The ``ra'' basis

A more transparent physical connection

- Introduce a new basis $\phi_r \equiv \dfrac{\phi_1 + \phi_2}{2},\ \phi_a \equiv \phi_1 - \phi_2$. The propagator is

$$\mathbf{D} = \begin{pmatrix} \langle \phi_r \phi_r \rangle & \langle \phi_r \phi_a \rangle \\ \langle \phi_a \phi_r \rangle & \langle \phi_a \phi_a \rangle \end{pmatrix} = \begin{pmatrix} D^{rr} & D^{R} \\ D^{A} & 0 \end{pmatrix}$$

The ``ra'' basis

A more transparent physical connection

- Introduce a new basis $\phi_r \equiv \frac{\phi_1 + \phi_2}{2}$, $\phi_a \equiv \phi_1 - \phi_2$. The propagator is

$$\mathbf{D} = \begin{pmatrix} \langle \phi_r \phi_r \rangle & \langle \phi_r \phi_a \rangle \\ \langle \phi_a \phi_r \rangle & \langle \phi_a \phi_a \rangle \end{pmatrix} = \begin{pmatrix} D^{rr} & D^R \\ D^A & 0 \end{pmatrix}$$

- **Two causal** and a **statistical** propagator (D_{rr} or *symmetric*). A vanishing entry $D_{rr} = \frac{1}{2}(D^> + D^<)$ is the anticommutator (for bosons).

$$D_{rr}(\omega) = \left(\frac{1}{2} + n_B(\omega)\right)\rho_B(\omega), \qquad S_{rr}(\omega) = \left(\frac{1}{2} - n_F(\omega)\right)\rho_F(\omega)$$

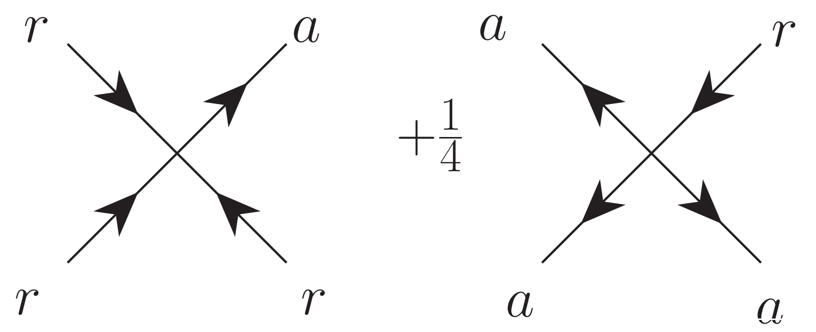
- Everything clearly determined by the retarded propagator and the statistical function
- Recall that the bare spf is $\rho_B(\omega) = 2\pi\epsilon(\omega)\delta(\omega^2 - E_k^2)$

The ``ra'' basis

A more transparent physical connection

- Introduce a new basis $\phi_r \equiv \dfrac{\phi_1 + \phi_2}{2}$, $\phi_a \equiv \phi_1 - \phi_2$. The vertices become

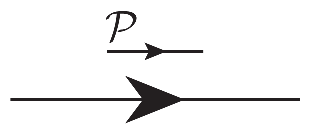
$$S_I(\phi_1) - S_I(\phi_2) = S_I\left(\phi_r + \frac{1}{2}\phi_a\right) - S_I\left(\phi_r - \frac{1}{2}\phi_a\right) \qquad S_I(\phi_1) - S_I(\phi_2) \propto \frac{1}{4!}\left(\phi_1^4 - \phi_2^4\right) = \frac{1}{2^2}\frac{1}{3!}\phi_a^3\phi_r + \frac{1}{3!}\phi_r^3\phi_a.$$



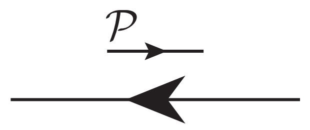
Standard vacuum vertex with one *a*, extra "triple *a*" vertex with 1/4 factor

- Graphical notation for the flow of causality, arrows point towards *r* fields

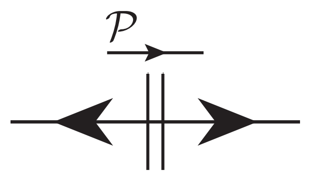
$D^R(\mathcal{P}) =$



$D^A(\mathcal{P}) =$

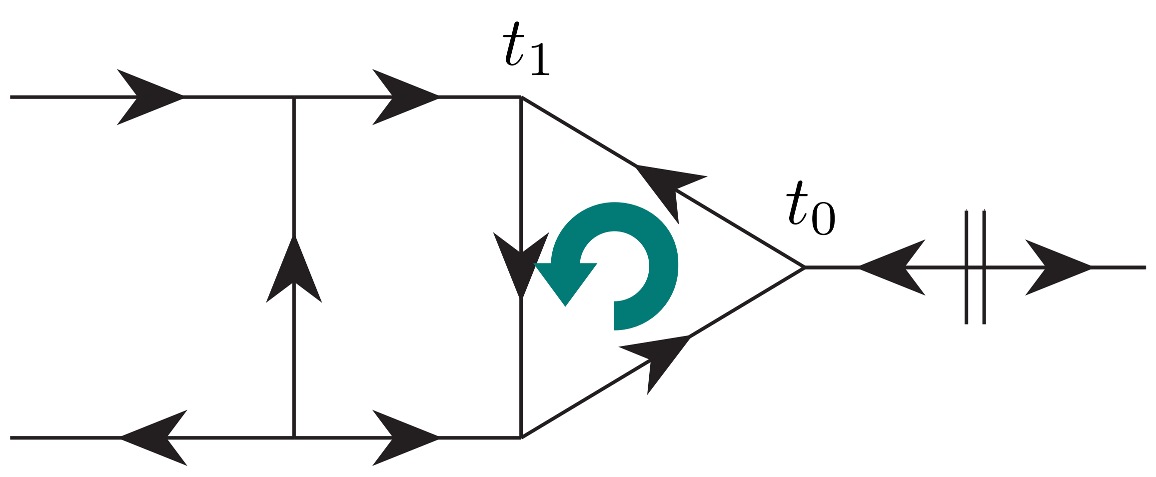


$D^{rr}(\mathcal{P}) =$

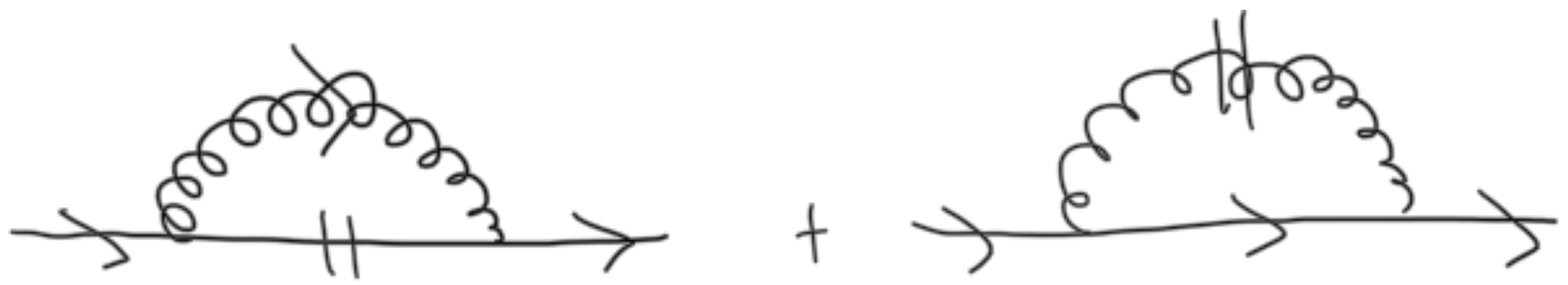


The ``ra'' basis

Causality

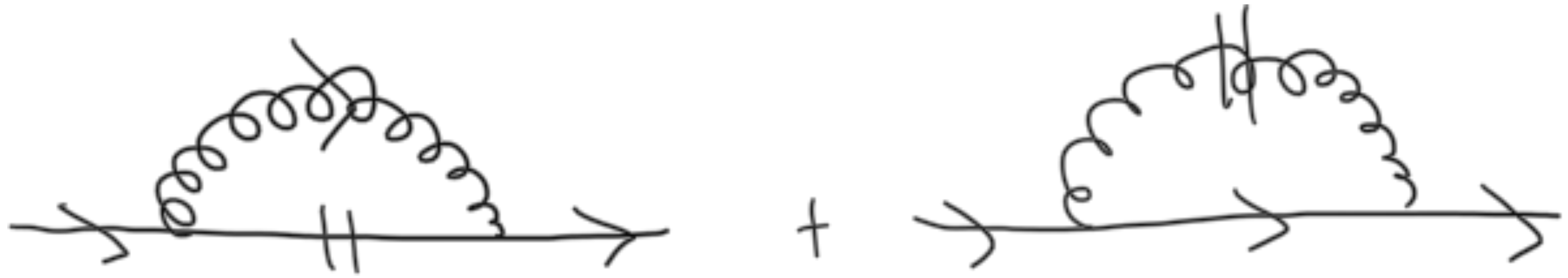


- Closed loops with a flow of causality vanish: require $t_1 > t_0$ and $t_0 > t_1$
- In momentum space: $\int_P D_R(P)D_R(P+Q)D_R(P+Q+K) = 0$
- Diagrams where *a* vertex is at latest time vanish
- Retarded self-energy very simple: correlation X causation

