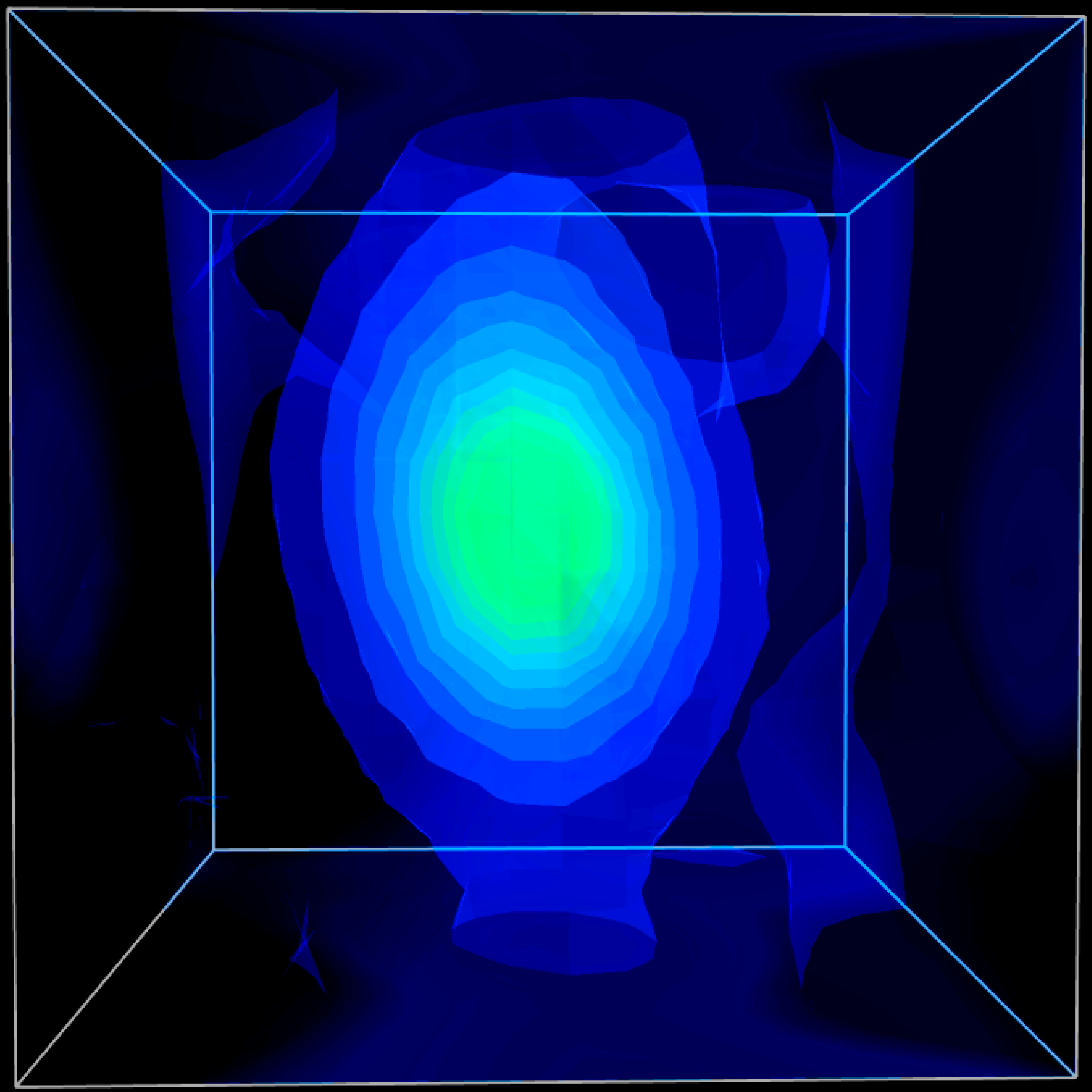
Electroweak Sphaleron in a magnetic field



JA, Kari Rummukainen [arxiv:2301.08626]

Contents:

- Why is sphaleron in a magnetic field interesting:
    1. Sphaleron has a magnetic dipole moment
    2. electroweak ‘phase transition’ gets modified in a magnetic field
- Measuring the sphaleron rate in a magnetic field
- Results

Electroweak Sphaleron

- EW chiral anomaly leads to non-conservation of baryon and lepton number:

$$3\Delta N_{CS} = \Delta B = \Delta L$$

- Chern-Simons numbers:

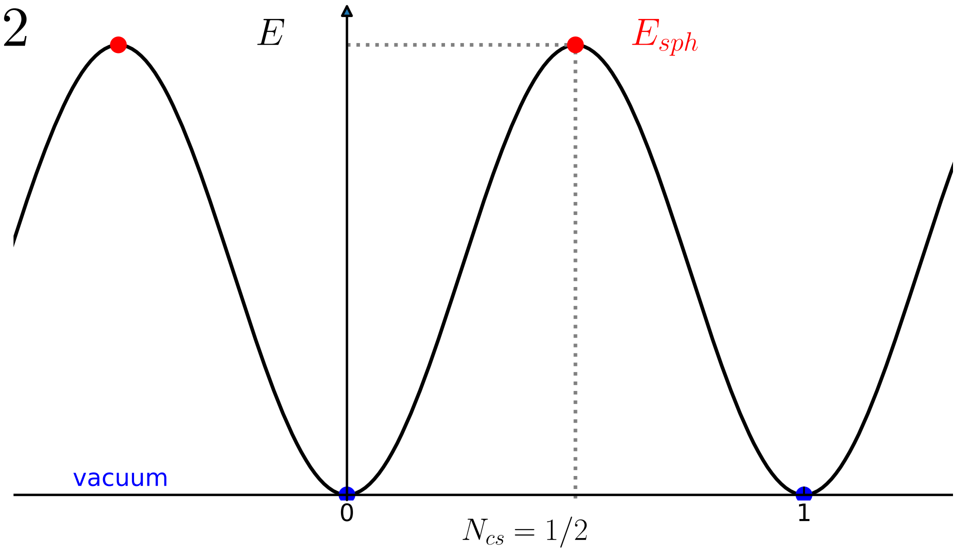
$$N_{\mathrm{CS}}(t) \equiv N_{\mathrm{CS}}^{W}(t) - N_{\mathrm{CS}}^{Y}(t)$$

- U(1): $N_{\mathrm{CS}}^{Y}(t) \equiv \frac{g'^2}{32\pi^2} \int_0^t \mathrm{d}t \int \mathrm{d}^3x \epsilon_{\alpha\beta\gamma\delta} B^{\alpha\beta} B^{\gamma\delta}$
  - Trivial in vacuum. Identical to zero.
- SU(2): $N_{\mathrm{CS}}^{W}(t) \equiv \frac{g^2}{32\pi^2} \int_0^t \mathrm{d}t \int \mathrm{d}^3x \epsilon_{\alpha\beta\gamma\delta} F^{\alpha\beta} F^{\gamma\delta}$
  - Non-trivial.

Sphaleron arises from non-trivial topology of SU(2)

- Infinitely many classically equivalent but topologically different vacua.
- **Sphaleron**: finite energy solution of classical EoMs separating two topologically distinct vacua.
- In vacuum: $N_{\rm CS}^{W}(t) = \text{integer}$
- At Sphaleron: $N_{\rm CS}^{W}(t) = \text{integer} + \frac{1}{2}$

$$3\Delta N_{CS} = \Delta B = \Delta L$$



Sphaleron rate

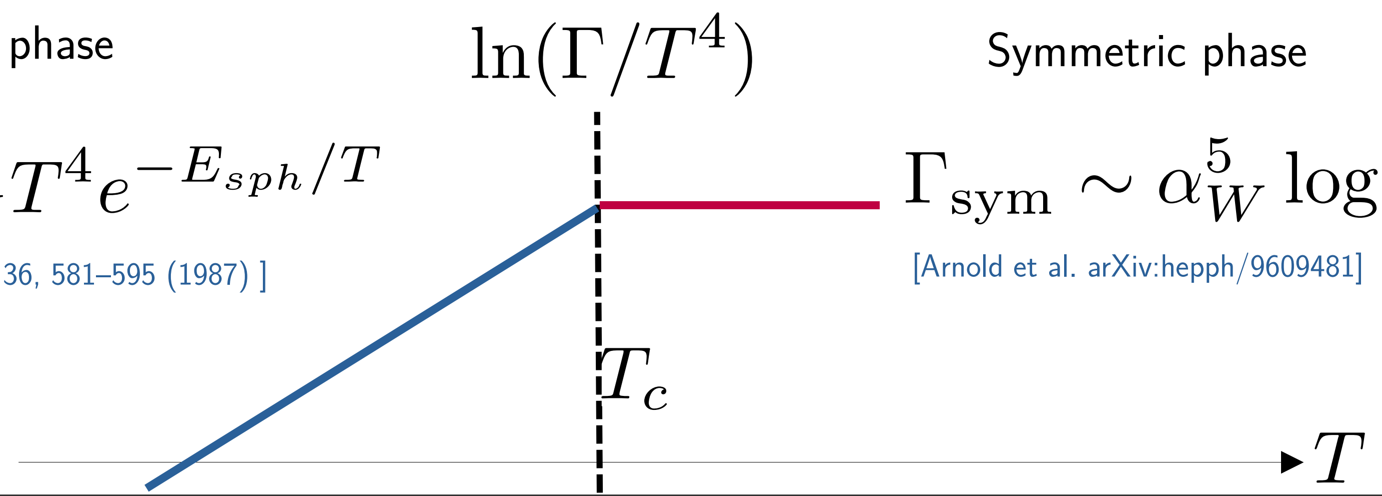
$$\Gamma = \lim_{V,t\to\infty} \frac{\langle N_{\mathrm{CS}}(t)^2 \rangle}{Vt}$$

- How fast sphaleron transitions are happening
- How the rate behaves through the transition is important for Baryo/Lepto-genesis
- Studied extensively in the past without U(1):[D'Onofrio et al. ArXiv:1404.3565, others...]

