Real Scalar Phase Transitions: Bubble Nucleation, Nonperturbatively

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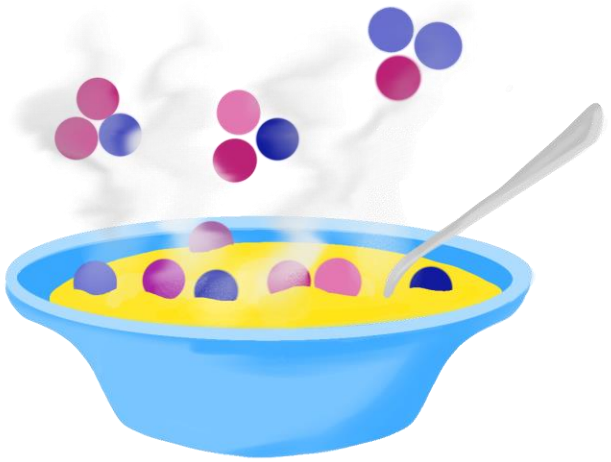
University of Helsinki & Helsinki Institute of Physics

Phase Transitions in the Early Universe

- Grand Unified Theories, Electroweak, QCD...
- In the Standard Model (SM) the electroweak PT is a crossover
- SM is incomplete → beyond SM (BSM) physics
- Things to look for: topological defects, bubbles from EWPT, … ?
- Something we could possibly detect: Gravitational waves?

Introduction

Universe at 100GeV



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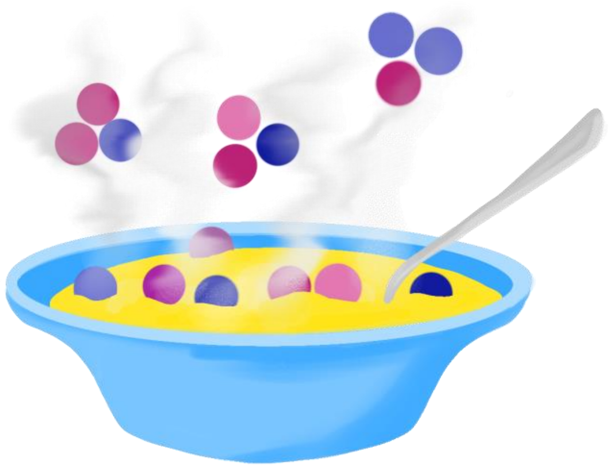

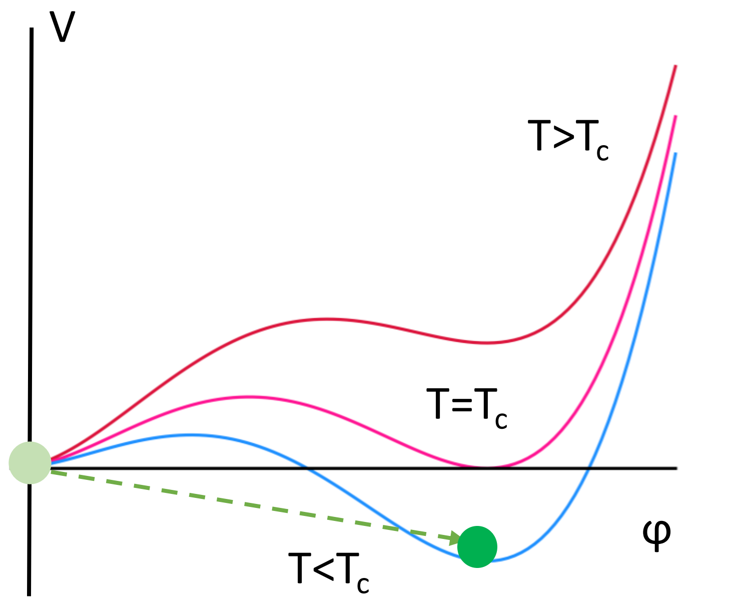

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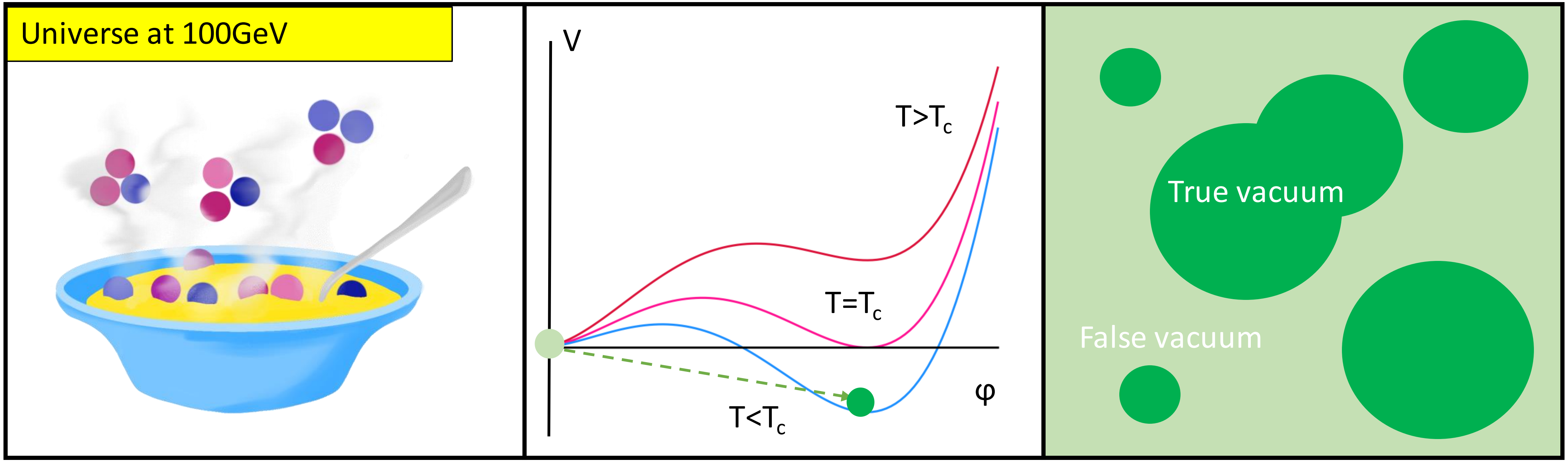

