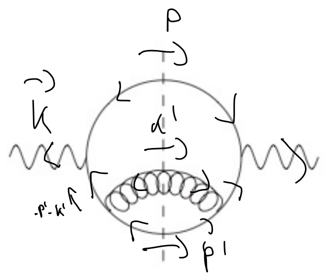


=

$$\Pi^>(\mathcal{P}) = \sum_n \frac{1}{n!}\left(\prod_n \int \frac{d^4 \mathcal{Q}_n}{(2\pi)^4}\right)(2\pi)^4\delta^4(\mathcal{Q}_1 + \ldots + \mathcal{Q}_n - \mathcal{P})$$
$$\times \mathcal{M}_{ar\ldots r}(\mathcal{P};\mathcal{Q}_1,\ldots,\mathcal{Q}_n)\mathcal{M}_{ar\ldots r}(-\mathcal{P};-\mathcal{Q}_1,\ldots,-\mathcal{Q}_n)$$
$$\times D^>(\mathcal{Q}_1)\ldots D^>(\mathcal{Q}_n), \qquad (55)$$

$$\Pi^{<\mu}_{\mu}(K) \propto e^2 Q^2 g^2 N_c \int \frac{d^4P}{(2\pi)^4} \int \frac{d^4P'}{(2\pi)^4} \int \frac{d^4K'}{(2\pi)^4} (2\pi)^4 \delta^{(4)}(P+P'+K'-K) \, \mathrm{Tr}\left[\gamma^\mu S^<(P) \gamma_\mu S_R(-K-P'-K') \gamma^\nu S^<(P') \gamma^\rho S_R(K+P'+K')\right] G^<_{\nu\rho}(K')$$

$+ - - -$ metric:

$$S_R(-Q) = \frac{i(-\not{Q})}{(-q^0+i\varepsilon)^2 - q^2} = -S_A(Q) \quad S_A(Q) = \frac{i\not{Q}}{(q^0-i\varepsilon)^2 - q^2} \qquad \gamma^\mu \not{Q} \gamma_\mu = -\gamma^\mu\gamma_\mu \not{Q} + 2 Q^\mu \gamma^\mu = -2\not{Q}$$

$$\Pi^{<\mu}_{\mu}(Q) \propto 2 e^2 Q^2 g^2 N_c \int \frac{d^4P}{(2\pi)^4} \int \frac{d^4P'}{(2\pi)^4} \int \frac{d^4K'}{(2\pi)^4} (2\pi)^4 \delta^{(4)}(P+P'+K'-K) \, \mathrm{Tr}\left[S^<(P) \quad S_A(+P'+K') \gamma^\nu S^<(P') \gamma^\rho S_R(+P'+K')\right] G^<_{\nu\rho}(K')$$

Feynman gauge: $G^<_{\nu\rho}(K') = -g_{\nu\rho}\left[\theta(-k'^0) + n_B|k'^0|\right] 2\pi\delta(K'^2)$

$$\Pi^{<\mu}_{\mu}(Q) \propto 4 e^2 Q^2 g^2 N_c \int \frac{d^4P}{(2\pi)^4} \int \frac{d^4P'}{(2\pi)^4} \int \frac{d^4K'}{(2\pi)^4} (2\pi)^4 \delta^{(4)}(P+P'+K'-K) \, \mathrm{Tr}\left[S^<(P) S_A(+P'+K') \quad S^<(P') \quad S_R(+P'+K')\right] \left[\theta(-k'^0) + n_B|k'^0|\right] 2\pi\delta(K'^2)$$

$$\mathrm{Tr}\left[\not{P}(+\not{P}'+\not{K}')\not{P}'(+\not{P}'+\not{K}')\right] = 4\left[2P\cdot(+P'+K')\, P'\cdot(+P'+K') - P\cdot P'(+P'+K')^2\right]$$

Recall that $K^2 = P^2 = K'^2 = P'^2 = 0$ (free massless propagators)

$$\Rightarrow 4\left[2(+P\cdot P' + P\cdot K')(+P'\cdot K') - 2P\cdot P'(+P'\cdot K')\right] = 8\, P\cdot K'\, P'\cdot K'$$

$$\Pi^{<\mu}_{\mu}(Q) \propto 32 e^2 Q^2 g^2 N_c \int \frac{d^4P}{(2\pi)^4} \int \frac{d^4P'}{(2\pi)^4} \int \frac{d^4K'}{(2\pi)^4} \frac{P\cdot K'\, P'\cdot K'}{(2P'\cdot K')_R (2P'\cdot K')_A} \left[\theta(-k'^0) + n_B|k'^0|\right] 2\pi\delta(K'^2) \left[\theta(-p^0) - n_F|p^0|\right] 2\pi\delta(P^2) \left[\theta(-p'^0) - n_F|p'^0|\right] 2\pi\delta(P'^2)$$

$$\times (2\pi)^4 \delta^{(4)}(P+P'+K'-K)$$

Hence we can have $p^0 = \pm p$, $k'^0 = \pm k'$, $p'^0 = \pm p'$. The positive solution causes the corresponding particle to be ingoing, the negative solution to be outgoing.

