Phase transitions, gravitational waves and 2-Higgs doublet model

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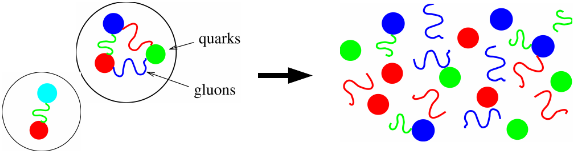
Electroweak symmetry breaking

- Electroweak symmetry breaking at $T \gtrsim 100\,\text{GeV}$ (in the Standard Model $T_c \approx 160\,\text{GeV}$).
- What is the order of the transition?
  - In the SM smooth cross-over
  - In some extensions of the SM a 1st order transition is possible
- Why 1st order phase transition is interesting?
  - Electroweak baryogenesis (EWBG)
  - **Gravitational waves**
    - Possibly observable with future GW detectors
    - ESA/NASA LISA (Laser Interferometer Space Antenna) mission, launch 2034
    - **A new probe for BSM physics and cosmology**
- In this talk:
  - How phase transitions can produce gravitational waves
  - How to use 3-d effective theory simulations to study phase transitions
  - Show results from 2-Higgs doublet study

No genuine phase transitions in the Standard Model:

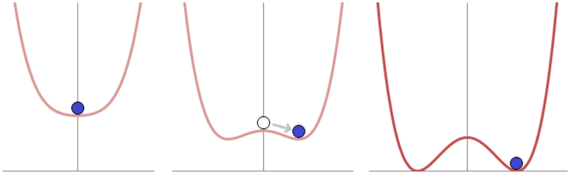
- QCD phase transition at $T \sim 170\,\text{MeV}$
  - Age of the universe $t \sim 10\mu\text{s}$
  - Quark-gluon plasma $\leftrightarrow$ hadrons
  - Smooth cross-over $\rightarrow$ no GWs produced
  - Lattice QCD simulations



Hadrons

Quark-gluon plasma

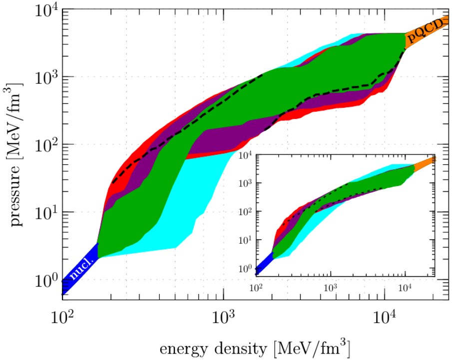
- Electroweak phase transition at $T = T_c \approx 160\,\text{GeV}$
  - $t \sim 10^{-11}\text{s}$
  - Higgs expectation value v becomes non-zero
  - Smooth cross-over $\rightarrow$ no GWs
  - At $T > T_c$, baryon number is not conserved!
  - Lattice effective theory simulations

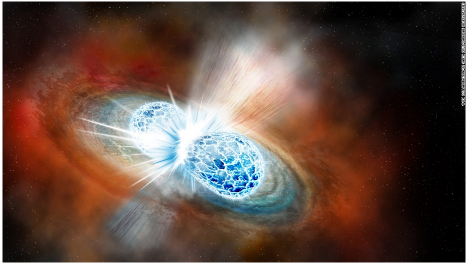


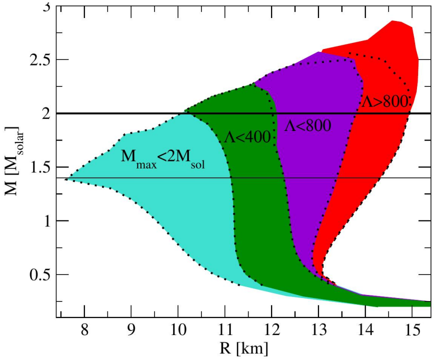
Gravitational waves and dense nuclear matter

- The single LIGO binary neutron star merger event already restricts neutron start mass - R -relation (red region)
- Dense QCD matter equation of state, notoriously hard to study theoretically

