University of Stavanger

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Complex potential at T>0

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+ HotQCD COLLABORATION

The IIIrd Nordic Lattice Meeting 2023

PHYS. REV. D 105, 054513 + UPCOMING STUDY

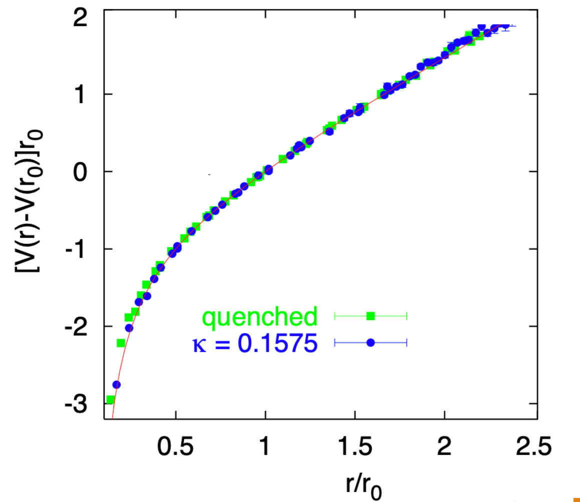
Introduction

Bound states of static quark and anti-quark pair: Probe for existence of Quark Gluon plasma in Heavy ion collisions (A Rothkopf, Physics Reports, Volume 858,2020,).

- Time evolution in Real-Time suffers from sign problem.
- If separation of scales is present: Use EFTs (NRQCD and pNRQCD): describe physics in form of potential?
- Potential corresponds to the Wilson coefficient of pNRQCD.

At T=0 Schrodinger like potential picture has been confirmed (G. Bali Phys.Rept. 343 (2001) 1-136).

$$i\ \partial_t W_\square(t,r) = \Phi(t,r) W_\square(t,r)$$
$$V(r) = \lim_{t\to\infty} \Phi(t,r)$$



Introduction

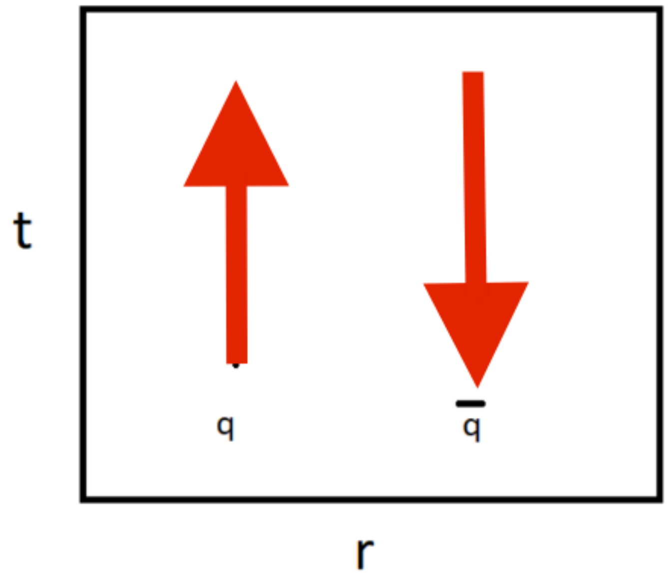
- It remains to be seen if the potential picture even holds for T>0, if it does how does the form need to be modified?
- In HTL regime there exists a complex potential with screened real part (M.Laine et. al JHEP 03 (2007), 054).
- Spectral function is a link between real and imaginary time: (A Rothkopf ,T Hatsuda, S Sasaki Phys.Rev.Lett. 108 (2012) 162001).

$$W_{\square}(r,t) = \int d\omega e^{i\omega t}\rho_{\square}(r,\omega) \quad \Longleftrightarrow \quad W_{\square}(r,\tau) = \int d\omega e^{-\omega\tau}\rho_{\square}(r,\omega)$$

- Computation of Spectral Function : Ill-posed Inverse problem
- Potential linked to the dominant peak position and width of Spectral Function (Y.Burnier and A. Rothkopf *Phys.Rev.D* 86 (2012) 051503)

Quenched Lattices

- Study on $32^3 \times N_\tau$ Lattices (A.Rothkopf , Y Burnier *Phys.Rev.D* 95 (2017) 5, 054511).
- Naive Wilson action used on anisotropic lattices $a_s = 0.097fm$. $a_s = 4a_\tau$ with $\beta = 6.1$ and T=0.78-1.4 Tc.
- Measure Wilson Line Correlators in Coulomb gauge.
- Bayesian BR method used for spectral reconstruction.



