The spectrum of QCD with one flavour – A Window for Supersymmetric Dynamics

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arXiv:2302.10514 (accepted PRD), arXiv:2212.06709 (PoS)

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DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

Outline

1. Introduction and Motivation
2. Numerical Set-up
3. Monitor possible sign problem
4. $N_f = 1$ QCD spectrum
5. Conclusions and Outlook

Introduction: large N_C limits and relation to SUSY

Compare two choices to take the $N_C \to \infty$ limit:

- 't Hooft [Nucl. Phys. B 72, 461 (1974)] :
  quark loops are suppressed; baryons become infinitely heavy
- Corrigan and Ramond (CR) [Phys. Lett. B 87, 73-74 (1979)]:
  1 fermion in two-index anti-symmetric representation of gauge group.
- CR limit with $N_f = 1$ Dirac fermion and $\mathcal{N} = 1$ supersymmetric Yang-Mills (sYM) have same number of dof.
- Bosonic spectrum of sYM and $N_f = 1$ identical in CR $N_C \to \infty$ limit [Armoni, Shifman, Veneziano 0302163, 0412203].
- For $N_C = 3$, two-index antisym. rep. coincides with conjugate rep.:
  $\Rightarrow N_f = 1$ **QCD**

Goal: **Study SUSY non-pert. via $N_f = 1$ QCD-like simulations!**

$N_f = 1$ QCD past and present

Relevant literature:

- *"Large N, Supersymmetry ...and QCD"*, Veneziano & Wosiek [0603045]
- *"One flavor QCD"*, Creutz [0609187]
- *"Hadron spectrum of QCD with one quark flavor"*, Farchioni et al. [0810.0161]
- *"Effective Lagrangians for Orientifold Theories"*, Sannino & Shifman [0309252] predict

$$\frac{m_P}{m_S} = 1 - \frac{22}{9N_C} - \frac{4}{9}\beta + \mathcal{O}(1/N_C^2)$$

- *"Predictions for orientifold theories from type 0' string theory"*, Armoni & Imeroni [0508107] predict

$$\frac{m_P}{m_S} = 1 - \frac{2}{N_C} + \mathcal{O}(1/N_C^2)$$

arXiv:2302.10514 (accepted PRD): Spectrum of massless $N_f = 1$ QCD.

arXiv:2212.06709 (PoS): Study the spectrum as N_C increases.

Peculiarities of simulating $N_f = 1$ QCD

Many assumptions we typically make have to be re-evaluated:

- Not a physical theory: No obvious way to set the scale?
  Use $N_f = 0$ and $N_f = 2$ gradient flow as a proxy!
- No chiral symmetry: How to define $m_q = 0$?
  Use fictitious pion as proxy!
- Single fermion: possible sign problem?
  Carefully study sign of the fermion determinant!
- No chiral symmetry: How big are finite size effects?
  Scan over range of different lattice sizes!

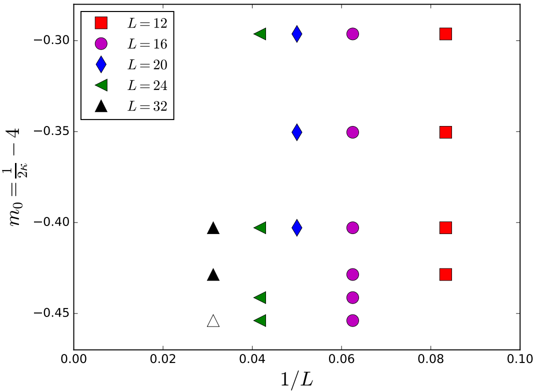
Need to compute $C(t) = \langle [\overline{q}\Gamma q](t)[\overline{q}\Gamma q](0)\rangle$