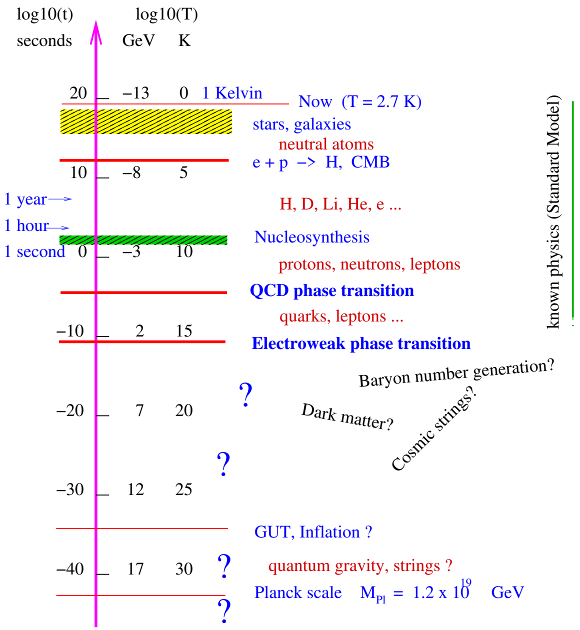
[Annala et al., 2017]



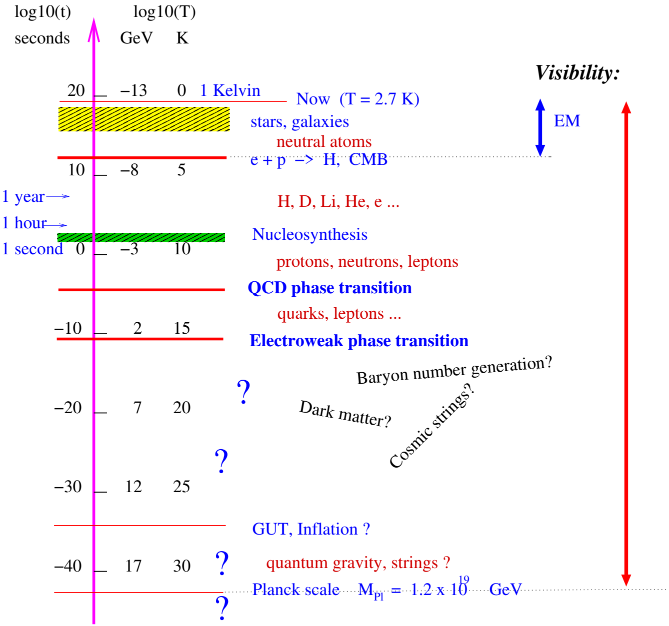




Peek into the early Universe with gravitational waves

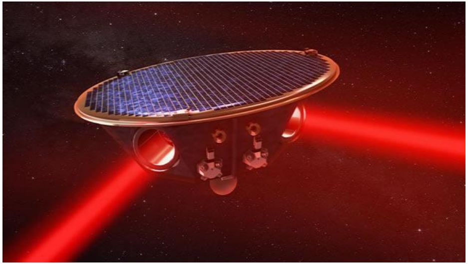


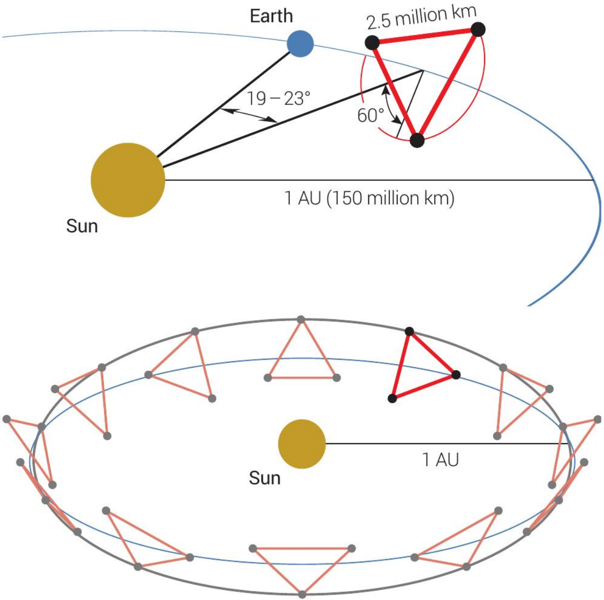
Peek into the early Universe with gravitational waves