Spectral Function Extraction using Bayesian Method

$$P[\rho|D,I] \propto P[D|\rho,I]P[\rho|I] = \exp[-L + \alpha S_{\mathrm{BR}}]$$

L is the usual quadratic distance used in chi-square fitting.

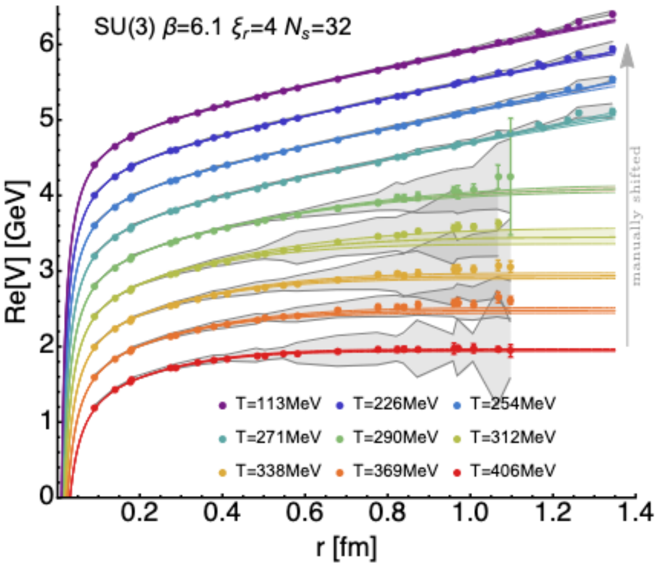
The prior probability $P(\rho|I) = \exp(\alpha S_{BR})$ acts as a regulator

$$\mathrm{S}_{\mathrm{BR}} = \int d\omega \big(1 - \tfrac{\rho(\omega)}{m(\omega)} + \log\big[\tfrac{\rho(\omega)}{m(\omega)}\big]\big).$$

- Look for the most probably spectrum by locating the extremum of the posterior $\frac{\delta}{\delta\rho}(L - \alpha S_{BR}) = 0.$

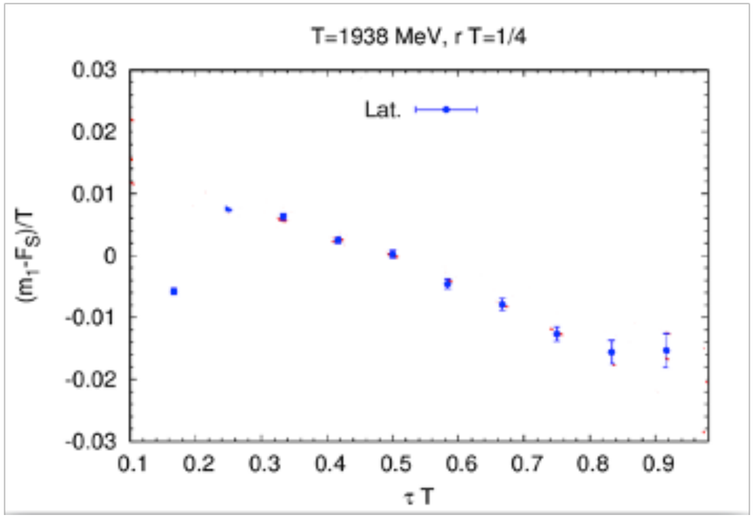
Quenched Lattices

- Study shows presence of a potential with a screened real part.
- Bayesian BR method used for spectral reconstruction.
- Potential is manually shifted.

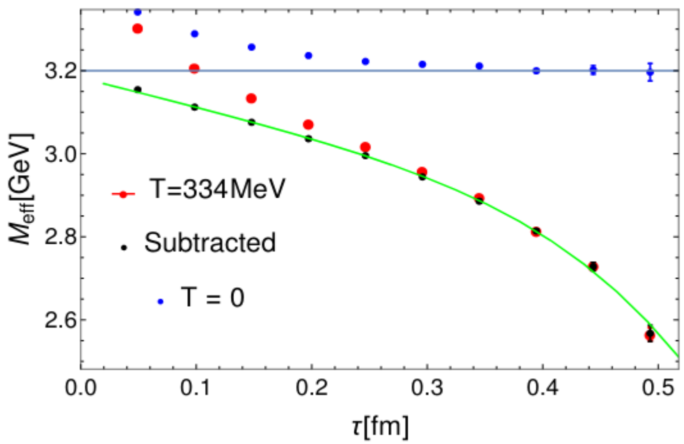


Study on Unquenched Lattices

- (2+1)-flavour QCD - configurations generated by HotQCD and TUMQCD collaborations.
- Using highly improved staggered quark (HISQ) action.
- Fix box approach; temp range - 140MeV to 2GeV.
- Non-monotonicity in Effective Masses; Non-positive spectral function.
- BR method NOT applicable.



