino irradiation $\rightarrow Y_p \uparrow$

Temp. at NS core ~ 10 MeV

BH + accretion \rightarrow Gamma Ray Burst

ejecta powered by dynamics

M_{ej} depends on the life time of HMNS

Processes in ejecta

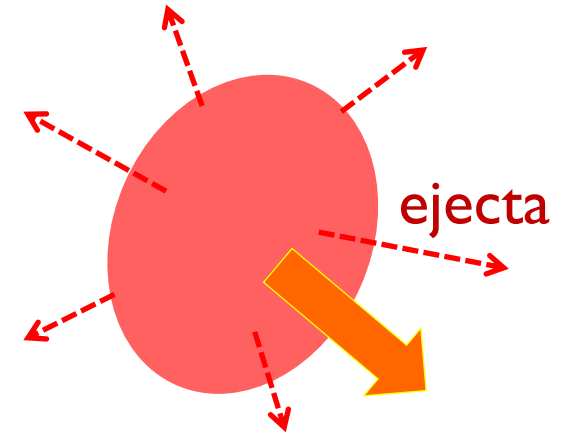
1) **decompression:**

$$M_{\text{ej}} \ll M_{\text{sun}}$$

→ weaker grav. attraction

→ expand (matter becomes dilute)

nuclear liquid → nuclei + neutrons & $Y_p \uparrow$

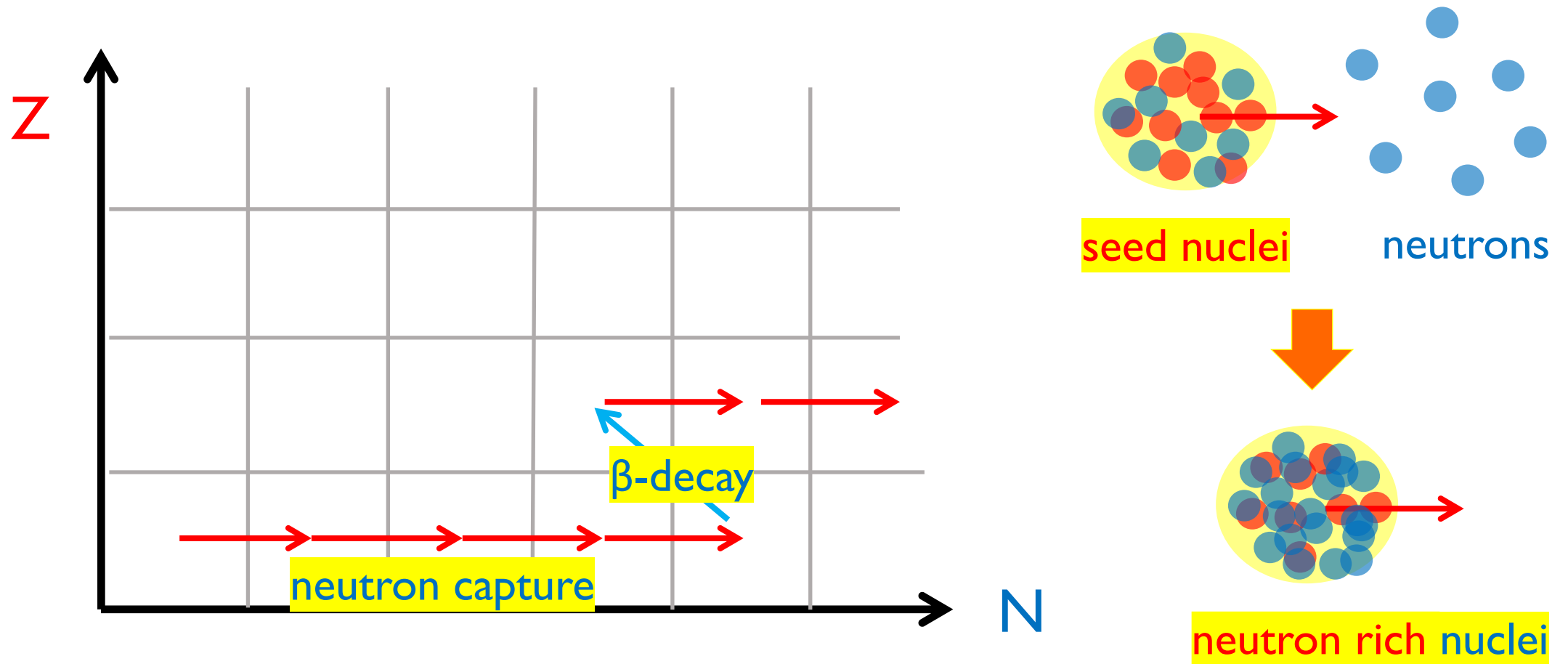


2) **r-process:** seed nuclei travel through neutron rich environment
depends on entropy, density, and Y_p (HMNS vs SMNS)

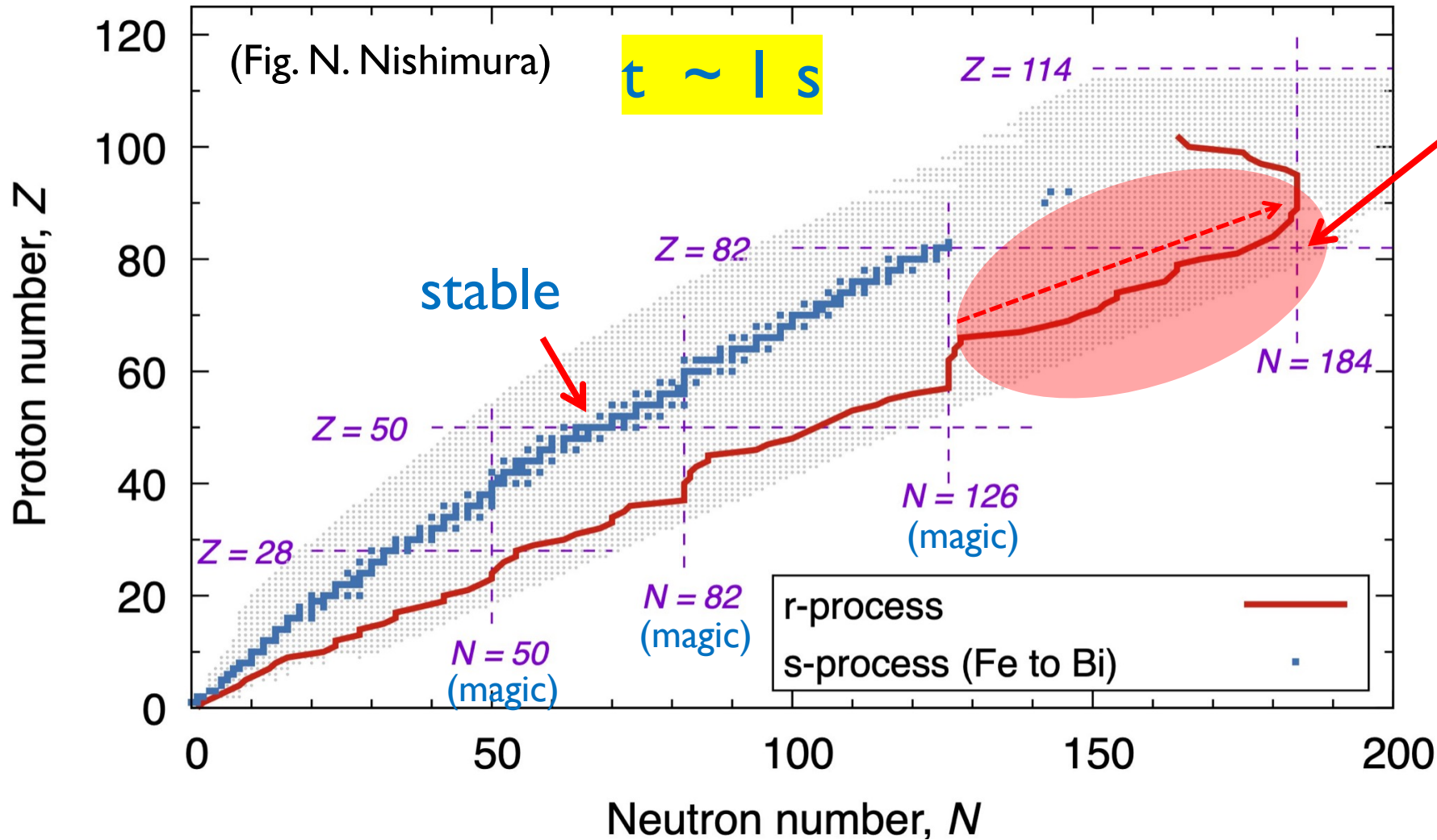
3) **radioactive decay:** decays of r-processed nuclei
thermal emission of photons → kilonova
(colors & time scales)