Broken phase

$$\Gamma_{\mathrm{brk}} \sim \alpha_W^4 T^4 e^{-E_{sph}/T}$$

[Arnold et al. Phys. Rev. D 36, 581–595 (1987)]



Symmetric phase

$$\Gamma_{\mathrm{sym}} \sim \alpha_W^5 \log(\alpha_W^{-1}) T^4$$

[Arnold et al. arXiv:hepph/9609481]

Why is sphaleron in a magnetic field interesting?

- 0. Magnetic fields could have been around during the electroweak phase transition.

1. Electroweak phase transition gets modified

- The details of the electroweak phase transition is changed by an external (hyper)magnetic field.
  - Crossover temperature is shifted to lower temperatures and the "strength" is modified.
  - Higgs expectation value lowered thus sphaleron rate is increased.

- At zero temperature large magnetic fields yield interesting phenomena:
  - Periodic lattice of vortices: Ambjørn-Olesen phase.

[Ambjørn & Olesen, Phys. Lett. B214, 565–569 (1988)]
[Chernodub et al. , 2206.14008]

- Have not been seen in finite T

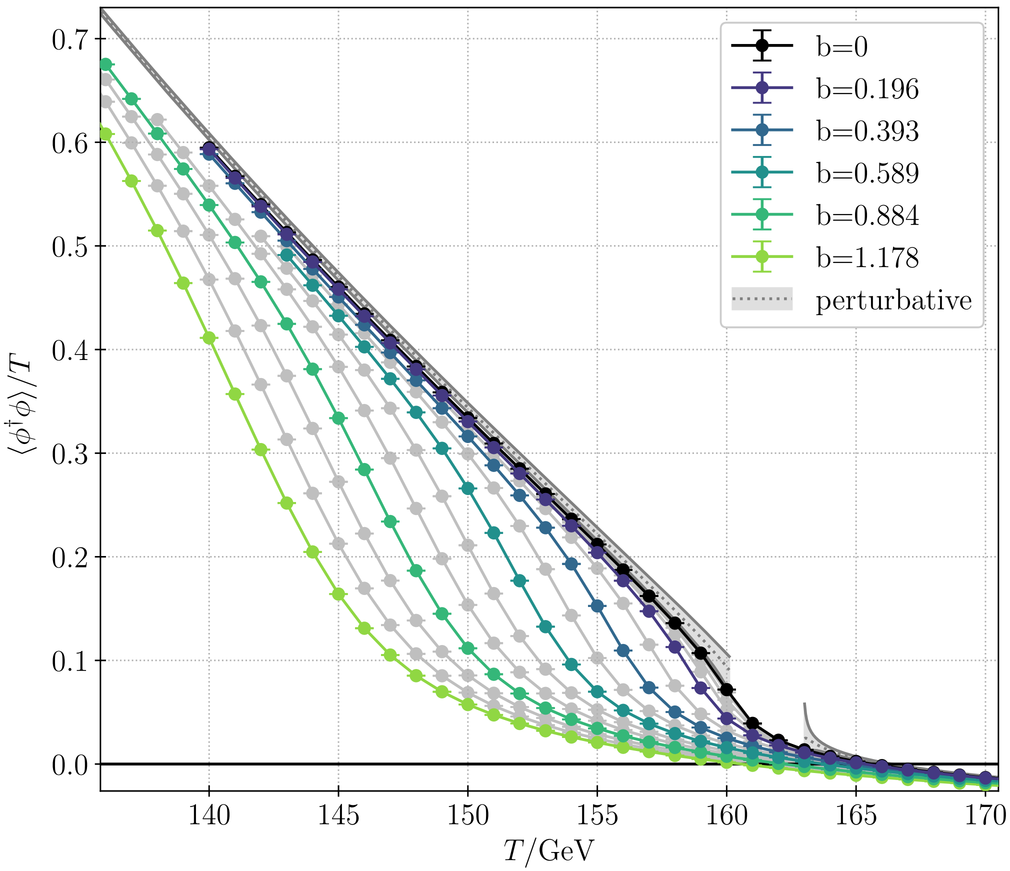
[Kajantie et al. , hep-lat/9809004]

EW transition shifts to lower temperature

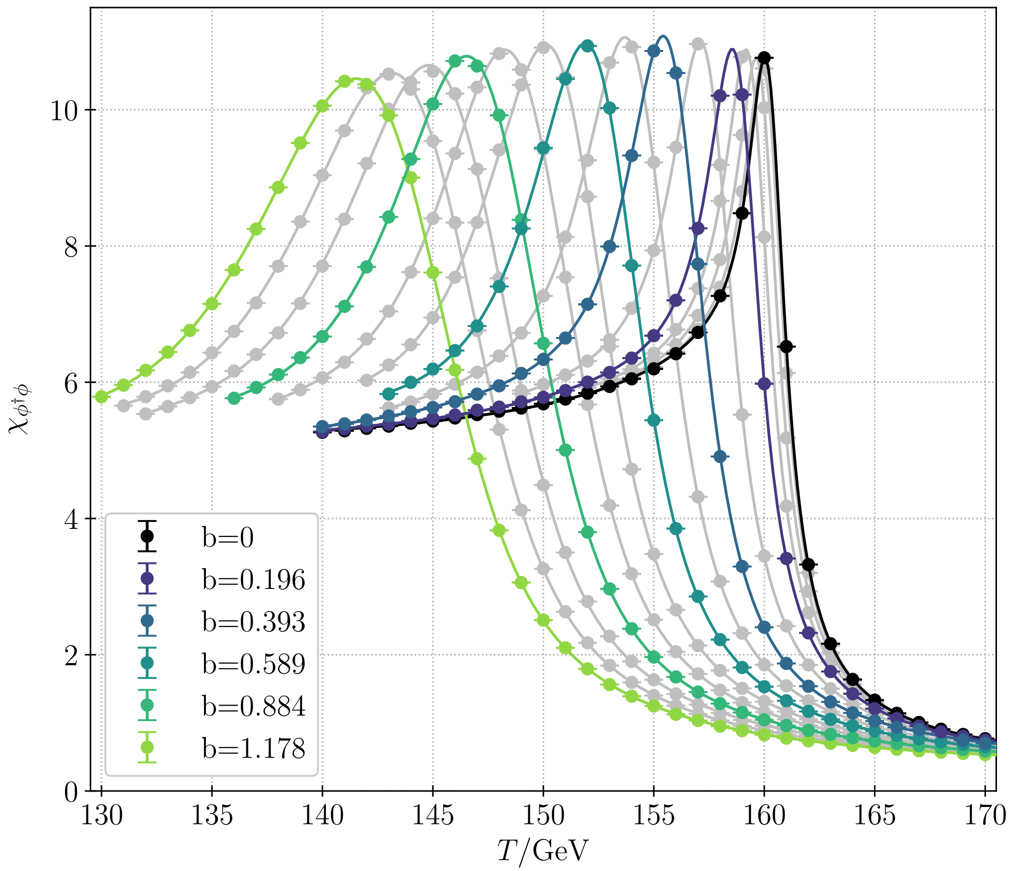
- With increased magnetic field the EW transition shifts to lower temperatures and the transition gets “wider”.