Spectral Function Model Fits



- Subtracting UV part using T=0 correlator results in linear behavior in m_{eff} consistent with gaussian peak.
- Parametrize Correlator as: $C_{sub} \approx \exp(-\Omega\tau + \frac{1}{2}\Gamma^2\tau^2) + \exp(-\omega^{cut}\tau)$

$$\rho_r(\omega, T) \quad = \quad A(T)\exp\left(-\frac{[\omega - \Omega(T)]^2}{2\Gamma^2(T)}\right) + A^{\mathrm{cut}}(T)\,\delta\left(\omega - \omega^{\mathrm{cut}}(T)\right)$$

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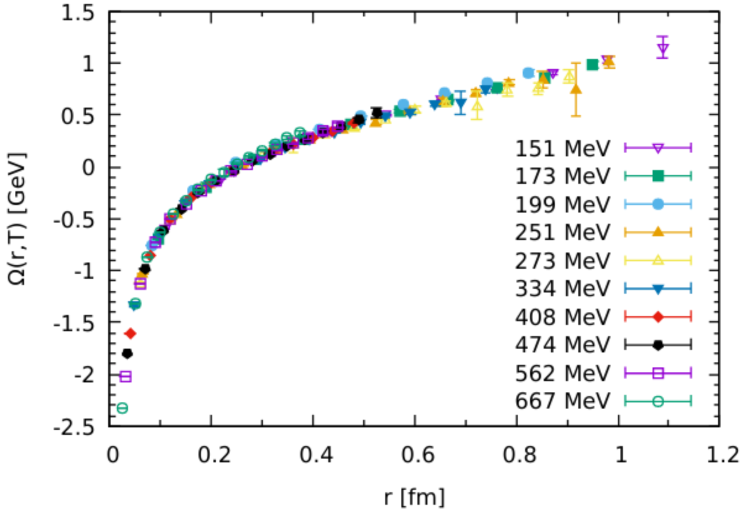

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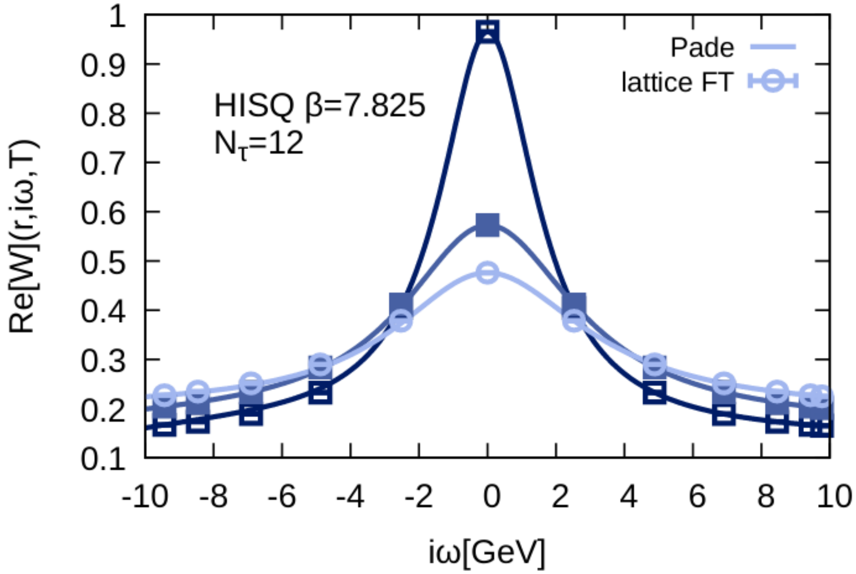
Pade' Interpolation

- Transform the Euclidean correlator into Matsubara frequency space.
- Implement Pade approximation in the form of continued fraction according to Schlessinger prescription (L. Schlessinger, Phys. Rev. 167, 1411 (1968)).
- This is interpolation of data and not fitting. Does not require minimization.
- Obtain pole structure from rational function: Directly related to the peak position (Ω) and width (Γ).