r-process (rapid neutron capture)

neutron capture rate \gg β -decay rate



r-process (rapid neutron capture)



to reach high mass

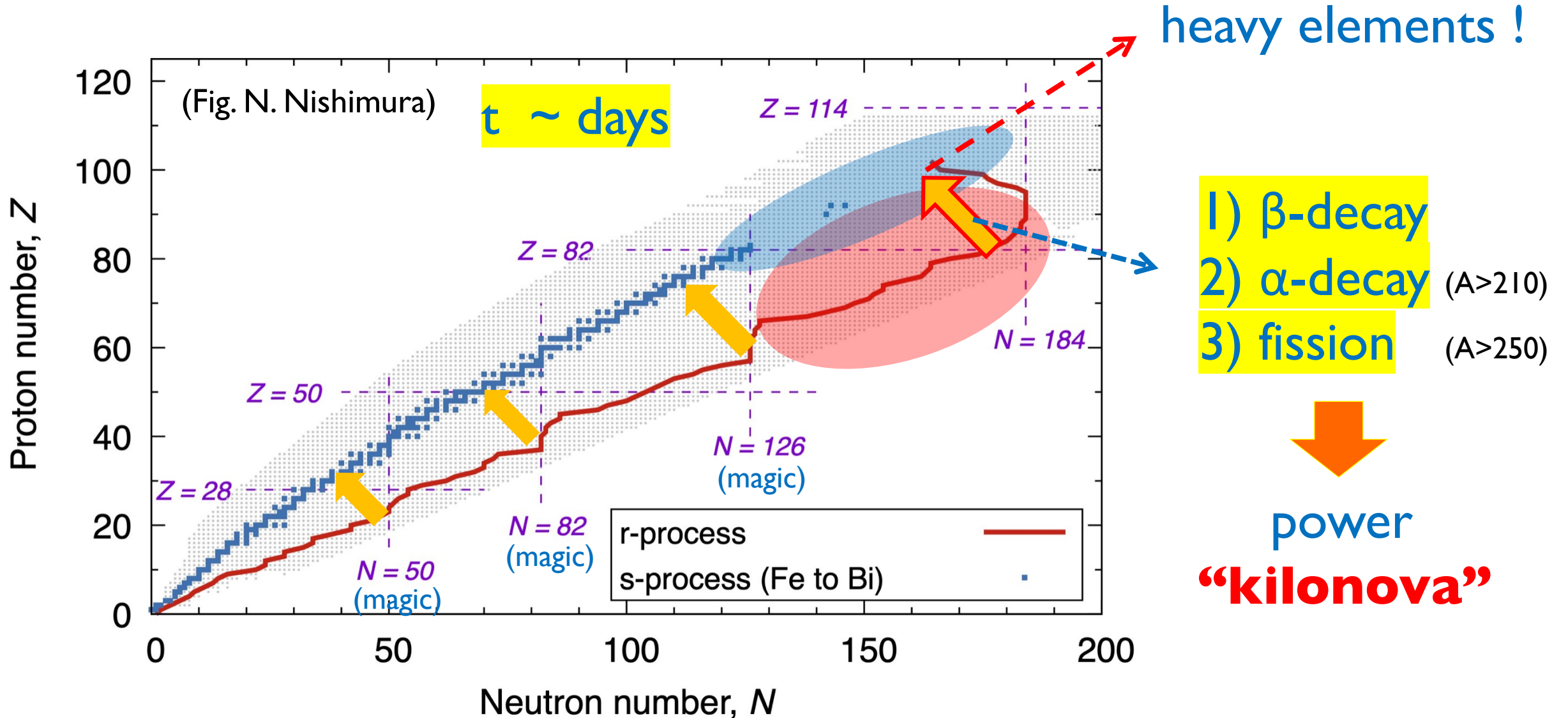
$Y_p < \sim 0.1$
needed

unique in
NS mergers!

(not in supernovae)

caveats:
magic num. in
neutron rich regime ?

radio active decays

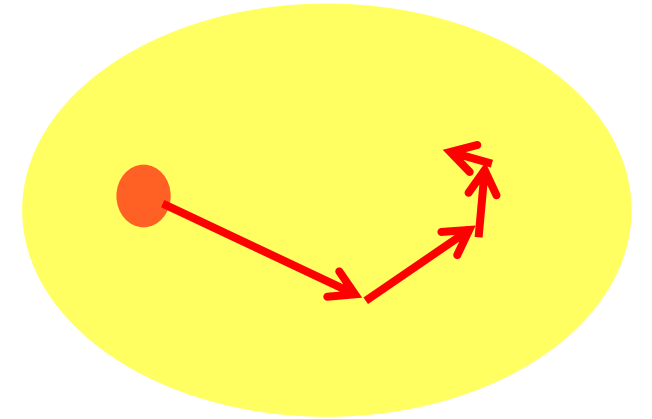


kilonova I) kinetic E to thermal E

decay products: **electrons** with $E \sim 0.1\text{-}1\text{ MeV}$

α -particles with $E \sim 5\text{ MeV}$