LISA gravitational wave mission

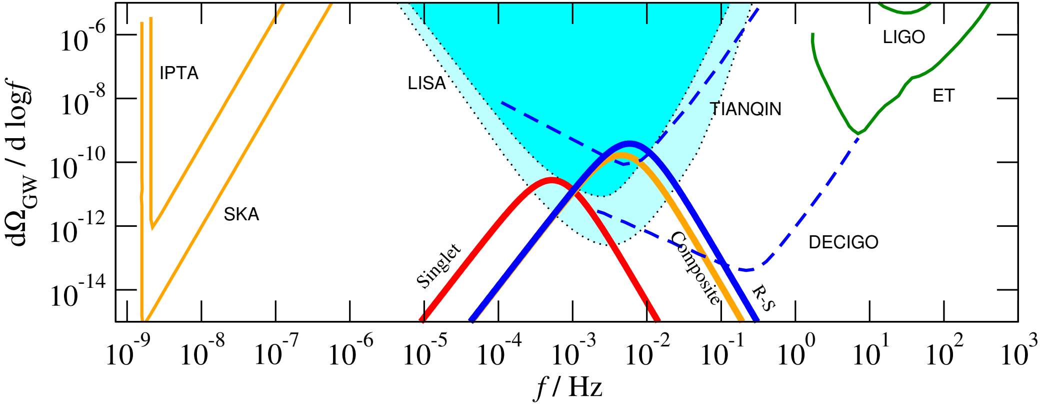
- "Free-flying" laser interferometer
- Strain sensitivity
  $\sim 10^{-21}$ @ 0.01 Hz
  ($\sim$ 1pm over 2.5 million km!)
- Launch 2034





LISA

- Frequency window of LISA is right for gravitational waves from the electroweak and above -eras.



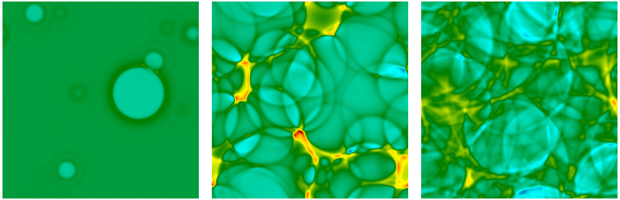
- Example signals from the LISA Cosmology Working Group report [Caprini et al. JCAP 03 (2020) 024]

1st order phase transitions

A first order phase transition proceeds through

a) *supercooling*
b) *critical bubble nucleation*
c) *bubble growth and collision* $\rightarrow$ gravitational waves
d) *sound waves, shocks, turbulence* $\rightarrow$ *gravitational waves*

If the latent heat of the transition and supercooling are large, the process is violent (cf. superheated water)



[Hindmarsh et al.]

Goal: take a set of Beyond-the-Standard-Model candidates (MSSM, 2HDM, ...) and calculate the gravitational wave spectrum observed @ LISA
Conversely: how to use LISA to constrain BSM models?

Calculating the gravitational wave production

We need to know, for a theory candidate:

i) Thermodynamics (✓)
  - equation of state, **latent heat, speed of sound**

ii) Critical bubble **nucleation rate** (✓)
  - Determines degree of supercooling, *characteristic length scale* $\longrightarrow$ frequency of the gravitational radiation

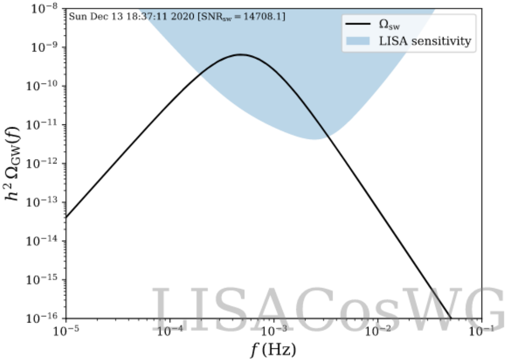
iii) Bubble wall - fluid interaction (?)
  - **bubble wall velocity**

} Microscopic QFT computation (analytical, numerical lattice)

iv) **Growth & collision of the bubbles, sound, shocks, turbulence** (✓?)
  - Requires numerical simulations
  - Relativistic hydrodynamics + scalar field, effective order parameter
    - ★ Scalar: Higgs in SM-like models, χ-condensate in strong dynamics ...
  - Large dynamical range, large volumes
  - Only a few relevant parameters: T_c, strength of the transition α, duration of the transition β, bubble wall velocity v_W, # of dof's g

✓ Coupling to gravity: transverse-traceless part of $T^{\mu\nu}$

GW signal at LISA:



Peak location, height, and shape of the spectrum depend on the transition parameters [Hindmarsh et al.].
Plot done with the nifty PTPlot tool which includes our current best knowledge of GW spectra and SNR (David Weir) http://www.ptplot.org/ptplot/

Effective theory simulations

- 3d effective theory simulations can be used to calculate static thermodynamic quantities: *critical temperature, equation of state, latent heat, speed of sound . . .*
- Real-time quantities – *phase transition wall propagation, critical bubble nucleation rate, sphaleron transition rate* – can be calculated in related effective theories/methods (not discussed here).