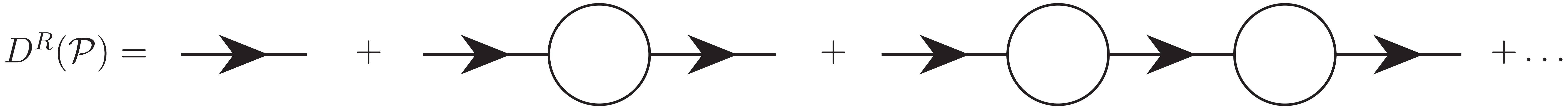


The ``ra'' basis

Resummation



- The lack of an *aa* propagator (at all orders!) makes the Schwinger-Dyson eq. for the retarded propagator diagonal



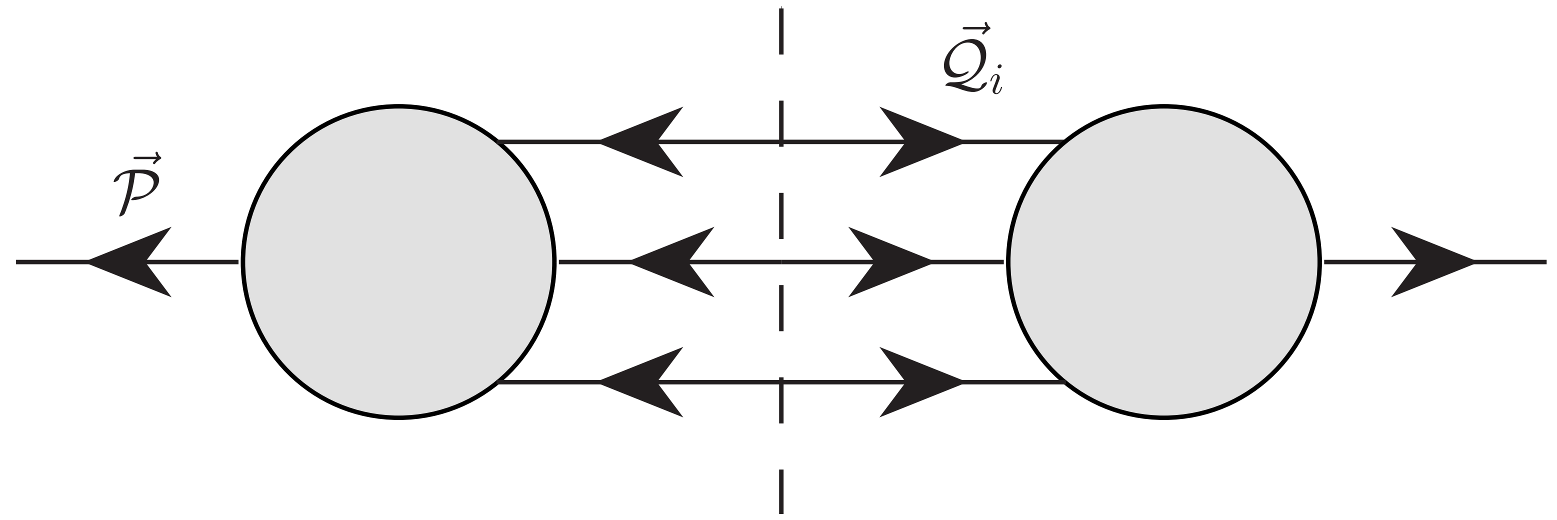
- Try that in the 12 basis…
- For the *rr* a bit more complicated. And there is KMS for self-energies too

$$\Pi^{aa}(\mathcal{P}) = \left(\frac{1}{2} \pm n(p^0)\right)\left(\Pi^R(\mathcal{P}) - \Pi^A(\mathcal{P})\right)$$

Cutting rules

From the Wightman self-energy to retarded amplitudes

$$\Pi^{>}(\mathcal{P}) = \sum_n \frac{1}{n!}\left(\prod_n \int \frac{d^4\mathcal{Q}_n}{(2\pi)^4}\right)(2\pi)^4\delta^4(\mathcal{Q}_1+\ldots+\mathcal{Q}_n-\mathcal{P})$$
$$\times \mathcal{M}_{ar\ldots r}(\mathcal{P};\mathcal{Q}_1,\ldots,\mathcal{Q}_n)\mathcal{M}_{ar\ldots r}(-\mathcal{P};-\mathcal{Q}_1,\ldots,-\mathcal{Q}_n)$$
$$\times D^{>}(\mathcal{Q}_1)\ldots D^{>}(\mathcal{Q}_n),$$



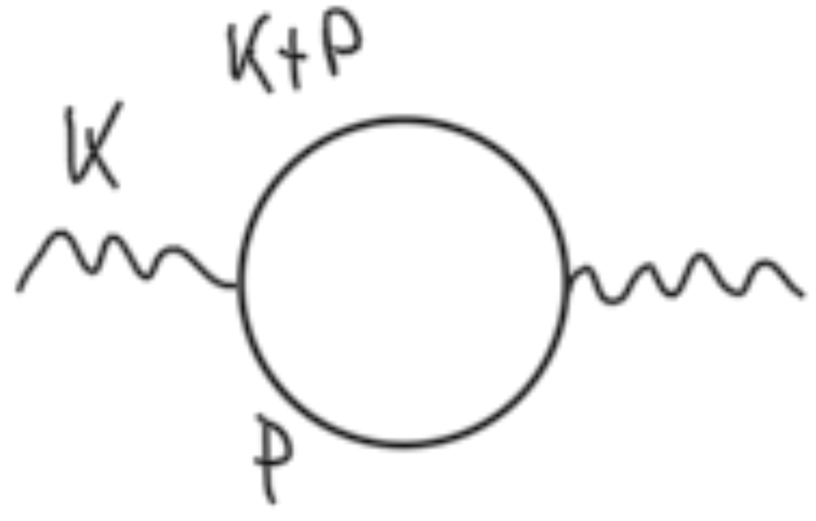
- Sum over all possible cuts. Each cut line is replaced by a Wightman propagator, amplitudes on both sides of the cut fully retarded (one *a*, all other *r*): finite-*T* generalisation of matrix elements squared
- Have to try it to actually see it work

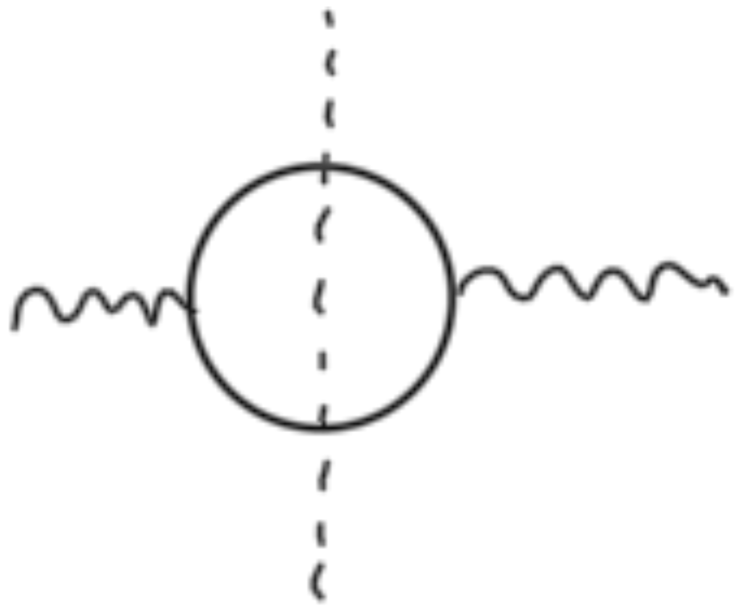
The *ra* basis with cutting rules

We are ready to go (and see the failure of the loop expansion)

$$\frac{dN_\gamma}{d^4X d^3k} \equiv \frac{d\Gamma_\gamma}{d^3k} \overset{k\|z}{=} \frac{-e^2}{(2\pi)^3 2k} \int d^4X e^{ik(t-z)} \langle J^\mu(0) J_\mu(X) \rangle$$

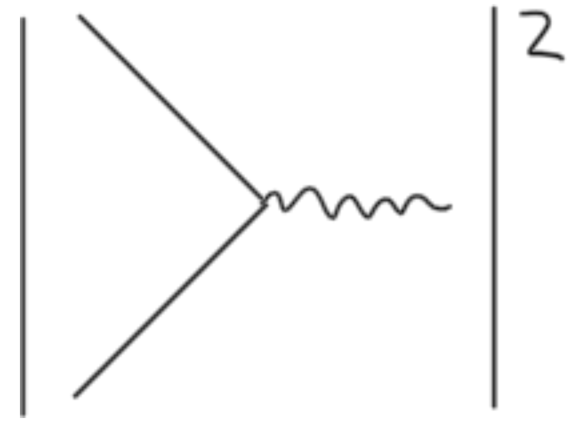
- This is a < Wightman function, $\Pi^<(K)$. We can use the cutting rule!