fission fragments with $E \sim 100\text{ MeV}$



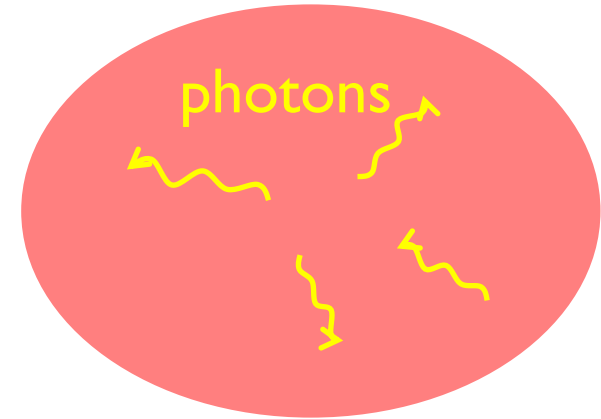
time scale
to **deposit kin. E**
(thermalization time scale)

$$t_{\text{th}} \approx \left(\frac{\overset{\text{stopping power}}{\downarrow} \sigma_{\text{st}}(E_i) \overset{\text{density of ions}}{\downarrow} N v_i}{E_i} \right)^{-1}$$

→ time scale \sim days

kilonova 2) opacity for photons

The thermalized ejecta have **thermal photons**



How do photons go **out of the ejecta** and reach us?