$\Rightarrow$ Disconnected contributions everywhere

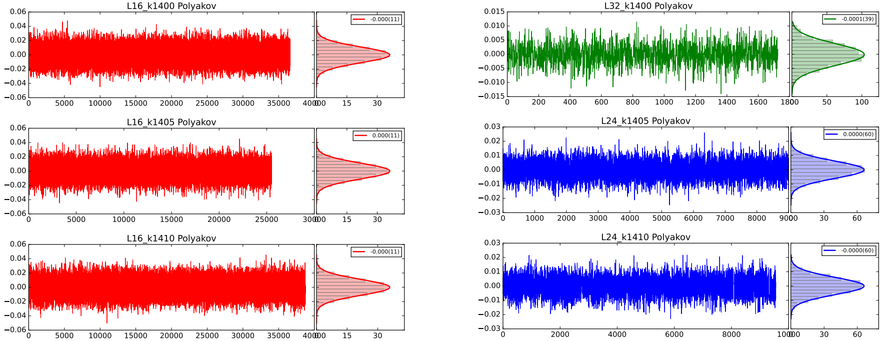
$\Rightarrow$ Require measurement strategy that computes these!

Numerical Set-up [OpenQCD]

- $N_f = 1,\ N_c = 3$
- Symanzik improved gauge action [1111.7054]
- tree-level improved Wilson fermions ($c_{SW} = 1$)
- RHMC algorithm [0608015]
- Single coupling ($\beta = 4.5$) [$a \approx 0.06\,\mathrm{fm}$ from gradient flow [1006.4518]]
- Range of box sizes (L) and quark masses (κ)
- Branched into replicas on larger ensembles

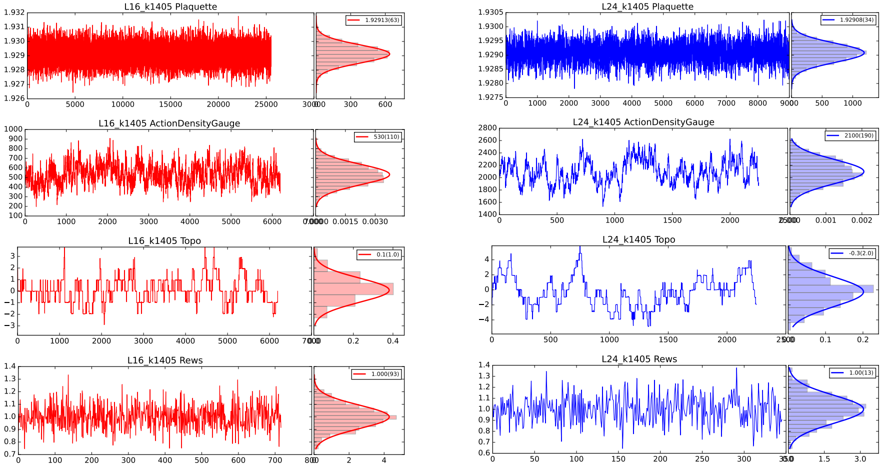


Monitor standard observables, e.g. tr(Polyakov loop)



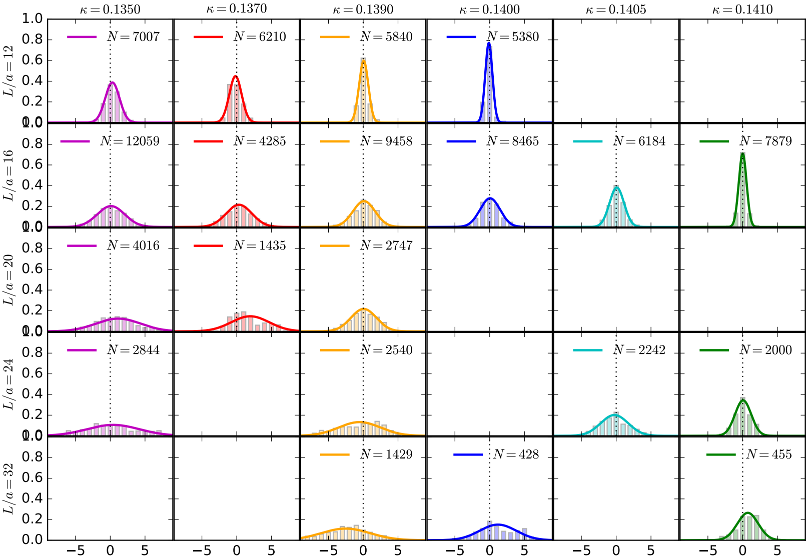
$\Rightarrow$ We are in the confined phase! ✓