$$B_Y^{4d} \simeq 2bT^2$$

Higgs expectation value

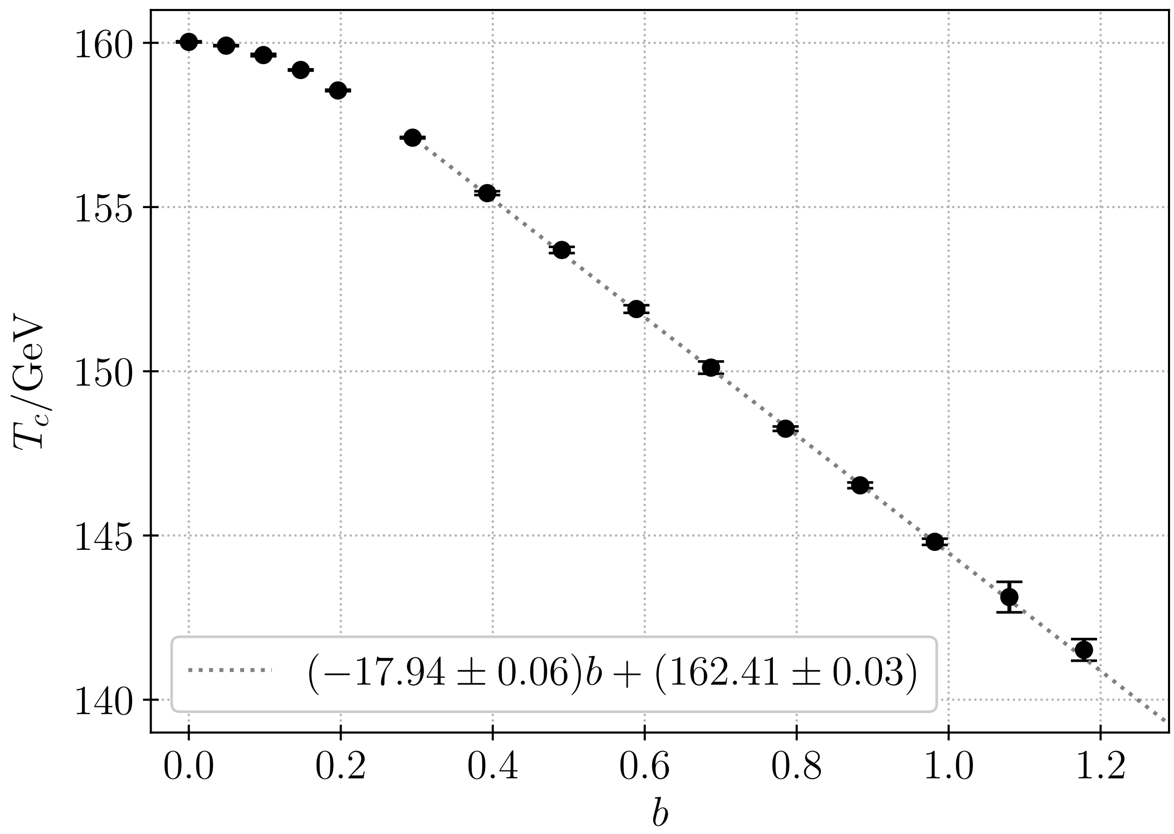


Higgs susceptibility



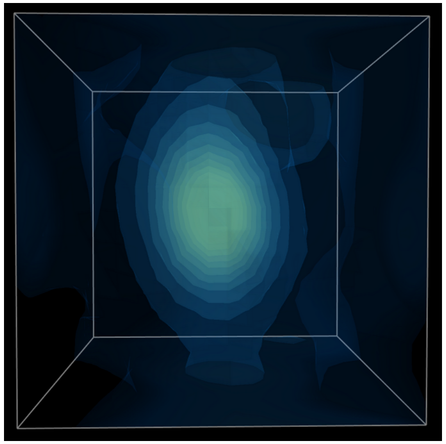
EW transition shifts to lower temperature

- With increased magnetic field the EW transition shifts to lower temperatures and the transition gets “wider”.
- $T_c =$ peak of the Higgs susceptibility

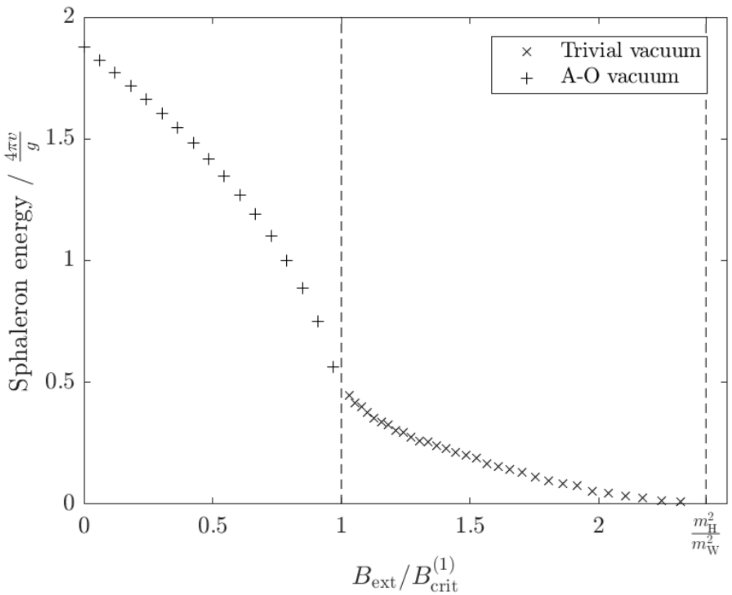


2. Sphaleron has a magnetic dipole moment

- In a magnetic field the energy of the sphaleron can be lowered.
  - For small magnetic fields a dipole interaction is expected: $\Delta E_{\mathrm{sph}} = -B_{\mathrm{ext}} \cdot \mu_{\mathrm{sph}}$
- Sphaleron gets elongated along the magnetic field.



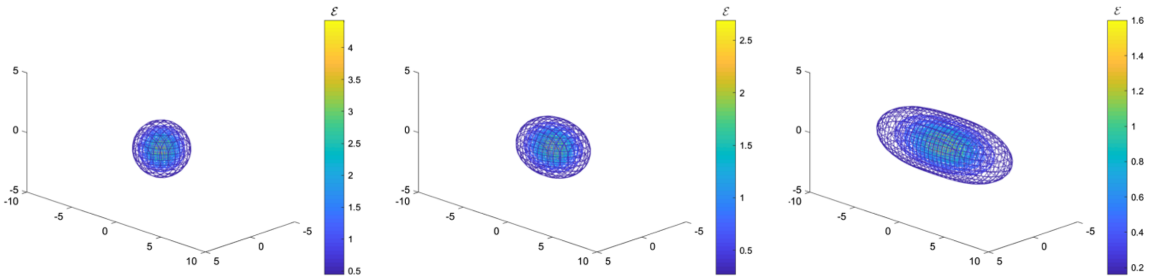
[L.-J. Ho & Rajantie, 2005.03125]:



Change to sphaleron energy at large fields:

Non dynamical simulations at zero temperature:

ELECTROWEAK SPHALERON IN A STRONG MAGNETIC FIELD PHYS. REV. D **102,** 053002 (2020)



3. U(1) CS number with external magnetic field

- With non-zero magnetic field the U(1) CS number also diffuses.
- Not restricted to integers in vacuum like the SU(2) counter part.
  - ↳ Can have any value.
- Can lead to baryon and lepton number change on its own.

[Figueroa et. al., 1707.09967]...

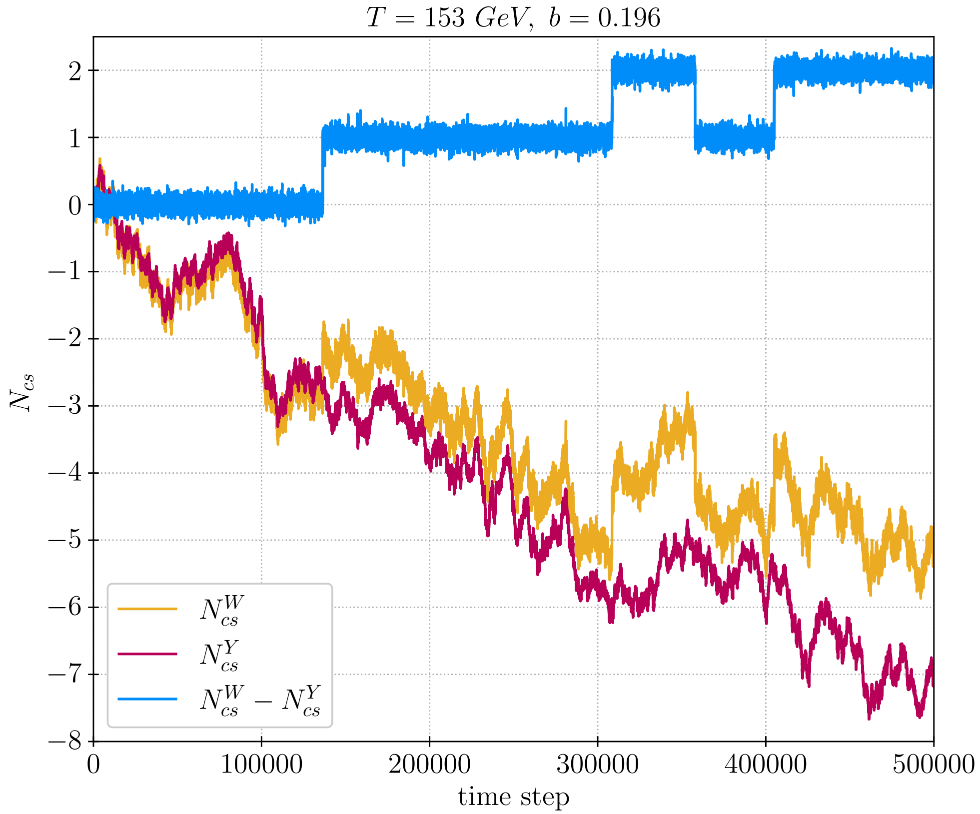
$$N_{\mathrm{CS}}(t) \equiv N_{\mathrm{CS}}^{W}(t) - N_{\mathrm{CS}}^{Y}(t),$$

