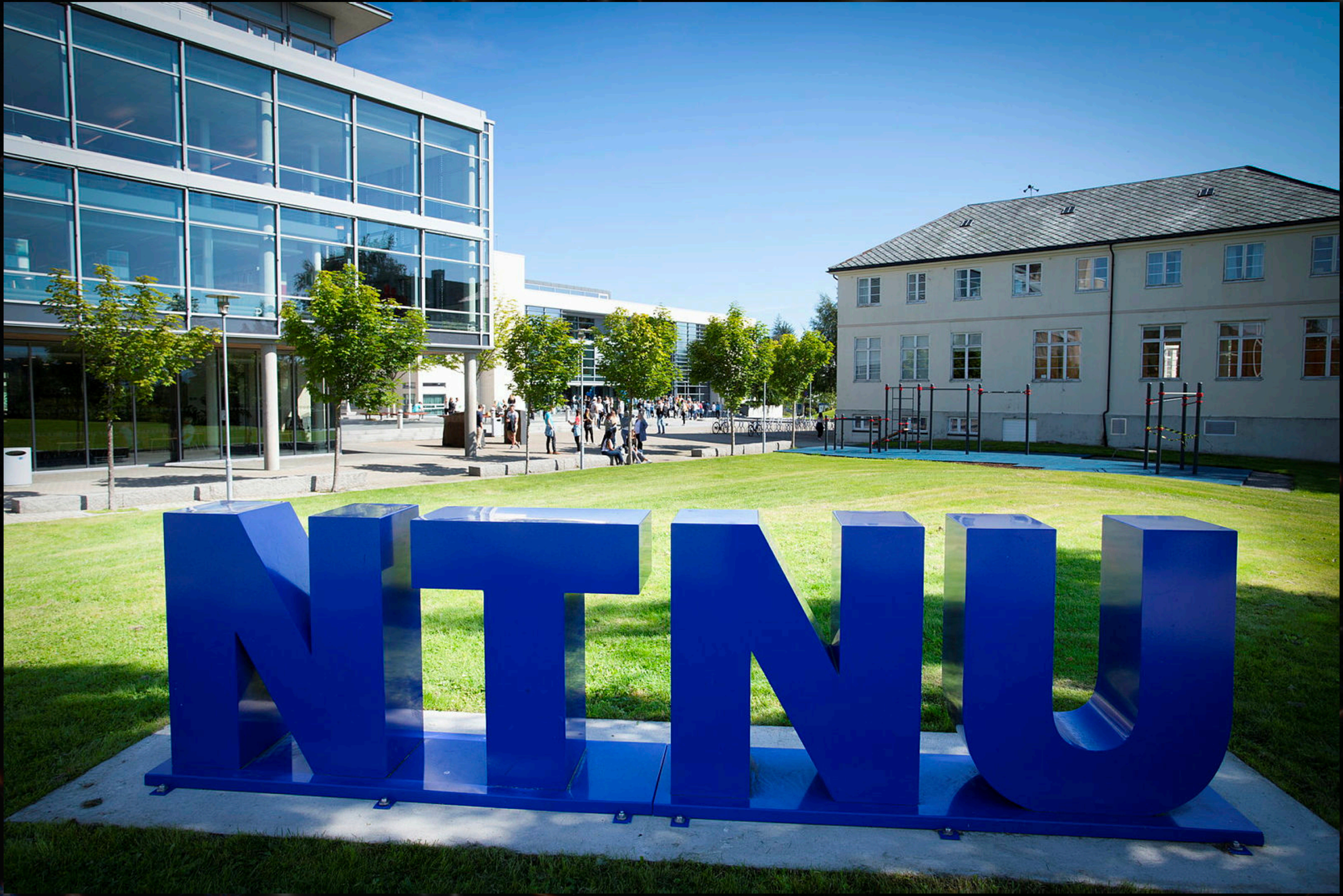
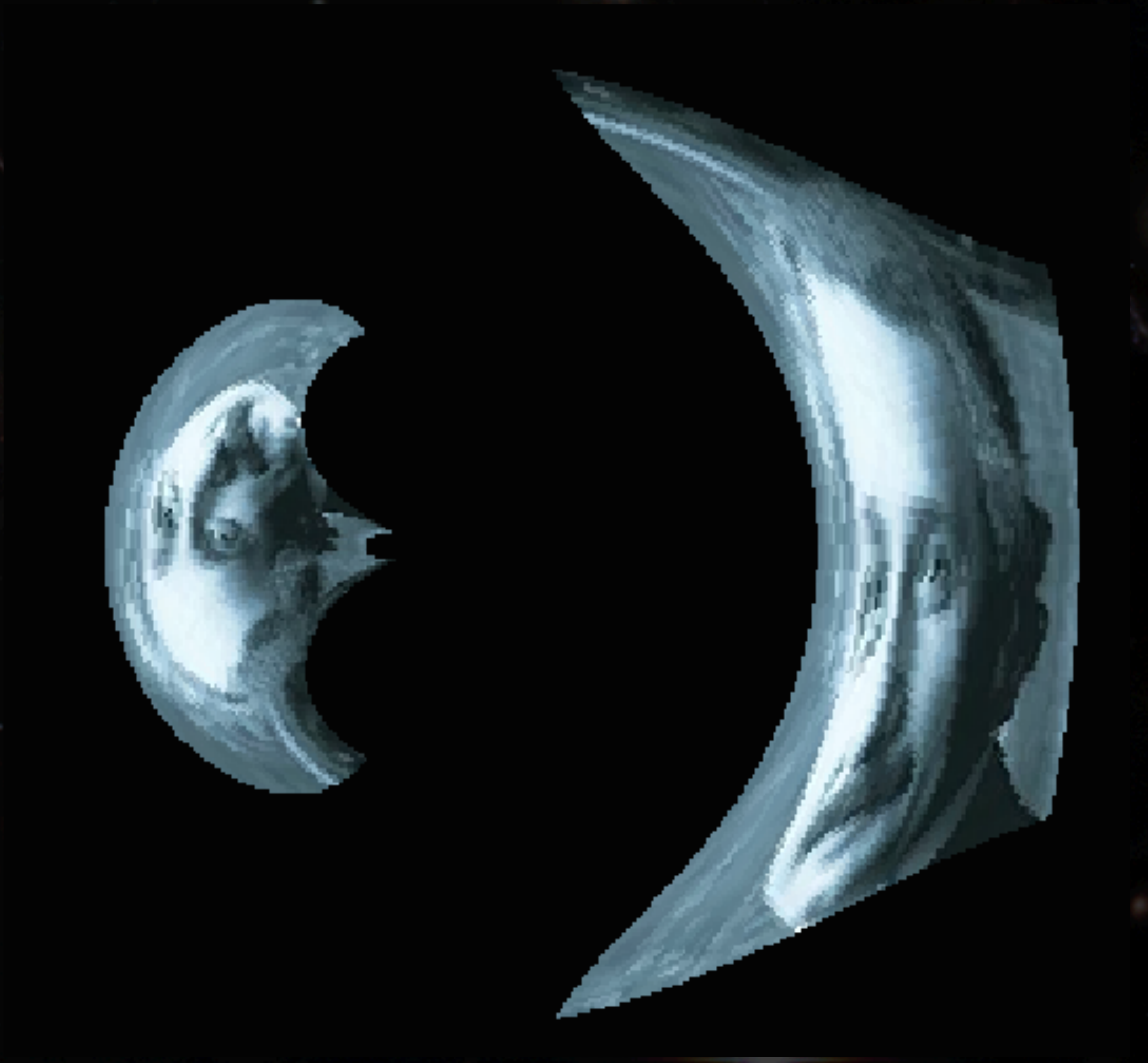

NTNU

HIGHER-ORDER LENSING

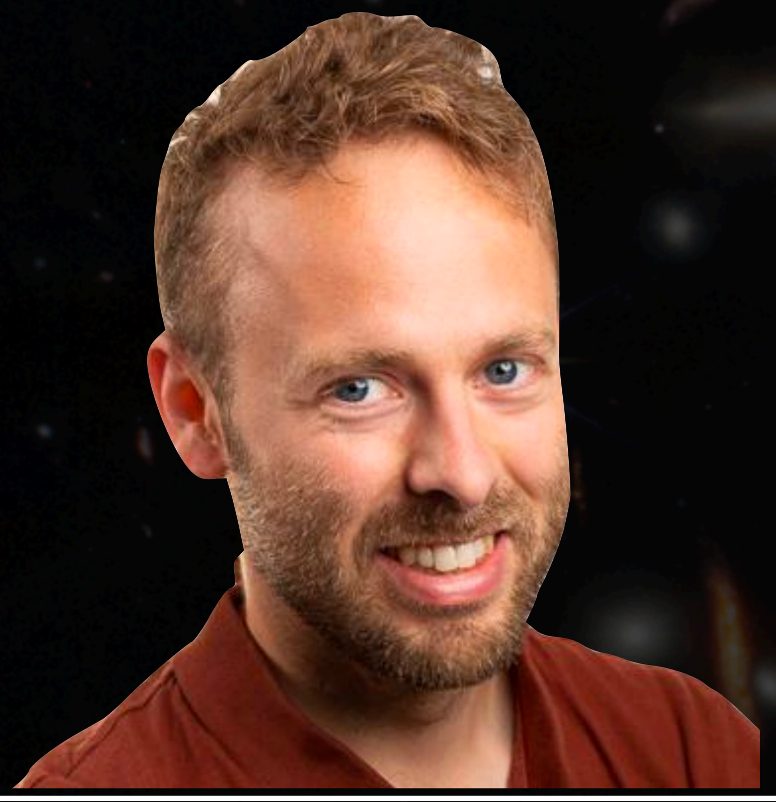


Group elements

NTNU
Faculty of Information Technology and Electrical Engineering
Department of ICT and Natural Sciences

Group elements

Hans George
(Computer guru)



Ben (Physicist)

Kenny (Physicist)

+ students

NTNU
Faculty of Information Technology
and Electrical Engineering
Department of ICT
and Natural Sciences