$\propto S^<(K+P)\, S^<(-P)$

$\propto \delta((K+P)^2)\,\delta(P^2) = 0$ for $K^2 = 0$

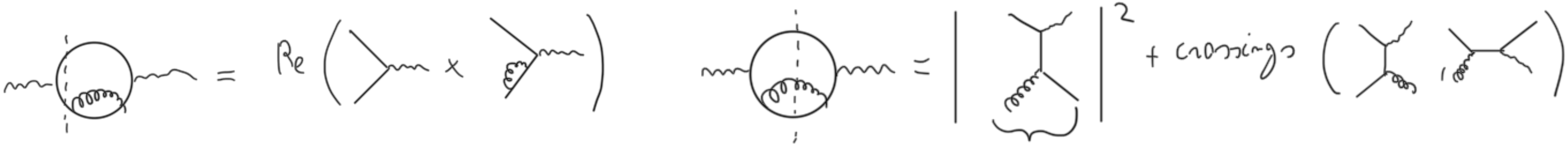


The *ra* basis with cutting rules

We are ready to go (and see the failure of the loop expansion)

$$\frac{d\Gamma_\gamma}{d^3k} = \frac{\Pi^<(\mathcal{K})}{(2\pi)^3 2k}, \qquad \Pi^<(\mathcal{K}) = \int d^4\mathcal{X} e^{-i\mathcal{K}\cdot\mathcal{X}} \left\langle J^\mu(0)\, J_\mu(\mathcal{X})\right\rangle$$

- At two-loops 

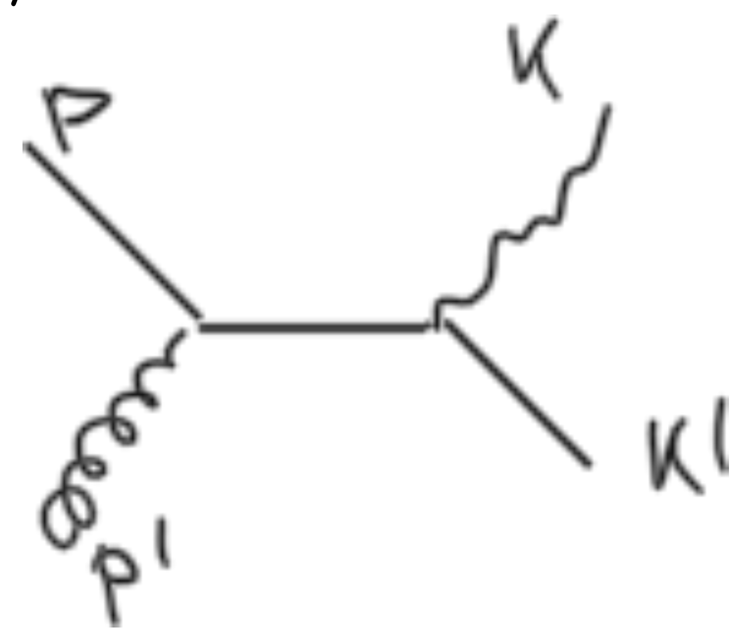


we are actually still tree-level, since the virtual corrections necessarily vanish.

The *ra* basis with cutting rules

We are ready to go (and see the failure of the loop expansion)

$$\frac{d\Gamma_\gamma}{d^3k} = \frac{\Pi^<(\mathcal{K})}{(2\pi)^3 2k}, \qquad \Pi^<(\mathcal{K}) = \int d^4\mathcal{X} e^{-i\mathcal{K}\cdot\mathcal{X}} \langle J^\mu(0)\, J_\mu(\mathcal{X})\rangle$$



- Doing all diagrams and cuts one finds

$$\Pi^<_{g^2\ \mathrm{naive}}(\mathcal{K}) \equiv \Pi^<(\mathcal{K})_{\mathrm{Compton}} + \Pi^<(\mathcal{K})_{\mathrm{annih}},$$

$$\Pi^<(\mathcal{K})_{\mathrm{Compton}} = e^2 \sum_{i=1}^{n_f} Q_i^2 \int \frac{d^3p\, d^3p'\, d^3k'}{(2\pi)^9\, 8\, p\, p'\, k'} (2\pi)^4 \delta^{(4)}(\mathcal{P}+\mathcal{P}'-\mathcal{K}-\mathcal{K}')\, 16 d_F C_F g^2 \left[\frac{-s}{t} + \frac{-t}{s}\right] n_{\mathrm{F}}(p) n_{\mathrm{B}}(p')(1-n_{\mathrm{F}}(k'))$$

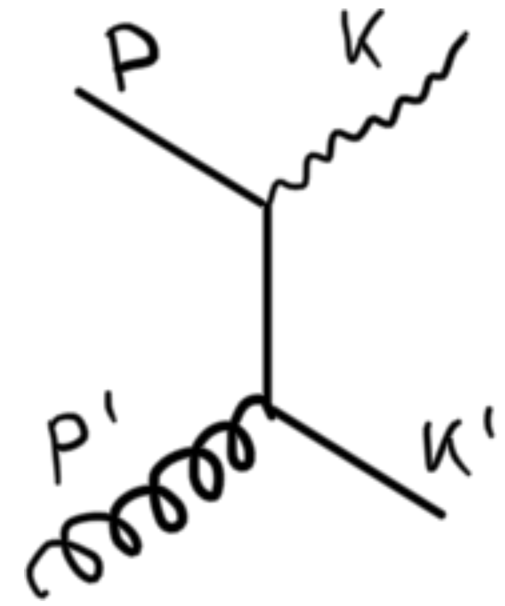
$$\Pi^<(\mathcal{K})_{\mathrm{annih}} = e^2 \sum_{i=1}^{n_f} Q_i^2 \int \frac{d^3p\, d^3p'\, d^3k'}{(2\pi)^9\, 8\, p\, p'\, k'} (2\pi)^4 \delta^{(4)}(\mathcal{P}+\mathcal{P}'-\mathcal{K}-\mathcal{K}')\; 8 d_F C_F g^2 \left[\frac{u}{t} + \frac{t}{u}\right] n_{\mathrm{F}}(p) n_{\mathrm{F}}(p')(1+n_{\mathrm{B}}(k'))$$

- This naive evaluation is equivalent to kinetic theory (gain term only)
- **Exercise**: work out one of these crossings directly from the cutting rules in Feynman gauge (see attachment). Bonus points if you try in 12 basis…

The *ra* basis with cutting rules

We are ready to go (and see the failure of the loop expansion)

$$\frac{d\Gamma_\gamma}{d^3k} = \frac{\Pi^<(\mathcal{K})}{(2\pi)^3 2k}, \qquad \Pi^<(\mathcal{K}) = \int d^4\mathcal{X} e^{-i\mathcal{K}\cdot\mathcal{X}} \langle J^\mu(0)\, J_\mu(\mathcal{X})\rangle$$



- Doing all diagrams and cuts one finds

$$\Pi^<_{g^2\,\mathrm{naive}}(\mathcal{K}) \equiv \Pi^<(\mathcal{K})_{\mathrm{Compton}} + \Pi^<(\mathcal{K})_{\mathrm{annih}},$$

$$\Pi^<(\mathcal{K})_{\mathrm{Compton}} = e^2 \sum_{i=1}^{n_f} Q_i^2 \int \frac{d^3p\, d^3p'\, d^3k'}{(2\pi)^9\, 8\, p\, p'\, k'} (2\pi)^4 \delta^{(4)}(\mathcal{P} + \mathcal{P}' - \mathcal{K} - \mathcal{K}')\, 16 d_F C_F g^2 \left[\frac{-s}{t} + \frac{-t}{s}\right] n_{\mathrm{F}}(p) n_{\mathrm{B}}(p')(1 - n_{\mathrm{F}}(k'))$$

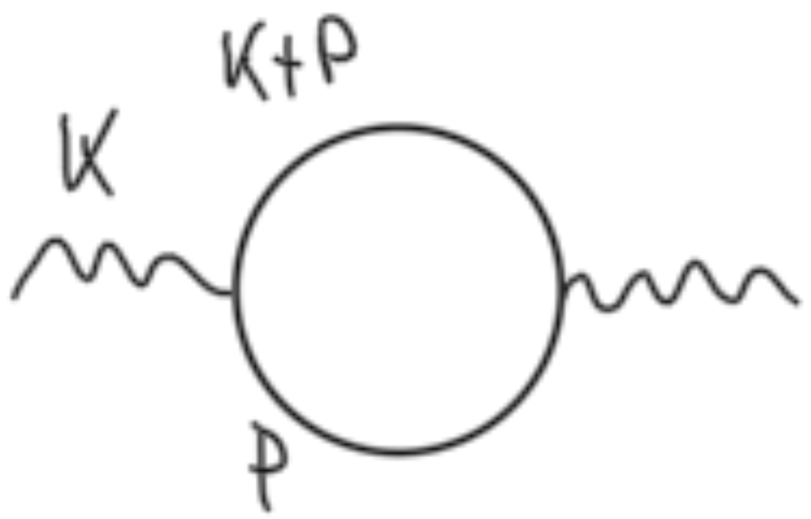
$$\Pi^<(\mathcal{K})_{\mathrm{annih}} = e^2 \sum_{i=1}^{n_f} Q_i^2 \int \frac{d^3p\, d^3p'\, d^3k'}{(2\pi)^9\, 8\, p\, p'\, k'} (2\pi)^4 \delta^{(4)}(\mathcal{P} + \mathcal{P}' - \mathcal{K} - \mathcal{K}')\; 8 d_F C_F g^2 \left[\frac{u}{t} + \frac{t}{u}\right] n_{\mathrm{F}}(p) n_{\mathrm{F}}(p')(1 + n_{\mathrm{B}}(k'))$$