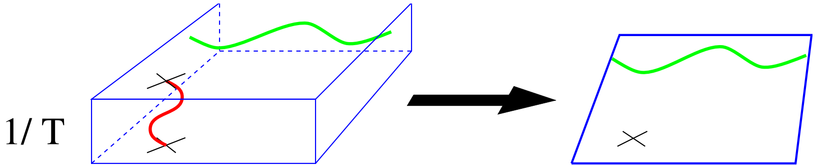
3d effective theory

- Phase transition in the SM and its extensions (MSSM, multiple Higgses) is at weak gauge coupling.
- However: at non-zero T infrared modes become non-perturbative for $k \lesssim g^2 T$ $\rightarrow$ perturbation theory accuracy limited. [Linde 80]

$\rightarrow$ **3d effective theory**

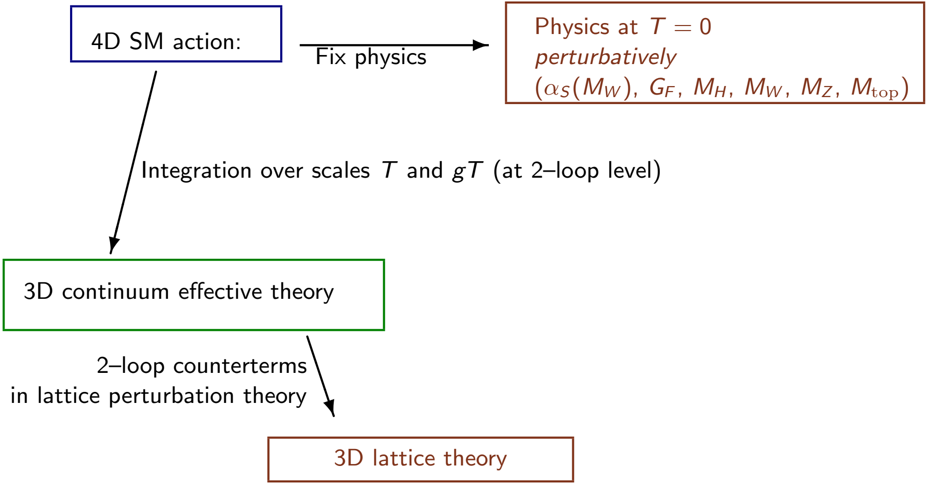
- Tool for perturbative and lattice computations
- Modes $p > g^2 T$ are perturbative (at weak coupling): can be integrated out in stages:
  1. $p \gtrsim T$: fermions, non-zero Matsubara frequencies

  $\rightarrow$ 3d theory (dimensional reduction)



  2. Electric modes $p \sim gT$
- Obtain a “magnetic theory” for modes $p \lesssim g^2 T$. Contains fully the non-perturbative thermal physics.

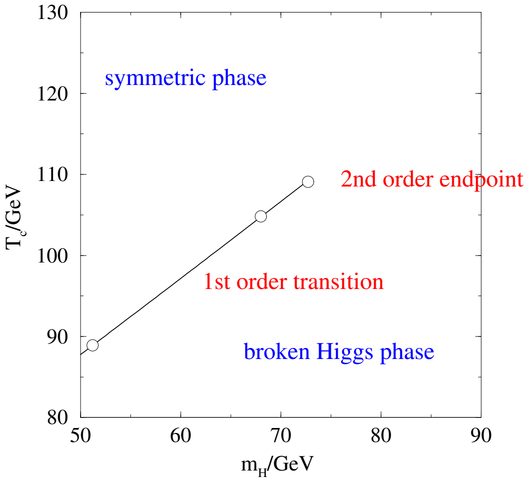
3d effective theory



Phase diagram of the Standard Model

- Effective theory was used very successfully for the SM 20+ years ago

→ No phase transition at all, smooth "cross-over" for $m_{\mathrm{Higgs}} \gtrsim 72\,\mathrm{GeV}$