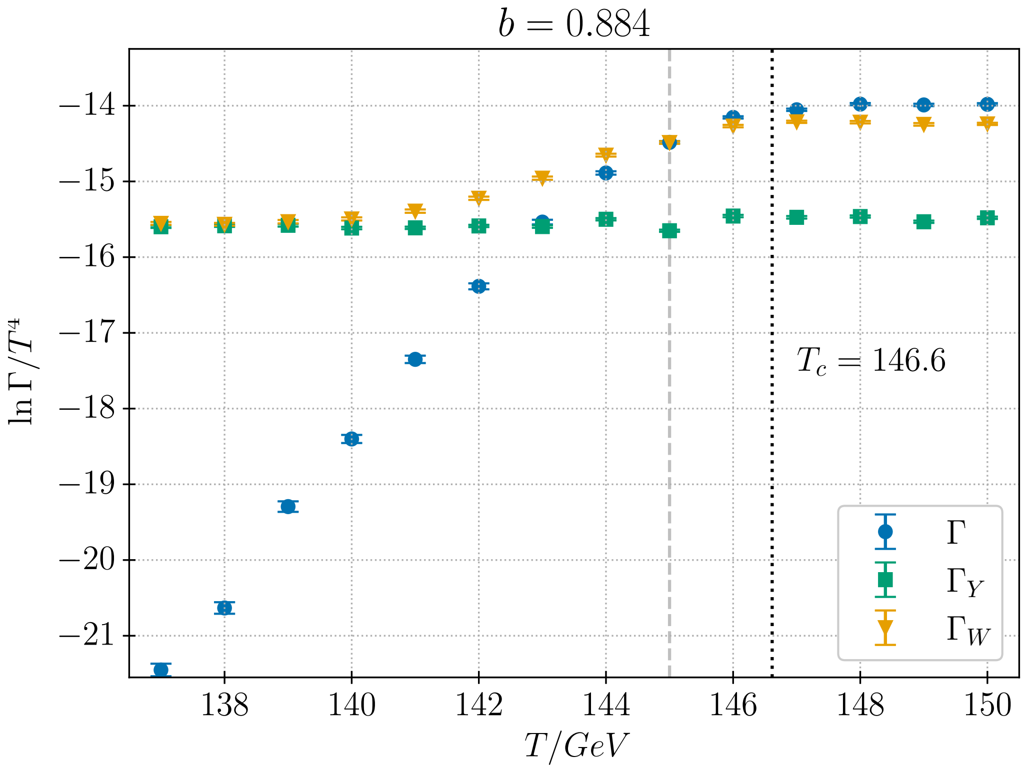
$$3\Delta N_{CS} = \Delta B = \Delta L,$$

$$\Gamma = \lim_{V,t\to\infty} \frac{\langle N_{\mathrm{CS}}(t)^2 \rangle}{Vt}$$

In the broken phase SU(2) and U(1) N_{CS} are not independent

- In the broken SU(2) and U(1) CS numbers are highly correlated, only the physical difference of the two gets suppressed, with non-zero magnetic field present.
- SU(2) Ncs on its own is not a good 'order parameter'





Dynamical lattice simulations of effective theory

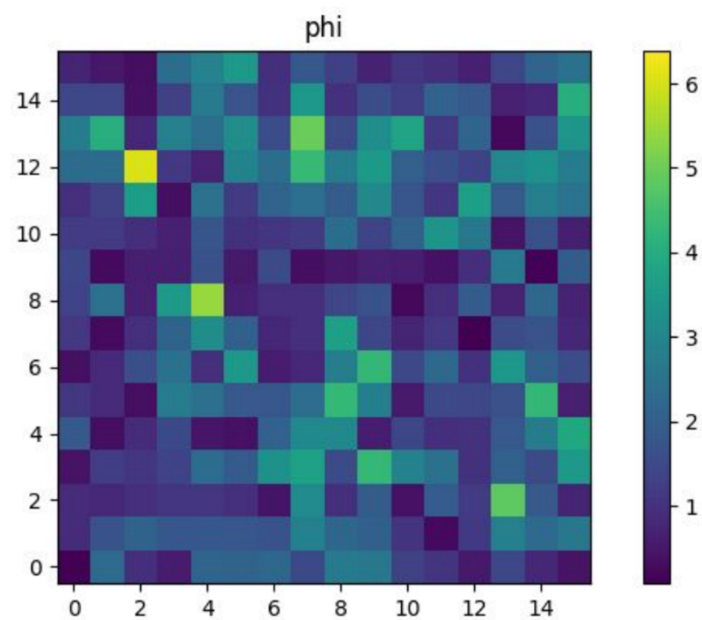
- Effective 3d theory: in finite T get hierarchy of scales $\pi T, gT, g^2T$
  - Temporal and fermionic fields integrated out and we are left with 3d bosonic theory. [Kajantie et. al., hep-ph/9508379]

$$L = \frac{1}{4}F_{ij}F_{ij} + \frac{1}{4}B_{ij}B_{ij} + (D_i\phi)^\dagger(D_i\phi) + m_3^2\phi^\dagger\phi + \lambda_3(\phi^\dagger\phi)^2$$

- Realtime: dynamics of SU(2) soft modes are described by Langevin equation. (In practice use heatbath update.) [Bödeker, arXiv:hep-ph/9801430.]
  - not true for U(1), correct way to describe the its time evolution is not clear. However, in the broken phase should not affect sphaleron rate.

Measuring the sphaleron rate

- Lattice definition of Ncs contains UV noise not related to sphaleron transitions.
- Sphaleron is big in lattice units $\sim 1/(g^2 T)$
- After every time step smooth the fields using gradient flow

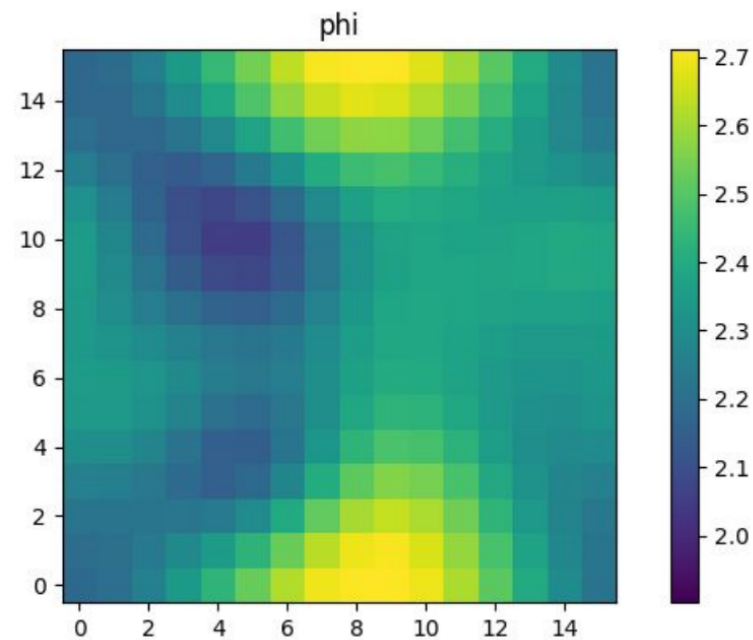


$$\frac{\partial \phi}{\partial \tau} = -\frac{\partial S}{\partial \phi}$$

Integrate flow time t

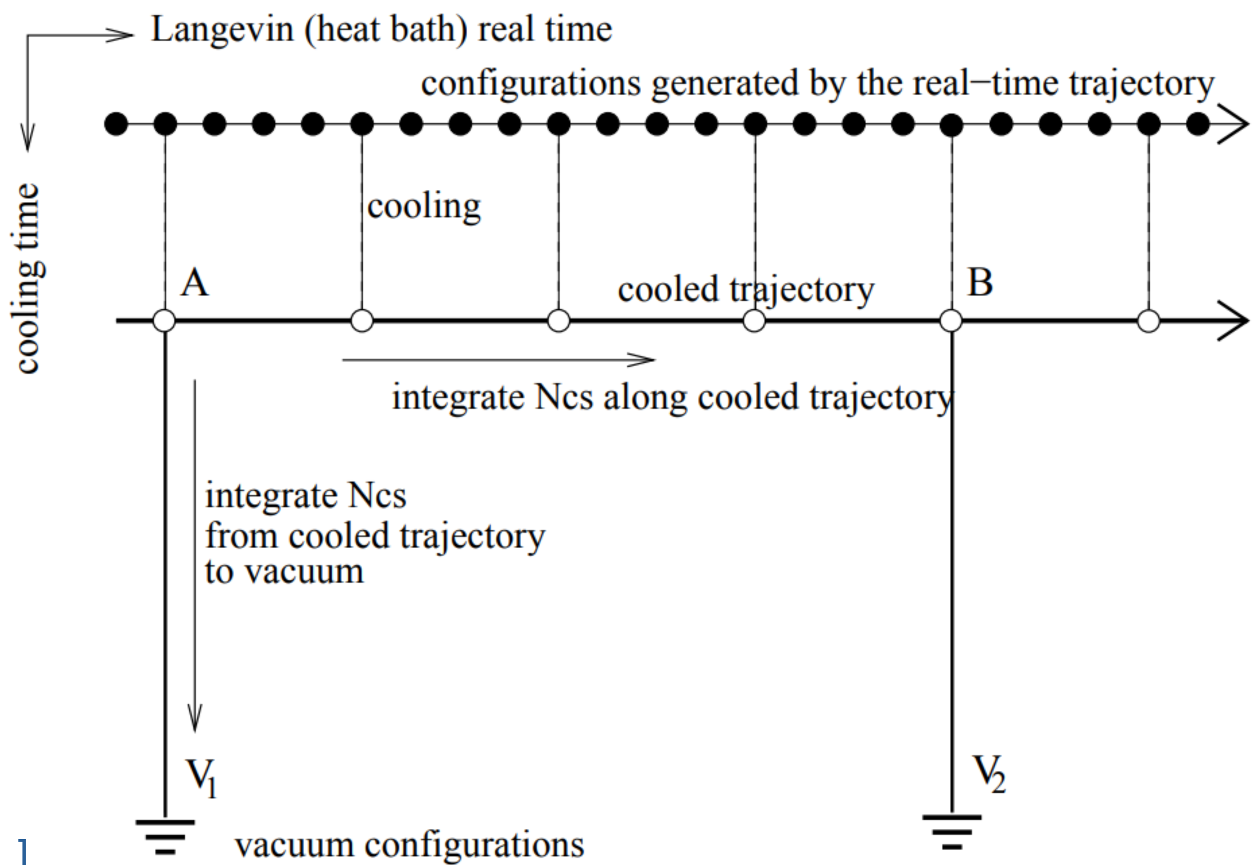
$t = 3$

$r \simeq 4$



Measuring the sphaleron rate

- Track the Ncs from the cooled fields.
- With a magnetic field we also need to cool the U(1) fields and compute its Ncs.