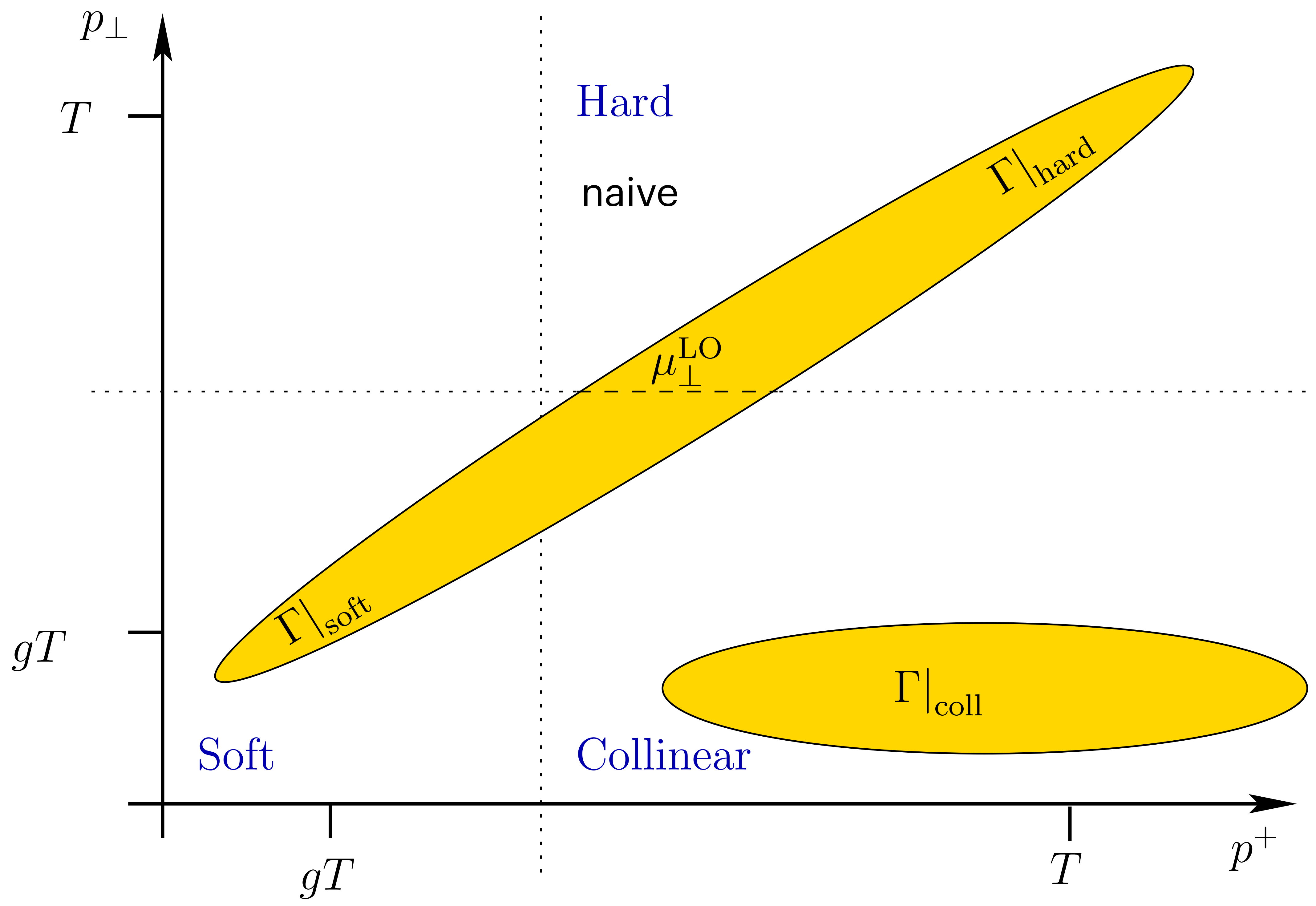
- The integrals can be worked out with the methods of **hep-ph/0111107**
- At small *t* and *u* we have a log IR divergence: breakdown of the loop expansion

The *ra* basis with cutting rules

We are ready to go (and see the failure of the loop expansion)

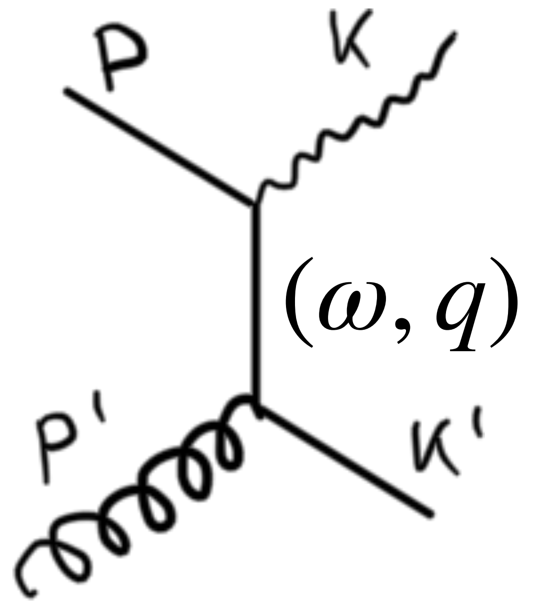
$$\frac{d\Gamma_\gamma}{d^3k} = \frac{\Pi^<(\mathcal{K})}{(2\pi)^3 2k}, \qquad \Pi^<(\mathcal{K}) = \int d^4\mathcal{X} e^{-i\mathcal{K}\cdot\mathcal{X}} \langle J^\mu(0)\, J_\mu(\mathcal{X})\rangle$$





Breakdown of the loop expansion

1) Soft modes



- When $t \ll s \sim T^2$ there is a logarithmic IR divergence.
  Exercise: show that it has this form (I might have screwed the prefactors)
  $$\Pi^<(K)^{\text{soft}}_{\text{compton}} = \frac{e^2 \sum_i Q_i d_F C_F g^2 n_{\text{F}}(k)}{2\pi^3} \int_{\text{soft}} dq \int_{-q}^{q} d\omega \int_0^\infty dp' \frac{p'}{q^2} n_{\text{B}}(p')(1 - n_{\text{F}}(p'))$$
- If there is an IR scale ($\ll T$) it will contribute to LO to this logarithmic phase space dq/q
- This scale is the scale gT where the first collective effects appear

Breakdown of the loop expansion

1) Soft modes

- In vacuum (at $m = 0$) $\Sigma(Q) \propto g^2 Q$
- Here we have an extra scale, T. Hence we can have $\Sigma_R(Q \ll T) \sim g^2T^2/Q$ (this will be shown explicitly soon)
- We have $Q \sim \Sigma_R(Q \ll T)$ for $Q \sim gT$: this is where the loop expansion breaks down. For gauge bosons $\Pi_R(Q \ll T) \sim g^2T^2$, same story
- How to deal with this breakdown? What are the physical consequences? And how is it related to the emergence of collectivity?
- To answer these question, introduce Hard Thermal Loops

Hard Thermal Loops

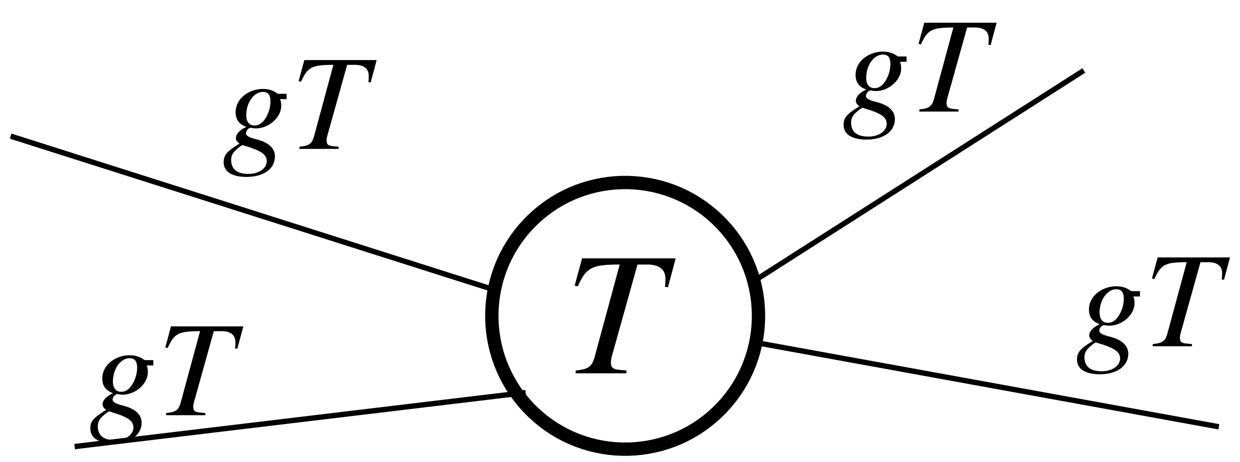
Emergence of collectivity

- Hard Thermal Loop (HTL) effective theory: a consistent, modern and gauge-invariant handling of these effects
- Originally introduced by Braaten and Pisarski [1, 2], Frenkel and Taylor [3, 4] and Taylor and Wong [5]. Their connection to a kinetic picture for the underlying hard modes has been illustrated in the review of Blaizot and Iancu [6]
  [1,2] 10.1016/0550-3213(90)90508-B, 10.1103/PhysRevD.45.R1827
  [3,4] 10.1016/0550-3213(90)90661-V, 10.1016/0550-3213(92)90480-Y
  [5] 10.1016/0550-3213(90)90240-E
  [6] hep-ph/0101103