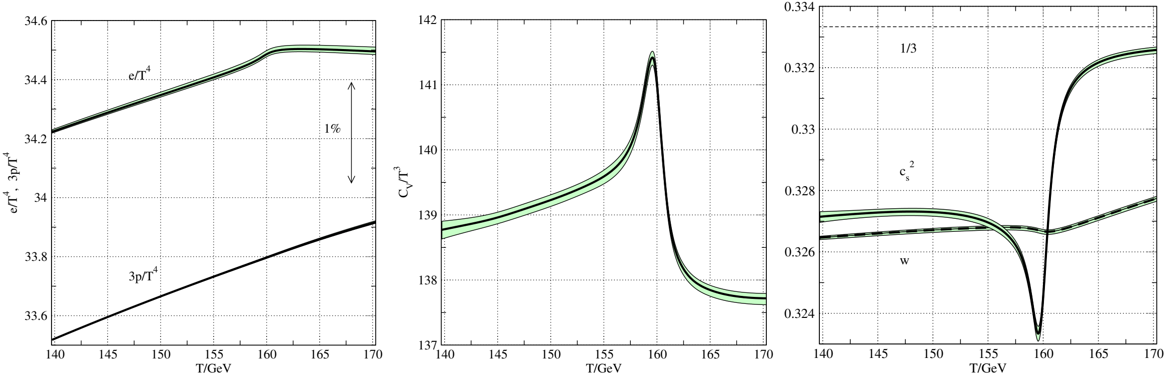
[Kajantie,Laine,K.R.,Shaposhnikov,Tsypin 95–98]

see also

[Csikor,Fodor, Heitger]

[Gürtler,Ilgenfritz,Schiller,Strecha]

Precision thermodynamics for the SM



[D'Onofrio, K.R 2015]

- Pseudocritical temperature $T_c = 159.6 \pm 0.1 \pm 1.5$ GeV
- Heat capacity $C_V = e'(T)$
- Speed of sound: $c_s^2 = p'/e'$
- EOS parameter $w = p/e$

Cross-over well defined, but very soft! Width of the transition region $\sim 3\,\text{GeV}$.

Beyond the Standard Model

First order phase transition *has been found in MSSM* [Laine,Nardini,K.R. 2013]. However, this is now excluded. Turn instead to 2-Higgs doublet model:

Lagrangian

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\mathrm{Tr}\,F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}\not{D}\Psi + |D_\mu\phi_1|^2 + |D_\mu\phi_2|^2 + (Y\bar{\Psi}_R\phi_2\Psi_L + \text{h.c.}) \\ &+ \mu_{11}^2\phi_1^\dagger\phi_1 + \mu_{22}^2\phi_2^\dagger\phi_2 + \left[\mu_{12}^2(\phi_1^\dagger\phi_2) + \text{h.c.}\right] \\ &+ \lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \left[\frac{\lambda_5}{2}(\phi_1^\dagger\phi_2)^2 + \text{h.c.}\right]\end{aligned}$$

- $F_{\mu\nu}$: U(1), SU(2) and SU(3) gauge; Ψ: SM fermions
- ϕ_i: two SU(2) scalars with hypercharge $+1$
- "Type-I" model: Yukawas couple only to ϕ_2 (less constrained than type-II)
- Terms of type $(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_2)$ not included: lead to large FCNCs
- Note: μ_{12}^2, $\lambda_5 \in C$: explicit CP violation. Strongly limited by neutron EDM measurements (ACME) $\rightarrow$ not sufficient for EW baryogenesis.

After 2-loop matching, we obtain 3d

3d effective Lagrangian:

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\mathrm{Tr}(F_{ij})^2 + D_i\phi^\dagger D_i\phi + \bar{\mu}_{11}^2\phi_1^\dagger\phi_1 + \bar{\mu}_{22}^2\phi_2^\dagger\phi_2 + \bar{\mu}_{12}^2\left(\phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1\right) \\ &+ \bar{\lambda}_1(\phi_1^\dagger\phi_1)^2 + \bar{\lambda}_2(\phi_2^\dagger\phi_2)^2 + \bar{\lambda}_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \bar{\lambda}_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\ &+ \frac{\bar{\lambda}_5}{2}\left((\phi_1^\dagger\phi_2)^2 + (\phi_2^\dagger\phi_1)^2\right),\end{aligned}$$

- F_{ij}: SU(2) gauge (U(1) has small effect and can be neglected)
- 3d mass parameters $\bar{\mu}^2 \propto \mathrm{GeV}^2$, couplings g_3^2, $\bar{\lambda} \propto \mathrm{GeV}$ are dimensionful $\rightarrow$ theory is **superrenormalizable**
- Parameters depend on 4d Lagrangian parameters and the temperature T.
- Starting point for both perturbative and lattice studies.
- Straightforward to put on the lattice. Superrenormalizability $\Rightarrow$ lattice counterterms (1- and 2-loop) known, rigorous curves of constant physics (up to $O(a)$). No tuning needed!
- Robust continuum limit (leading errors $O(a)$). Lattice gauge coupling

$$\beta_G \equiv \frac{4}{g_3^2 a} \approx \frac{4}{g_W^2 a}.$$

Symmetry breaking

In the original 4d theory, the symmetry breaking pattern is

$$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \rho_1 + i\eta_1) \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \rho_2 + i\eta_2) \end{pmatrix},$$