[hep-ph/9805264, ...]

Multicanonical method

- Sample with probability: $P_W \sim e^{-S+W(N_{cs}^W, N_{cs}^Y)}$
  - Weight function W chosen to favor sphalerons
- Run simulation sampling with the weighted distribution to get canonical distribution $P = P_W e^{-W}$

[D'Onofrio et. al., 1207.0685, refs therein…]

$$\Gamma = \frac{P(|N_{cs} - \frac{1}{2}| < \epsilon/2)}{\epsilon V} \langle \left| \frac{\Delta N_{cs}}{\Delta t} \right| \rangle d$$

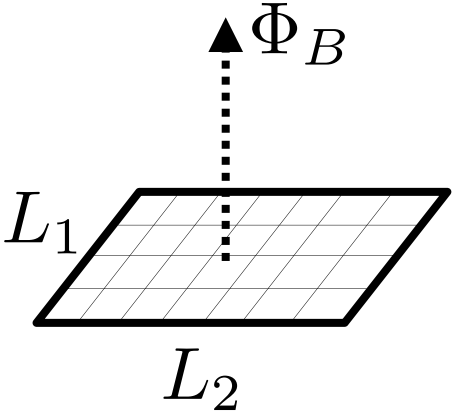
statistical dynamical

- Multicanonical method becomes very inefficient with large volumes
  - Ncs is a global quantity: increasing lattice size does not help with statistics.

External hypermagnetic field on the lattice

- Maintaining translational invariance on a lattice with a periodic boundary conditions requires to quantize the external magnetic field into flux quanta: $g_3' \Phi_B = 4\pi n_b, \quad n_b \in \mathbb{Z}$
- Magnetic flux density:

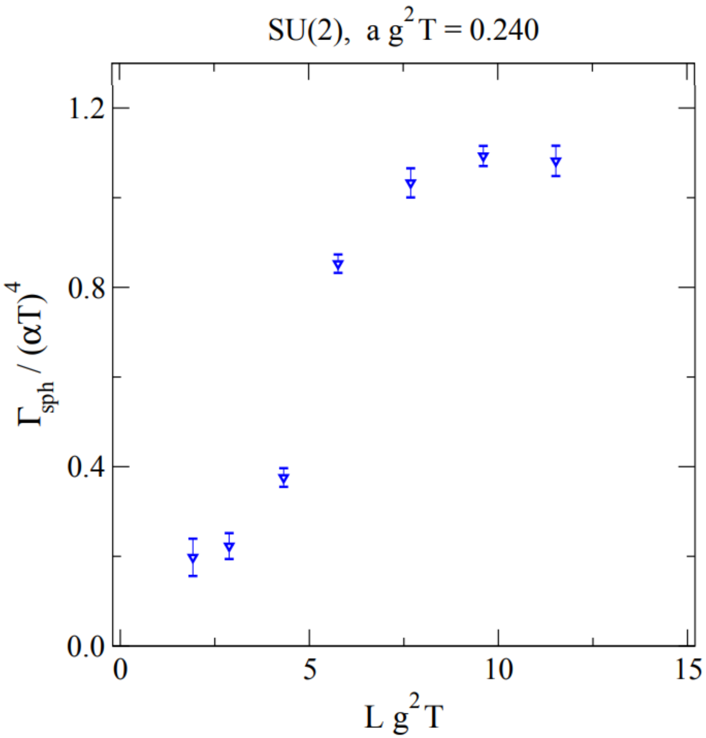
$$b \equiv \frac{g_3' B_Y^{3d}}{g_3^4} = \frac{4\pi n_b}{L_1 L_2} \left(\frac{1}{g_3^2 a} \right)^2$$



$$B_Y^{4d} \simeq (g'/g^4) b T^2 \approx 2bT^2$$

Volume & Lattice spacing dependence

- Volume dependence very mild after $L = 8/g_3^2$

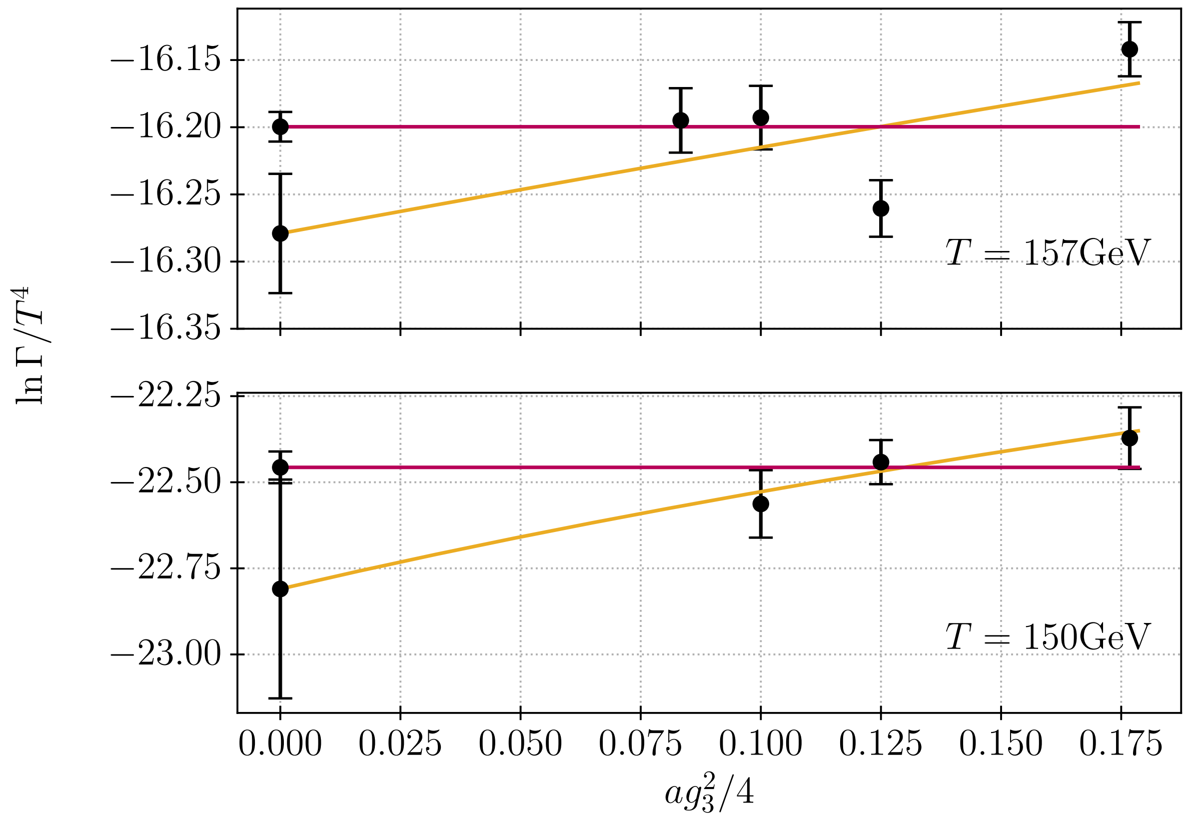


[Laine et al., 2209.13804]

[Moore et al. 1011.1167]

[Moore&Rummukainen, hep-ph/9906259.]

- Continuum limit is computationally very expensive



[Arnold et al. Phys. Rev. D 36, 581–595 (1987)]