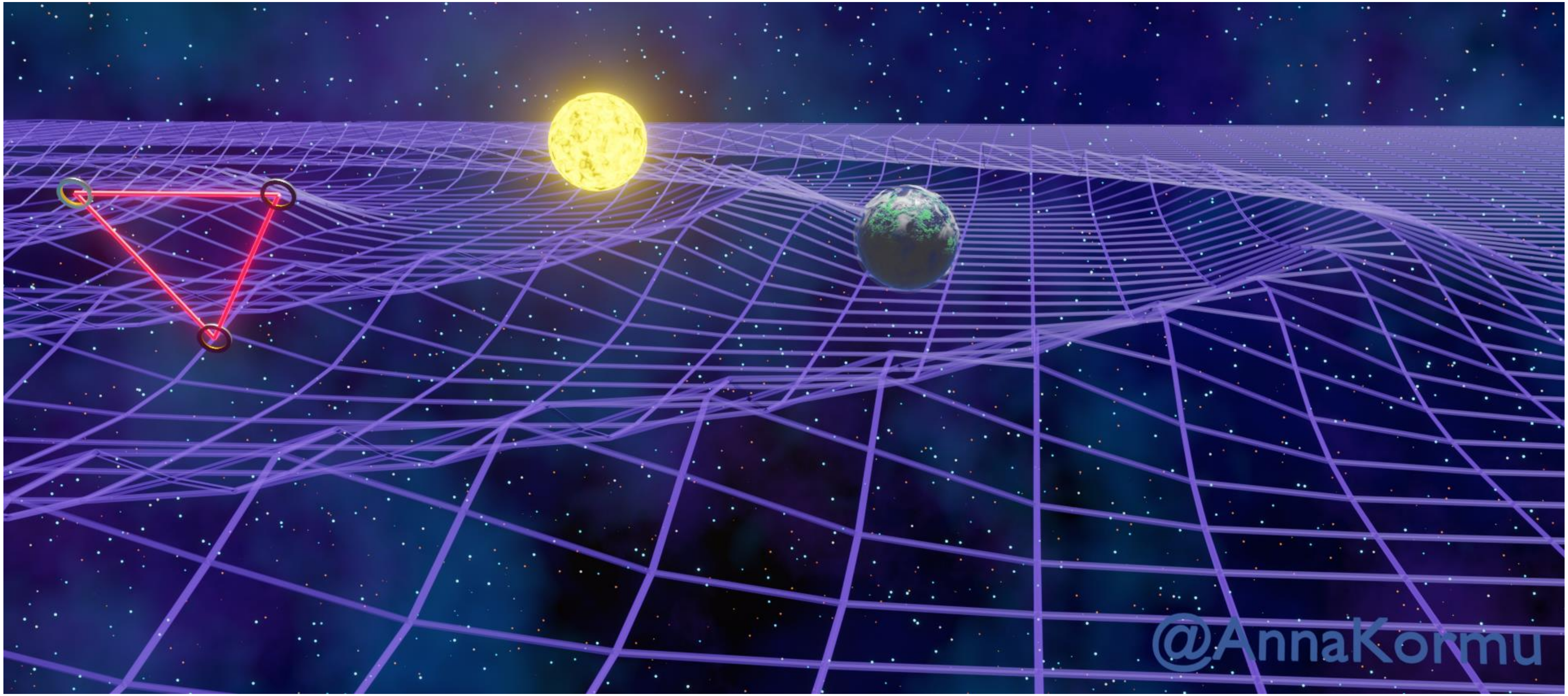

Bubbles collide in the early universe plasma, sourcing gravitational waves

Fate of the False Vacuum

- Semiclassical solution (Coleman 1977)

Volume averaged nucleation rate

$$\frac{\Gamma}{V} = A\, e^{-S_{3,b}(T)/T}$$

Prefactor

3D action

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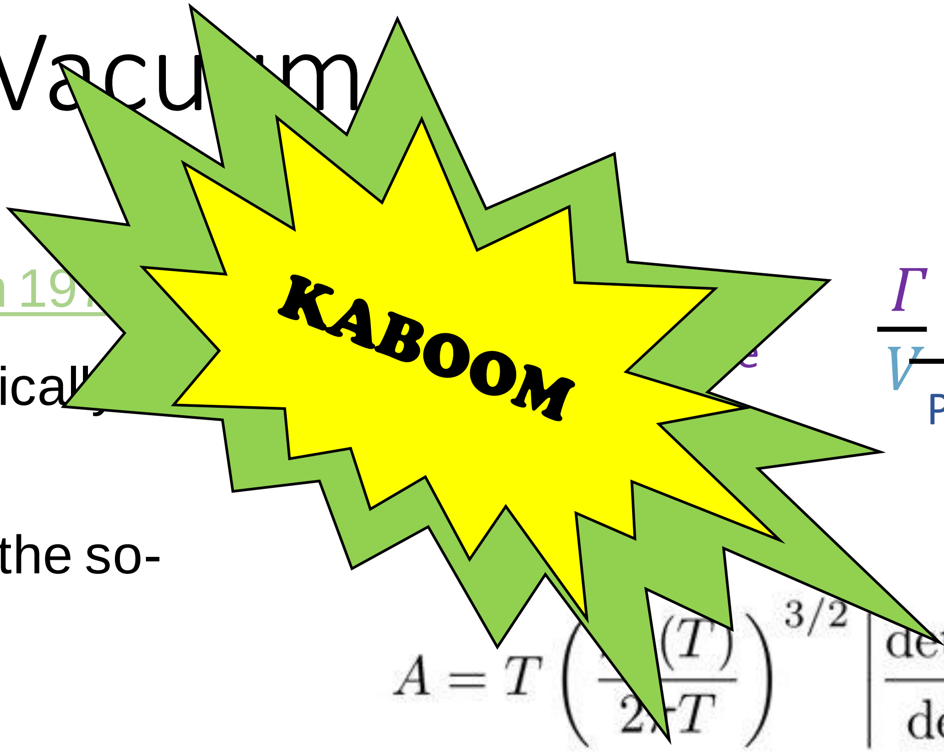