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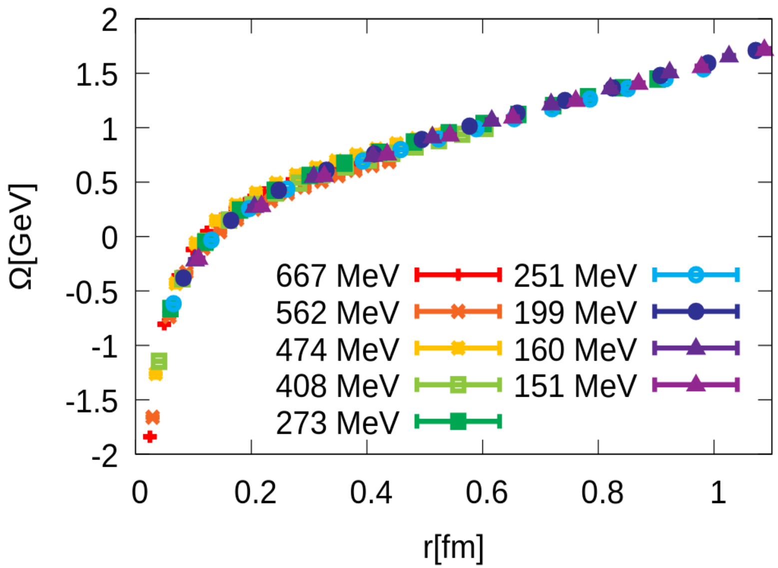
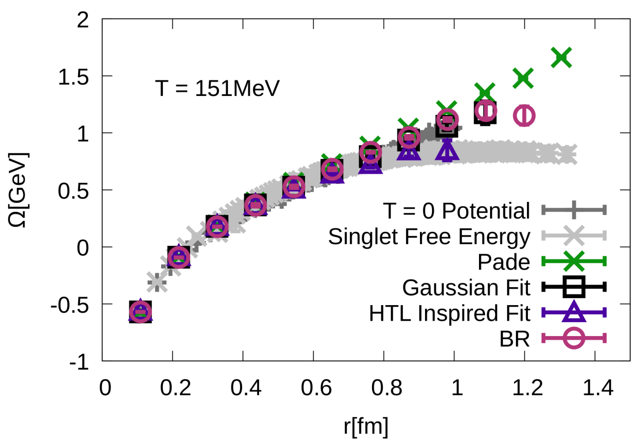
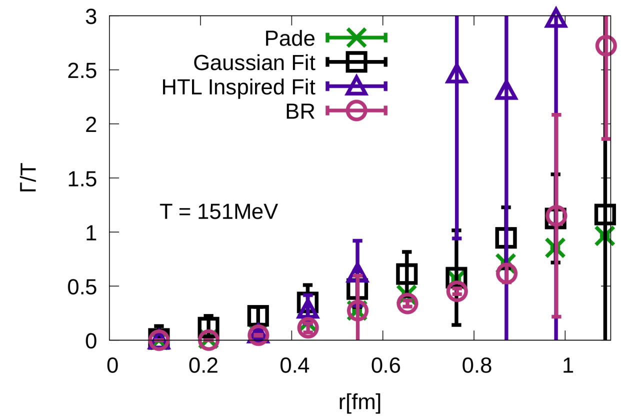
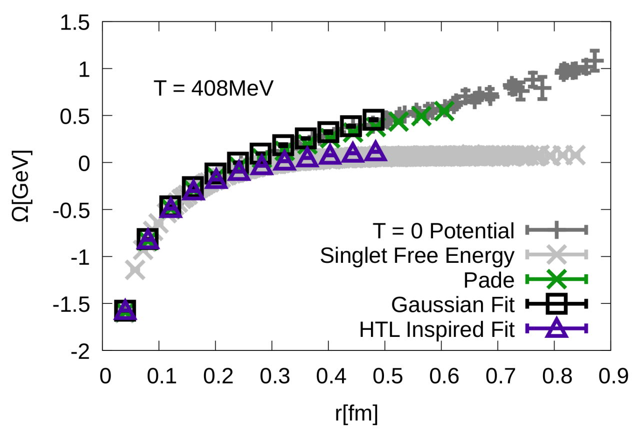
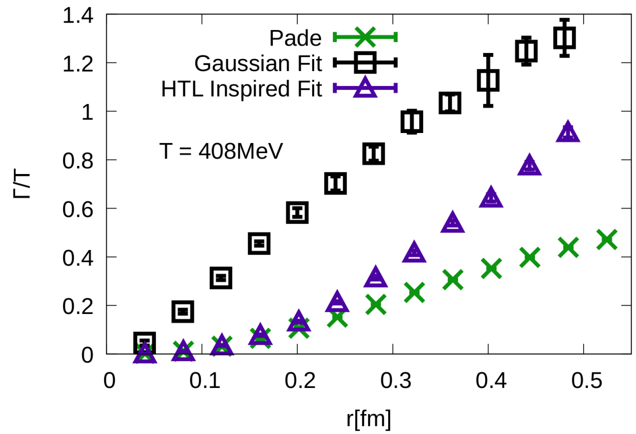