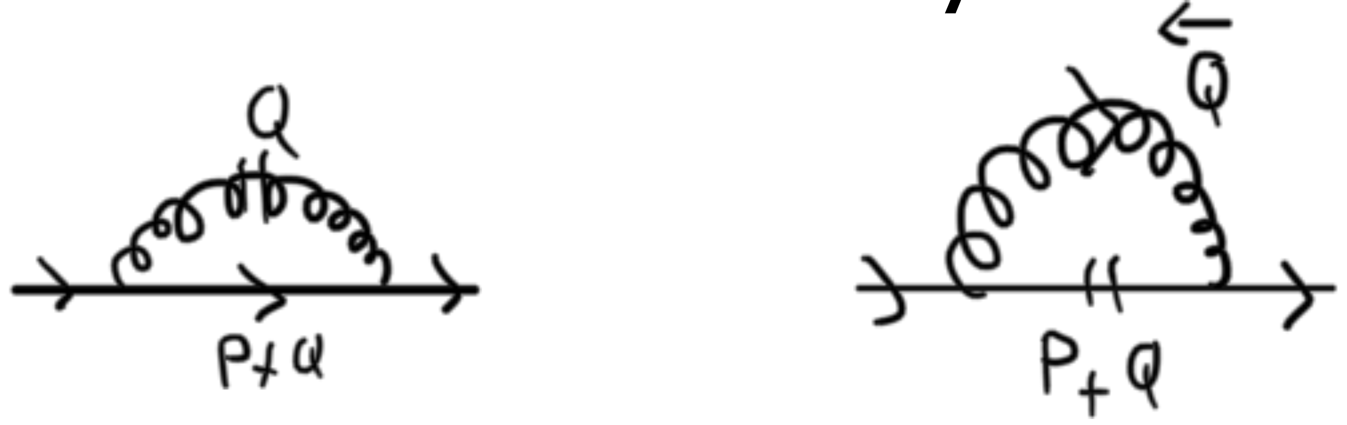
Hard Thermal Loops

Diagrammatics and connection to kinetics

- Diagrammatically: HTLs=g^2T^2-proportional gauge-invariant amplitudes with $n \geq 2$ external soft lines and thermal ("hard") loop momentum



- Let us derive the quark two-point HTL. From the Feynman rules we have

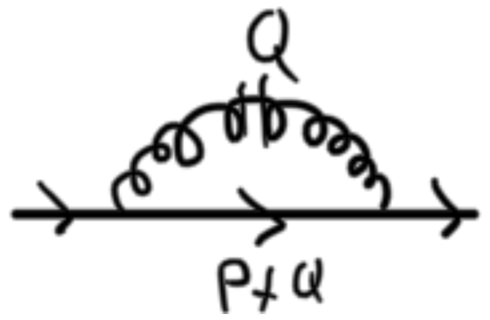


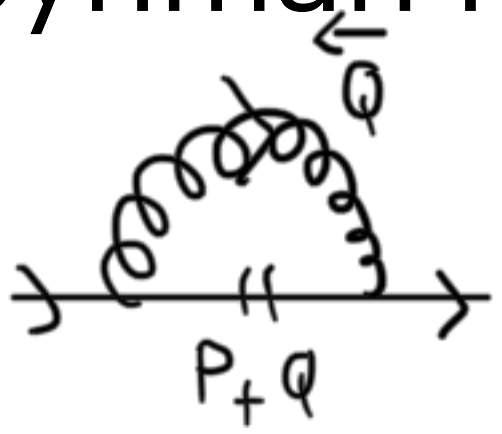
$$-i\Sigma^R(\mathcal{Q}) = (-ig)^2 C_F \int \frac{d^4\mathcal{P}}{(2\pi)^4}\gamma^\mu \left[S^R(\mathcal{P}+\mathcal{Q})G^{rr}_{\mu\nu}(\mathcal{P}) + S_{rr}(\mathcal{P}+\mathcal{Q})G^A_{\mu\nu}(\mathcal{P})\right]\gamma^\nu,$$

Hard Thermal Loops

Diagrammatics and connection to kinetics

- Let us derive the quark two-point HTL. From the Feynman rules we have





$$-i\Sigma^R(\mathcal{Q}) = (-ig)^2 C_F \int \frac{d^4\mathcal{P}}{(2\pi)^4}\gamma^\mu \left[S^R(\mathcal{P}+\mathcal{Q})G^{rr}_{\mu\nu}(\mathcal{P}) + S_{rr}(\mathcal{P}+\mathcal{Q})G^A_{\mu\nu}(\mathcal{P})\right]\gamma^\nu,$$

- This is the one-loop self-energy without approximations
- Use bare propagators (copypaste from my review, mostly+ metric) and throw away vacuum part

$$\Sigma^R(\mathcal{Q}) \quad = \quad g^2 C_F \int \frac{d^4\mathcal{P}}{(2\pi)^4}\gamma^\mu(\not{\mathcal{P}}+\not{\mathcal{Q}})\gamma_\mu \left[\frac{n_\mathrm{B}(|p^0|)2\pi\delta(\mathcal{P}^2)}{(\mathcal{P}+\mathcal{Q})^2 - i\epsilon(p^0+q^0)} - \frac{n_\mathrm{F}(|p^0+q^0|)2\pi\delta((\mathcal{P}+\mathcal{Q})^2)}{\mathcal{P}^2 + i\epsilon p^0}\right]$$

Hard Thermal Loops

Diagrammatics and connection to kinetics

- Shift the second term into the same form as the first

$$\Sigma^R(\mathcal{Q}) = g^2 C_F \int \frac{d^4\mathcal{P}}{(2\pi)^4} \frac{4\pi\delta(\mathcal{P}^2)}{(\mathcal{P}+\mathcal{Q})^2 - i\epsilon(p^0+q^0)} \left[(\not{\mathcal{P}} + \not{\mathcal{Q}})\, n_{\mathrm{B}}(|p^0|) + \not{\mathcal{P}}\, n_{\mathrm{F}}(|p^0|)\right]$$

- We now expand for $Q \ll P$. At first order (unlike for gluons) we find HTL

$$\Sigma^R(\mathcal{Q}) = g^2 C_F \int \frac{d^4\mathcal{P}}{(2\pi)^4} 2\pi\delta(\mathcal{P}^2) \left(n_{\mathrm{B}}(|p^0|) + n_{\mathrm{F}}(|p^0|)\right) \frac{\not{\mathcal{P}}}{\mathcal{P}\cdot\mathcal{Q} - i\epsilon p^0} \qquad \Sigma^R(\mathcal{Q}) = g^2 C_F \int \frac{d^4\mathcal{P}}{(2\pi)^4} 2\pi\delta(\mathcal{P}^2) \left(n_{\mathrm{B}}(|p^0|) + n_{\mathrm{F}}(|p^0|)\right) \frac{\not{v}}{v\cdot\mathcal{Q} - i\epsilon}$$

with $v \equiv P/p^0$. Factorisation of angular part. Finally

$$\Sigma^R(\mathcal{Q}) = \frac{m_\infty^2}{2} \int \frac{d\Omega_v}{4\pi} \frac{\not{v}}{v\cdot\mathcal{Q} - i\epsilon}$$

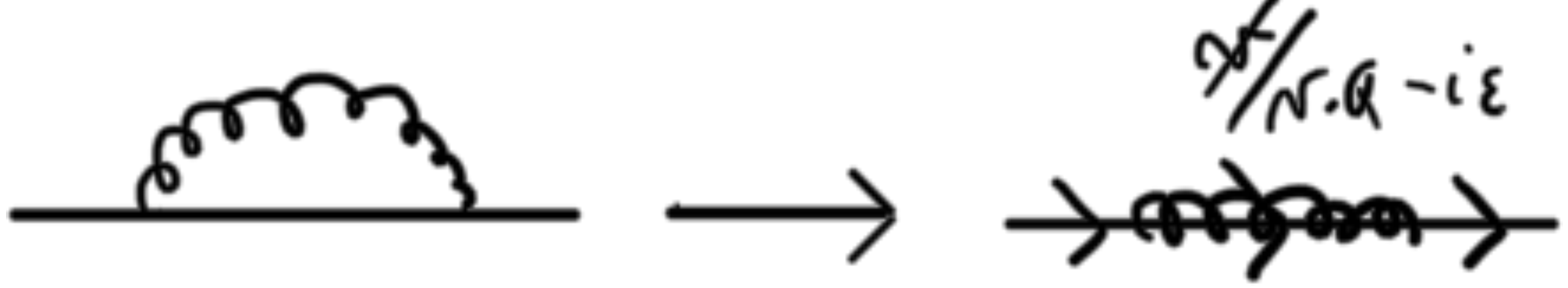
$m_\infty^2 = g^2 C_F T^2/4$ is the *asymptotic mass* of the quark. $\Sigma_R(Q \ll T) \sim g^2T^2/Q$