Structure of talk

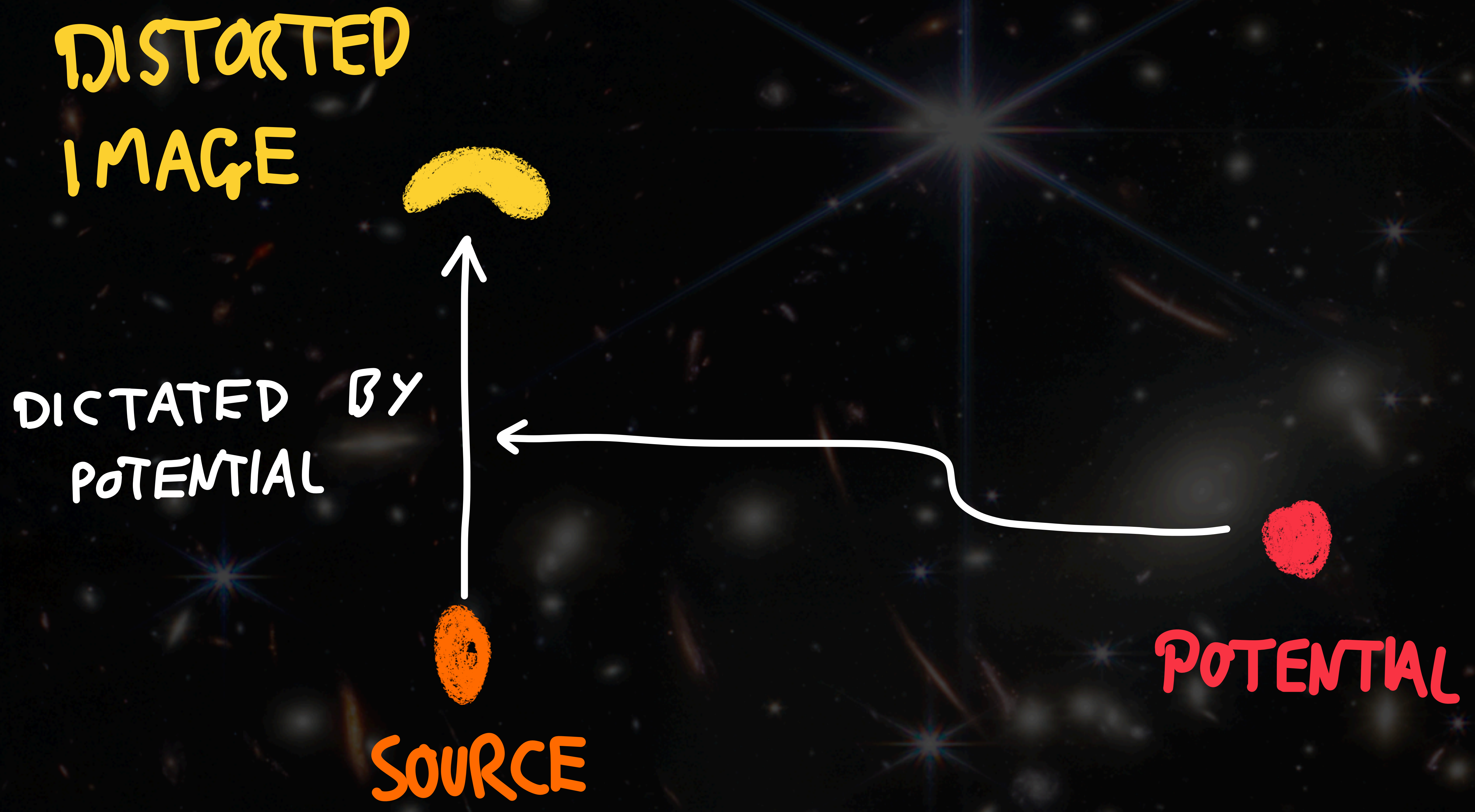
Idea

Formalism

Results

Idea


DISTORTED IMAGE
SOURCE
POTENTIAL
BANANA!

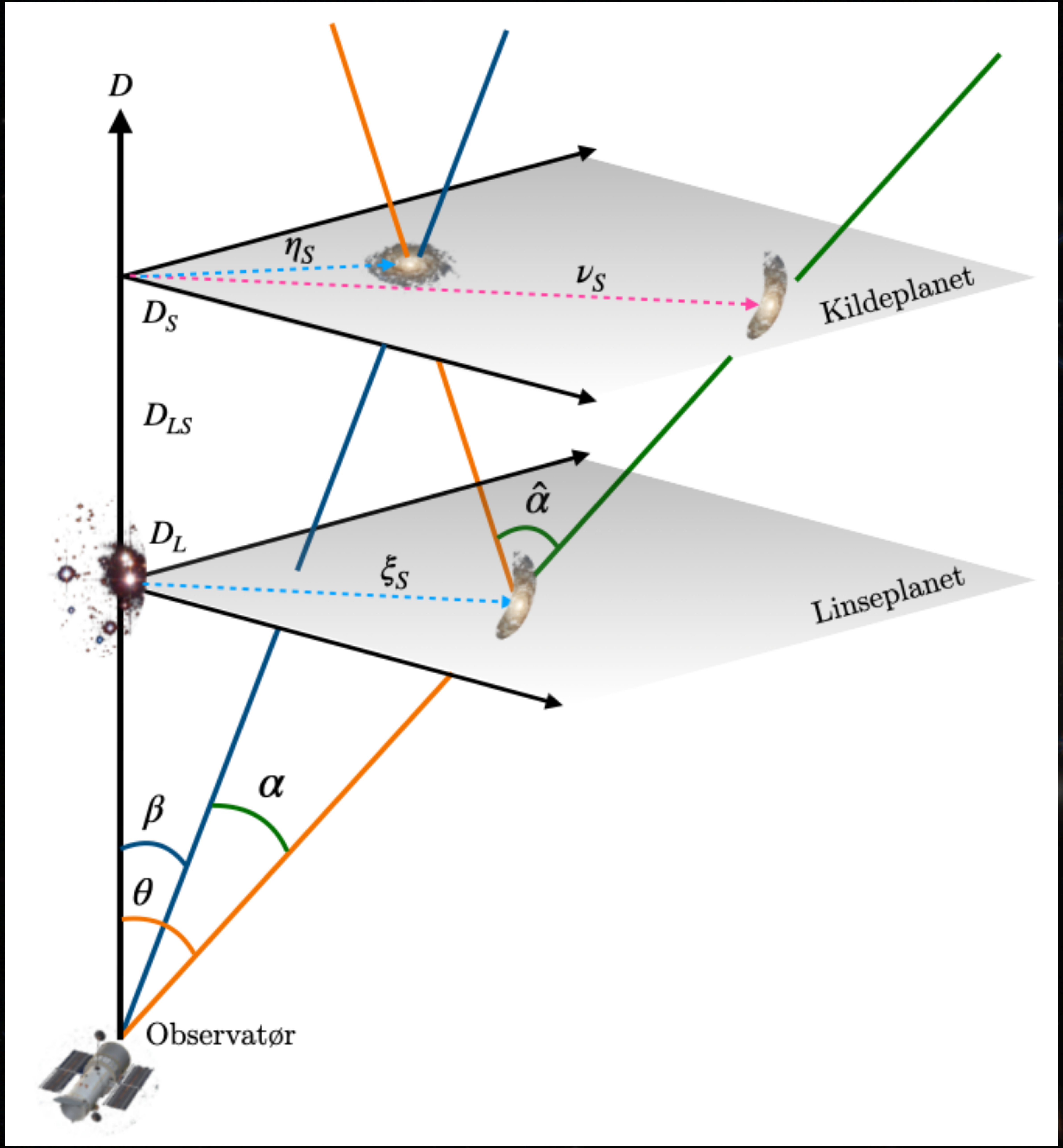


Can we go the other way around?!

DISTORTED IMAGE
ML
SOURCE
ψ
POTENTIAL
Knowledge of DM distribution!

Training phase
DISTORTED IMAGE
ML TRAINING ②
SOURCE
① CALCULATE FOR KNOWN ψ
POTENTIAL
DISTORTED IMAGE
③ ML APPLIC.
SOURCE
OBTAIN ψ
POTENTIAL
Real-data phase

Formalism



$$\boldsymbol{\beta} = \boldsymbol{\theta} - \boldsymbol{\alpha}(\boldsymbol{\theta})$$

$$\boldsymbol{\beta} = \boldsymbol{\theta} - \boldsymbol{\nabla}\psi(\boldsymbol{\theta})$$

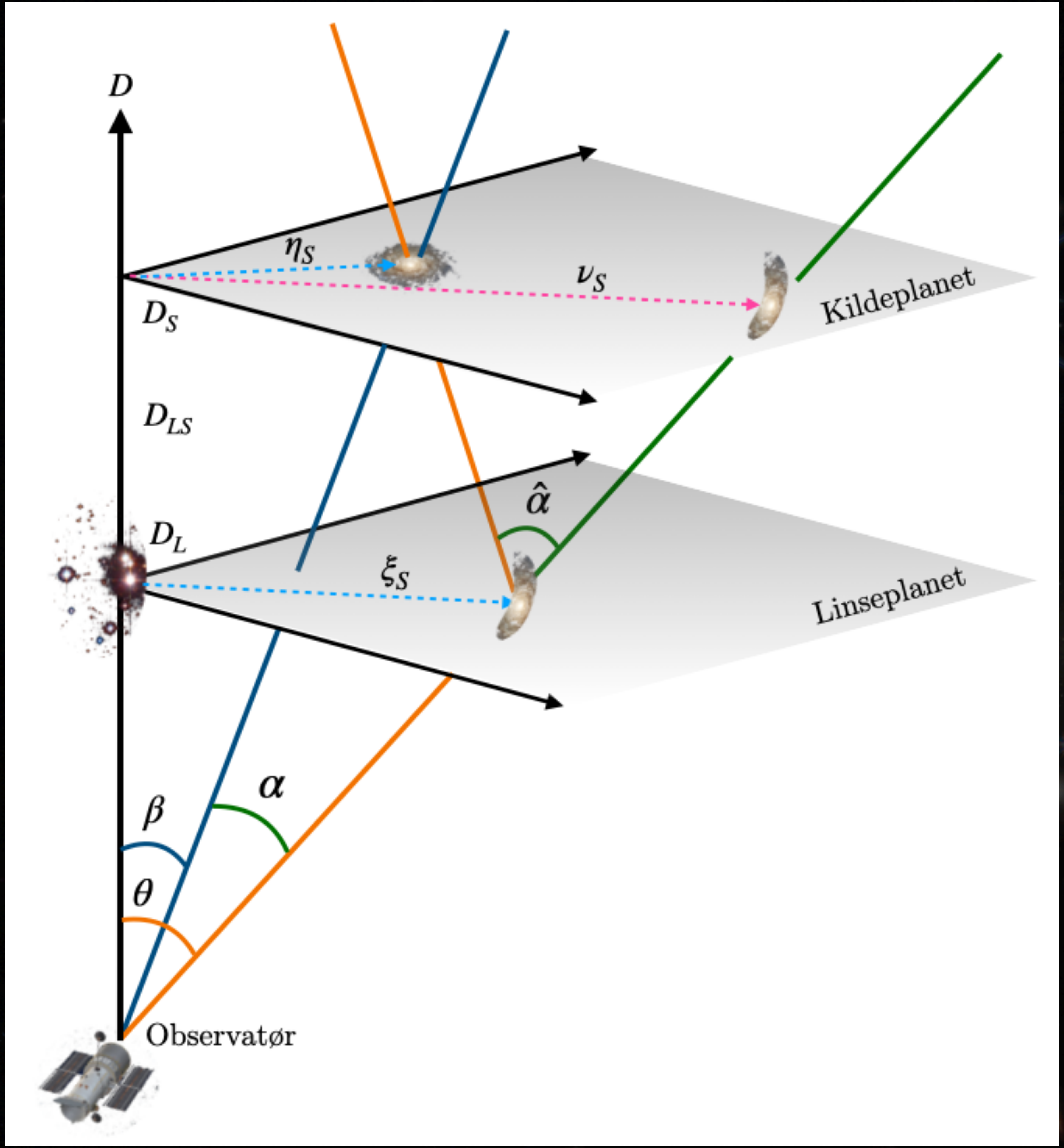
$$\mathrm{d}\boldsymbol{\beta} = A^{-1}\mathrm{d}\boldsymbol{\theta}$$

$$A^{-1}(\boldsymbol{\theta}) \equiv \frac{\partial \boldsymbol{\beta}}{\partial \boldsymbol{\theta}} = \begin{bmatrix} 1 - \psi_{xx} & \psi_{xy} \\ \psi_{yx} & 1 - \psi_{yy}. \end{bmatrix}$$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad , \quad \mathbf{R}_{-} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad , \quad \mathbf{R}_{/} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$A^{-1} = (1-\kappa)I + \gamma_{+}\boldsymbol{R}_{-} + \gamma_{\times}\boldsymbol{R}_{/}$$

$$\kappa(\theta) = \frac{1}{2}(\psi_{yy} + \psi_{xx}),$$
$$\gamma_{+} = \frac{1}{2}(\psi_{yy} - \psi_{xx}),$$
$$\gamma_{\times} = \psi_{xy}.$$

