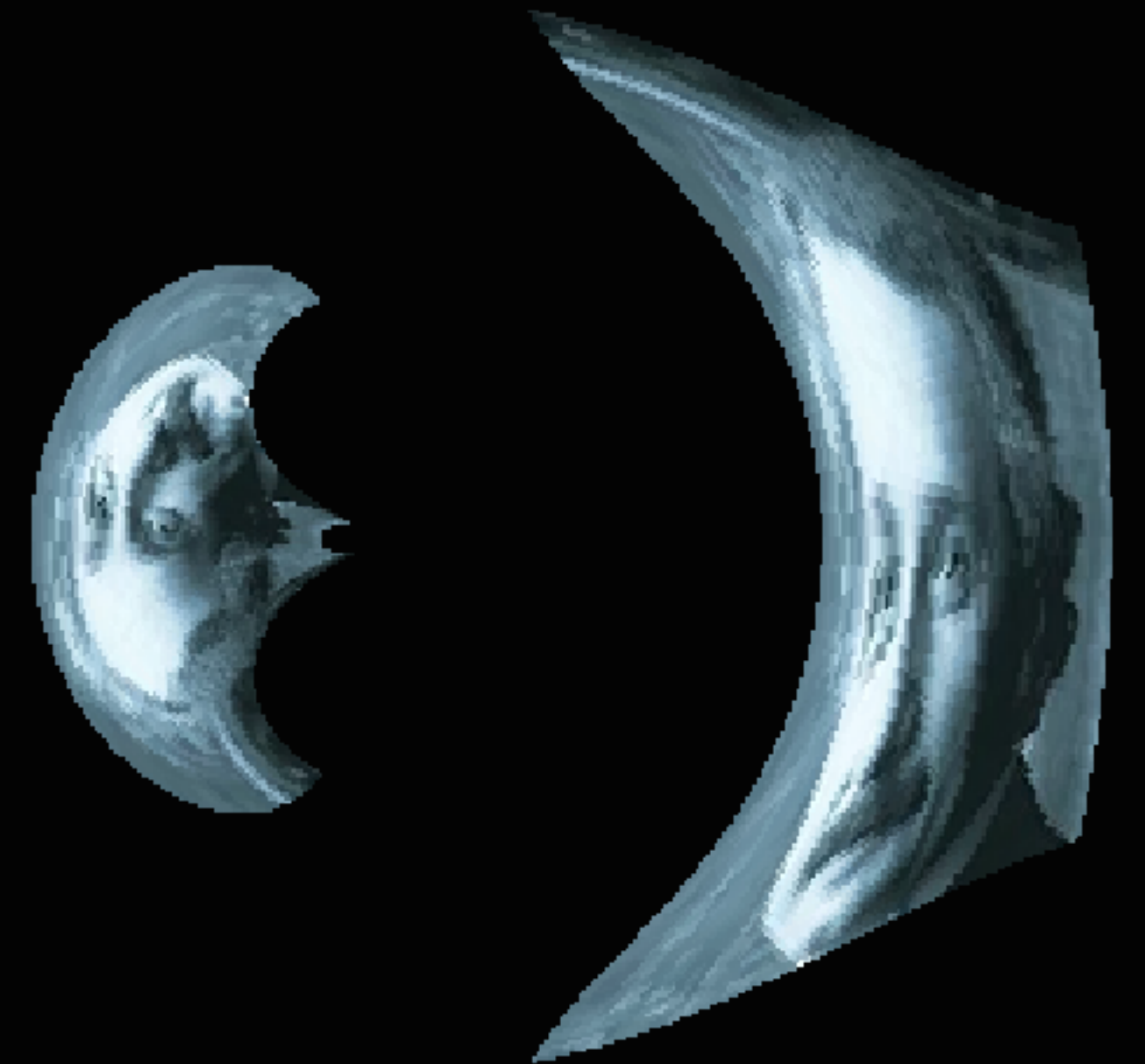


HIGHER-ORDER LENSING

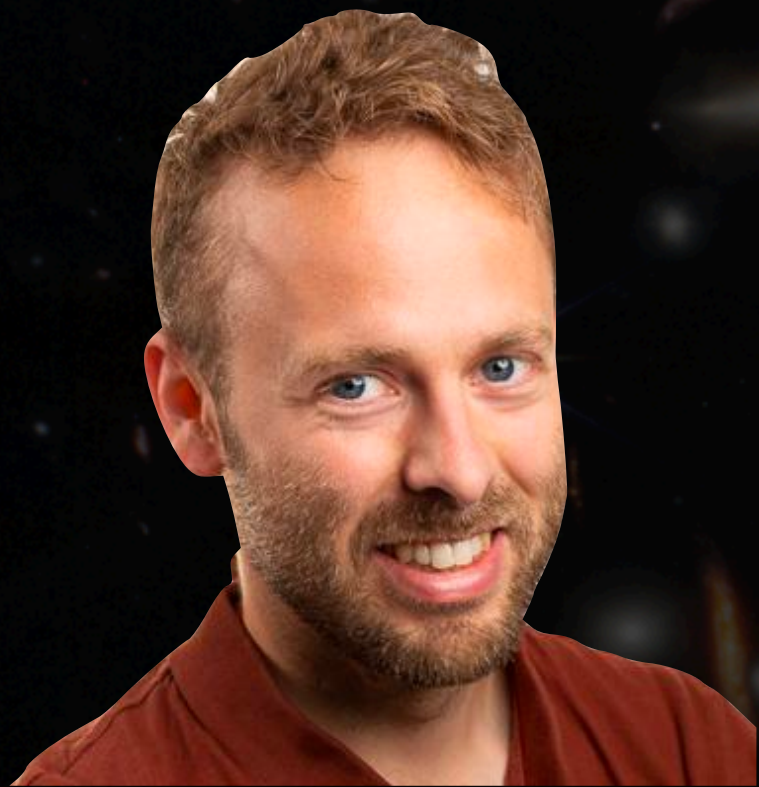


Group elements



Faculty of Information Technology
and Electrical Engineering
Department of ICT
and Natural Sciences

Group elements



Ben (Physicist)



Hans George
(Computer guru)



Kenny (Physicist)