Hard Thermal Loops

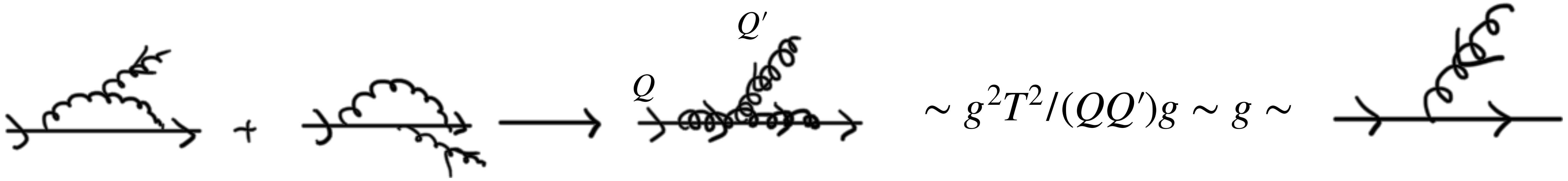
Diagrammatics and connection to kinetics

$$\Sigma^R(\mathcal{Q}) = \frac{m_\infty^2}{2}\int \frac{d\Omega_v}{4\pi}\frac{\not{v}}{v\cdot\mathcal{Q} - i\epsilon}$$

- This is the quark 2-point HTL. Simple structure: m_∞^2 times the angular average of the propagator for the induced fermion source, which is the effective structure that emerges. Original loop not resolved



- In Fourier space $1/v\cdot Q \to 1/v\cdot\partial$. Gauge invariance then suggests $1/v\cdot D$, which is confirmed by calculations of higher-point functions



HTL resummation needed not just in two-point function

Hard Thermal Loops
Effective Lagrangian and resummation

- All n-point HTLs with two external quark lines and n-2 gluon lines are generated by

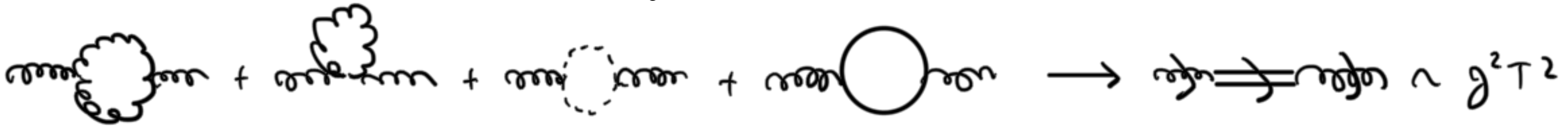
$$\delta\mathcal{L}_f = i\frac{m_\infty^2}{2}\,\overline{\psi}\int\frac{d\Omega_v}{4\pi}\frac{\not{v}}{v\cdot D}\,\psi$$

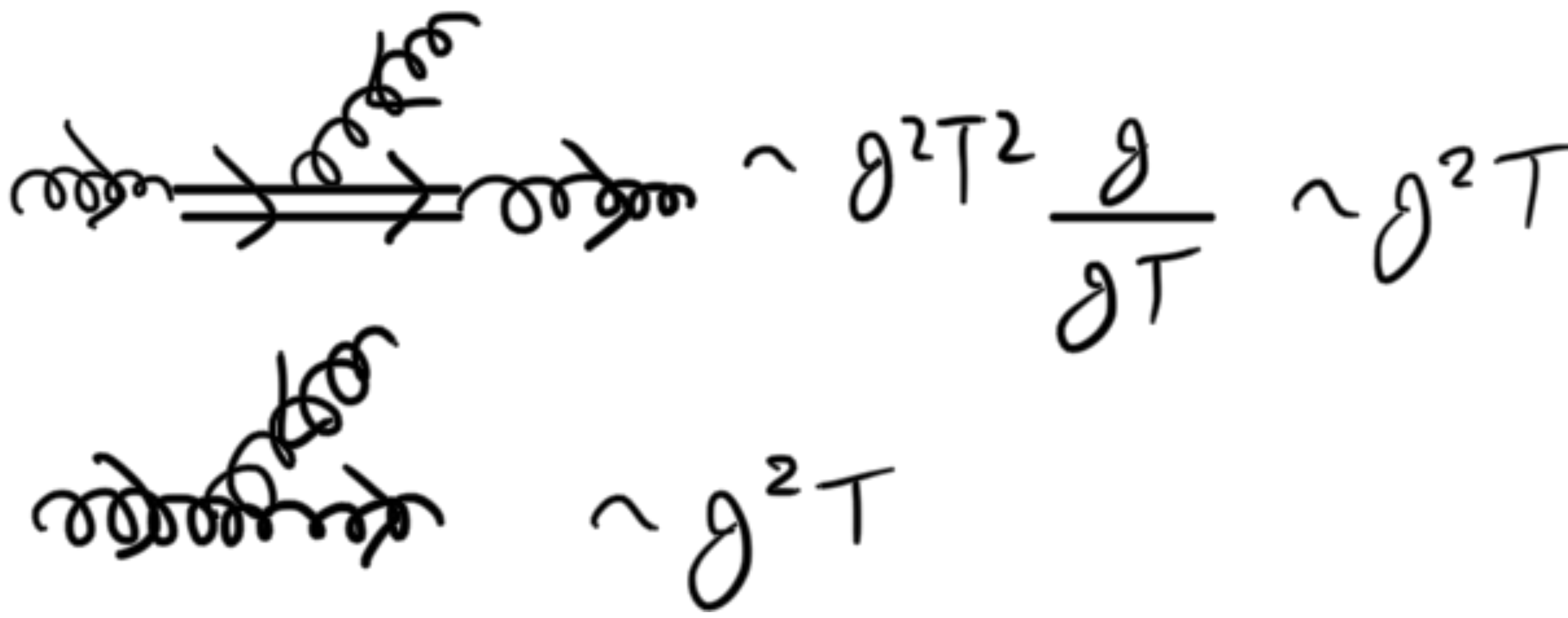
  No HTLs with more than 2 quark lines exist

- A similar derivation finds for the $n \geq 2$ all-gluon HTL

$$\delta\mathcal{L}_g = \frac{m_D^2}{2}\mathrm{Tr}\int\frac{d\Omega_v}{4\pi}F^{\mu\alpha}\frac{v_\alpha v_\beta}{(v\cdot D)^2}\,F^{\beta}_{\mu}$$

$m_D^2 = g^2T^2(N_c/3 + N_f/6)$ is the Debye mass





Hard Thermal Loops

Effective Lagrangian and resummation

- For practical higher-loop (typically beyond LO) in the HTL theory, this business of eikonal propagators and effective Feynman rules in the *ra* formalism is ideal. See my review and the original paper by Caron-Huot (0710.5726) for more detail
- The power-counting that emerges is non-trivial and requires some getting used to. Dealing with loop-level HTLs requires even more getting used to, analytically and numerically…
- NLO heavy quark momentum diffusion, Caron-Huot Moore 0801.2173
  Cold Quark matter at N3LO: soft contribution Gorda et al 2103.07427

Hard Thermal Loops

Resummations and collective modes

- To see the emergence of collectivity, consider resummed propagators
- Gluons: now two independent structures, versus one in vacuum. Longitudinal and transverse to spatial **q**, both transverse to Q. They are $\Pi_L(Q) = (1 - q_0^2/q^2)\Pi^{00}(Q)$, $\Pi_T(Q) = (\delta^{ij} - \hat{q}^i\hat{q}^j)\Pi^{ij}(Q)/2$
- From the rules/Lagrangian

$$\Pi_R^{\mu\nu}(\mathcal{Q}) = m_D^2 \int \frac{d\Omega_v}{4\pi}\left(\delta_0^\mu\delta_0^\nu + v^\mu v^\nu \frac{q^0}{v\cdot\mathcal{Q} - i\epsilon}\right) \qquad \Pi_R^{00}(Q) = m_D^2\left(1 - \frac{q^0}{2q}\ln\frac{q^0 + q + i\epsilon}{q^0 - q + i\epsilon}\right) \qquad \Pi_T^R(Q) = \frac{m_D^2}{2} - \frac{\Pi_L^R(Q)}{2}$$

the logs come from the angular average of the induced source (kinetic) propagator

Hard Thermal Loops

Collective gluonic modes

- Resummed Coulomb gauge propagators (physical features gauge inv.)

$$G_R^{00}(Q) = \frac{i}{q^2 + m_D^2\left(1 - \frac{q^0}{2q}\ln\frac{q^0+q+i\epsilon}{q^0-q+i\epsilon}\right)}$$

$$G_R^{ij}(Q) \equiv (\delta^{ij} - \hat{q}^i\hat{q}^j)G_R^T(Q) = \left.\frac{i(\delta^{ij} - \hat{q}^i\hat{q}^j)}{q_0^2 - q^2 - \frac{m_D^2}{2}\left(\frac{q_0^2}{q^2} - \left(\frac{q_0^2}{q^2} - 1\right)\frac{q^0}{2q}\ln\frac{q^0+q}{q^0-q}\right)}\right|_{q^0=q^0+i\epsilon}$$

- In the time-like sector plasmons: collective excitations with modified dispersion relation. At vanishing momentum $G_R^{00}(q^0) = G_R^L(q^0) = G_R^T(q^0) = \frac{i}{(q^0+i\epsilon)^2 - \frac{m_D^2}{3}}$ three propagating, massive modes! Plasma oscillations

At large $q \gg m_D$, longitudinal mode has exponentially small residue, transverse modes have $q_0^2 - q^2 = m_D^2/2 = M_\infty^2$ asymptotic mass.