small $Y_p (< \sim 0.1)$ ejecta

large $Y_p (> 0.2-0.3)$ ejecta

heavy r-processed nuclei

light r-processed nuclei only

more open channels for the final states

less open channels for the final states

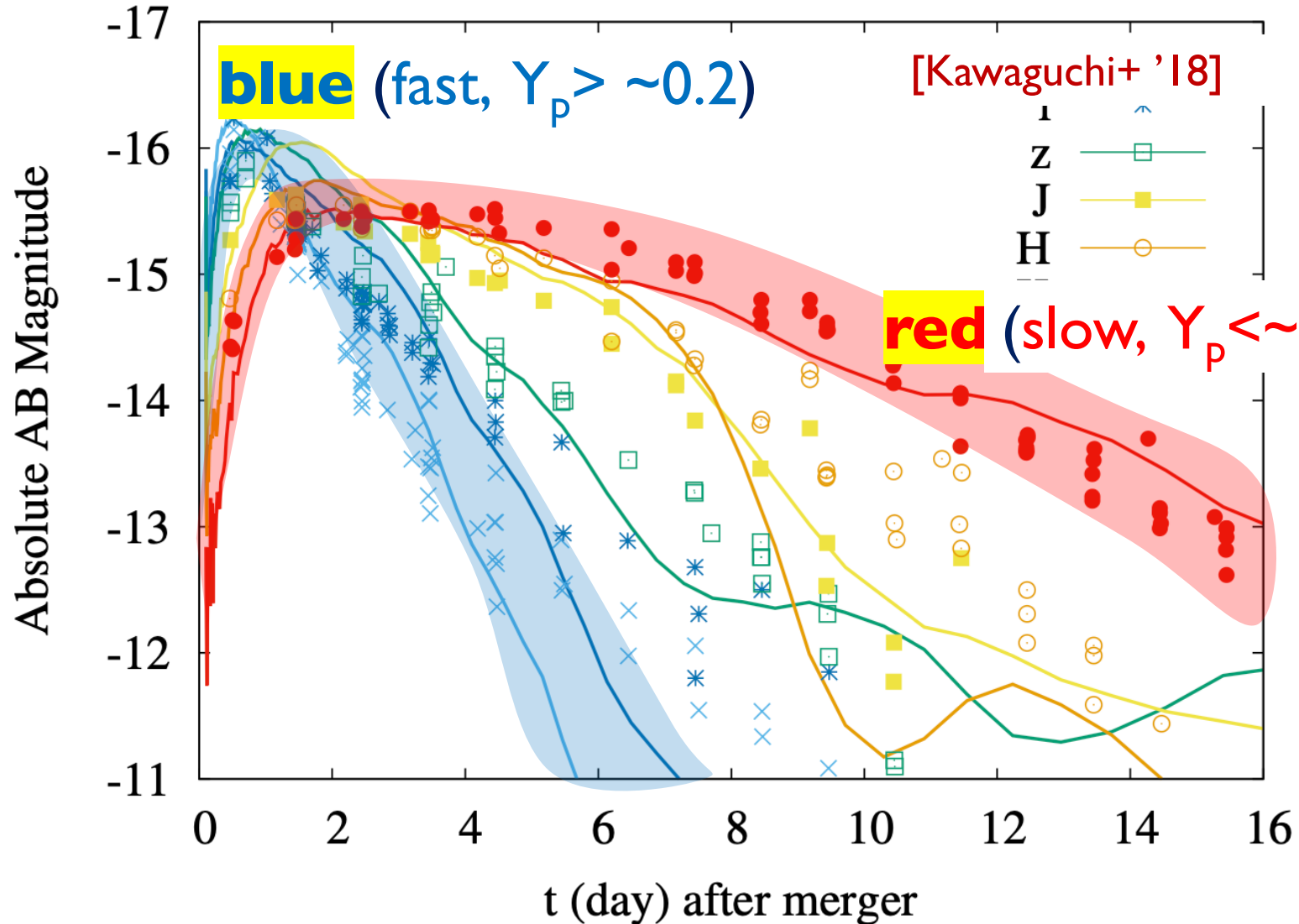
→ **more** absorption of photons

→ **less** absorption of photons

Red kilonova (slow: \sim weeks)

Blue kilonova (fast: \sim days)