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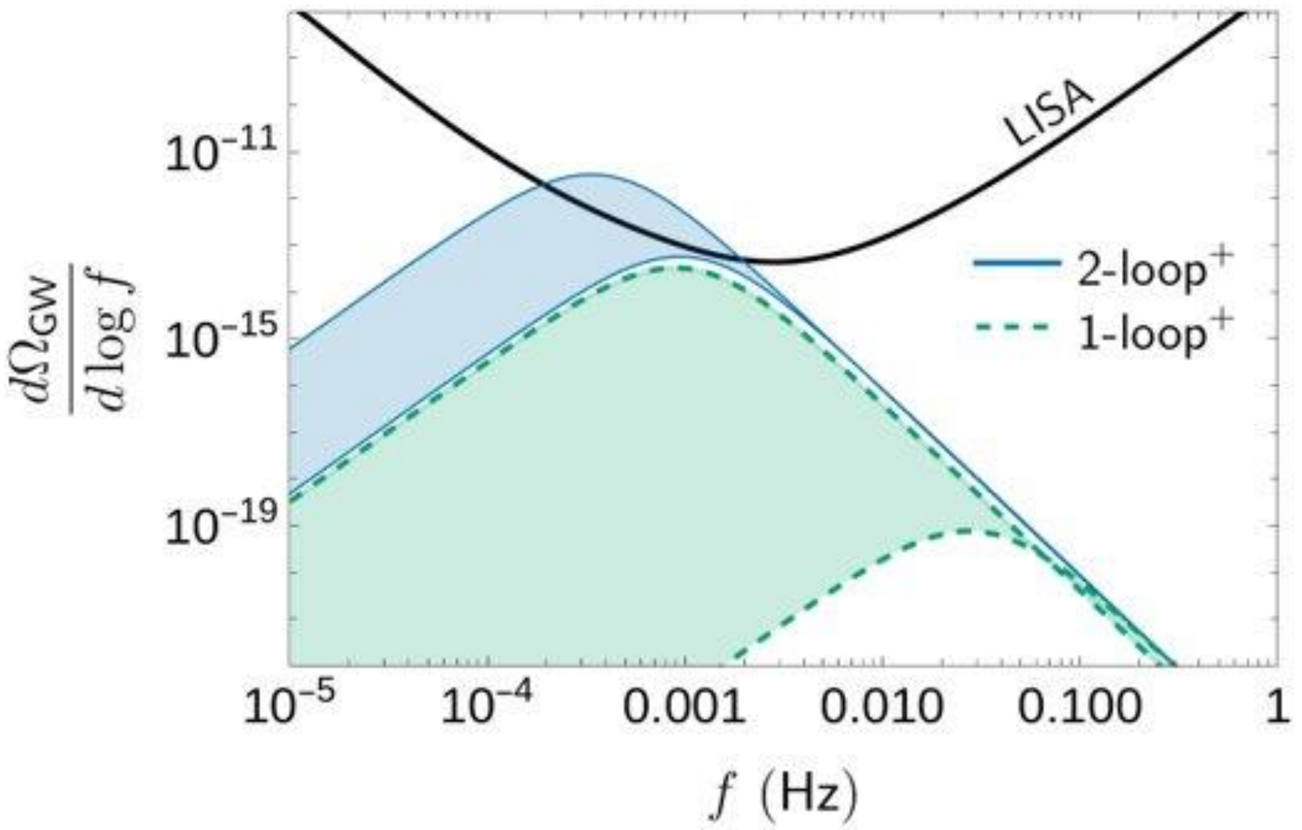
- Introduces uncertainty! How accurate are our cosmological predictions?



Gould, Tenkanen arXiv:2104.04399

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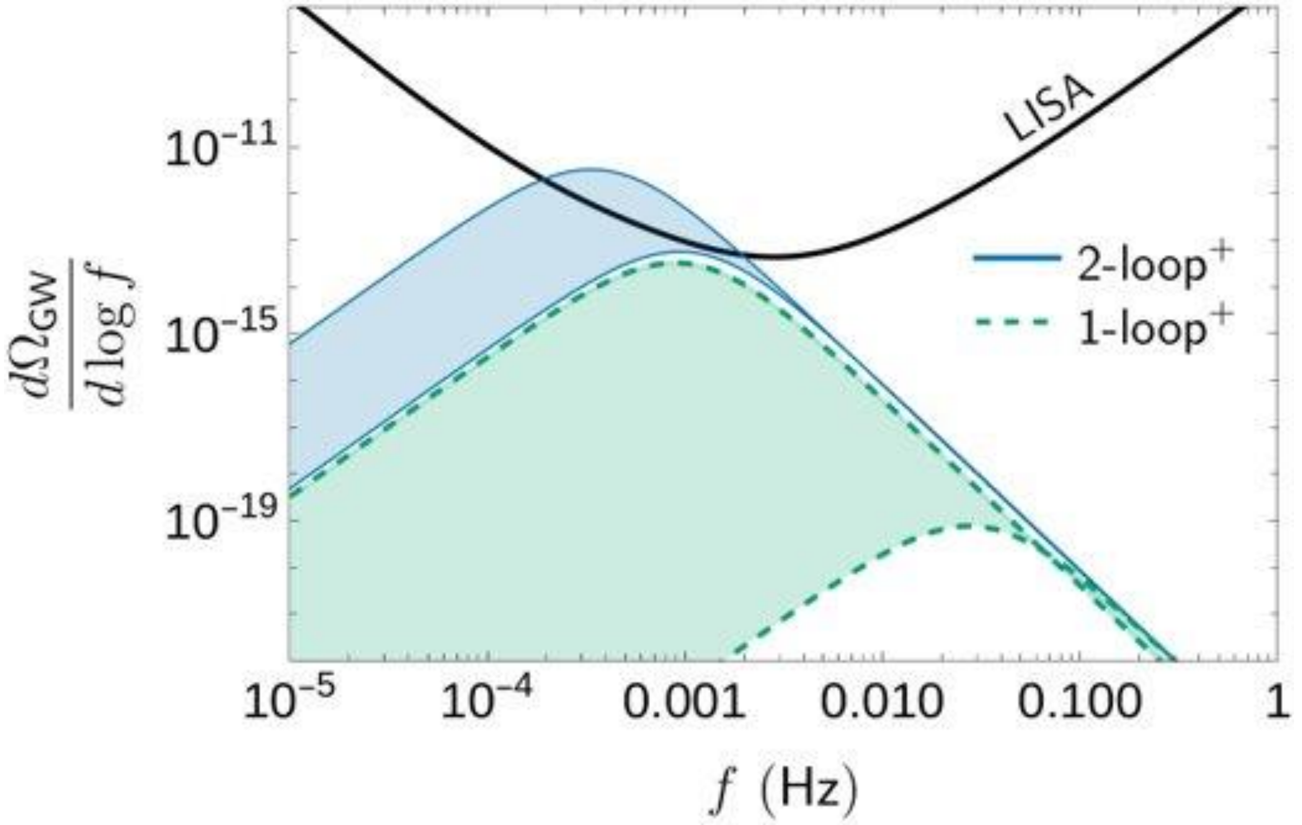
- Moore, Rummukainen & Tranberg introduce a simulation method (hep-lat/0103036, hep-ph/0009132)