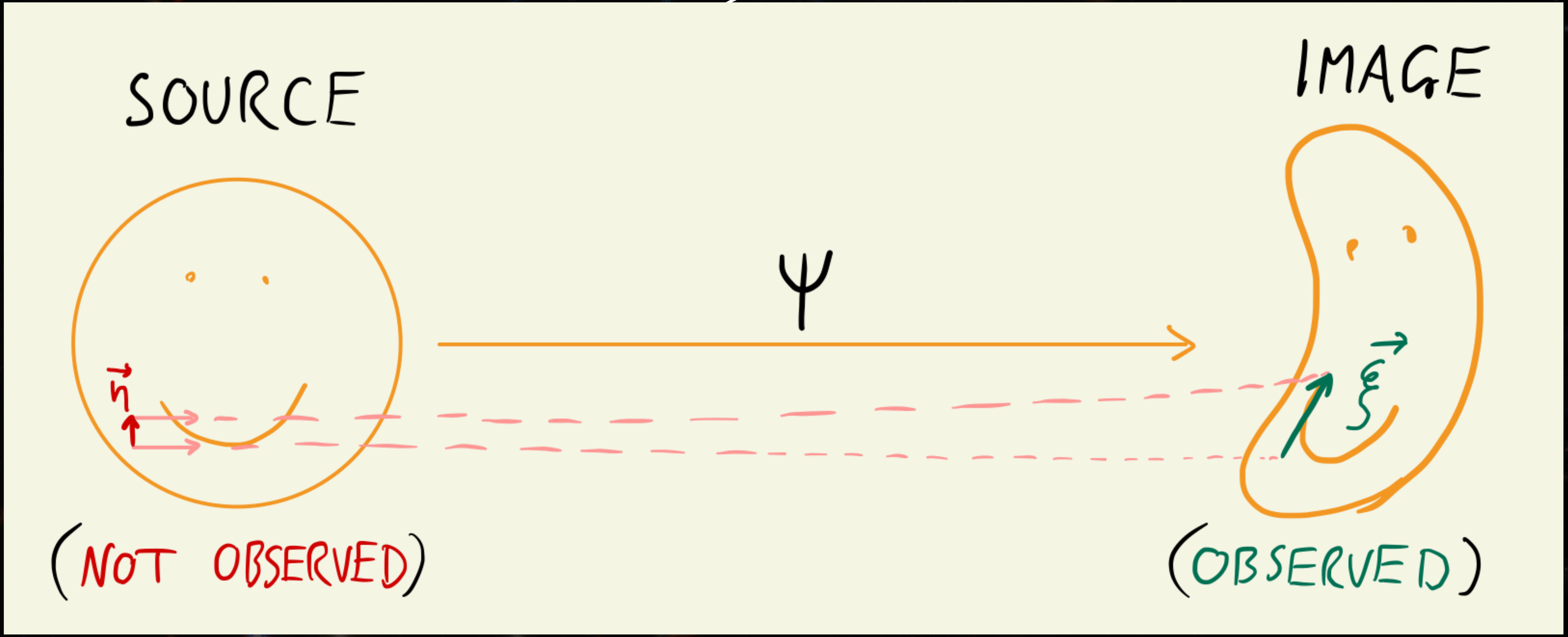


$$\mathrm{d}\boldsymbol{\beta} = A^{-1}\mathrm{d}\boldsymbol{\theta}$$

$$A^{-1} = (1-\kappa)I + \gamma_+ \boldsymbol{R}_- + \gamma_\times \boldsymbol{R}_/$$

$$\kappa(\theta) = \frac{1}{2}(\psi_{yy} + \psi_{xx}),$$
$$\gamma_+ = \frac{1}{2}(\psi_{yy} - \psi_{xx}),$$
$$\gamma_\times = \psi_{xy}.$$

Geodesic deviation



Geodesic deviation

$$\ddot{\boldsymbol{\eta}} - \mathcal{R}(\psi)\boldsymbol{\eta} = \mathcal{O}^1(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}})$$

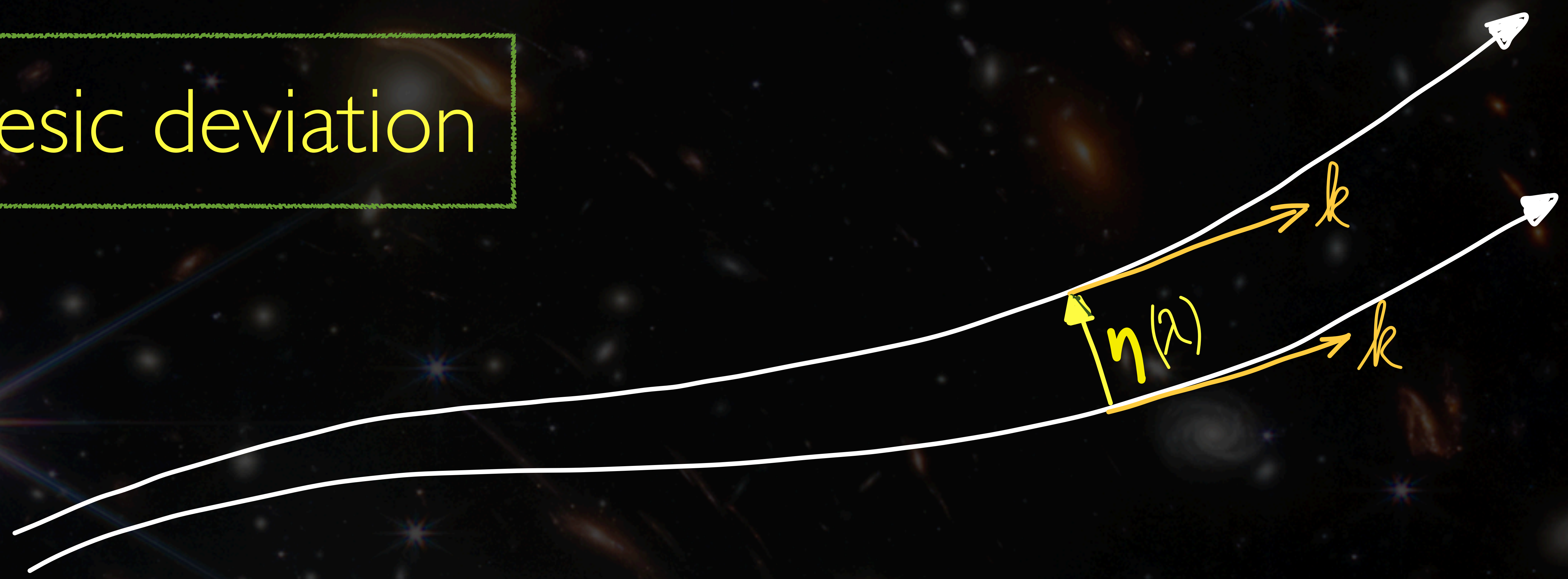
→ convergence+ shear

$$\ddot{\boldsymbol{\eta}} - \mathcal{R}(\psi)\boldsymbol{\eta} = \mathcal{F}(\psi)$$

→ Shear +
flexion+2.flexion+…inf

leading-order screen-space derivatives

Geodesic deviation



Roulette formalism

$$\ddot{\boldsymbol{\eta}} - \mathcal{R}(\psi)\boldsymbol{\eta} = \mathcal{F}(\psi)$$

leading-order screen-space derivatives

Solution via series expansion

$$\boldsymbol{\eta}(\boldsymbol{\xi}) = \sum_{m=1}^{\infty} \frac{\epsilon^m}{m!} \hat{\boldsymbol{\eta}}^{(m)}(\boldsymbol{\xi})$$

→

$$\eta^A_{(m)} = \mathcal{M}^A_{B_1 \cdots B_m} \xi^{B_1} \dots \xi^{B_m}$$

$$\mathcal{M}_{AB_1 \cdots B_m} \hat{\xi}^{B_1} \dots \hat{\xi}^{B_m} = \sum_{s=0}^{m+1} \frac{[1-(-1)^{m+s}]}{4} \times \left(\left[\left[C^+ \alpha_s^m + \bar{\beta}_s^m \right] \mathbf{R}_- + \left[C^+ \beta_s^m - \bar{\alpha}_s^m \right] \mathbf{R}_/ \right] \mathbf{p}_{s-1} + \left[\left[C^- \alpha_s^m - \bar{\beta}_s^m \right] \mathbf{I} + \left[C^- \beta_s^m + \bar{\alpha}_s^m \right] \varepsilon \right] \mathbf{p}_{s+1} \right),$$

$$C^{\pm} = 1 \pm \frac{s}{m+1}.$$

$$\mathbf{p}_{(s)} = \cos s\theta \mathbf{e}_x + \sin s\theta \mathbf{e}_y.$$

$$\varepsilon_- = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \mathbf{R}_- = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \mathbf{R}_/ = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Our job is therefore to determine $\alpha_s^m, \beta_s^m, \bar{\alpha}_s^m, \bar{\beta}_s^m$

$$\alpha_s^m = -2^{-\delta_{0s}} \chi^{m+1} \sum_{k=0}^{m} \binom{m}{k} \left(\mathcal{C}_s^{m(k)} \partial_{\mathrm{X}} + \mathcal{C}_s^{m(k+1)} \partial_{\mathrm{Y}} \right) \partial_{\mathrm{X}}^{m-k} \partial_{\mathrm{Y}}^{k} \psi,$$

$$\beta_s^m = -\chi^{m+1} \sum_{k=0}^{m} \binom{m}{k} \left(\mathcal{S}_s^{m(k)} \partial_{\mathrm{X}} + \mathcal{S}_s^{m(k+1)} \partial_{\mathrm{Y}} \right) \partial_{\mathrm{X}}^{m-k} \partial_{\mathrm{Y}}^{k} \psi,$$

$$\mathcal{C}_s^{m(k)} = \frac{1}{\pi} \int_{-\pi}^{\pi} \mathrm{d}\theta \, \sin^k \theta \cos^{m-k+1} \theta \cos s\theta,$$

$$\mathcal{S}_s^{m(k)} = \frac{1}{\pi} \int_{-\pi}^{\pi} \mathrm{d}\theta \, \sin^k \theta \cos^{m-k+1} \theta \sin s\theta.$$