Standard observables, e.g. $\kappa = 0.1405$, $L/a = 16$ vs 24



*Reweighting factors are normalised and only measured on those configurations (where valence measurements take place)

Monitor standard observables, e.g. topological charge



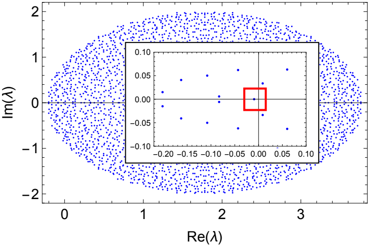
The sign problem in $N_f = 1$ QCD

- **Single Wilson fermion**: $\det(D) < 0$ possible

$\Rightarrow$ **Need to monitor sign of the fermion determinant**

- Check where **real eigenvalues** of D change sign.

[Plot courtesy of F. Ziegler]



How to check for negative determinant signs?

Note: zero eigenvalue of D $\quad\Rightarrow\quad$ zero eigenvalue of $Q \equiv \gamma_5 D$.

$$D\,|\psi_i\rangle = 0\,|\psi_i\rangle \Rightarrow Q\,|\psi_i\rangle = (\gamma_5 D)\,|\psi_i\rangle = \gamma_5 0\,|\psi_i\rangle = 0\,|\psi_i\rangle$$

Easier to analyse zero eigenvalues of Hermitian matrix Q instead

$$Q\,|\psi_i\rangle = \lambda_i\,|\psi_i\rangle$$

- eigenvalue $\lambda_i(m_0)$,
- chirality $\chi_i(m_0) = \langle\psi_i|\,\gamma_5\psi_i\rangle\,(m_0)$ (for $|\lambda| \ll 1$: $\propto \frac{d\lambda}{dm_0}$)

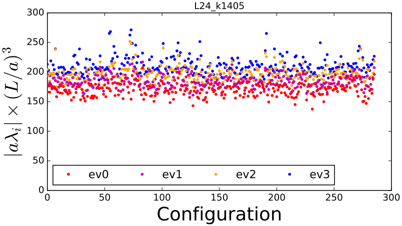
Monitoring the sign problem: Recipe

- Need to identify cases where the fermion determinant is negative $\Rightarrow D$ has odd number of negative eigenvalues.
- Determine mass m_0^* for which $\lambda_i(m_0^*) = 0$.
- Odd number of $m_0^* > m_0 \Rightarrow \det(D) < 0$

Recipe

1. Compute eigenvalue λ and chirality χ (of Q).
2. Determine cases where λ and $\chi(\propto \frac{d\lambda}{dm_0})$ have opposite signs.
3. On these configs, compute eigenvalues and eigenvectors for scan of partially quenched data points.
4. Perform *tracking analysis* [Schaefer & Mohler 2003.13359] and check for zero crossings.

Monitoring the sign problem: initial scan



- $L/a = 24$, $\kappa = 0.1405$: Clear separation of $|\lambda_i|$ from zero. $\Rightarrow$ Safe ✓
- Expect most problematic ensembles near κ_{crit}.
- Scan the other $\kappa = 0.1405$ and $\kappa = 0.1410$ ensembles.

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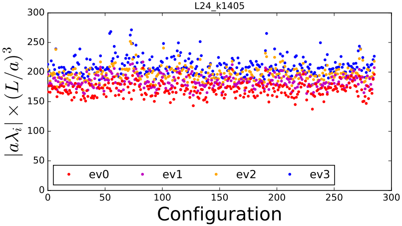