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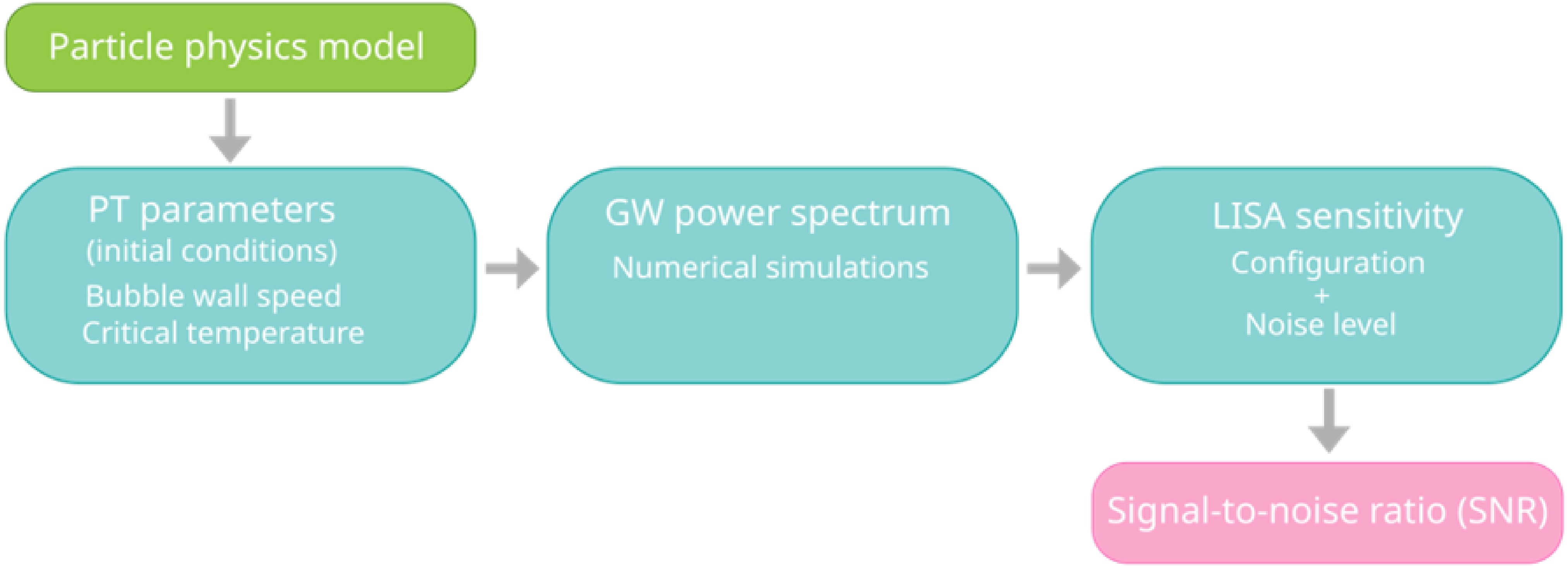
LISA pipeline



Modified from Caprini et al. arXiv:1910.13125

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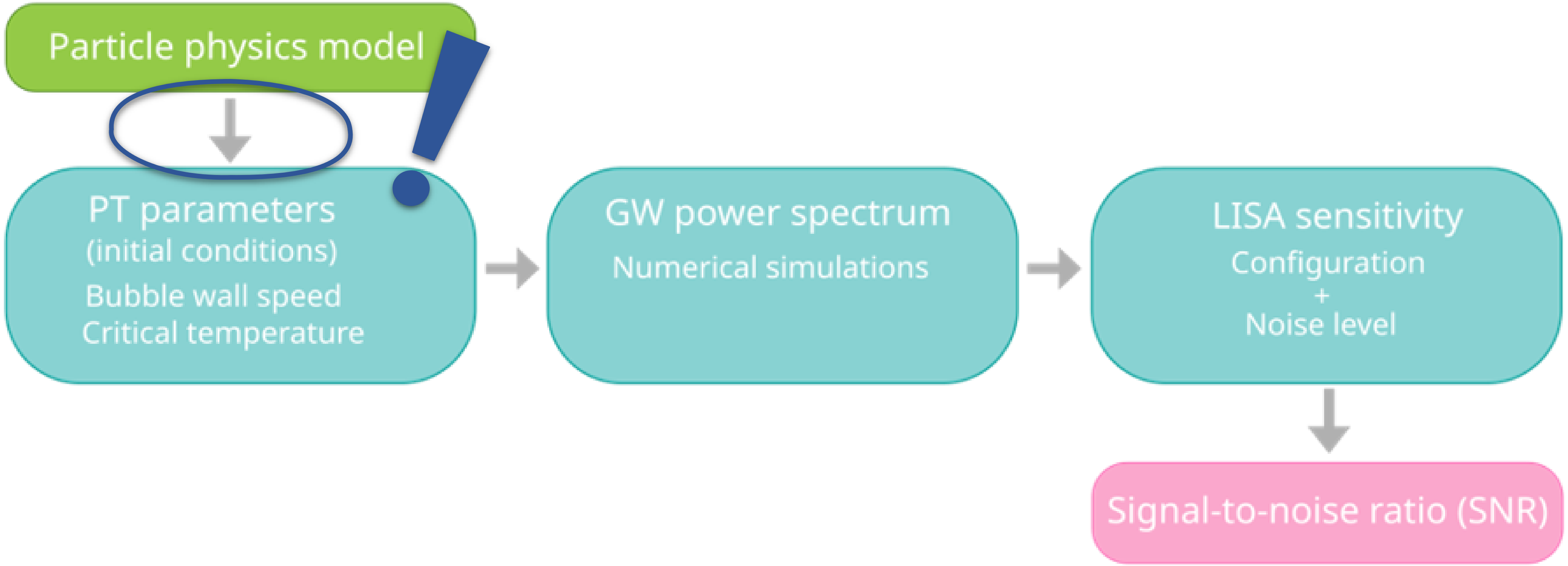

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The Real Scalar Theory

Gould, arXiv:2101.05528

- Toy model possessing key features of BSM models
  - Potential has a tree-level barrier
  - Strong phase transition
- Dimensional reduction 4D cont → 3D cont → 3D lattice (imaginary time, high temp)

$$S_{\text{lat}} = \sum_x a^3 \left[-\frac{1}{2} Z_\phi \phi_x (\nabla^2_{\text{lat}} \phi)_x + \sigma_{\text{lat}} \phi_x + \frac{1}{2} Z_\phi Z_m m^2_{\text{lat}} \phi_x^2 + \frac{1}{3!} g_{\text{lat}} \phi_x^3 + \frac{1}{4!} Z_\phi^2 \lambda_{\text{lat}} \phi_x^4 \right]$$

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See Oliver Gould's talk!