Determine these by machine learning!

$$\alpha_s^m = -2^{-\delta_{0s}} \chi^{m+1} \sum_{k=0}^{m} \binom{m}{k} \left(\mathcal{C}_s^{m(k)} \partial_{\mathrm{X}} + \mathcal{C}_s^{m(k+1)} \partial_{\mathrm{Y}} \right) \partial_{\mathrm{X}}^{m-k} \partial_{\mathrm{Y}}^{k} \psi,$$

$$\beta_s^m = -\chi^{m+1} \sum_{k=0}^{m} \binom{m}{k} \left(\mathcal{S}_s^{m(k)} \partial_{\mathrm{X}} + \mathcal{S}_s^{m(k+1)} \partial_{\mathrm{Y}} \right) \partial_{\mathrm{X}}^{m-k} \partial_{\mathrm{Y}}^{k} \psi.$$

$$\kappa = \alpha_0^1 \quad , \quad \gamma_+ = \alpha_2^1 \quad , \quad \gamma_\times = \beta_2^1$$

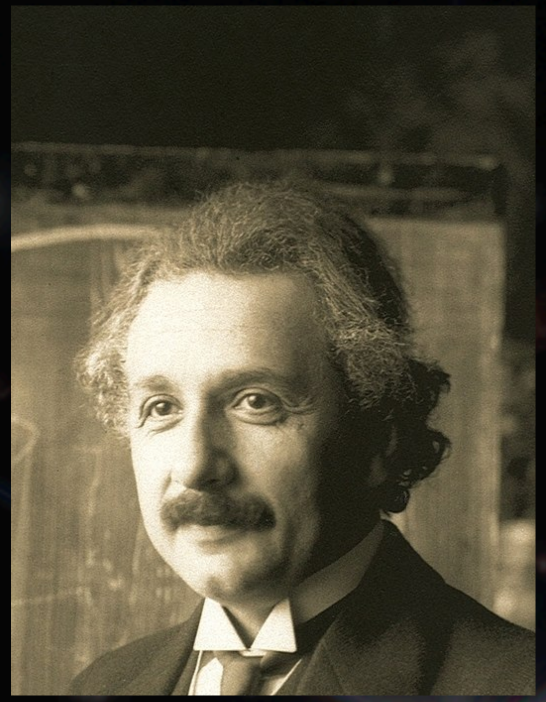
$$\mathcal{C}_s^{m(k)} = \frac{1}{\pi} \int_{-\pi}^{\pi} \mathrm{d}\theta \sin^k \theta \cos^{m-k+1} \theta \cos s\theta,$$

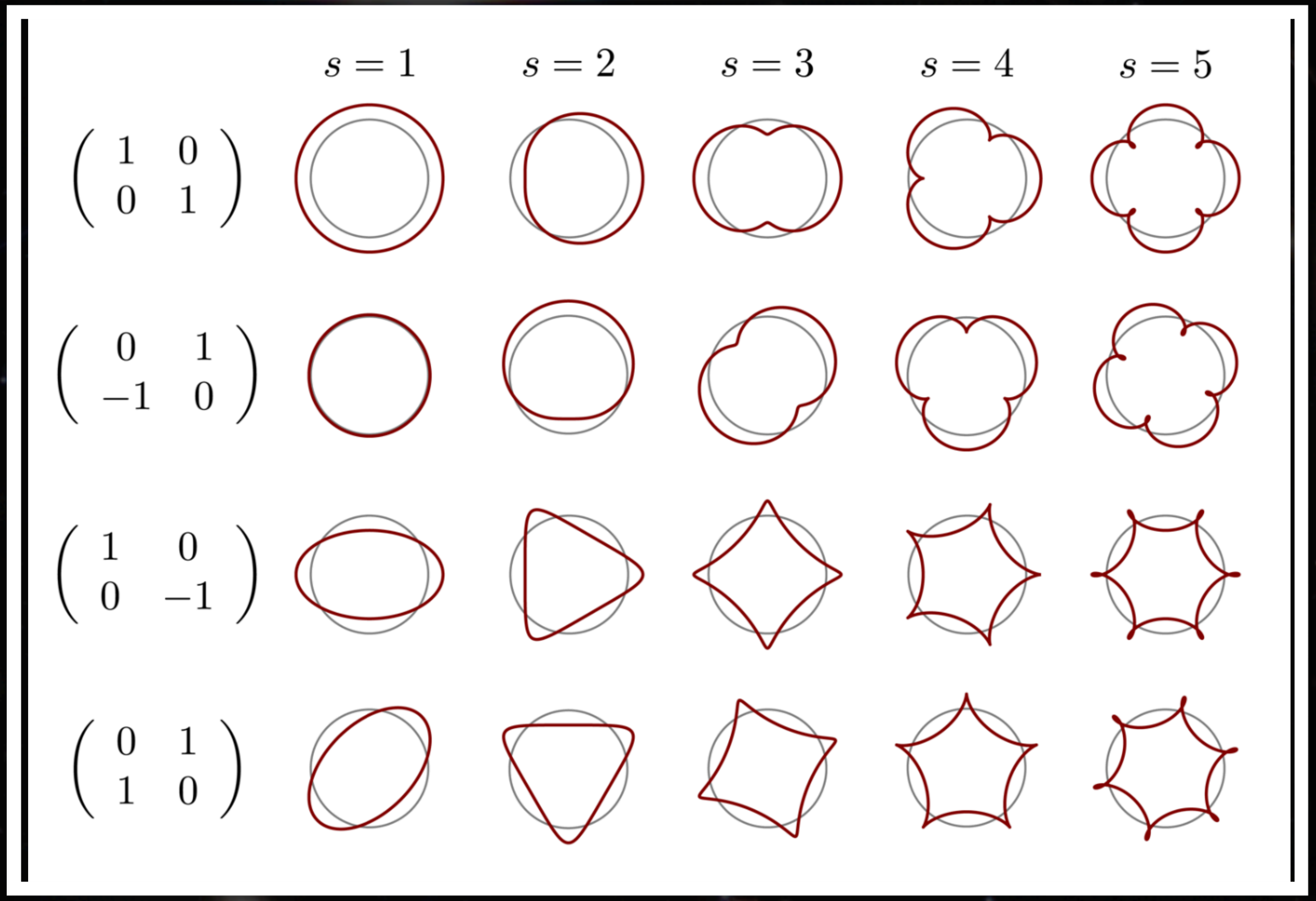
$$\mathcal{S}_s^{m(k)} = \frac{1}{\pi} \int_{-\pi}^{\pi} \mathrm{d}\theta \sin^k \theta \cos^{m-k+1} \theta \sin s\theta.$$

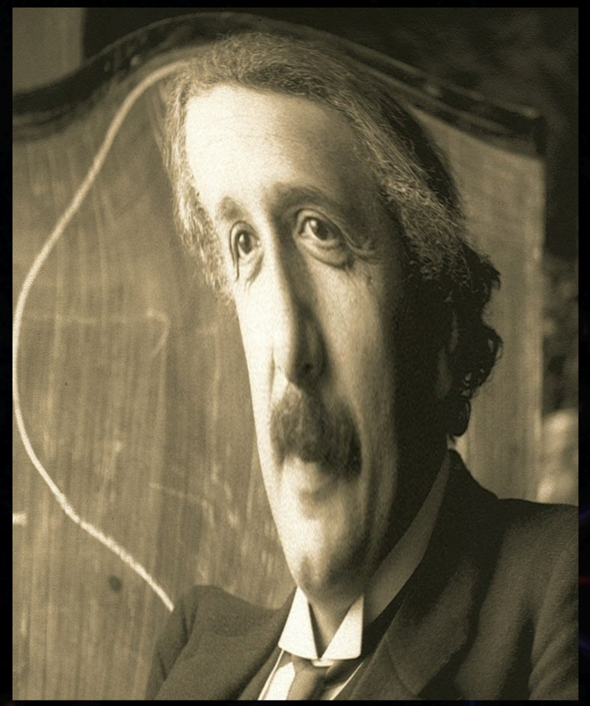
$$\varepsilon_{-} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad , \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad , \quad \mathbf{R}_{-} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad , \quad \mathbf{R}_{/} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\mathbf{p}_{(s)} = \cos s\theta \mathbf{e}_x + \sin s\theta \mathbf{e}_y .$$

$$\boldsymbol{\eta}_{\circ} \longrightarrow \boldsymbol{\eta}_{\circ} + \alpha \boldsymbol{R} \cdot \boldsymbol{p}_s$$



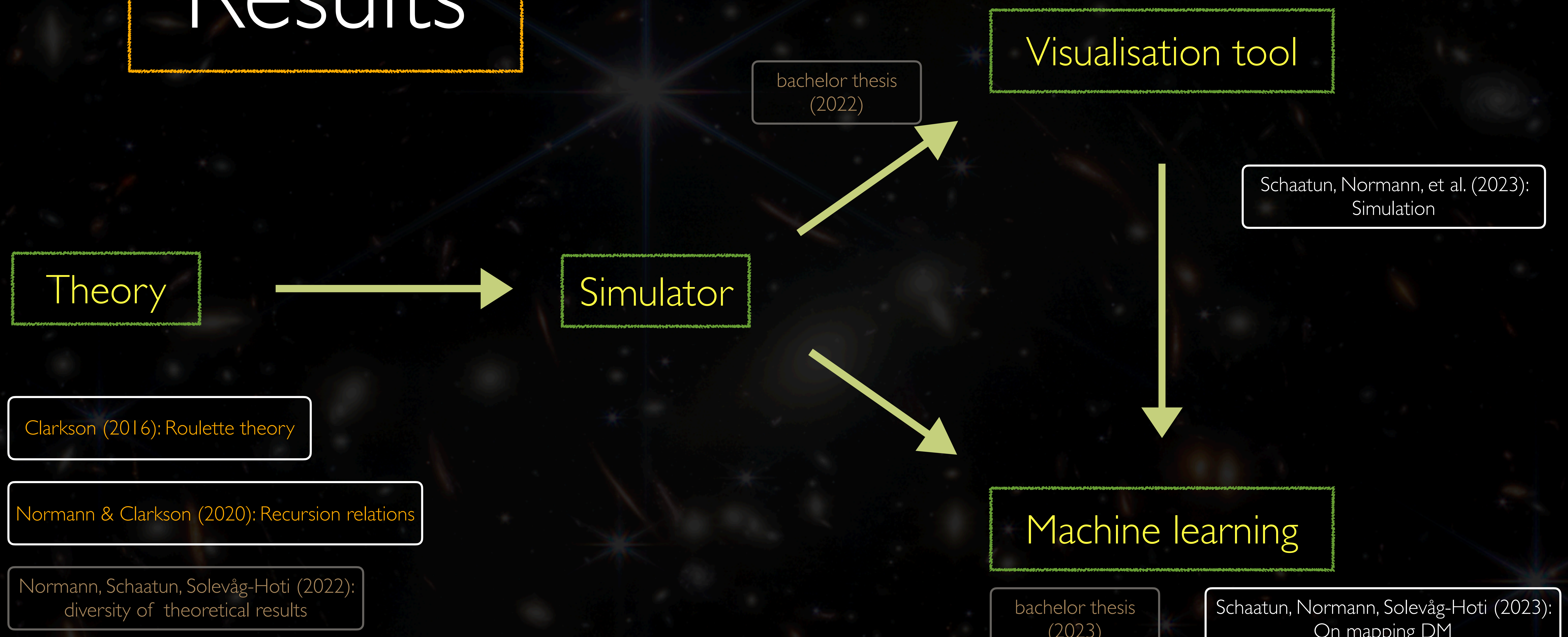




$$\boldsymbol{\eta}_\circ \longrightarrow \boldsymbol{\eta}_\circ + \alpha \boldsymbol{R} \cdot \boldsymbol{p}_s$$