In-between: numerical solution

Hard Thermal Loops

Collective gluonic modes

- Resummed Coulomb gauge propagators (physical features gauge inv.)

$$G_R^{00}(Q) = \frac{i}{q^2 + m_D^2\left(1 - \frac{q^0}{2q}\ln\frac{q^0+q+i\epsilon}{q^0-q+i\epsilon}\right)}$$

$$G_R^{ij}(Q) \equiv (\delta^{ij} - \hat{q}^i\hat{q}^j)G_R^T(Q) = \left.\frac{i(\delta^{ij} - \hat{q}^i\hat{q}^j)}{q_0^2 - q^2 - \frac{m_D^2}{2}\left(\frac{q_0^2}{q^2} - \left(\frac{q_0^2}{q^2} - 1\right)\frac{q^0}{2q}\ln\frac{q^0+q}{q^0-q}\right)}\right|_{q^0=q^0+i\epsilon}$$

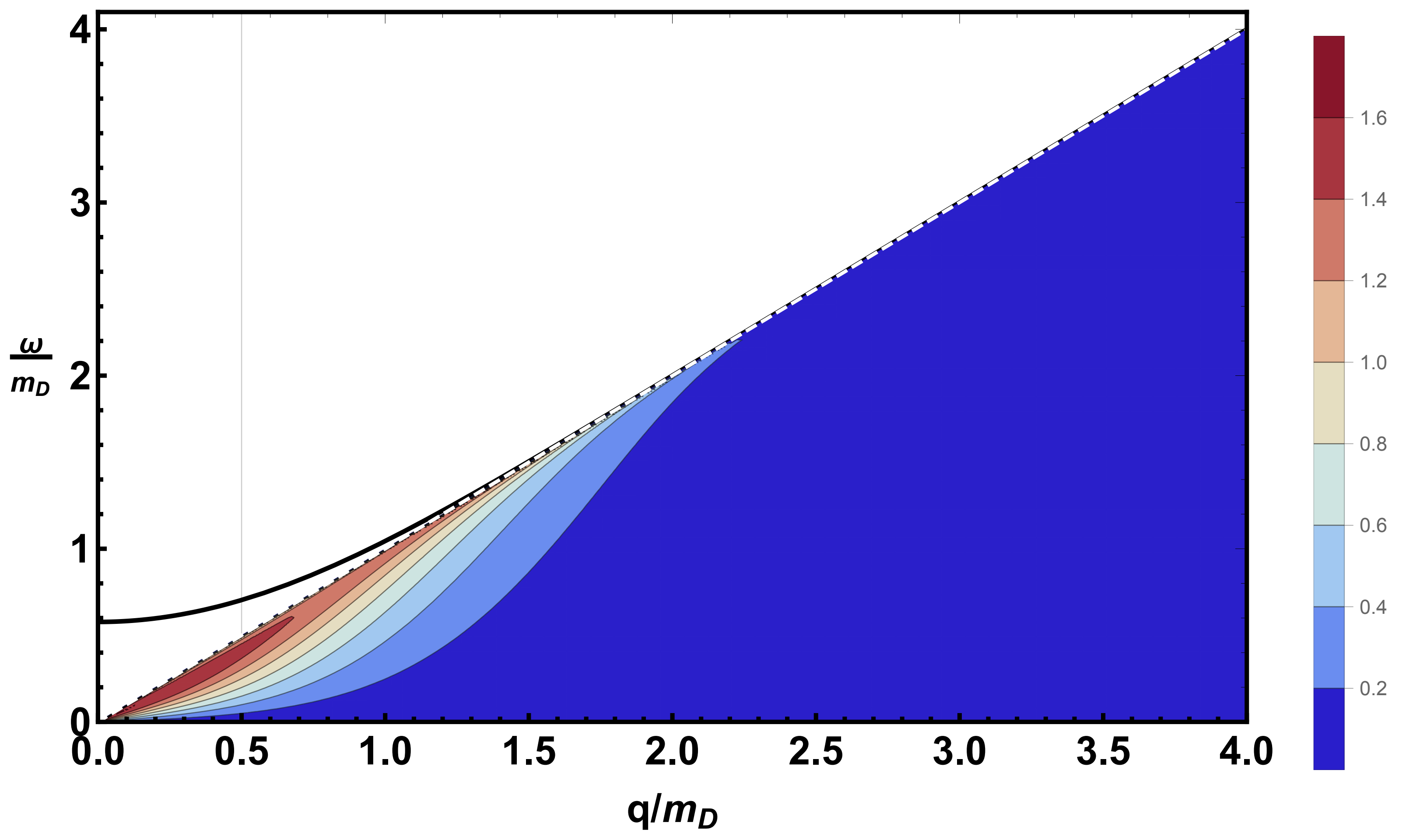
- In the space-like sector Landau damping: the branch cuts in the logarithms create a non-zero imaginary part in the denominators, a non-zero spectral function! The coupling of the soft modes to the induced current damps them!
- At zero frequency (time-independent): Debye screening of (chromo)electrostatic modes

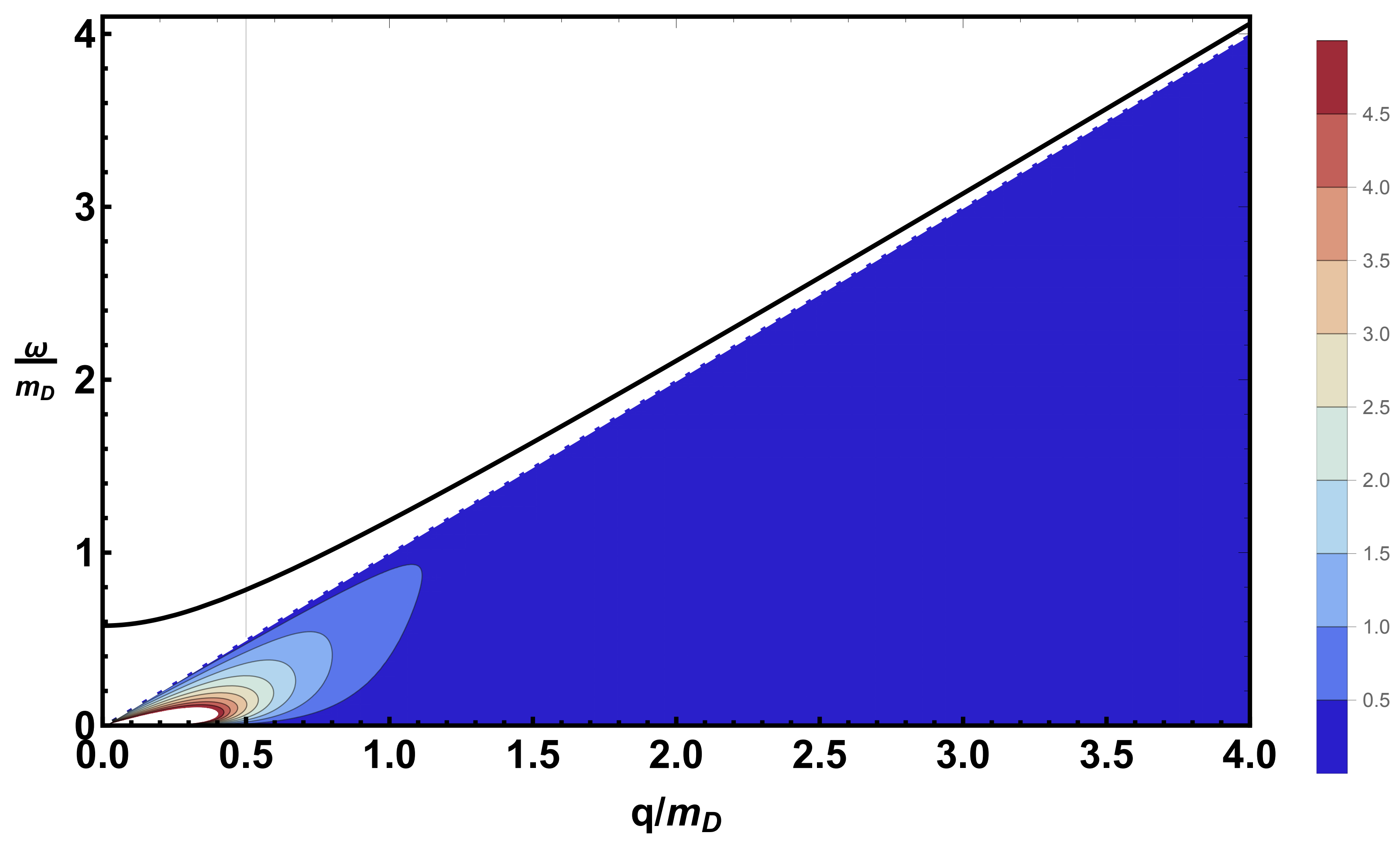
Hard Thermal Loops

Collective gluonic modes

$$G_R^{00}(Q) = \frac{i}{q^2 + m_D^2\left(1 - \frac{q^0}{2q}\ln\frac{q^0+q+i\epsilon}{q^0-q+i\epsilon}\right)}$$

$$G_R^{ij}(Q) \equiv (\delta^{ij} - \hat{q}^i\hat{q}^j)G_R^T(Q) = \frac{i(\delta^{ij} - \hat{q}^i\hat{q}^j)}{q_0^2 - q^2 - \frac{m_D^2}{2}\left(\frac{q_0^2}{q^2} - \left(\frac{q_0^2}{q^2} - 1\right)\frac{q^0}{2q}\ln\frac{q^0+q}{q^0-q}\right)}$$