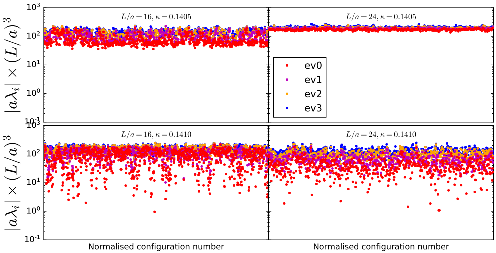

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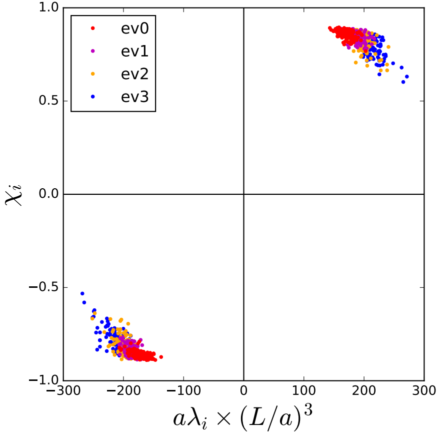
- Behaviour "worse" as $\kappa \uparrow$
- Behaviour "worse" as $L \downarrow$

Monitoring the sign problem: Identifying suspicious cases

- NE,SW: $\Rightarrow \lambda \times \frac{d\lambda}{dm} > 0 \Rightarrow |\lambda| \uparrow$ as $m \uparrow$. Safe region ✓
- NW,SE: $\Rightarrow \lambda \times \frac{d\lambda}{dm} < 0 \Rightarrow |\lambda| \downarrow$ as $m \uparrow$. Dangerous region ✗



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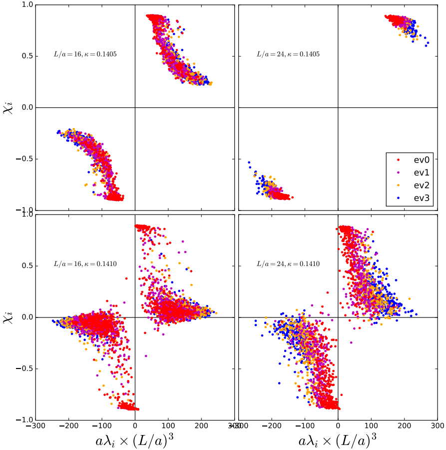

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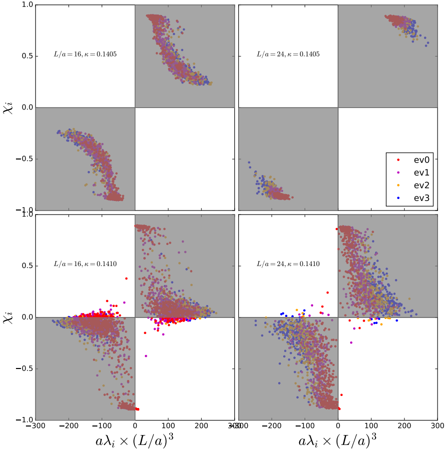


- $L/a = 24, \kappa = 0.1405$ safe! ✓
- $L/a = 16, \kappa = 0.1405$ safe! ✓
- $L/a = 16, \kappa = 0.1410$ Some suspicious cases ✗
- $L/a = 24, \kappa = 0.1410$ Some suspicious cases ✗

Particularly dangerous when $|\chi| \gg 0$
$\Rightarrow$ Need to run the *tracking analysis* on the suspicious cases!

Monitoring the sign problem: tracking suspicious cases

Investigate the suspicious cases using the tracking analysis [2003.13359].
Assume λ_i and ψ_i vary slowly and smoothly with m.