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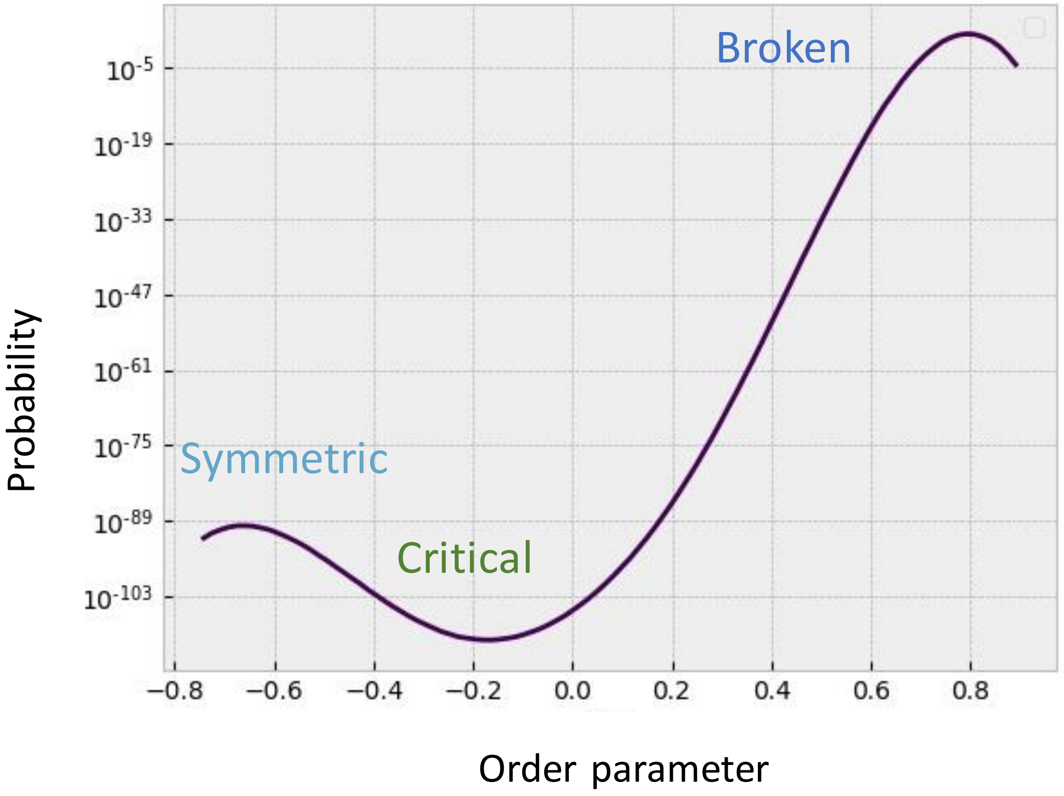
Bubble nucleation, nonperturbatively

1. Pick an order parameter that behaves differently in the two phases
2. Simulate the probability of being in the critical bubble configuration
3. Perform real time evolution to determine whether the critical bubble tunnels or not
4. Calculate the total nucleation rate, dynamical prefactor × probability info

Bubble nucleation, nonperturbatively



3

Perform real time evolution to determine whether the critical bubble tunnels or not

4

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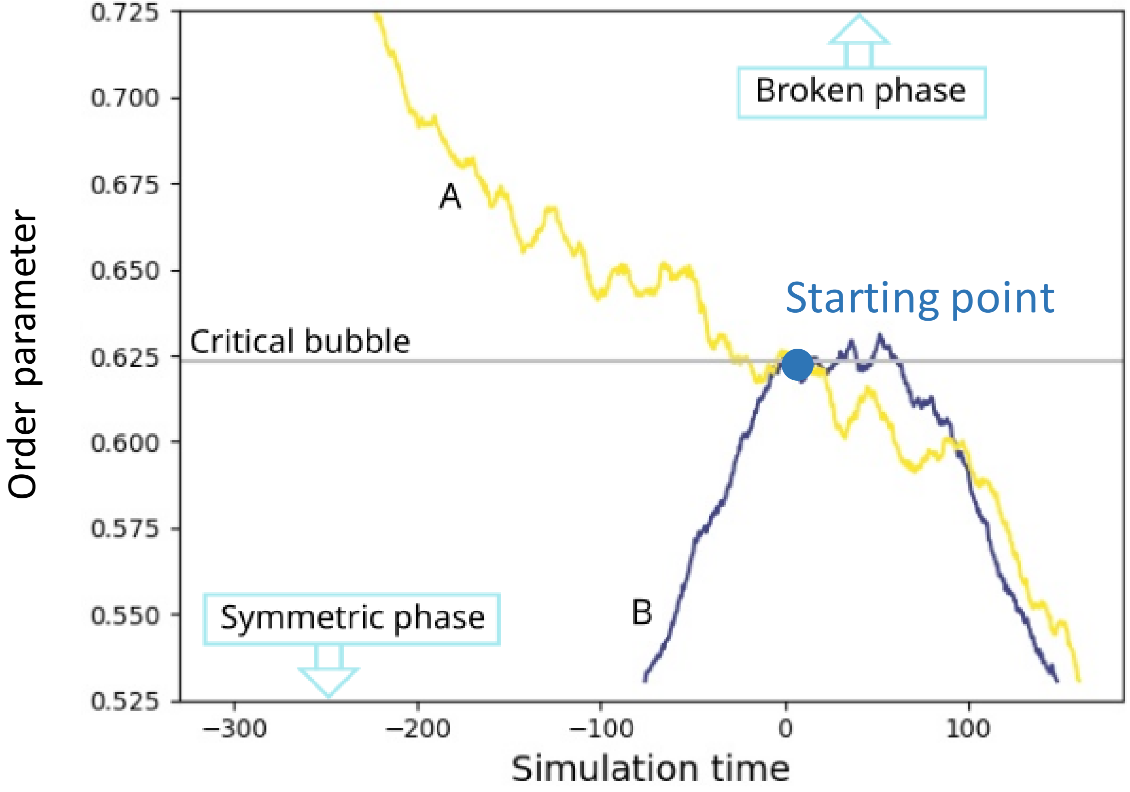
Bubble nucleation, nonperturbatively