Scalar excitations: neutral Higgses h, H, CP-odd A, charged $H^\pm$

$$h = -\sin\alpha\rho_1 + \cos\alpha\rho_2, \qquad H = -\cos\alpha\rho_1 - \sin\alpha\rho_2,$$
$$H^\pm = -\sin\beta\phi_1^\pm + \cos\beta\phi_2^\pm, \qquad A = -\sin\beta\eta_1 + \cos\beta\eta_2.$$

2 mixing angles $\tan\beta = v_2/v_1$, α.
Input parameters – fix the (4d) pole masses:

- G_F, M_W, M_Z, $M_h = M_{\rm Higgs}$, $M_{\rm top}$ (same as in SM)
- M_H, M_A, $M_{H^\pm}$, $\tan\beta$, $\cos(\beta-\alpha)$, μ_{12}^2 (New)

In the "alignment limit" $\cos(\beta-\alpha) = 0$ the coupling of h to the SM particles is exactly as the SM Higgs.

Benchmark points

2HDM is less constrained than MSSM by collider phenomenology (but still pretty limited).
Strong phase transition $\Rightarrow$ large scalar couplings λ_i

$\rightarrow$ problems in perturbation theory; accuracy of eff. 3d description?

$\rightarrow$ Landau pole is close

BM1	M_H 66 GeV	M_A 300 GeV	$M_{H^\pm}$ 300 GeV	μ_{12} 0 GeV	$(\lambda_3+\lambda_4+\lambda_5)/2$ 1.07×10^{-2}	λ_1 0.01	Λ_0 91 GeV
BM2	M_H 150 GeV	M_A 350 GeV	$M_{H^\pm}$ 350 GeV	μ_{12} 80 GeV	$\cos(\beta-\alpha)$ -0.02	$\tan\beta$ 2.75	Λ_0 265.018 GeV

- BM1: "Inert doublet model", studied perturbatively by [Laine, Meyer, Nardini 2017].
  - Here $v_1 = 0$ (only ϕ_2 breaks)
  - H is a dark matter candidate (long-lived)
- BM2: Approaches model studied by [Dorsch, Huber, Konstandin, No 2017] but with more restricted λ_i. Features a strong transition, possibly producing observable gravitational waves.

Simulation volumes

BM1

β_G	Volumes, $L_x \times L_y \times L_z$		
10	$18^2 \times 72$	$20^2 \times 80$	$24^2 \times 96$
12	$20^2 \times 96$	$24^2 \times 96$	$28^2 \times 120$
14	$28^2 \times 84$	$28^2 \times 140$	
16	$24^2 \times 96$	$32^2 \times 120$	$32^2 \times 162$
	$38^2 \times 162$		
20	$24^2 \times 112$	$32^2 \times 132$	$38^2 \times 156$
24	$34^2 \times 156$	$42^2 \times 172$	$42^2 \times 200$
32	$42^2 \times 200$	$48^2 \times 192$	$54^2 \times 216$

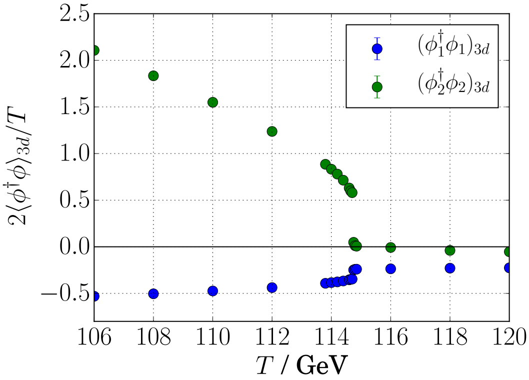
BM2

β_G	Volumes, $L_x \times L_y \times L_z$		
20	$32^2 \times 132$	$38^2 \times 156$	$42^2 \times 168$
24	$34^2 \times 156$	$42^2 \times 172$	$48^2 \times 182$
28	$42^2 \times 168$	$48^2 \times 192$	$54^2 \times 200$
32	$48^2 \times 192$	$54^2 \times 216$	$58^2 \times 240$

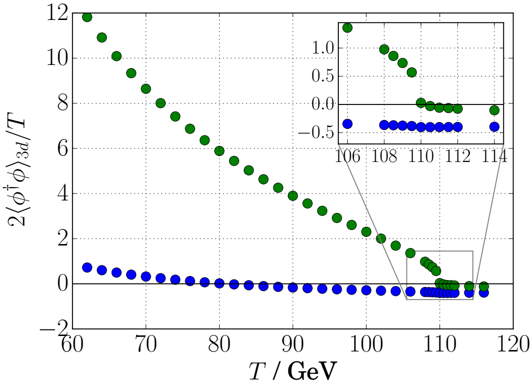
Large $\beta_G \leftrightarrow$ small lattice spacing a. For BM2, some modes are so heavy that large β_G is necessary.
Multicanonical simulations and cylindrical geometry are used to obtain surface tension σ.