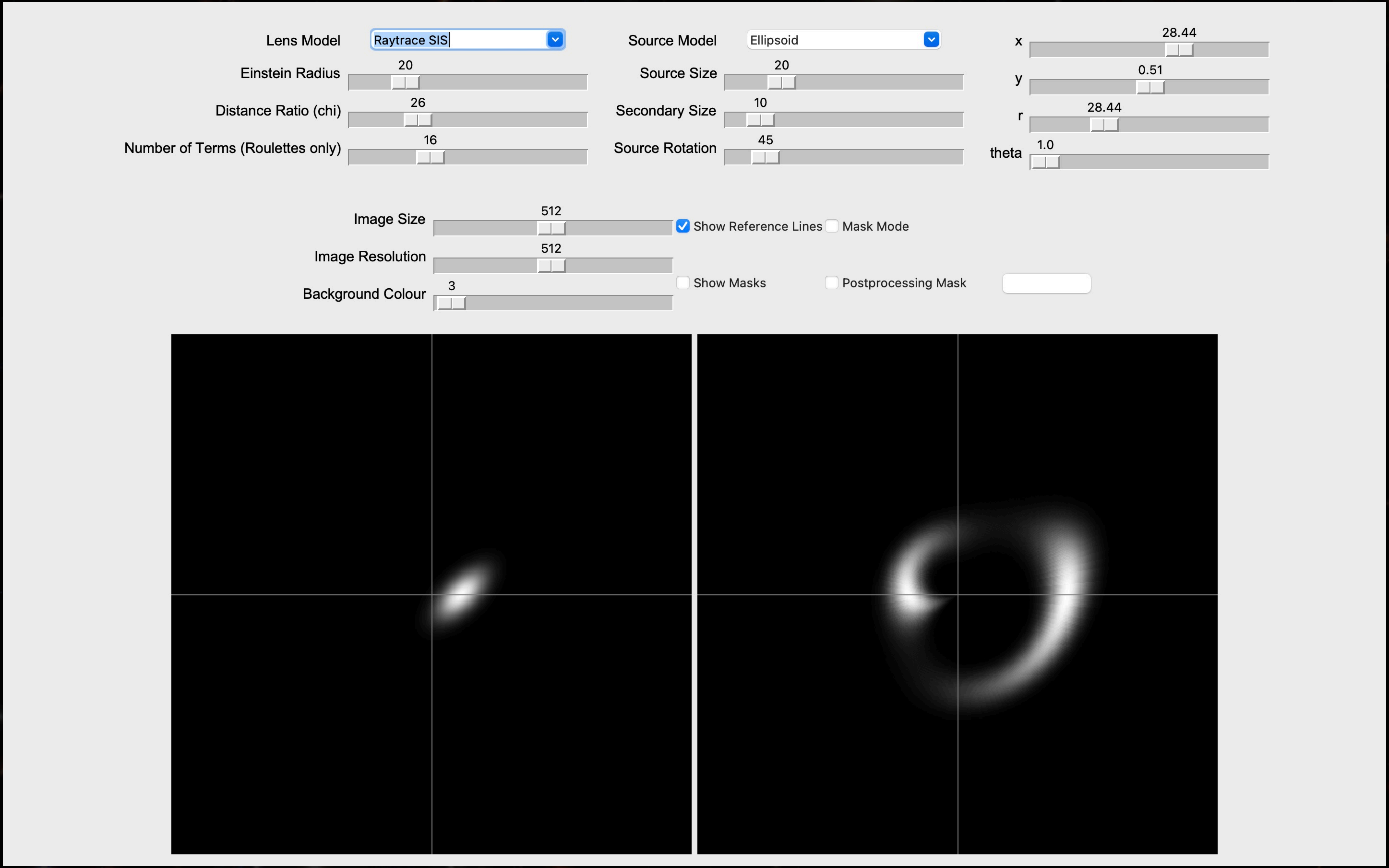
Results

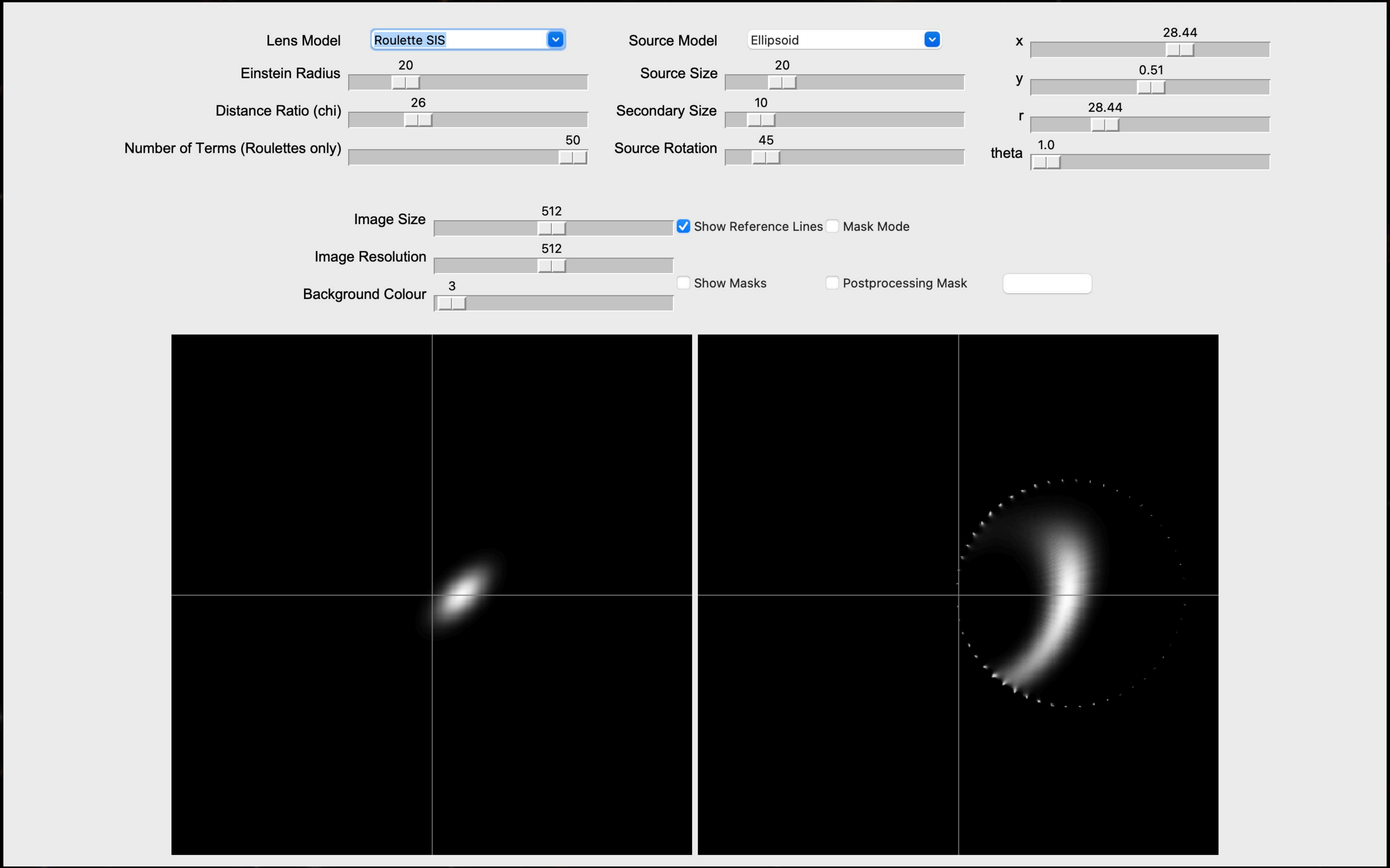
Results



Theory
Simulator
bachelor thesis (2022)
Visualisation tool
Schaatun, Normann, et al. (2023): Simulation
Machine learning
bachelor thesis (2023)
Schaatun, Normann, Solevåg-Hoti (2023): On mapping DM
Clarkson (2016): Roulette theory
Normann & Clarkson (2020): Recursion relations
Normann, Schaatun, Solevåg-Hoti (2022): diversity of theoretical results

Visualisation tool


Lens Model
Raytrace SIS
Source Model
Ellipsoid
x
28.44
Einstein Radius
20
Source Size
20
y
0.51
Distance Ratio (chi)
26
Secondary Size
10
r
28.44
Number of Terms (Roulettes only)
16
Source Rotation
45
theta
1.0
Image Size
512
Show Reference Lines
Mask Mode
Image Resolution
512
Show Masks
Postprocessing Mask
Background Colour
3