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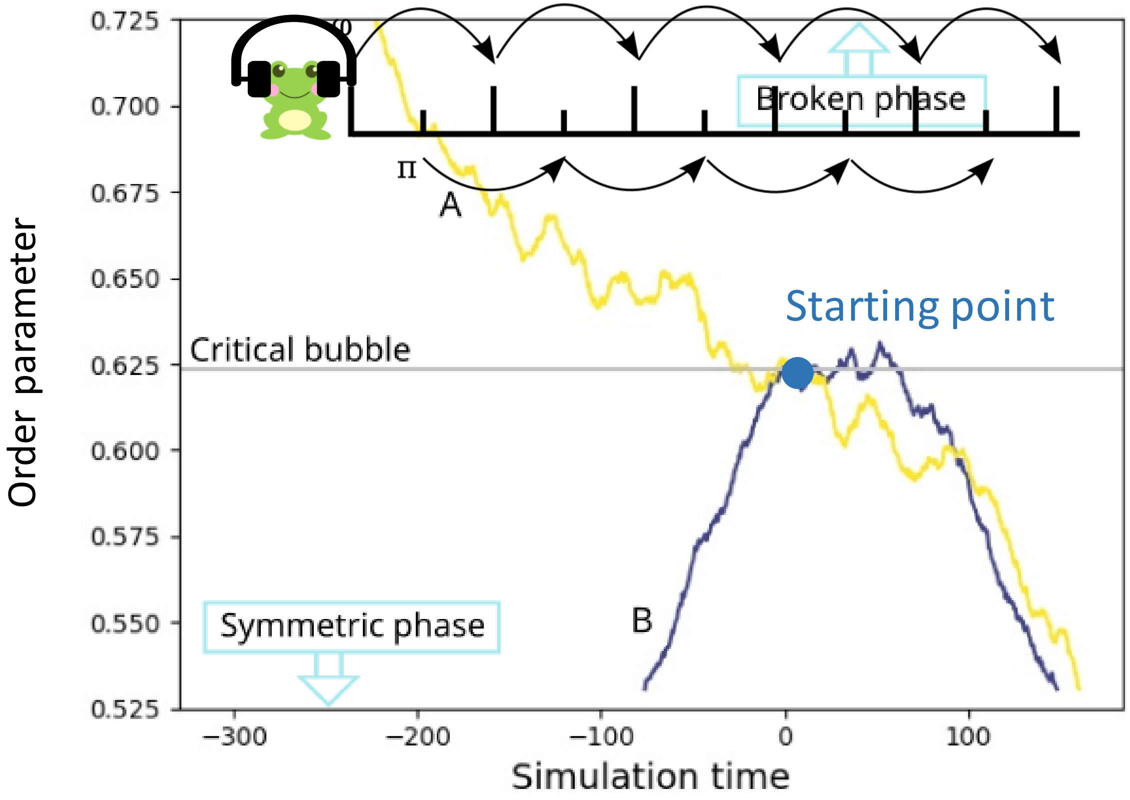
Bubble nucleation, nonperturbatively

Stochastic Hamiltonian eqs + Langevin



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Bubble nucleation, nonperturbatively

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Pick an order parameter that behaves differently in the two phases

$$\Gamma V = \frac{1}{2} P_C^{\epsilon} \left\langle \left| \frac{\Delta\theta(\alpha)}{\Delta t} \right| \times \mathbf{d}^{\alpha} \right\rangle$$

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Calculate the total nucleation rate, dynamical prefactor × probability info

Bubble nucleation, nonperturbatively

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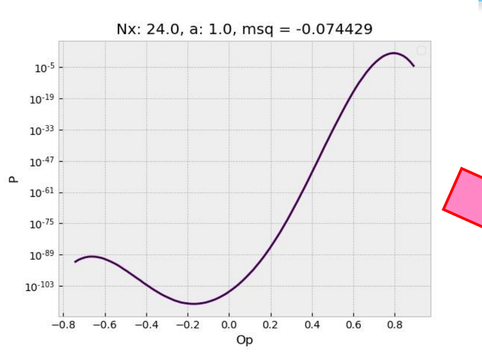

Flux, rate of change of op

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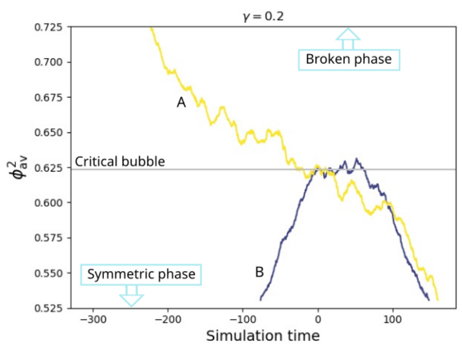
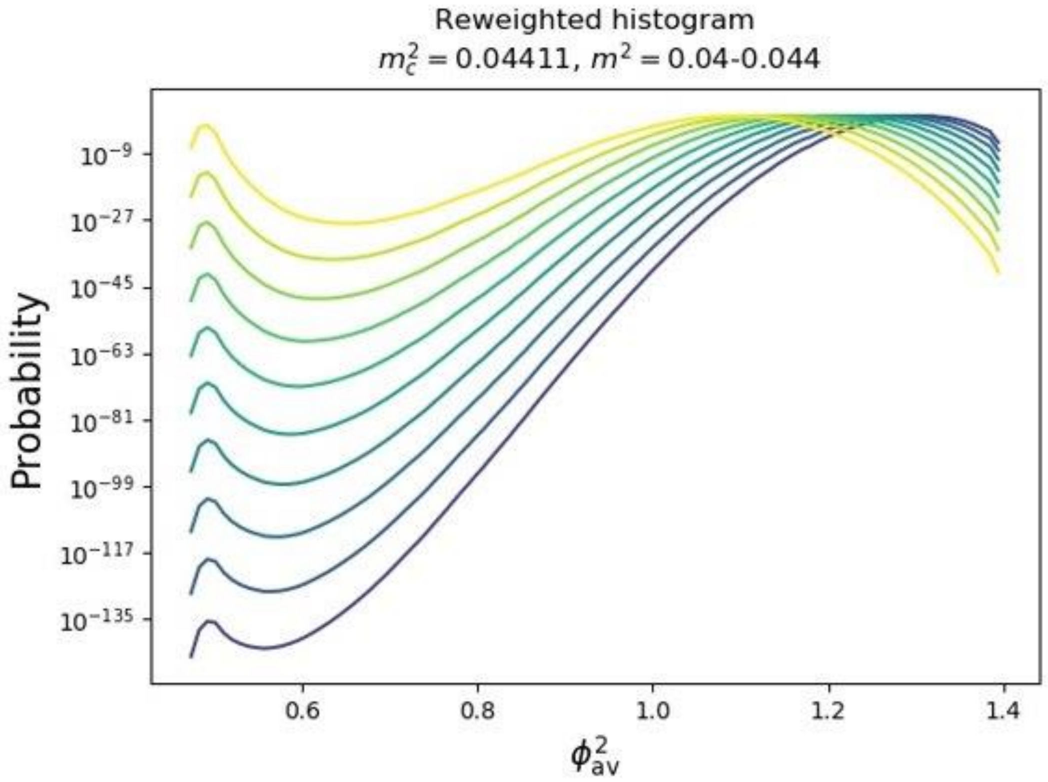