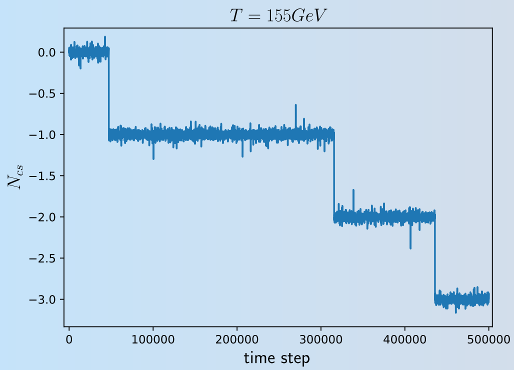
Broken phase | Symmetric phase

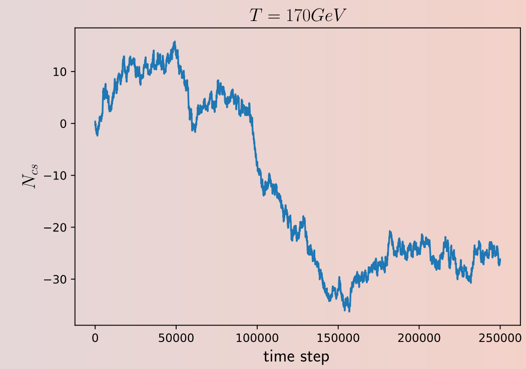
[Arnold et al. arXiv:hepph/9609481]

$$\Gamma_{\mathrm{brk}} \sim \alpha_W^4 T^4 e^{-E_{sph}/T}$$

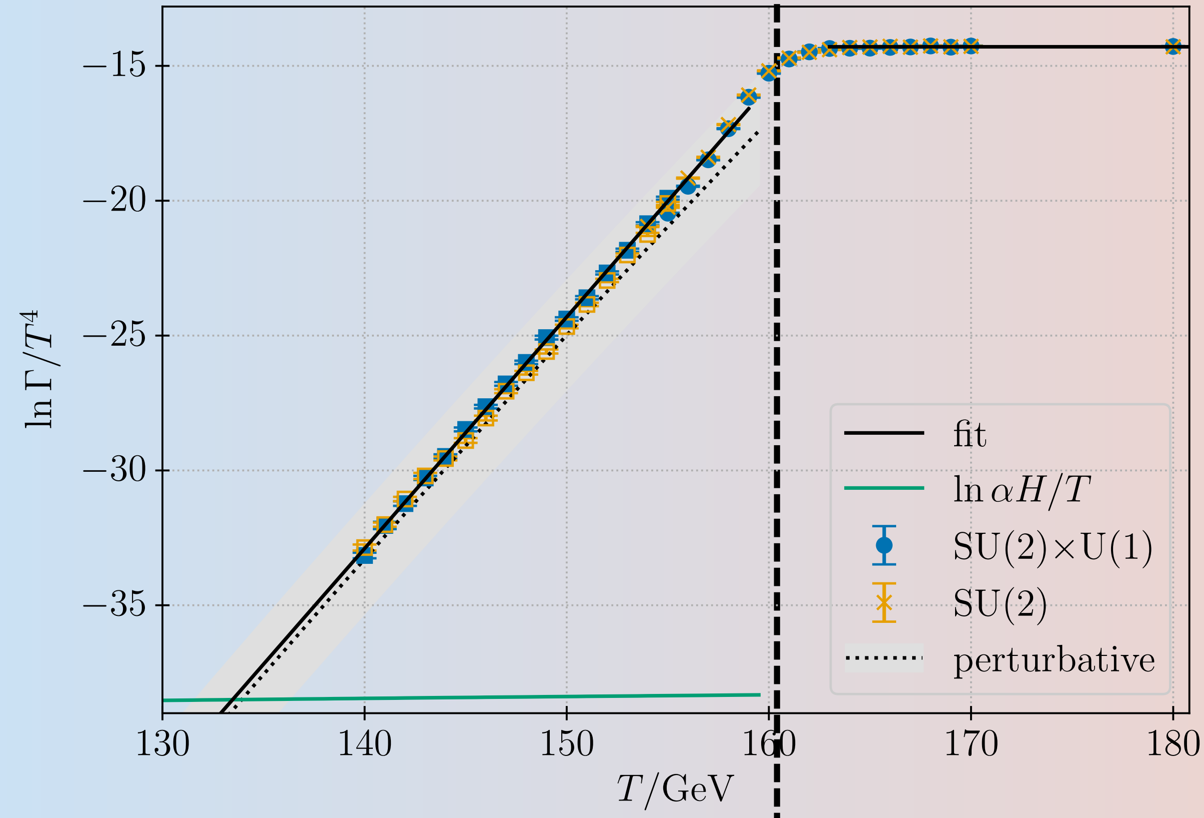
$$T_c \simeq 160\,\mathrm{GeV}$$

$$\Gamma_{\mathrm{sym}} \sim \alpha_W^5 \log(\alpha_W^{-1}) T^4$$





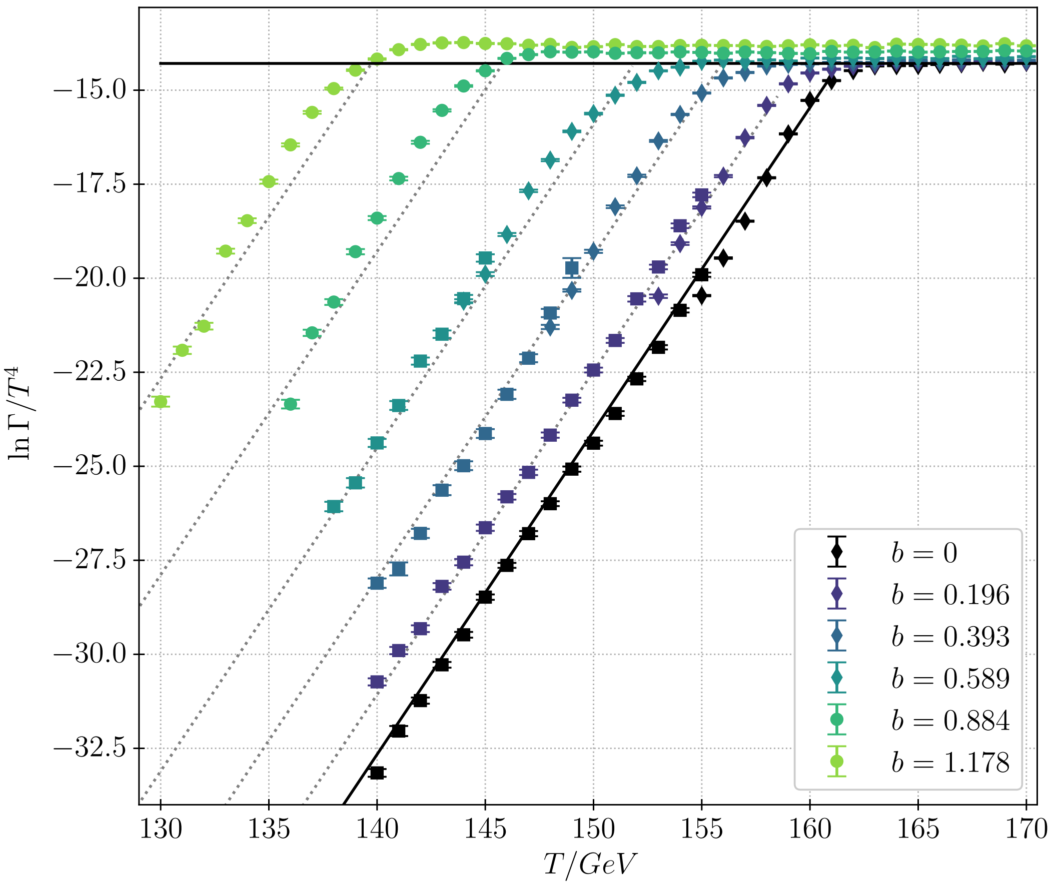
(no magnetic field)

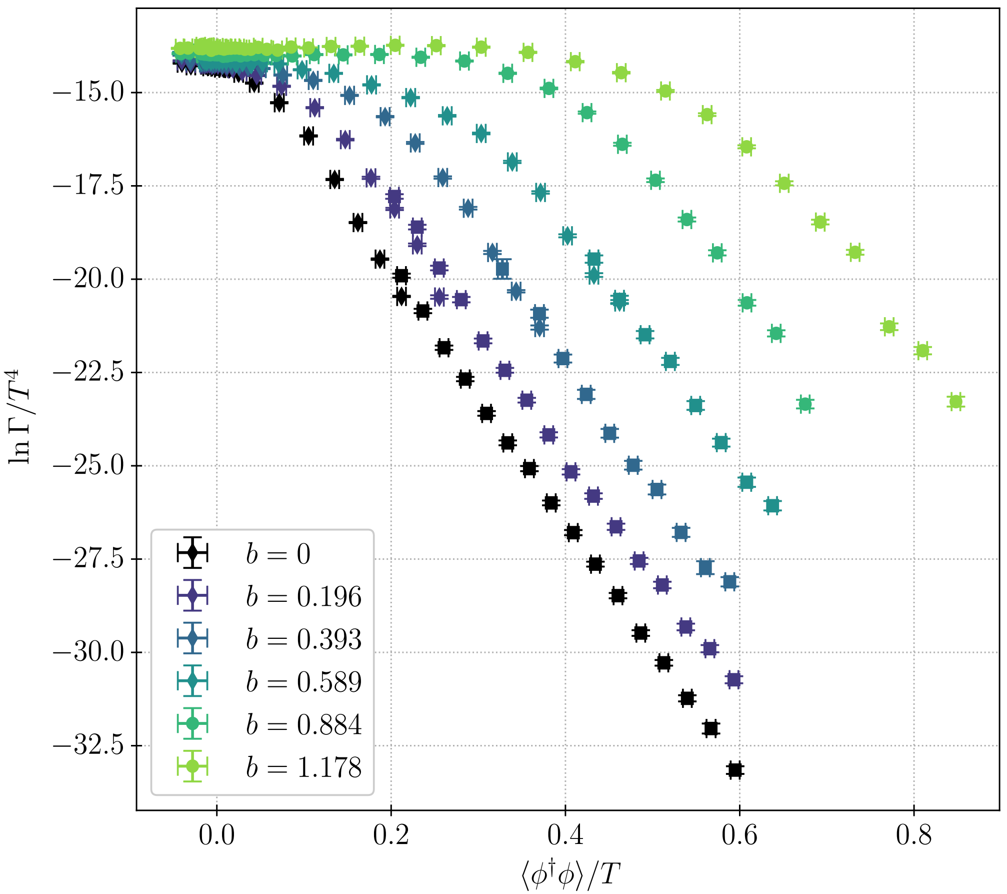


$$\ln(\Gamma_{\mathrm{brk}}/T^4) = (0.86 \pm 0.01)T/\mathrm{GeV} - (153.1 \pm 0.9)$$