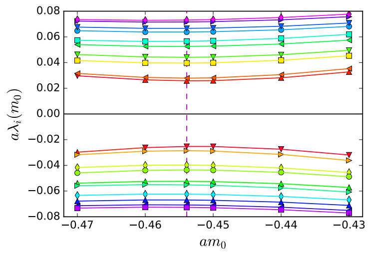
1. Find $O(10)$ lowest eigenvalues λ_i and eigenvectors ψ_i at values m and $m + \delta m$ (i labels the eigenvalue)
2. Construct the square matrix of overlap factors

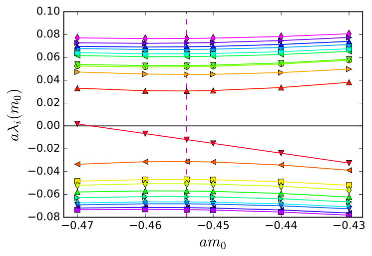
$$M_{ij}(m, m + \delta m) \equiv \langle \psi_i(m) | \, \psi_j(m + \delta m) \rangle$$

3. Find $\max |M_{ij}| \Rightarrow$ **"λ_i at m continues as λ_j at $m + \delta m$."**
4. Remove the corresponding row and column and repeat until all $\lambda_i(m)$ are matched up with some $\lambda_j(m + \delta m)$
5. Iterate for different m until all $|\lambda_i|$ are moving away from zero.
6. Count # eigenvalues that crossed zero between m_0 and $m \gg m_0$

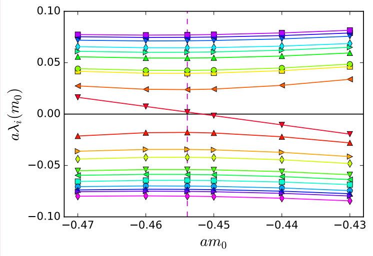
Monitoring the sign problem: tracking analysis results

positive determinant





negative determinant



- Important check!
- Very few cases and ensembles do not enter final analysis
- No correlation to other observables observed (acceptance/autocorrelation)

⇒ **Our simulations are safe ✓**

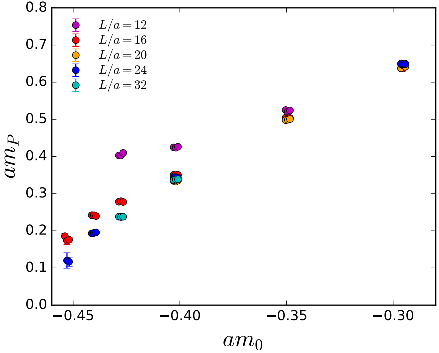
Measurement set-up

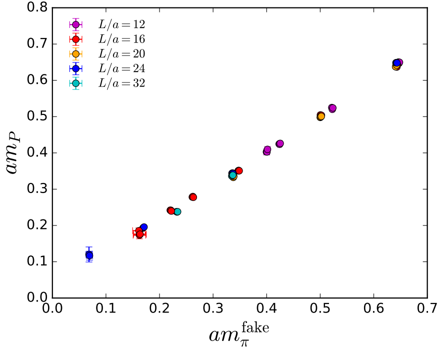
- Exact distillation set-up using Laplacian Heaviside smearing (LapH) Obtain different smearings for operators by varying number of distillation eigenvalues (N_{ev}) *post*-data production
- Includes all disconnected pieces!
- $q\bar{q}$-type operators $\mathcal{P}$, $\mathcal{I}$, $\mathcal{S}$ with quantum numbers of Pseudoscalar (P), Vector (V) and Scalar (S) and scalar glueball-type operator $\mathcal{G}$ constructed purely gluonically
- Reweighted and vacuum subtracted correlation functions

$\Rightarrow$ Simultaneous correlated multi-exponential fits to several N_{ev}.
$\Rightarrow$ Assess systematics/stability by varying which N_{ev} enter.

Defining the massless limit

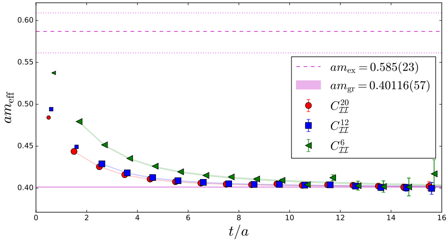
- Interested in the meson spectrum in massless limit
- Simulate fictitious pion (i.e. connected part only) with mass m_π^{fake} to define massless limit [1809.09117]:



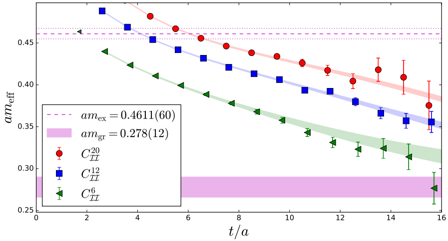


Vector spectrum surprises I: Correlator fits

$L/a = 32\,,\ \kappa = 0.1390$



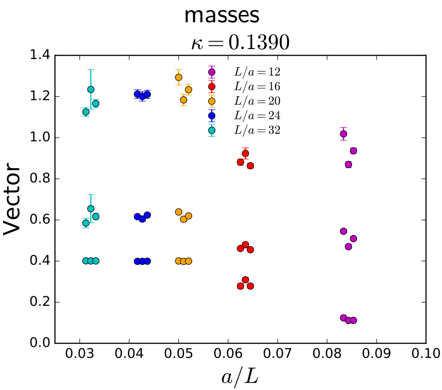
$L/a = 16\,,\ \kappa = 0.1390$

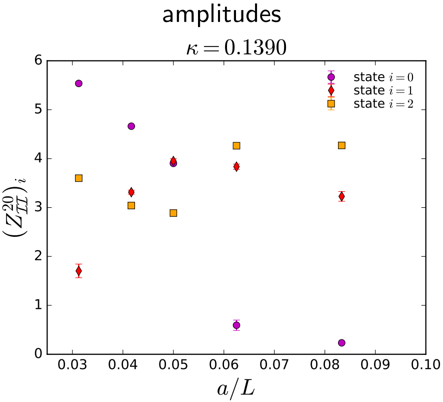


Qualitatively different behaviour for two point functions:

- Slower approach to the plateau
- Precision between ground and 1st excited state reversed
- Lighter state present in $L/a = 16$ than $L/a = 32$, despite same κ
- Persists for different combinations of N_{ev} entering the fit

Vector spectrum surprises II: Volume behaviour



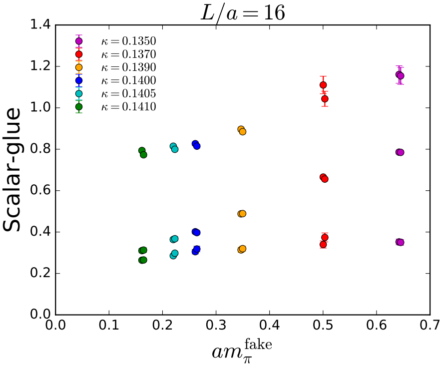


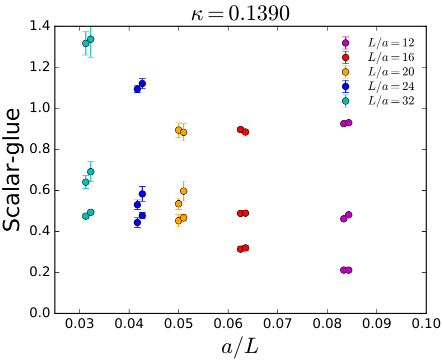
- Lightest state on small volumes has small overlap with correlator
- Additional light state volume dependent
- 1st excited state on small L/a similar to ground state of larger L/a

$\Rightarrow$ Unphysical FV state. Identify 1st excited state with state of interest.

Scalar spectrum results

- Glueball state: expected to be mass and volume insensitive
- $q\bar{q}$-like state: strong mass dependence, volume insensitive



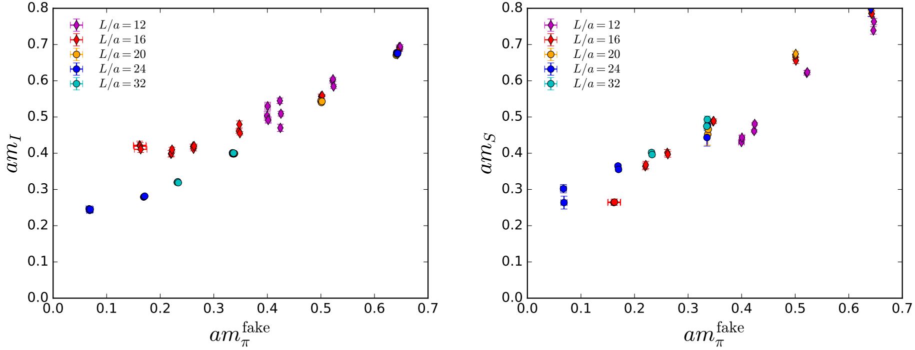


- One mass insensitive state (?), one mass dependent state ($q\bar{q}$)
- One L indep. state ($am \sim 0.45$)
- One L dep. state