Reweighting



- Simulations are computationally expensive → use reweighting the order parameter histogram at different parameter points

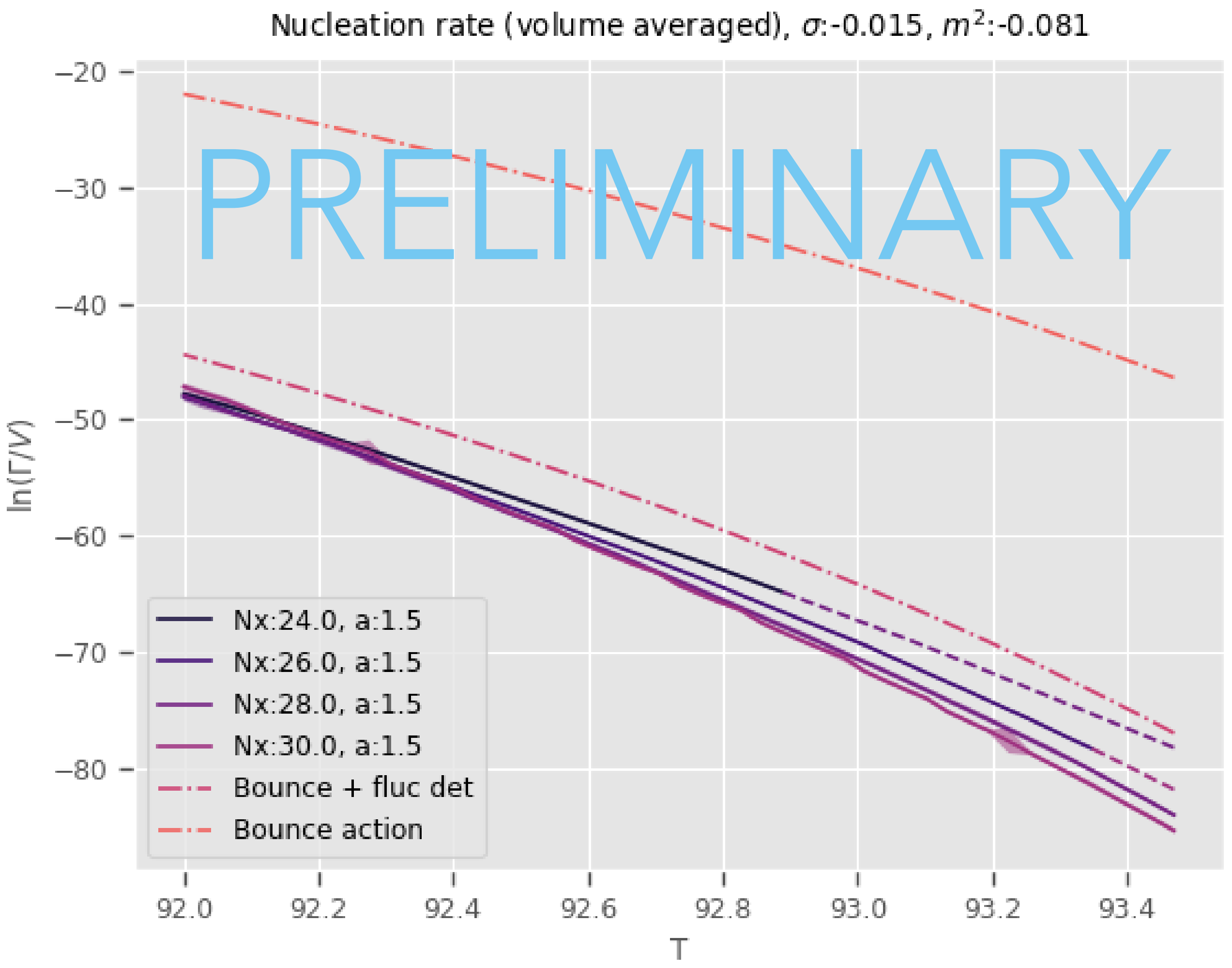
- In our case we reweight in two parameters

$$P_{m_2^2}(\phi_{\mathrm{av}}^2) \propto \exp\left[-\frac{V}{2}(m_1^2 - m_2^2)\phi_{\mathrm{av}}^2\right] P_{m_1^2}(\phi_{\mathrm{av}}^2)$$

Results

Volume averaged nucleation rate vs. the perturbative calculation results as a function of temperature T (GeV)