$$\Gamma_{\mathrm{sym}}/T^4 \simeq (13.9 \pm 0.1)\alpha_W^5$$

Sphaleron rate in a magnetic field

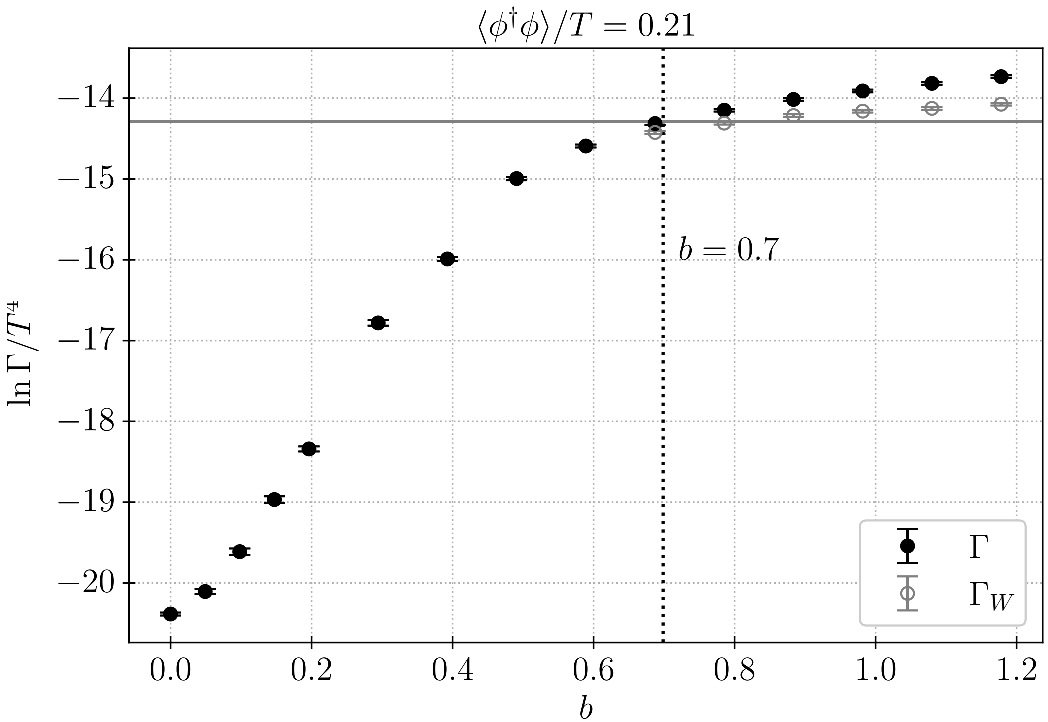


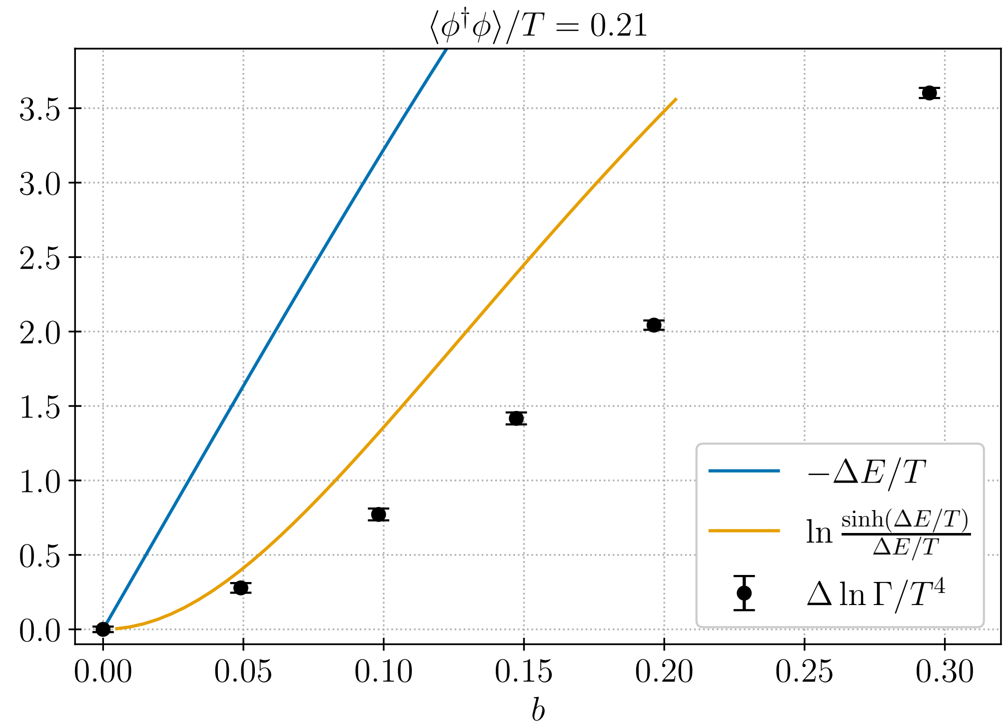


- Black solid lines: b=0 fits
- Grey dotted: b=0 fit shifted according to the shift of the pseudo critical temperature.

- For small external fields the sphaleron dipole moment has bigger effect compared to the changing Higgs expectation value.

Rate vs magnetic field at constant vev

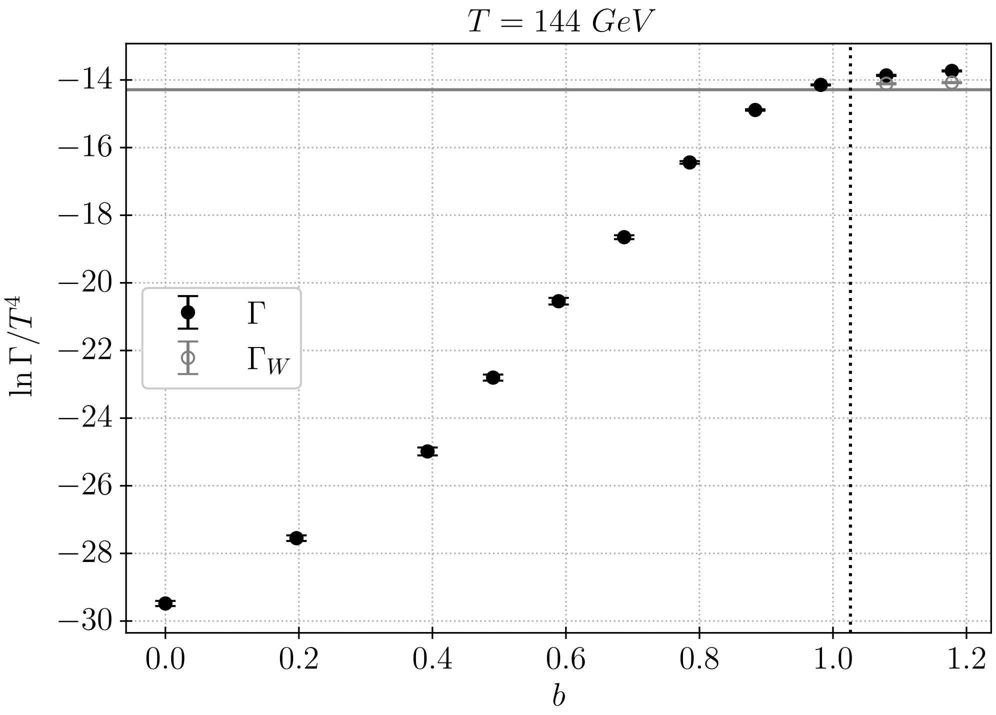


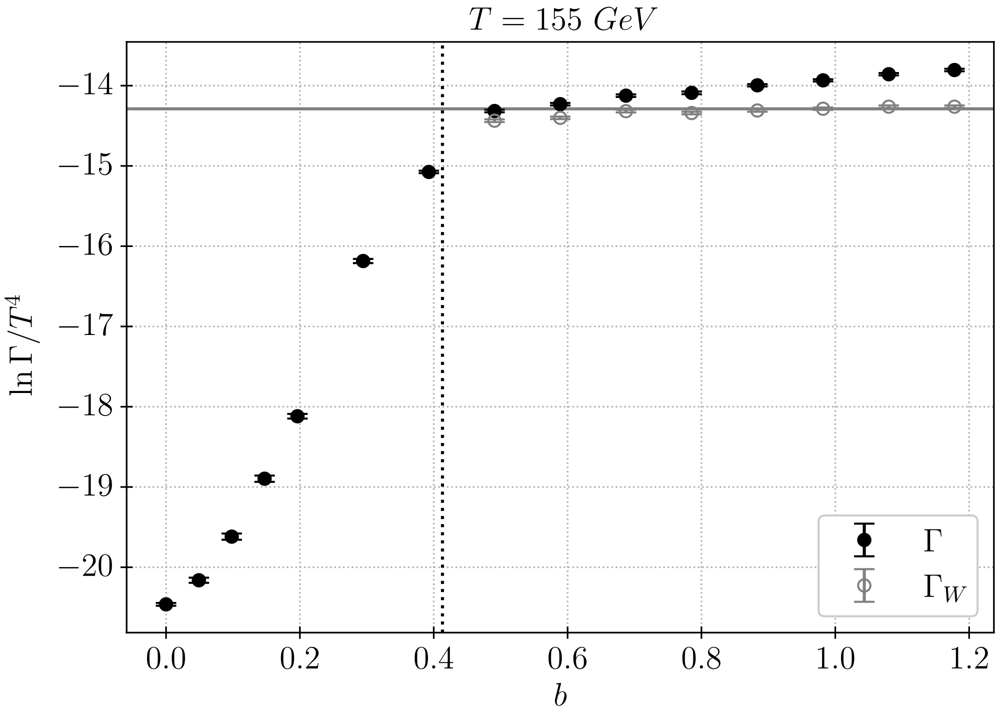


Solid line: semi analytical computation assuming dipole&spherical approximations

Rate vs magnetic field at constant T

- In the symmetric phase the full rate is slightly faster than its components alone $N_{\rm CS}^{W} - N_{\rm CS}^{Y}$
- In symmetric phase SU(2) rate does not change significantly.