Condensates

BM1

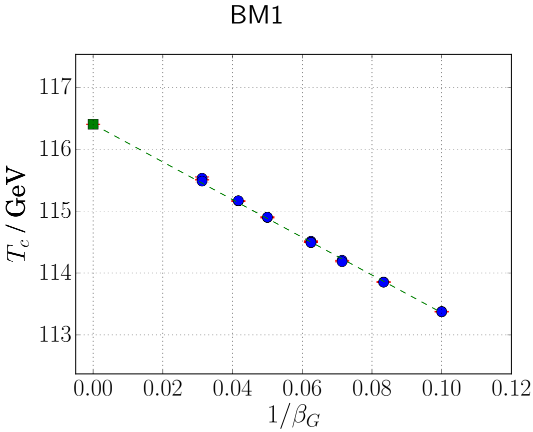


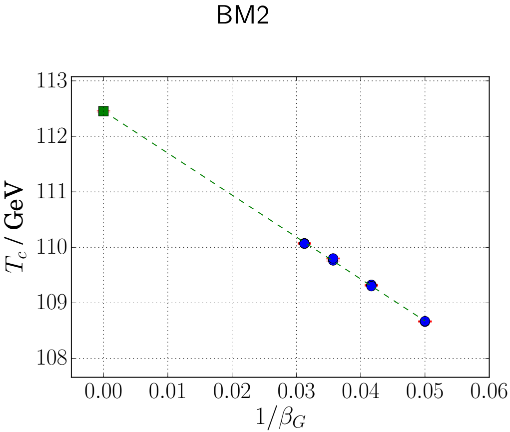
BM2



Both have a discontinuity in $\langle\phi_2^\dagger\phi_2\rangle$.
At $T = 0$ $\langle\phi_2^\dagger\phi_2\rangle/\langle\phi_1^\dagger\phi_1\rangle = \tan^2\beta \approx 7.5$: slow evolution towards $T = 0$ value.

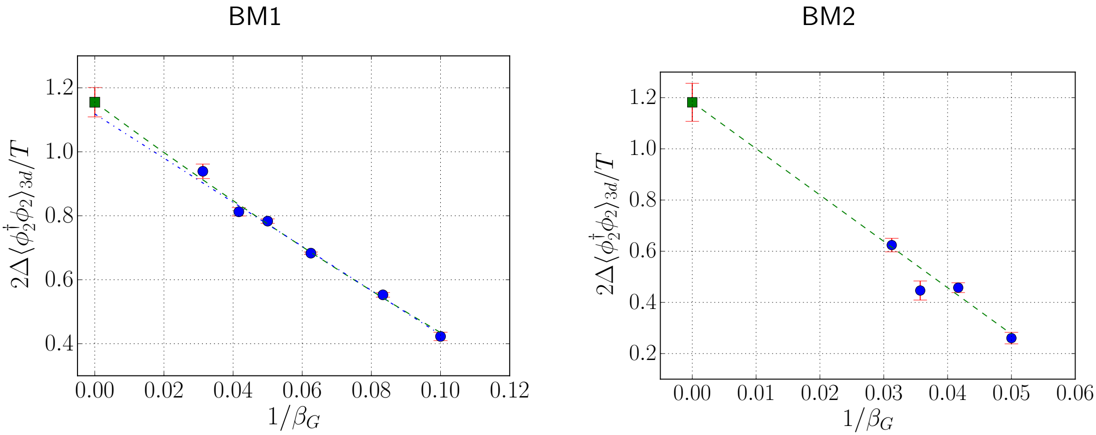
Critical temperature





BM1: $T_c = 116.402 \pm 0.005$ GeV
BM2: $T_c = 112.454 \pm 0.015$ GeV

Order parameter discontinuity



BM1: $\Delta\phi/T = 1.07 \pm 0.02$
BM2: $\Delta\phi/T = 1.09 \pm 0.03$

Order parameter discontinuity

We can also obtain the *latent heat* L and the *surface tension* σ:

$$\frac{L}{T^4} = \frac{T^2}{V}\Delta\left(\frac{\partial}{\partial T}\ln Z\right),$$