Lattice spacing fixed, varying physical volume



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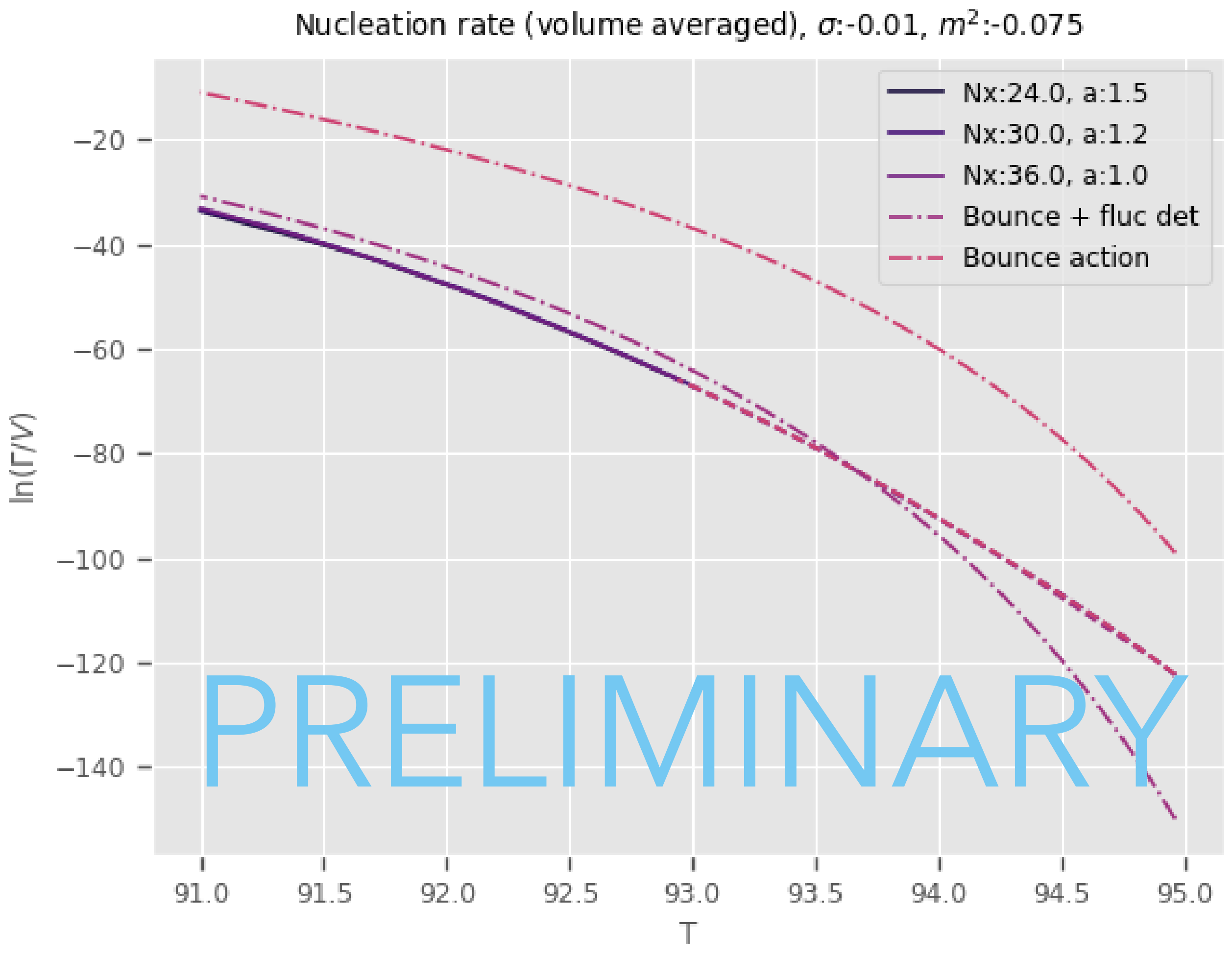
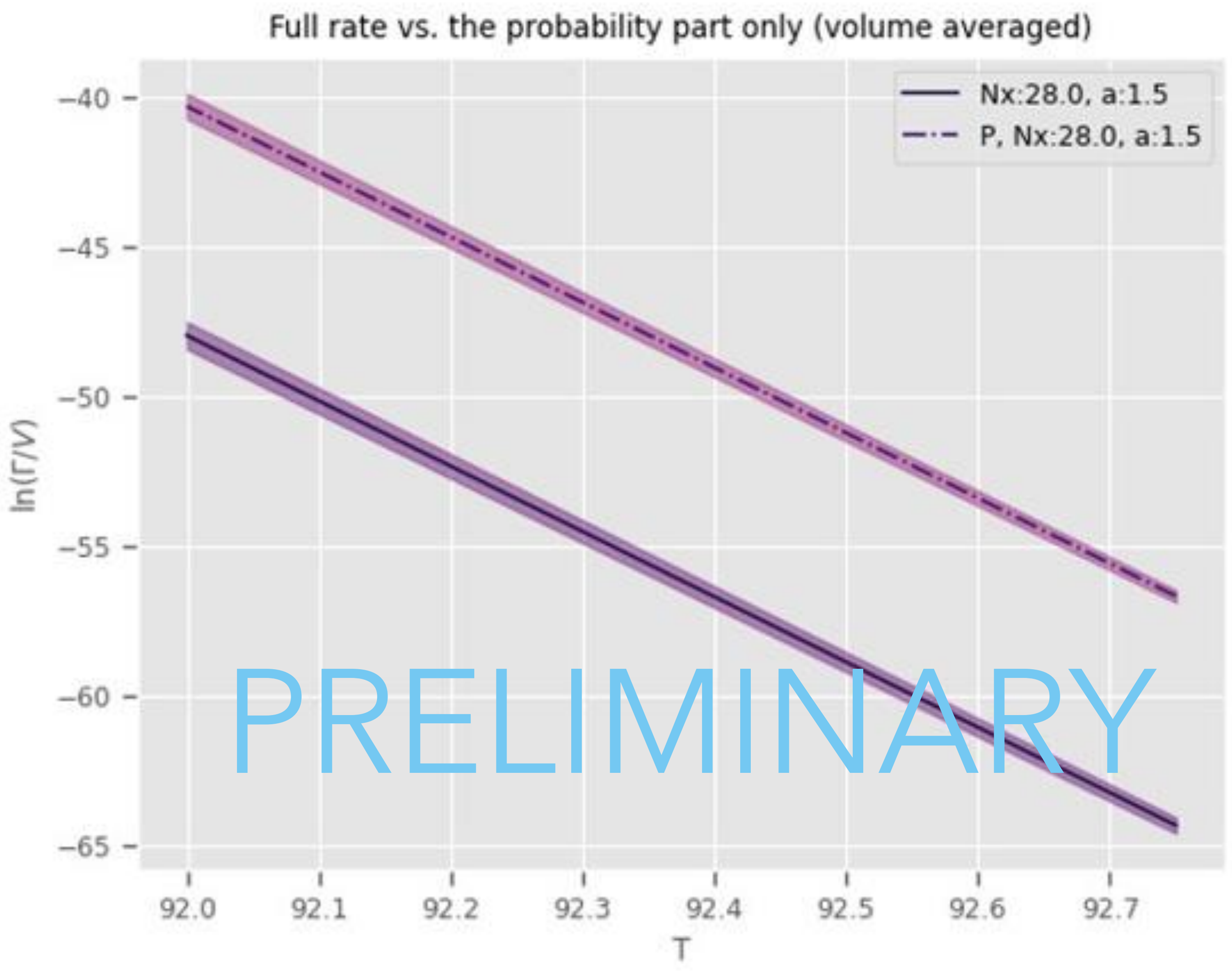
Keeping physical volume at 36^3



Results

Volume averaged probability part only vs. full volume averaged nucleation rate with the prefactor as a function of temperature T



Why does this matter?

- Allows us to calibrate the uncertainty in PT parameters when obtained from perturbative results
- Our simulations show us a suppression of the nucleation rate by a factor of 20 compared to the one loop estimate
- Accurate computations of the nucleation rate are crucial for calculating e.g. the GW power spectrum
- Method and results can be applied to other theories

One-bubble takeaway

There can be large uncertainties in nucleation rates calculated from the bounce action