Lens Model
Roulette SIS
Source Model
Ellipsoid
x
28.44
Einstein Radius
20
Source Size
20
y
0.51
Distance Ratio (chi)
26
Secondary Size
10
r
28.44
Number of Terms (Roulettes only)
50
Source Rotation
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theta
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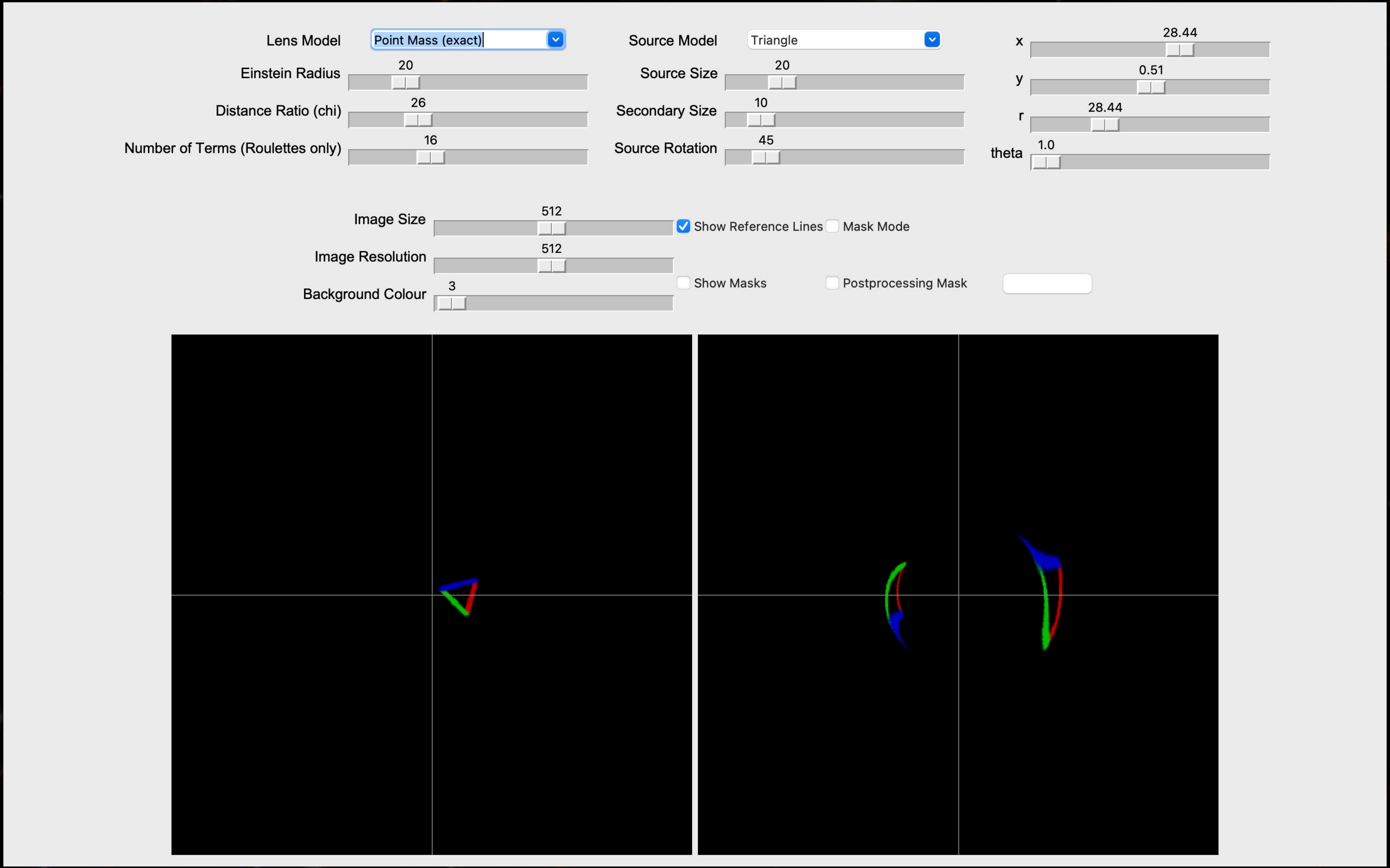
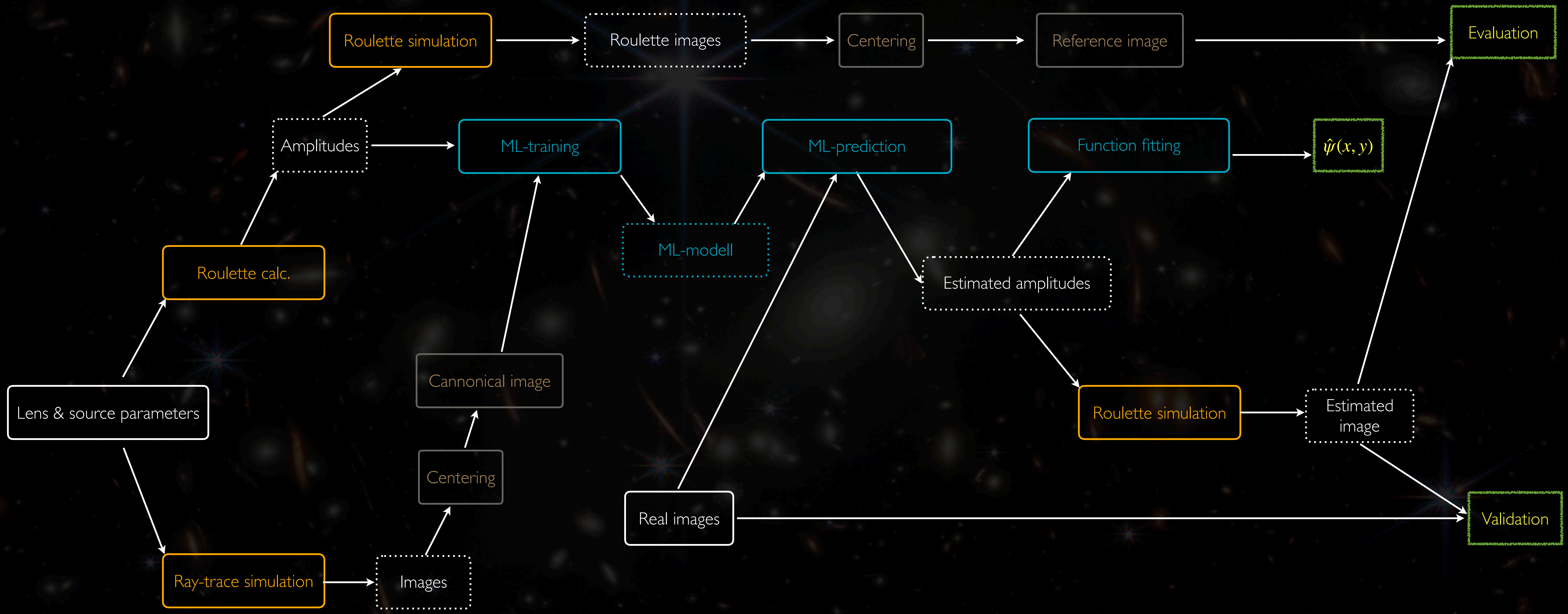
Image Size
512
Show Reference Lines
Mask Mode
Image Resolution
512
Background Colour
3
Show Masks
Postprocessing Mask

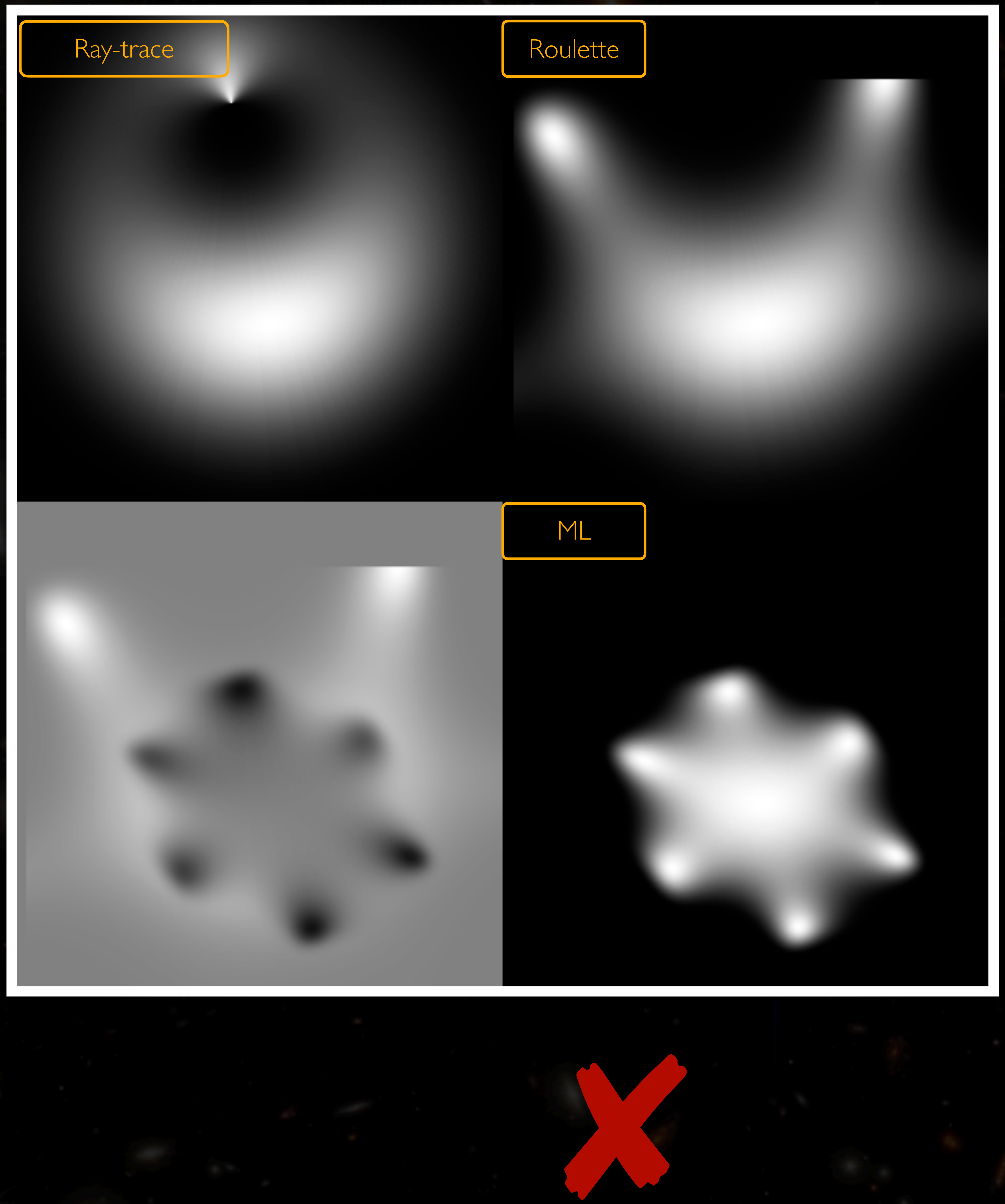

Lens Model
Point Mass (exact)
Source Model
Triangle
x
28.44
Einstein Radius
20
Source Size
20
y
0.51
Distance Ratio (chi)
26
Secondary Size
10
r
28.44
Number of Terms (Roulettes only)
16
Source Rotation
45
theta
1.0
Image Size
512
Show Reference Lines
Mask Mode
Image Resolution
512
Show Masks
Postprocessing Mask
Background Colour
3