Volume dependence $\Rightarrow$ not glueball! $\Rightarrow$ Unphysical FV state

Extracted spectrum

- Repeat identification of lowest-lying mass dependent ($q\bar{q}$) state on each ensemble.



$\Rightarrow$ Consistent results from different volumes for large masses

$\Rightarrow$ Clear finite size effects as quark mass is reduced

It remains to extrapolate $m_\pi^{\mathrm{fake}} \to 0$

Extrapolation to the massless limit

TARGET: m_P/m_S in the massless limit. Two options:

1. ratio of limits

$$\frac{\lim_{m_\pi^{\mathrm{fake}}\to 0} m_P}{\lim_{m_\pi^{\mathrm{fake}}\to 0} m_S}$$

2. limit of ratios

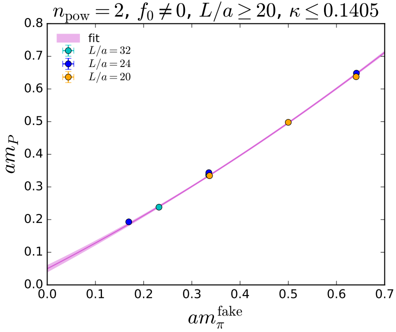
$$\lim_{m_\pi^{\mathrm{fake}}\to 0} \frac{m_P}{m_S}$$

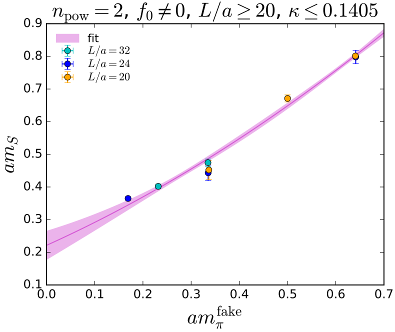
Try both with simple fit ansatz (M either a mass or a ratio):

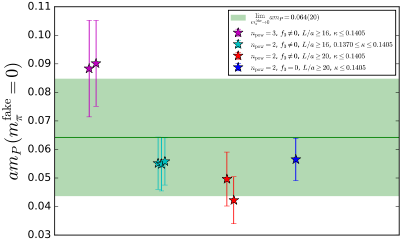
$$M(m_\pi^{\mathrm{fake}}, L) = \left[\sum_{i=0}^{N} c_i \left(m_\pi^{\mathrm{fake}}\right)^i\right]\left[1 + f_0 e^{-m_P L}\right]$$

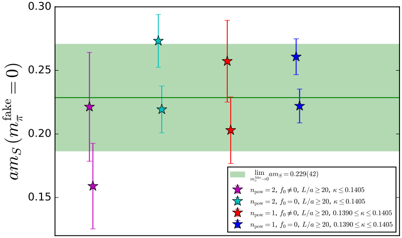
<u>Aside:</u> Does i run over even and odd, or only over even values?

Ratio of limits

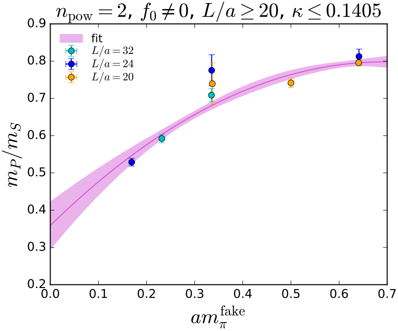


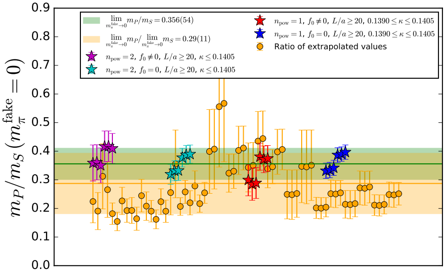






Limit of Ratios





- Green band and stars: variations of "Limit of Ratios".
- yellow points: all mutual combinations of "Ratio of Limits".