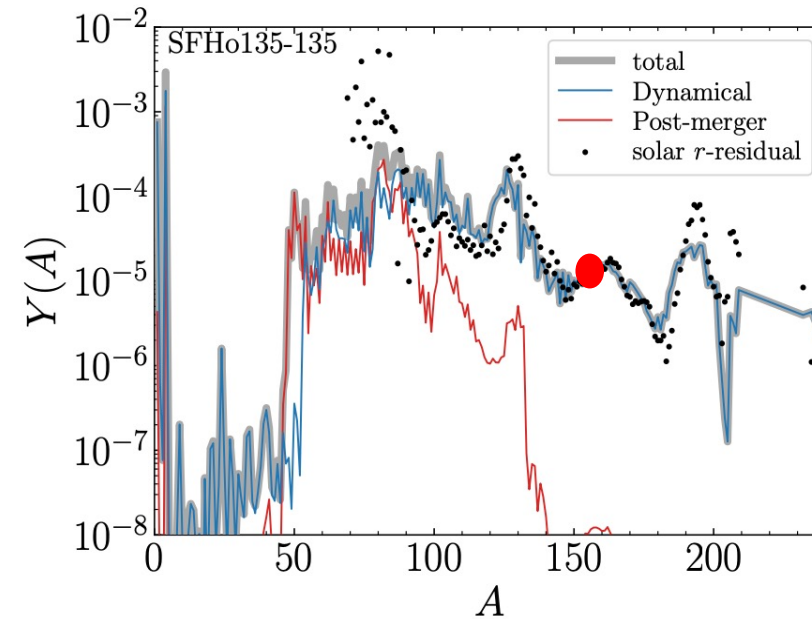
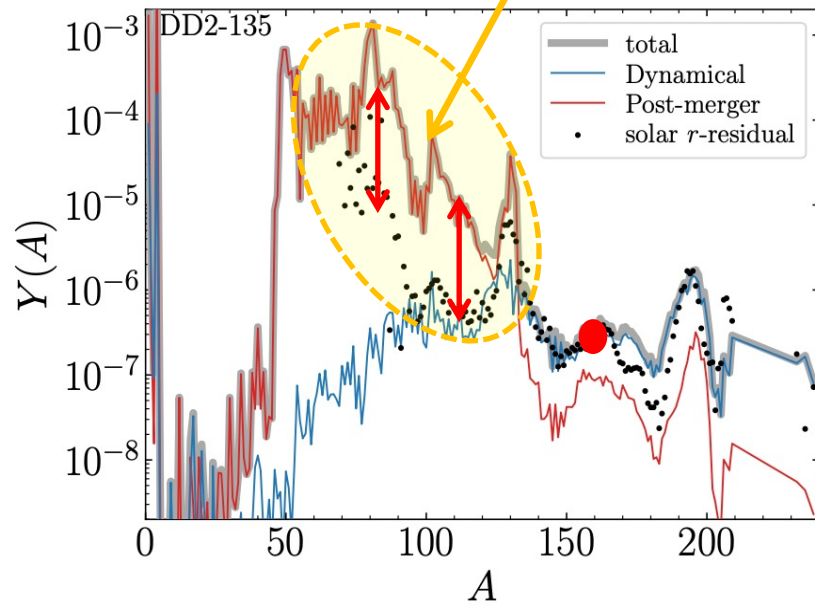