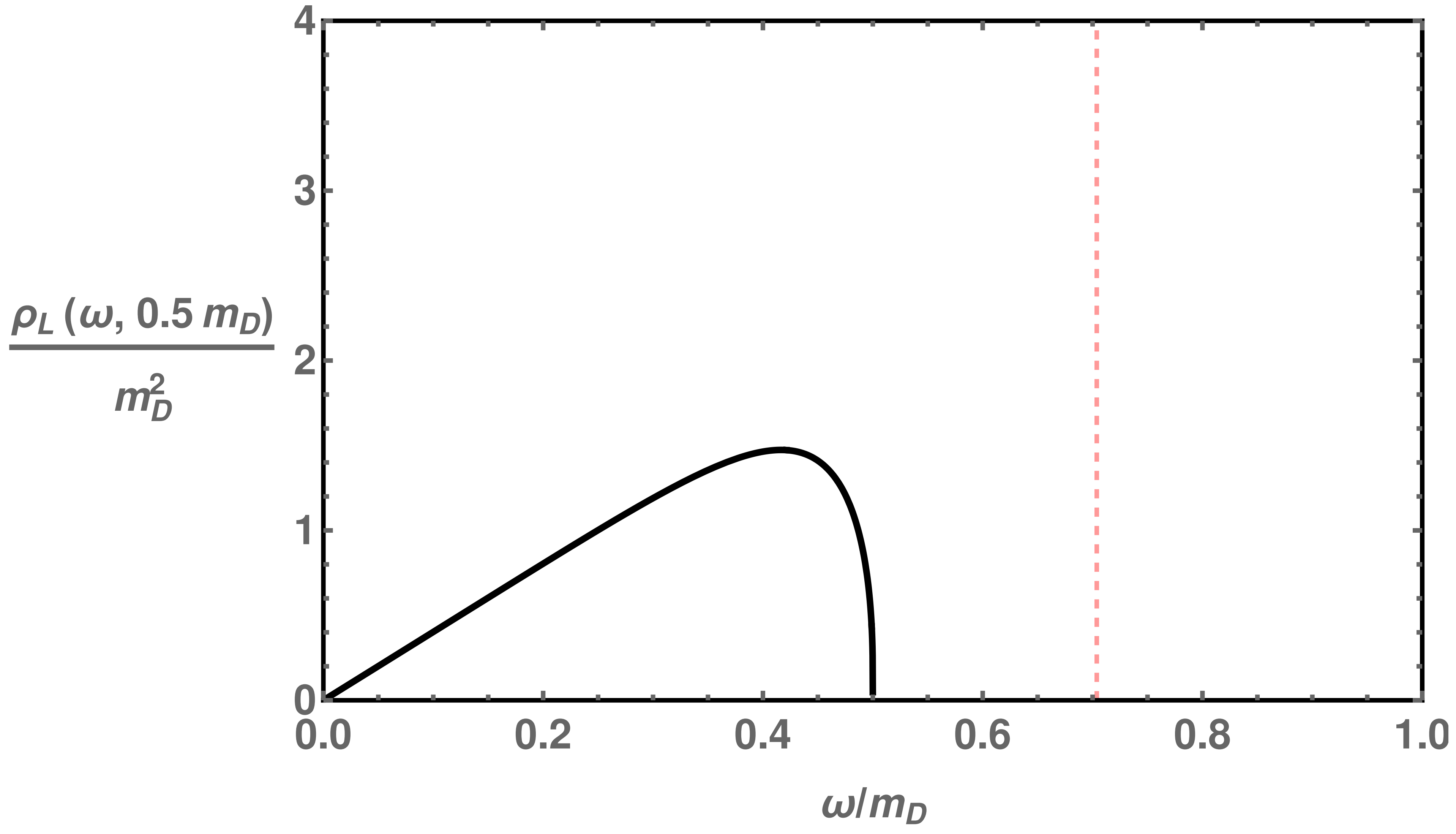


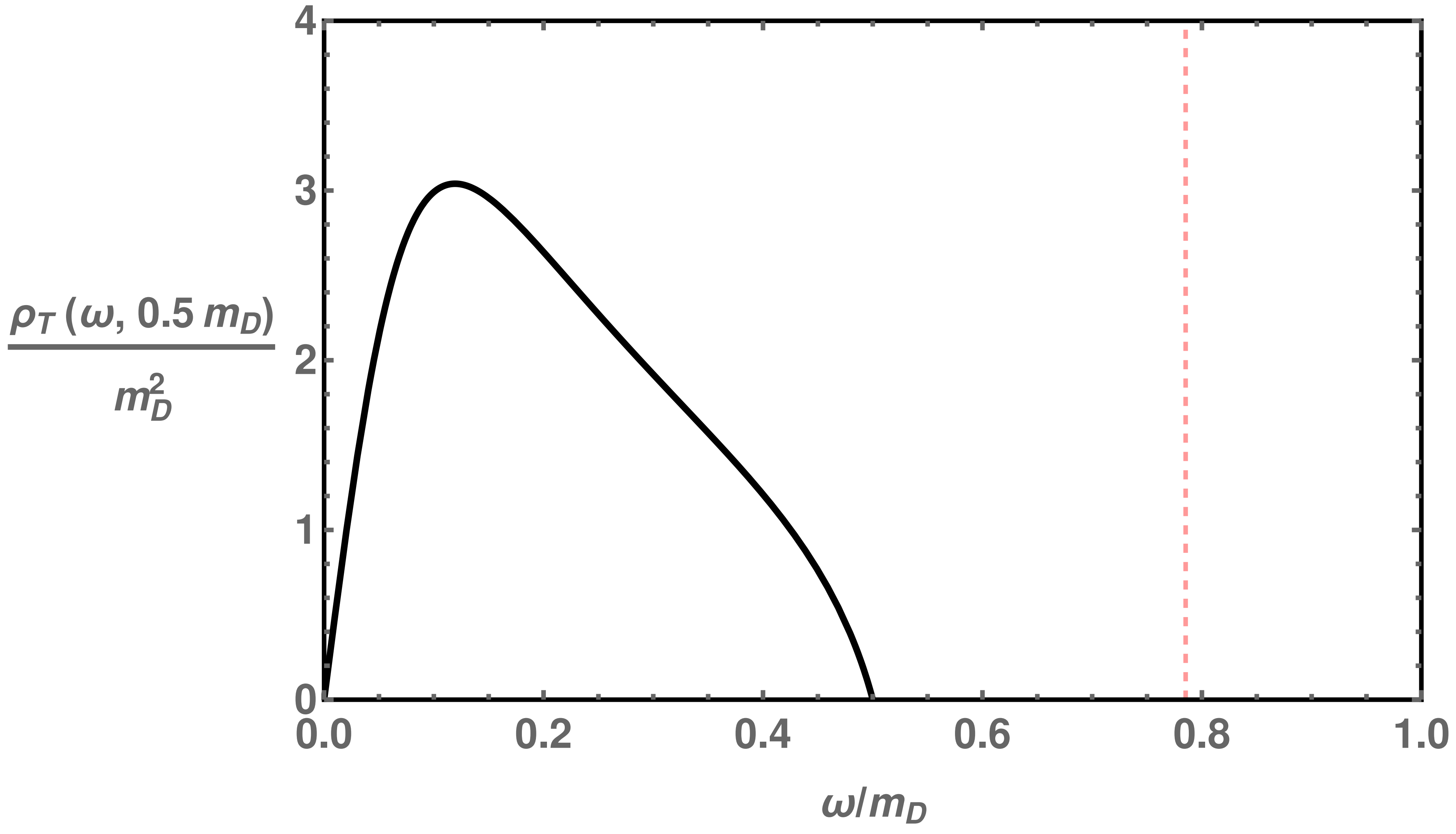


Hard Thermal Loops
Collective gluonic modes

$$G_R^{00}(Q) = \frac{i}{q^2 + m_D^2\left(1 - \frac{q^0}{2q}\ln\frac{q^0+q+i\epsilon}{q^0-q+i\epsilon}\right)}$$

$$G_R^{ij}(Q) \equiv (\delta^{ij} - \hat{q}^i\hat{q}^j)G_R^T(Q) = \frac{i(\delta^{ij} - \hat{q}^i\hat{q}^j)}{q_0^2 - q^2 - \frac{m_D^2}{2}\left(\frac{q_0^2}{q^2} - \left(\frac{q_0^2}{q^2} - 1\right)\frac{q^0}{2q}\ln\frac{q^0+q}{q^0-q}\right)}$$





- Plasmons appear as delta functions in the spf. That is because their pole position is $\mathcal{O}(gT)$, their width is $\mathcal{O}(g^2T)$, to be determined within HTL thy