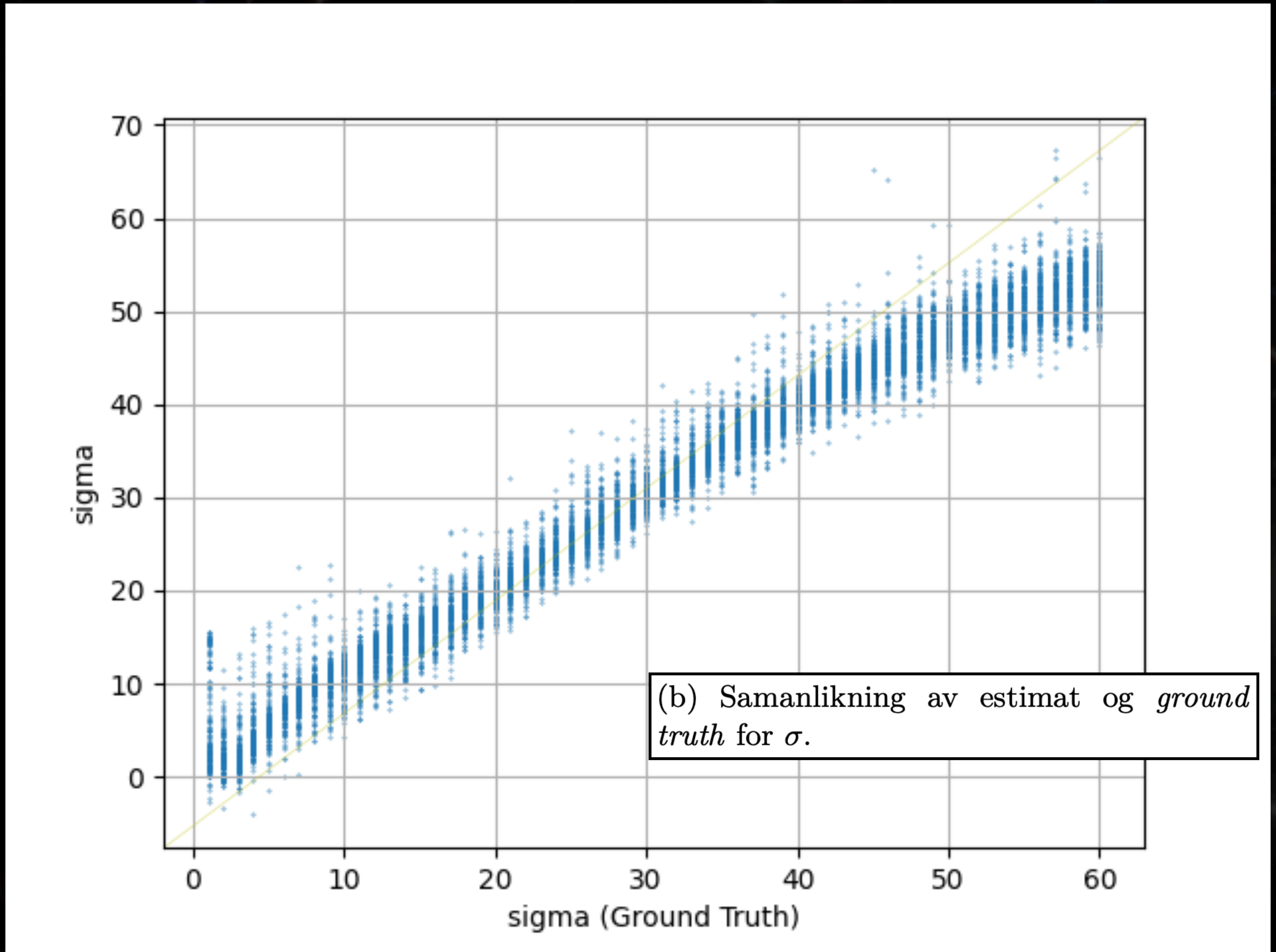
ML

MACHINE LEARNING

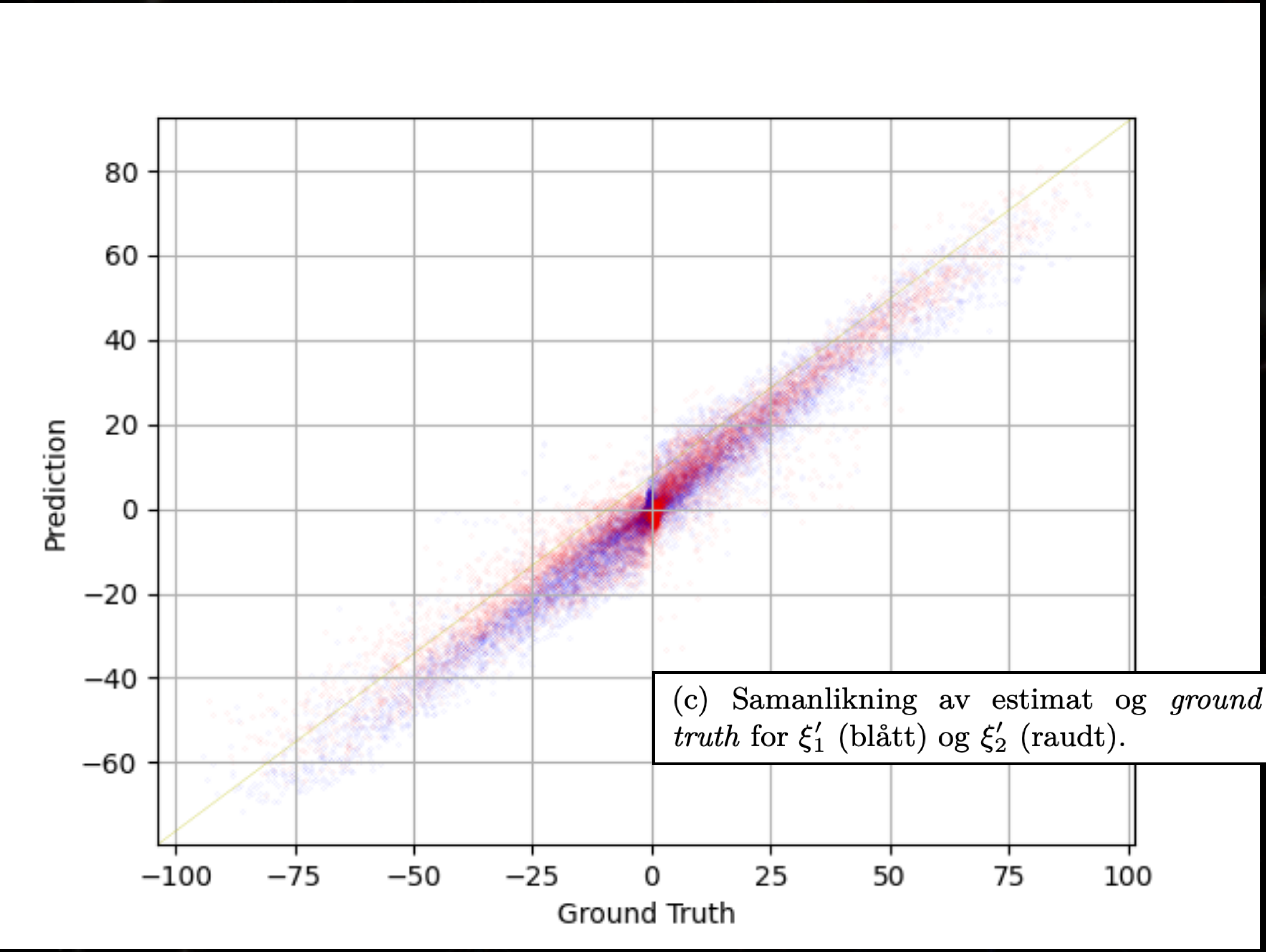


MACHINE LEARNING

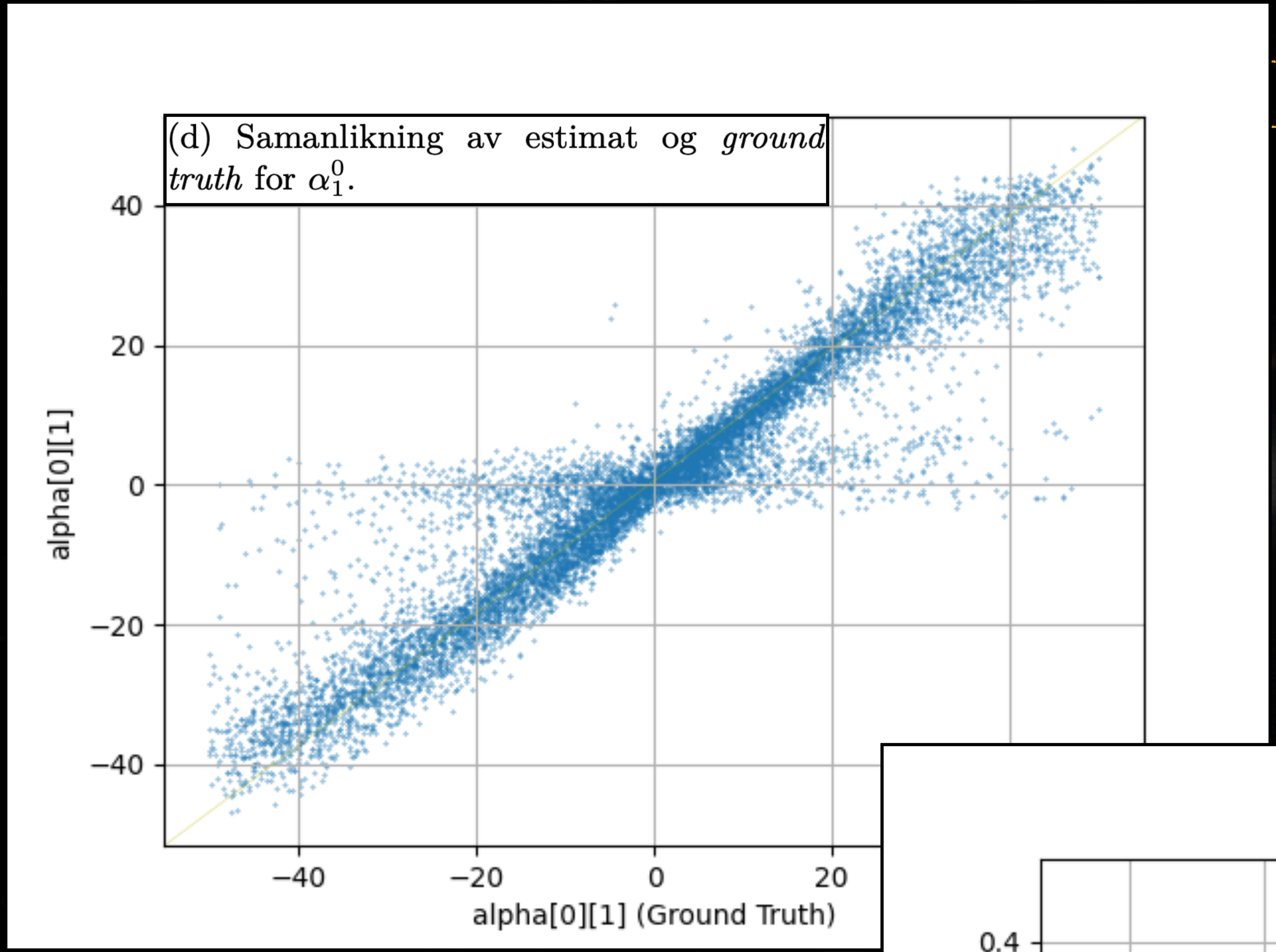




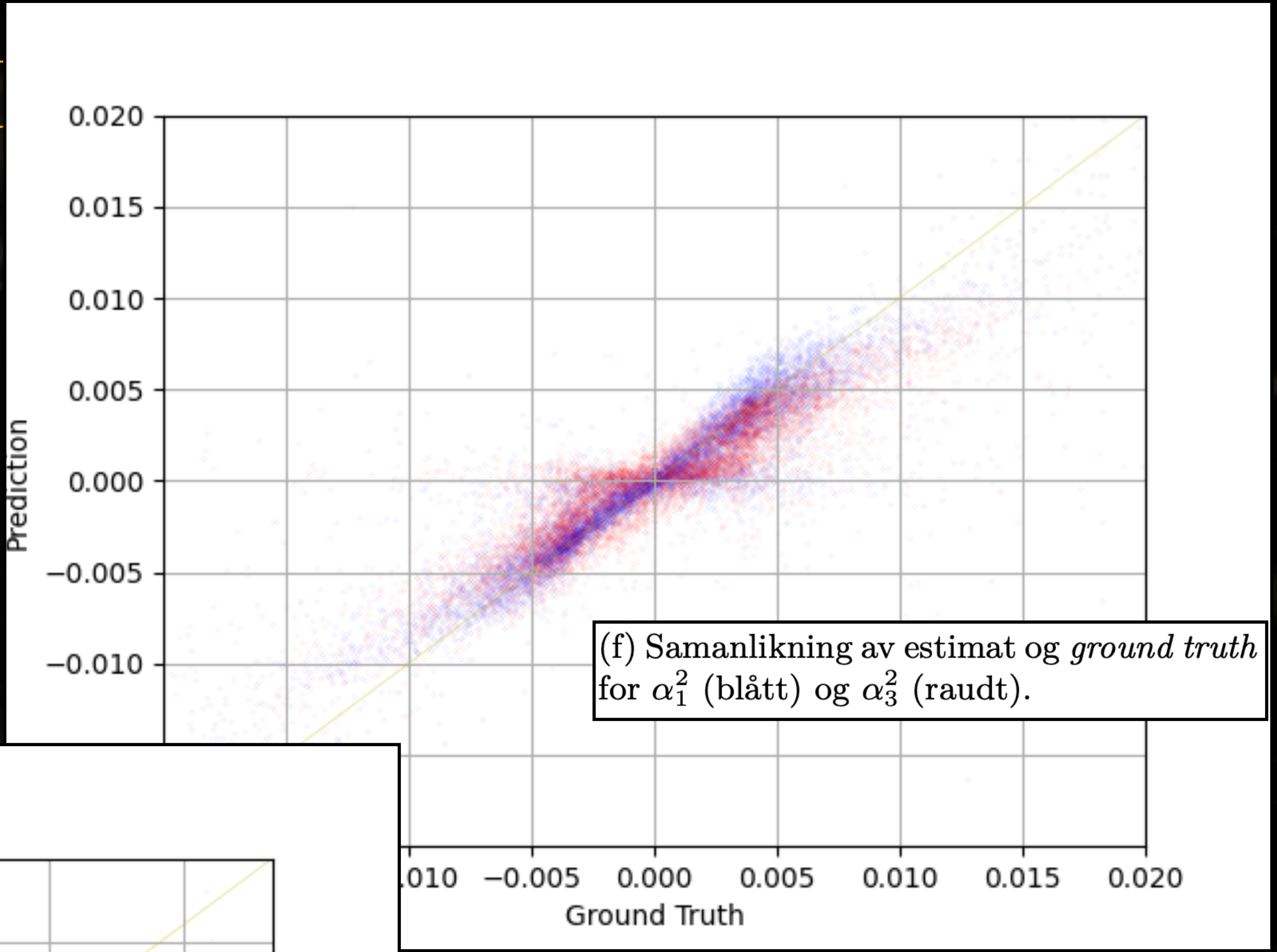
(b) Samanlikning av estimat og *ground truth* for σ.



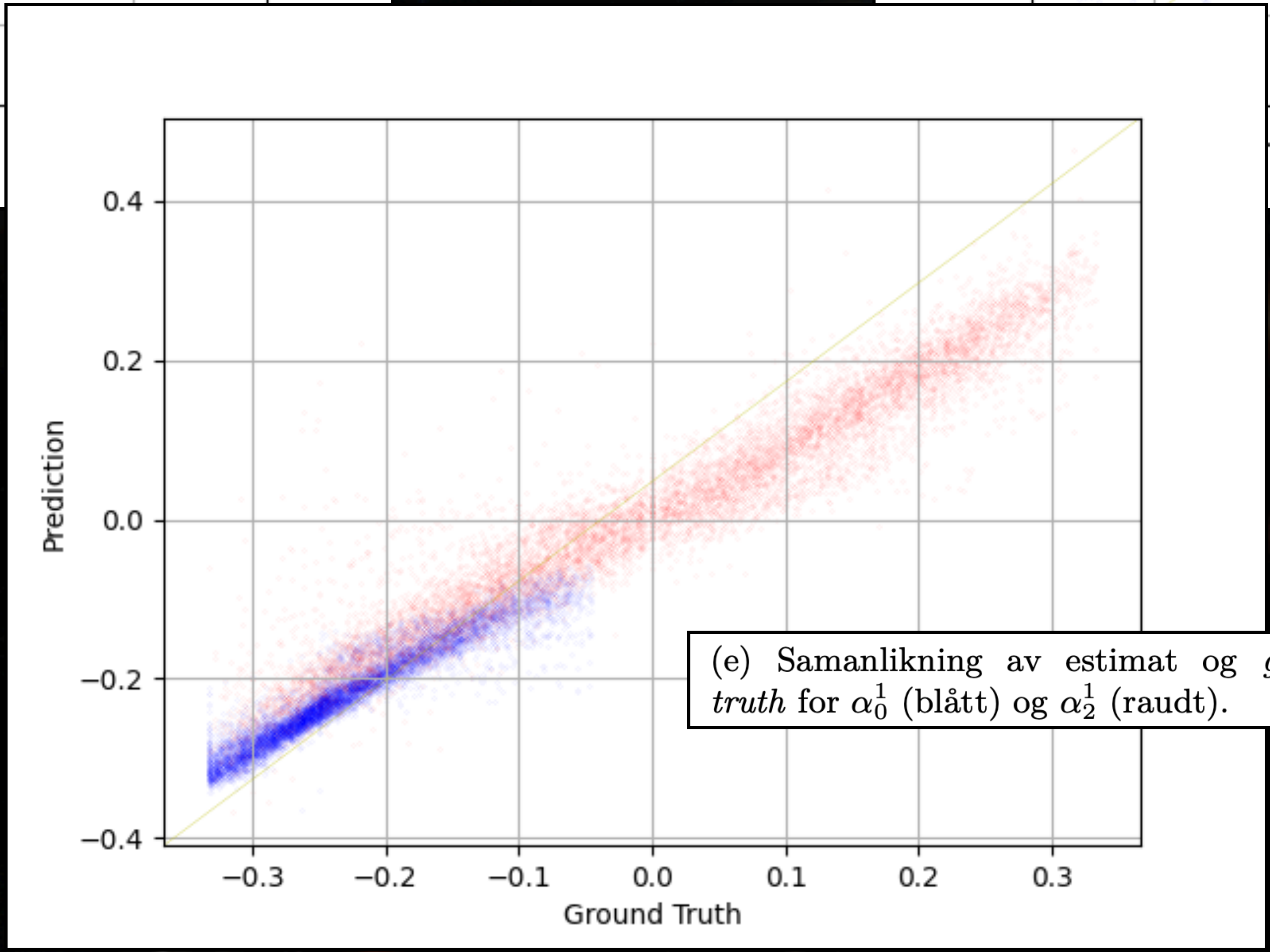
(c) Samanlikning av estimat og *ground truth* for ξ'_1 (blått) og ξ'_2 (raudt).



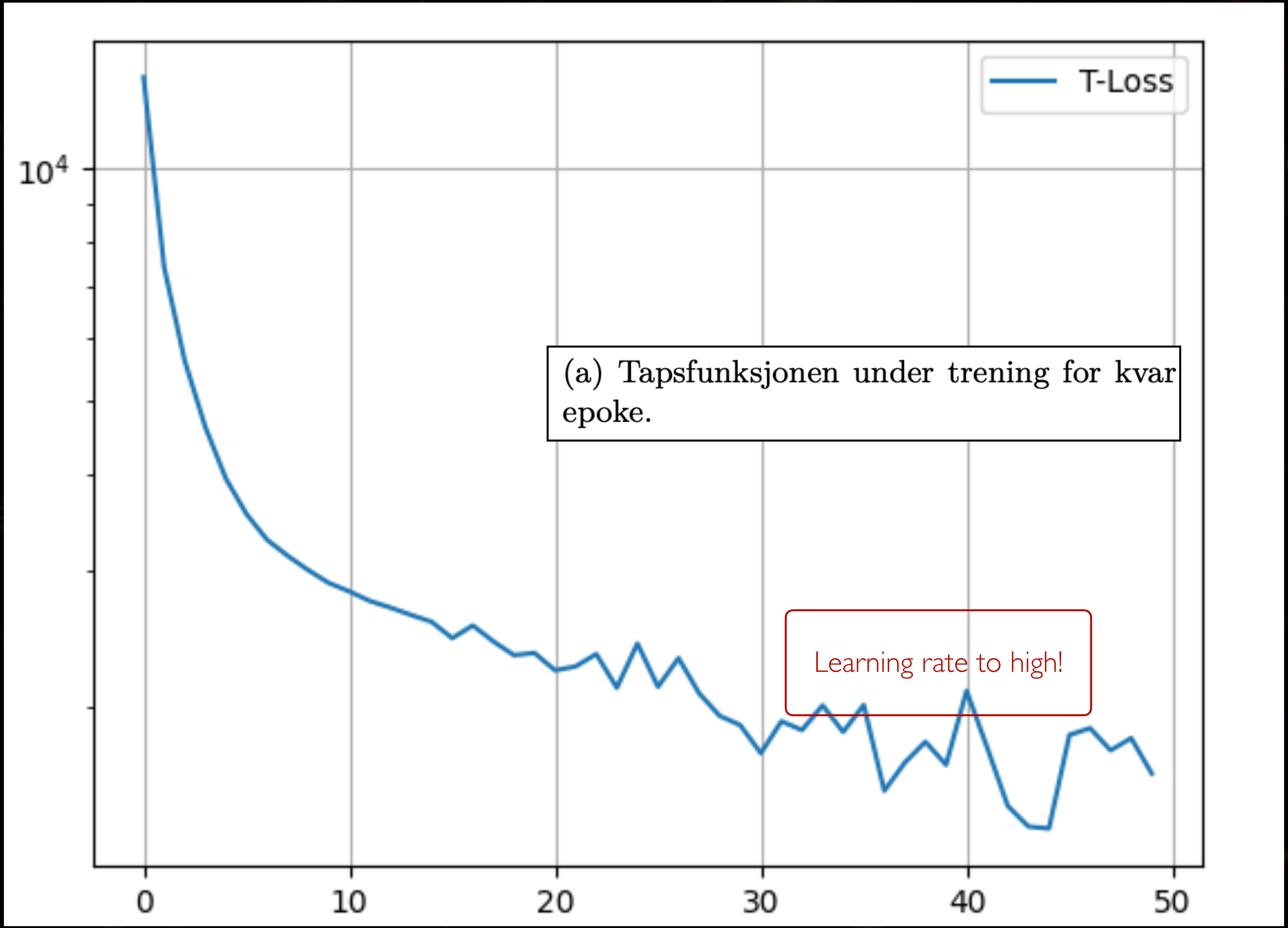
(d) Samanlikning av estimat og *ground truth* for α_1^0.



(f) Samanlikning av estimat og *ground truth* for α_1^2 (blått) og α_3^2 (raudt).



(e) Samanlikning av estimat og *ground truth* for α_0^1 (blått) og α_2^1 (raudt).



(a) Tapsfunksjonen under trening for kvar epoke.

Future work

Theory

Roulette 2.0

Cluster lenses

Visualisation

Critical curves / caustics

ML

systematics

Real data

Thank you for your attention!

References

- GitHub: CosmoAI-AES
- Conference paper for ECMS (arXiv:2303.11824)