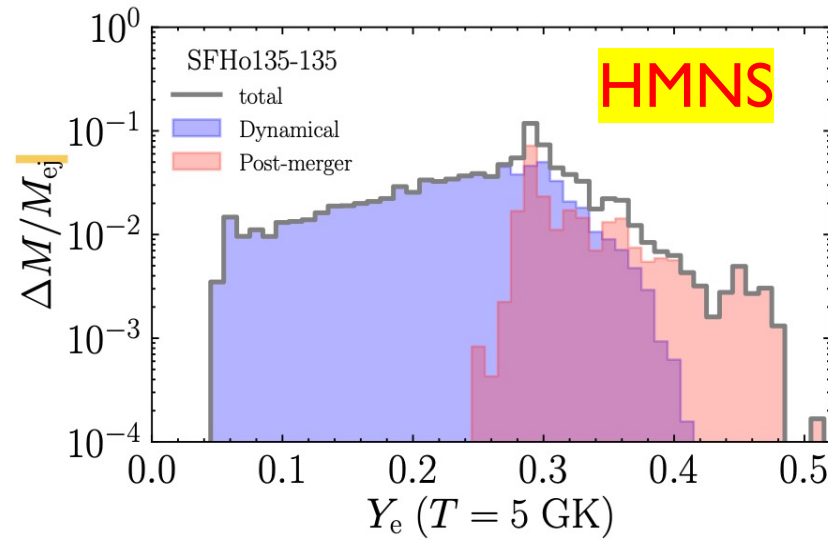
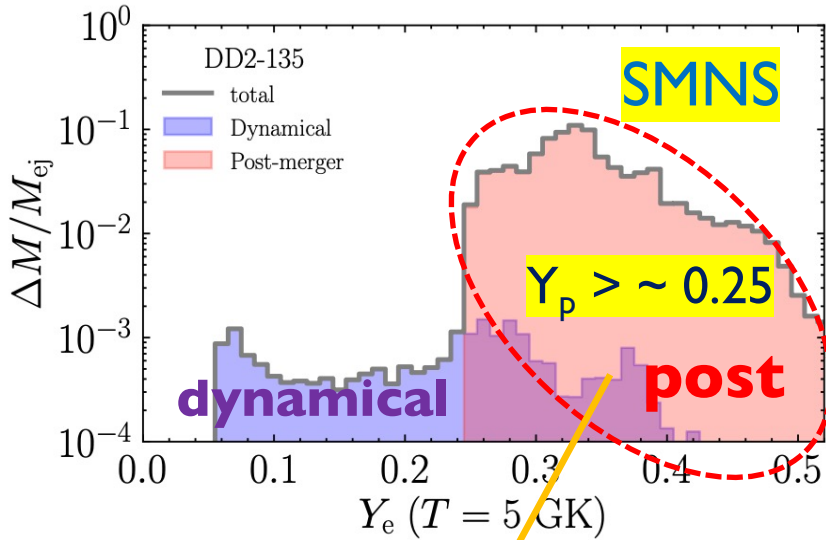
kilonova light curves of GW170817