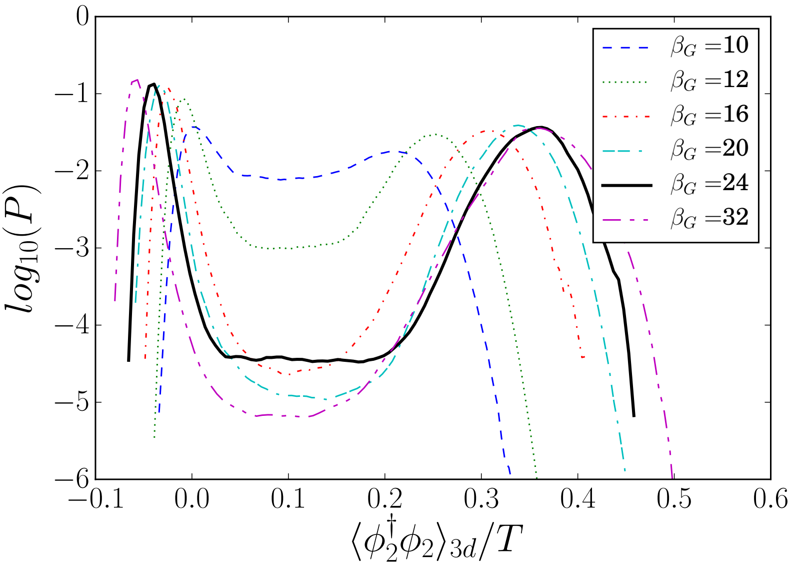
$$= -\frac{1}{VT^2}a^3\Delta\left\langle \Phi_1^\dagger\Phi_1\frac{dm_{11}^2}{dT} + \Phi_2^\dagger\Phi_2\frac{dm_{22}^2}{dT} + \Phi_1^\dagger\Phi_2\frac{dm_{12}^2}{dT} + \text{h.c.} + (\Phi_1^\dagger\Phi_1)^2\frac{d\bar{\lambda}_1}{dT} + \ldots\right\rangle,$$

$$\frac{\sigma}{T} = \frac{1}{2A}\ln\frac{P_{\max}}{P_{\min}} + \text{finite volume corrections},$$

where P_{max} is the maximum of probability distribution and A area.

	L/T_c^4	σ/T_c^3
BM1	0.603 ± 0.023	0.0270 ± 0.0013
BM2	0.807 ± 0.051	0.0204 ± 0.0045

Probablity distributions



Comparison with perturbative calculations

	Method	T_c/GeV	L/T_c^4	ϕ_c/T_c	L/GeV^4
BM1	1-loop Parwani resum.	134.0 ± 8.75	0.396 ± 0.002	1.01 ± 0.06	1.27×10^8
	1-loop A-E resum.	142.4 ± 6.88	0.33 ± 0.02	1.00 ± 0.07	1.37×10^8
	2-loop V_{eff} in 3d	111.6 ± 2.30	0.57 ± 0.10	0.98 ± 0.09	0.89×10^8
	3d lattice	116.40 ± 0.005	0.60 ± 0.02	1.08 ± 0.02	1.11×10^8
BM2	1-loop Parwani resum.	142.6 ± 18.0	0.29 ± 0.04	0.91 ± 0.06	1.19×10^8
	1-loop A-E resum.	162.5 ± 21.0	0.20 ± 0.03	0.88 ± 0.05	1.36×10^8
	2-loop V_{eff} in 3d	104.9 ± 2.30	0.61 ± 0.10	0.97 ± 0.06	0.74×10^8
	3d lattice	112.5 ± 0.01	0.81 ± 0.05	1.09 ± 0.03	1.29×10^8

- V_{eff} in 3d relies on the same 3d effective theory than 3d lattice
- Lattice does not suffer from IR problems: much smaller errors.
  - Note: errors do not include theoretical errors due to truncation or parameter uncertainty
- Lattice is needed to reduce the uncertaintly in perturbative analysis

Conclusions:

- Gravitational waves provide a new way to probe the early Universe and BSM physics.
- Phase transitions are a powerful source of gravitational waves.
- 3d effective theory simulations can be used to study thermodynamics of the transition

$\rightarrow$ Part of the mapping out the theory space of suitable BSM theory candidates.

- Standard Model: no transition
- Strong transitions seen in MSSM (excluded by now), 2HDM
- Effective theory method applicable to many "Higgs-like" models