Comparison of Results









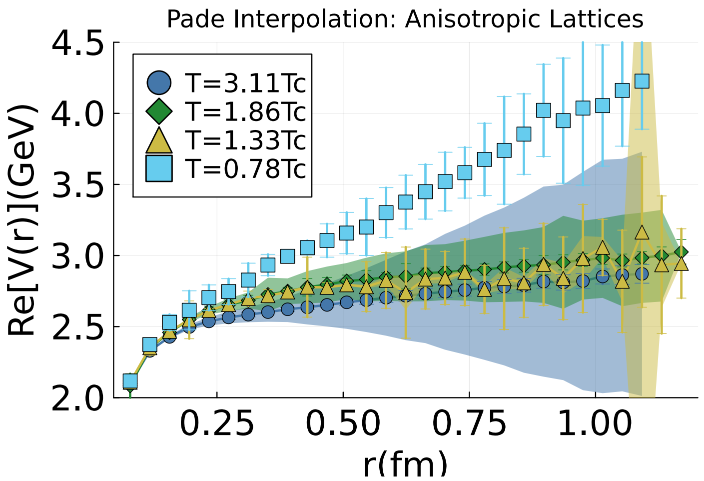
Quenched Lattices again

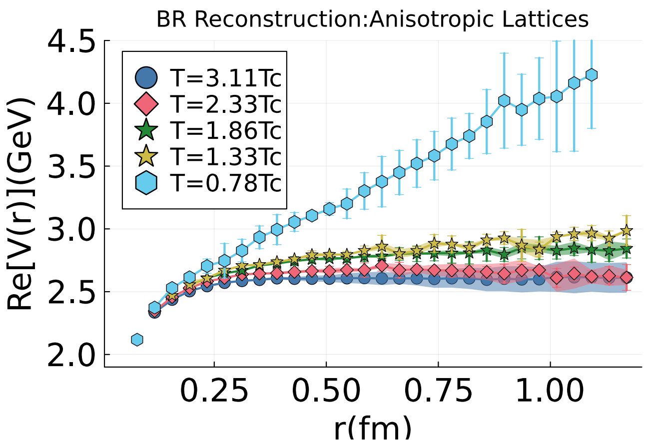
- We see that from the Gaussian fits and Pade' peak position (Ω) for HISQ is temperature independent:; Quite puzzling.
- New results different from previous studies of Quenched Lattices and full QCD:; Different methods used.
- Need further investigation:: Check new methods with Bayesian reconstruction.