- **confirmed** the diversity of Y_p :
 Y_p from \sim **0.01** to ~ 0.4

- fit well with **predictions** of **HMNS** + kilonova
 [Metzger+, Kasen+, Tanaka+,...]

Abundance of **r-processed** nuclei [Fujibayashi+ '22]



observations

solar abundance

(known to be universal)

normalized at $A = 153$

(abs. values are not universal)

SMNS:

ejecta: **post** \gg dynamical

too much light elements

HMNS:

better agreement
& **systematics**

Short- vs long-lived NS (HMNS vs SMNS)

In HMNS scenarios for GW170817,

sGRB, kilonovae, r-process nuclei,...

can be described *within standard mechanisms*

In SMNS scenarios, one needs to introduce *extra discussions*
for *each* of them to explain GW170817

(not covered in this lecture)

For now, the HMNS scenario seems more likely for GW170817:

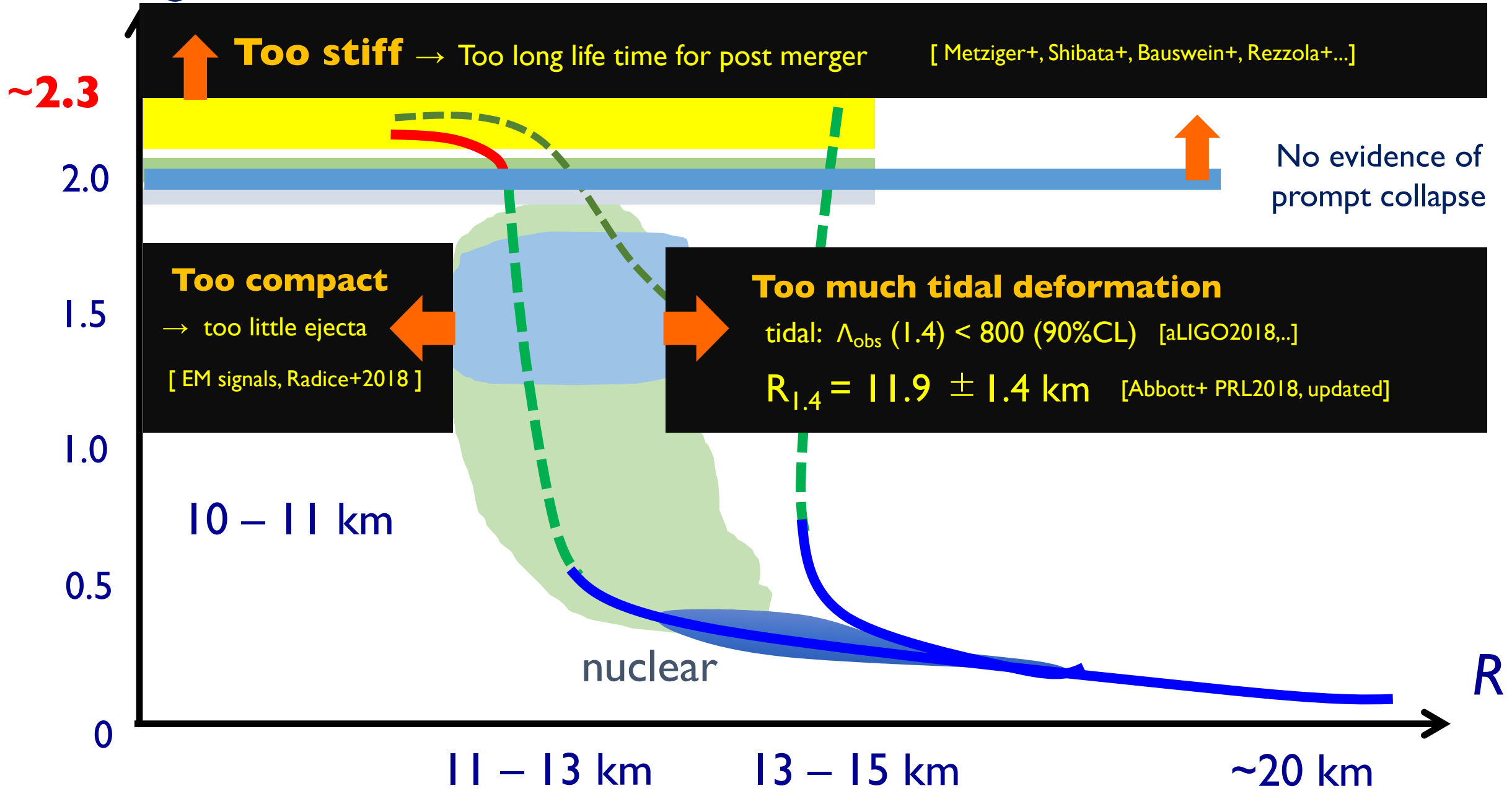


$$M_{\text{TOV}} < 2.28-2.32$$

(But if HMNS is incorrect, a real big deal !)

M/M_{\odot}

GW170817 + GRB170817A + AT2017gfo



Summary of Lecture 2

- Gravitational waves from fluctuating energy quadrupole
- GW170817, **pre**-mergers, tidal deformability $\rightarrow R_{1.4}$
- GW170817, **post**-mergers, EM-counterparts

Single event already yields a lot of info

in next 10 years, detections will be daily events

Lect. 3 : From hadrons to quarks in NS