$\Rightarrow$ Limit of Ratios is more precise and less steep as a function of quark mass. Take this as preferred prescription and view "Ratio of Limits" as sanity check.

$$\frac{m_P}{m_S}(N_C = 3) = 0.356(54)$$

Aside: Even vs Odd powers of m_π^{fake} (example: m_P/m_I)

$$\text{Recall: } M(m_\pi^{\mathrm{fake}}, L) = \left[\sum_{i=0}^{N} c_i \left(m_\pi^{\mathrm{fake}}\right)^i\right]\left[1 + f_0 e^{-m_P L}\right]$$

- Most general: even + odd powers
- partially quenched $\Rightarrow$ ext. χ sym. group $\Rightarrow m_0 \propto \left(m_\pi^{\mathrm{fake}}\right)^2$.

are our masses light enough for this?

Aside: Even vs Odd powers of $m_\pi^{\rm fake}$ (example: m_P/m_l)

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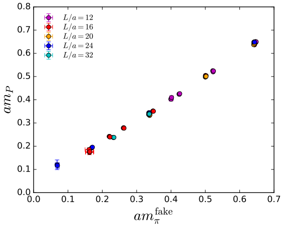
If so, expect clear difference between m_P and $m_\pi^{\rm fake}$ ✗



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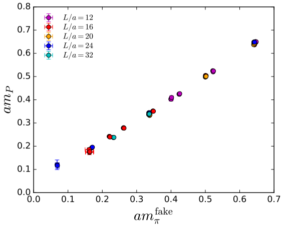
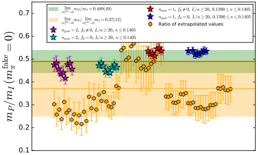
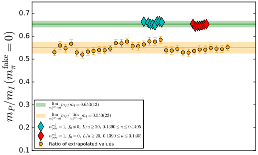


odd + even



only even



- for only even: only fits with large $m_\pi^{\rm fake}$-cuts survive p-value criterion
- results for "only even" are discrepant for two methods

Discussion

Recall: Sannino & Shifman [0309252] predict ($\beta = O(1/N_C) > 0$)

$$\frac{m_P}{m_S}(N_C) = 1 - \frac{22}{9N_C} - \frac{4}{9}\beta + O(1/N_C^2) \Rightarrow \frac{m_P}{m_S}(3) \lesssim 0.185 + O(1/3^2)$$

Armoni & Imeroni [0508107] predict

$$\frac{m_P}{m_S}(N_C) = 1 - \frac{2}{N_C} + \mathcal{O}(1/N_C^2) \Rightarrow \frac{m_P}{m_S}(3) = 1/3 + O(1/3^2)$$

$$\text{we find:} \qquad \frac{m_P}{m_S}(3) = 0.356(54)$$

- Reasonable agreeement, despite N_C ✓
  $\Rightarrow$ Need to move to larger N_C to test this!
- New prediction for pseudoscalar/vector ratio:

$$\frac{m_P}{m_I}(3) = 0.489(49)$$

Conclusions and Outlook

Summary

- Studied $N_f = 1$ QCD at fixed β as function of L/a and κ
- Tracking analysis to monitor sign problem
- predicted ratios of P,S,I in χ-limit

Outlook [Lattice Proceedings: 2212.06709]

- Extend to $N_C > 3$ (using "HighRep" on GPU & CPU)
- Compare tree-level $O(a)$-improved to un-improved.
- Rescaled β to stay at roughly same lattice spacing but need to retune κ
- Exploring parameter space $(L, \kappa) \rightarrow$

m_π^{fake} for $N_C > 3$