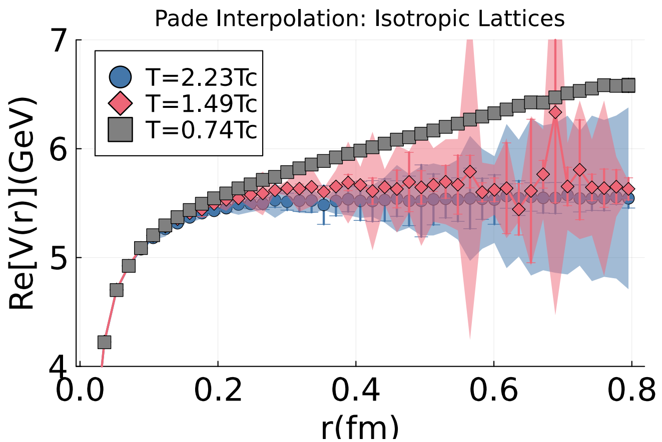
Lattice Setup

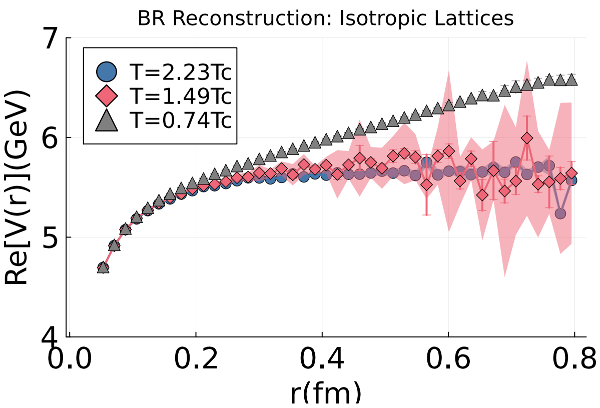
- Use anisotropic lattices: $64^3 \times N_\tau$ lattices using Wilson action.
- $a_s = 4a_\tau = 0.039fm$
- $N_\tau = 24 - 96$; scan temperature range of 0.78Tc-3.11 Tc.
- Gauge Fix to Coulomb Gauge and measure Wilson Lines using SimulateQCD code.
- Some isotropic lattices: $96^3 \times N_\tau$.
- $N_\tau = $ 16,24,48 (T=2.23,1.49,0.74 Tc).

Preliminary Results





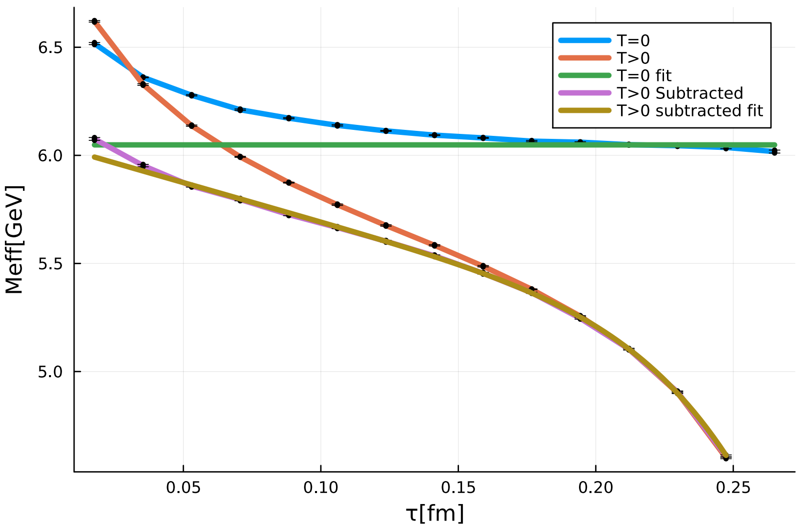




Model fits?

- Small τ behavior is not temperature independent.
- Interpretation of continuum subtraction is still unclear.
- Linear behavior is still observed.
- Fits are a work in progress.

$$T = 2.23Tc(N_\tau = 16)$$



Summary

- Spectral functions of Wilson line correlators encode the real and imaginary part of the complex potential between static quark-antiquark pairs.
- Analysis of spectral structure can be done with different methods.
- We see that from the Gaussian fits and Pade' peak position (Ω) is temperature independent in HISQ lattices: Results puzzling and different from studies in Quenched Lattices.
- Preliminary analysis on Quenched lattices suggests a screened potential.
- Efforts to carry out model fits on Quenched lattices still ongoing.

HTL inspired fits

Peak position (Ω) and width (Γ) interpreted as the real and imaginary part of thermal static energy Es (D. Bala and S. Datta, Phys. Rev. D 101, 034507(2020)).

$$E_s(r,T) = \lim_{t\to\infty} i\frac{\partial \log W(r,t,T)}{\partial t} = \Omega(r,T) - i\Gamma(r,T).$$

W (r, t, T) is the Fourier transform of the spectral function ρ_r (r, ω,T)

$$m_{eff}(r, n_\tau = \tau/a)a = \log\left(\frac{W(r,n_\tau,N_\tau)}{W(r,n_\tau+1,N_\tau)}\right)$$

$$= \Omega(r,T)\,a - \frac{\Gamma(r,T)aN_\tau}{\pi}\,\log\left[\frac{\sin(\pi n_\tau/N_\tau)}{\sin(\pi(n_\tau+1)/N_\tau)}\right]$$