Coupled to energy-momentum conservation (the $\delta^{(4)}$) this requires two particles to be incoming, 1 outgoing.

Consider explicitly the case $p^0 = p$, $p'^0 = p'$, $k'^0 = -k'$. To make contact with eq. (62) in my review I do $K' \to -K'$ (4-vector)

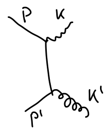
$$\Pi^{<\mu}_{\mu}(Q) \propto 8 e^2 Q^2 g^2 N_c \int \frac{d^4P}{(2\pi)^4} \int \frac{d^4P'}{(2\pi)^4} \int \frac{d^4K'}{(2\pi)^4} \frac{P\cdot K'}{P'\cdot K'} \left[\theta(+k'^0) + n_B|k'^0|\right] 2\pi\delta(K'^2) \left[\theta(-p^0) - n_F|p^0|\right] 2\pi\delta(P^2) \left[\theta(-p'^0) - n_F|p'^0|\right] 2\pi\delta(P'^2)$$

$$\times (2\pi)^4 \delta^{(4)}(P+P'-K'-K)$$

And thus $p^0 = p$, $p'^0 = p'$, $k'^0 = +k'$

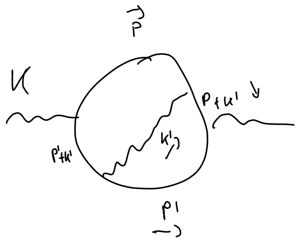
$$\Longrightarrow \quad 8e^2Q^2g^2N_c\int\frac{d^3p}{(2\pi)^3}\int\frac{d^3p'}{(2\pi)^3}\int\frac{d^3k'}{(2\pi)^3}\,\frac{1}{8pp'k'}\;\frac{P\cdot K'}{P'\cdot K'}\Bigg|_{p^0=p,\;p'^0=p',\;k'^0=k'}\left[1+n_B(k')\right]n_F(p)\,n_F(p')\,(2\pi)^4\delta^{(4)}(P+P'-K'-K)$$

This is



$$\Rightarrow\quad s=(P+P')=(K+K')^2=2P\cdot P'=2K\cdot K'$$
$$t=(K-P)^2=(K'-P')^2=-2P\cdot K=-2P'\cdot K'$$
$$u=(K-P')^2=(P-K')^2=-2P\cdot K'=-2K\cdot P'$$

$$8e^2Q^2g^2N_c\int\frac{d^3p}{(2\pi)^3}\int\frac{d^3p'}{(2\pi)^3}\int\frac{d^3k'}{(2\pi)^3}\,\frac{1}{8pp'k'}\;\frac{u}{t}\left[1+n_B(k')\right]n_F(p)\,n_F(p')\,(2\pi)^4\delta^{(4)}(P+P'-K'-K)$$



$$-\mathrm{Tr}\left[\gamma^\mu(\not P+\not K')\gamma^\nu\not P\gamma_\mu(\not P'+\not K')\gamma_\nu\not P'\right]$$

$$\gamma^\mu\not a\gamma^\nu=-\not a\gamma^\mu\gamma^\nu+2a^\mu\gamma^\nu=2a^\mu\gamma^\nu+\not a\gamma^\nu\gamma^\mu-2g^{\mu\nu}\not a=2a^\mu\gamma^\nu-2g^{\mu\nu}\not a-\gamma^\nu\not a\gamma^\mu+2a^\nu\gamma^\mu$$

$$\Rightarrow -\mathrm{Tr}\left[\gamma^\nu(\not P+\not K')\gamma^\mu\not P\gamma_\mu(\not P'+\not K')\gamma_\nu\not P'\right]+2\left[\gamma^\nu\not P(\not P+\not K')(\not P'+\not K')\gamma_\nu\not P'\right]$$

$$+2\,\mathrm{Tr}\left[\gamma^\mu\not P\gamma_\mu(\not P'+\not K')(\not P+\not K')\not P'\right]-2\,\mathrm{Tr}\left[(\not P+\not K')\not P\gamma_\mu(\not P'+\not K')\gamma^\mu\not P'\right]$$

$$=4\Big\{-\mathrm{Tr}\left[(\not P+\not K')\not P(\not P'+\not K')\not P'\right]-\mathrm{Tr}\big[\overset{A}{\not P}\,\overset{B}{(\not P+\not K')}\,\overset{C}{(\not P'+\not K')}\,\overset{D}{\not P'}\big]-\mathrm{Tr}\big[\overset{A}{\not P}\,\overset{C}{(\not P'+\not K')}\,\overset{B}{(\not P+\not K')}\,\overset{D}{\not P'}\big]+\mathrm{Tr}\left[(\not P+\not K')\not P(\not P'+\not K')\not P'\right]\Big\}$$

$$=16\Big\{-A\cdot B\;C\cdot D-A\cdot D\;B\cdot C+A\cdot C\;B\cdot D\;-A\cdot C\;B\cdot D-A\cdot D\;B\cdot C+A\cdot B\;C\cdot D\Big\}$$

$$(P+K')\cdot(P'+K')=P\cdot P'+P\cdot K'+P'\cdot K'=\frac{1}{2}(s+t+u)=0$$

(↳ have not flipped K' yet here!)