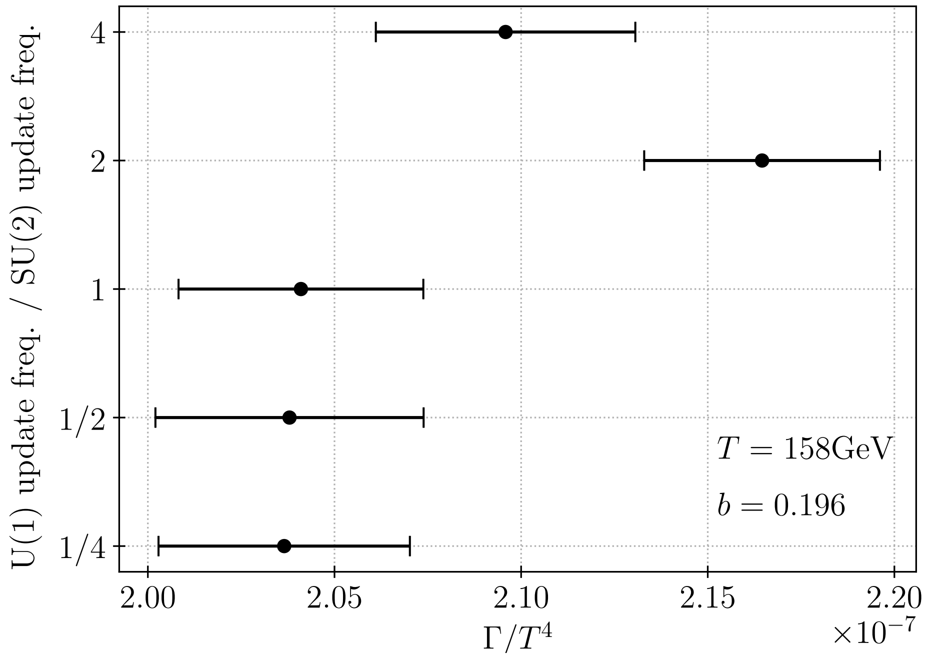


Conclusions

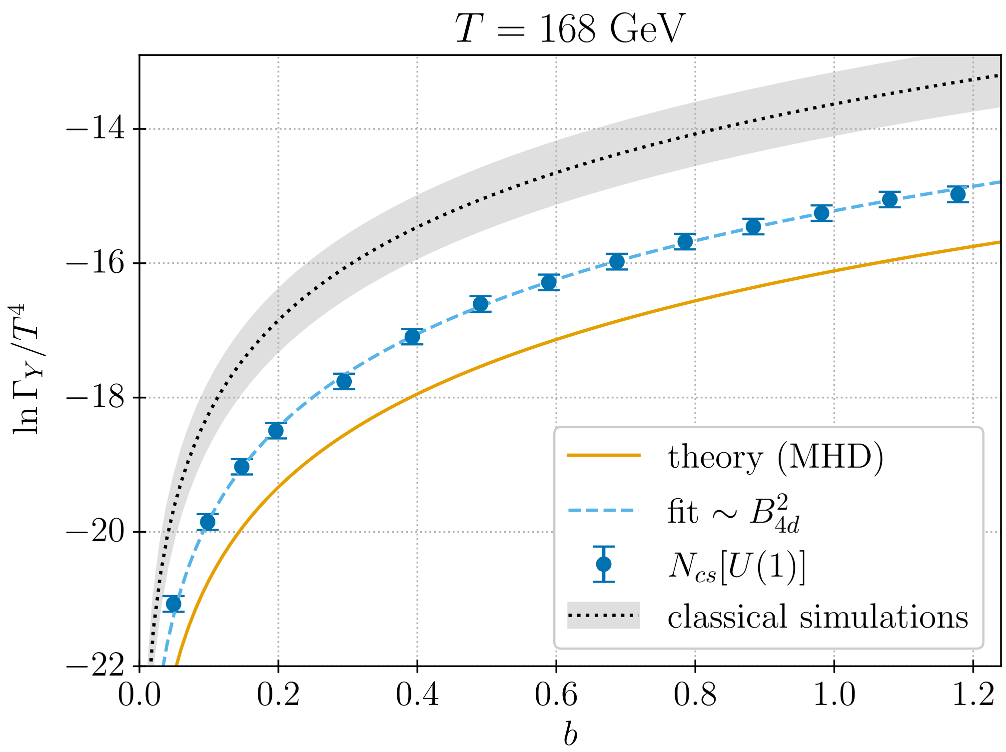
- We performed first dynamical lattice simulations investigating effects of external magnetic field on the sphaleron rate.
- Verified that U(1) does not change the result when magnetic field is zero.
- Sphaleron has a magnetic dipole moment and its energy can be lowered in a magnetic field.
  - For small fields the dipole moment gives the biggest effect. Ultimately the shifting of the pseudocritical temperature dominates.
- Electroweak transition shifts to lower temperatures with increased external magnetic field $B_Y = 0...2T^2: \quad T_c = 160...145\ \mathrm{GeV}$
- Comparison to semianalytical result: simple dipole approximation works only for very small fields and quickly becomes invalid.

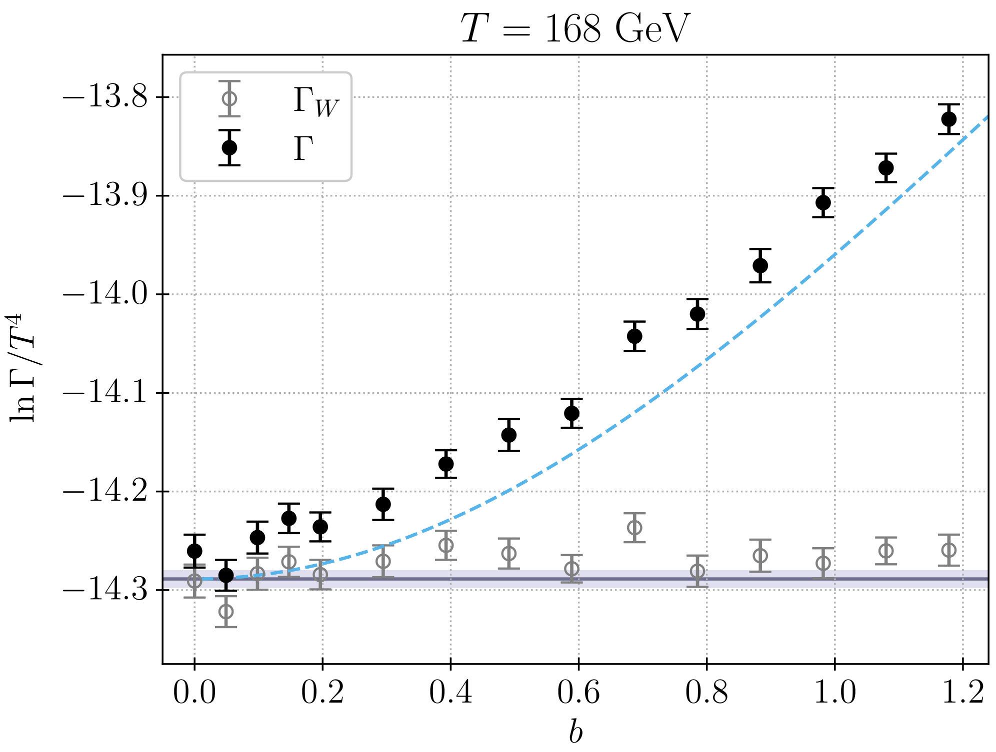
Backup

- Sphaleron size $\sim 1/(g^2T)$
- U(1) modes with wavelength of sphaleron size has time scale also $\sim 1/(g^2T)$ and are weakly coupled $g'^2 \ll g^2$
- SU(2) is over damped, time scale $\sim 1/(g^2T)^2$



The U(1) rate





- (MHD), classical simulations [Daniel G. Figueroa et al. arxiv:1707.09967, arxiv:1904.11892]

Backup

$$\Delta \ln \Gamma / T^4 \sim \ln \left\{ \int \frac{\mathrm{d}\Omega}{4\pi} \exp \left[-\frac{\mu_{\mathrm{sph}} B_c^{4d}}{T} \cos\theta \right] \right\} \simeq \ln \frac{\sinh(\Delta E/T)}{\Delta E/T} \ .$$