+ students

Structure of talk

Idea

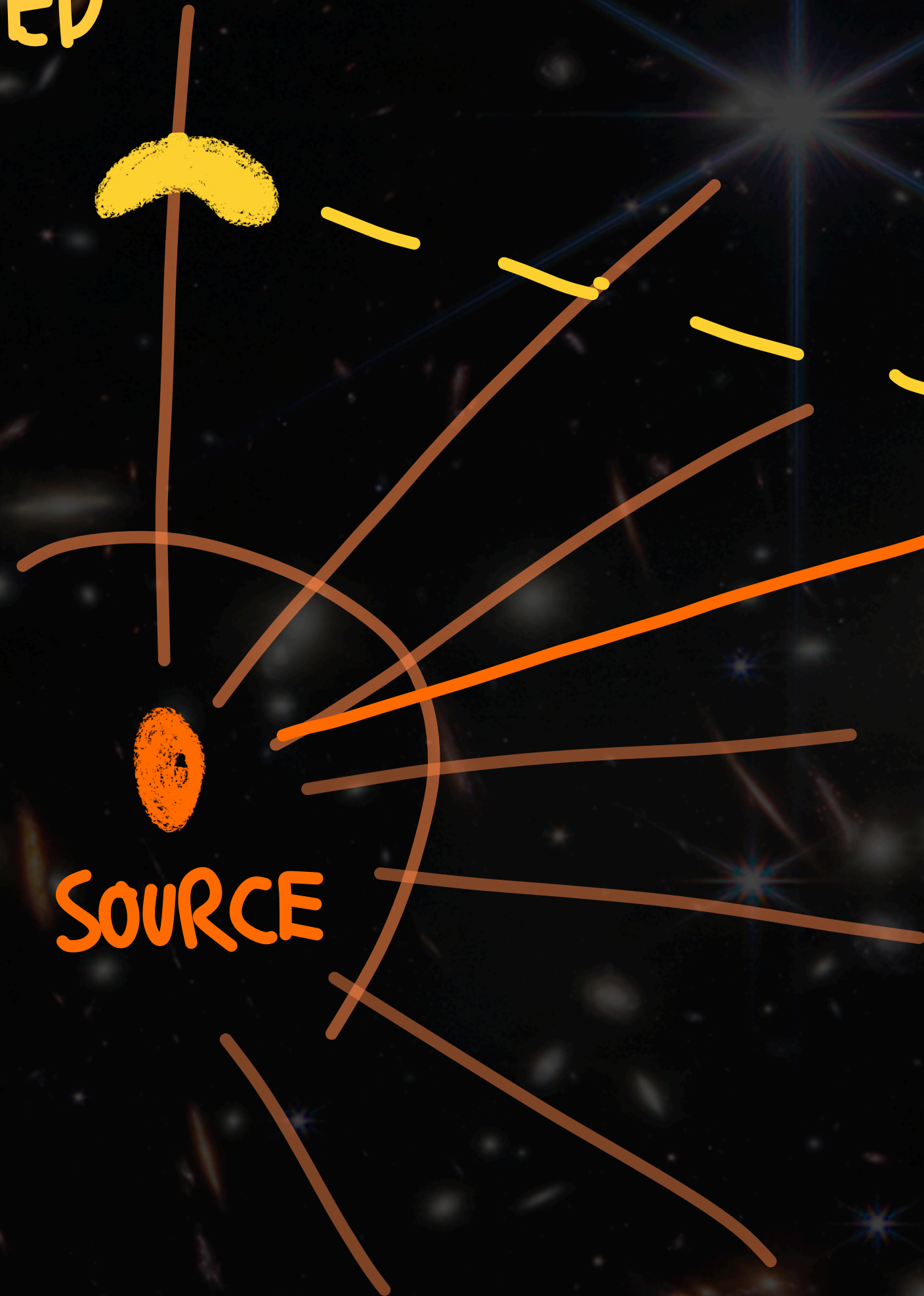
Formalism

Results



Idea

DISTORTED
IMAGE



POTENTIAL

BANANA!
/



DISTORTED
IMAGE

Dictated BY
POTENTIAL

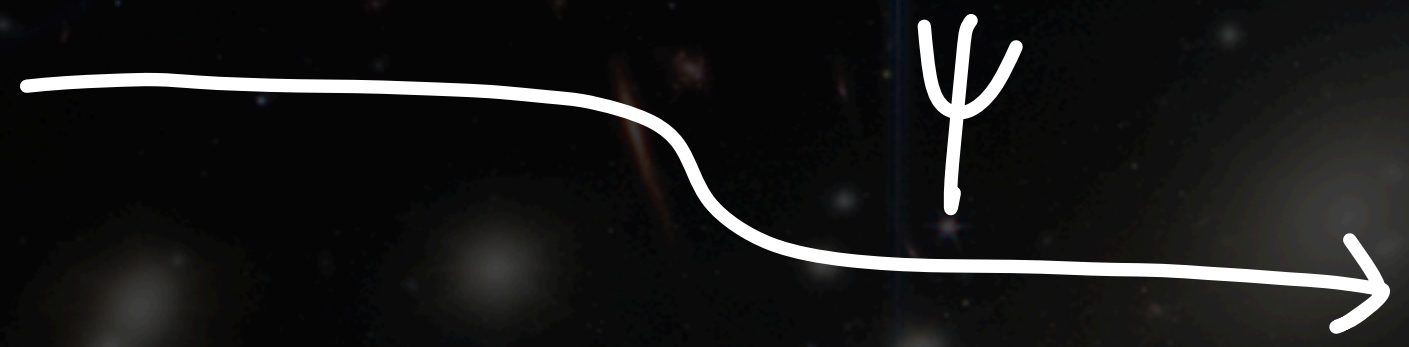
SOURCE

POTENTIAL

Can we go the other
way around?!



DISTORTED
IMAGE



Knowledge of DM
distribution!



DISTORTED
IMAGE

③

ML APPLIC.



SOURCE

OBTAIN ψ

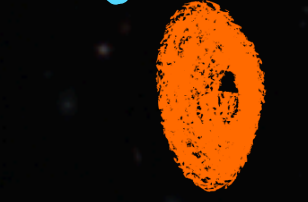


POTENTIAL

Training phase

DISTORTED
IMAGE

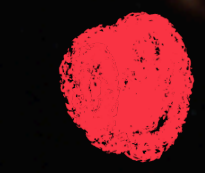
ML
TRAINING ②



SOURCE

①

CALCULATE
FOR KNOWN ψ

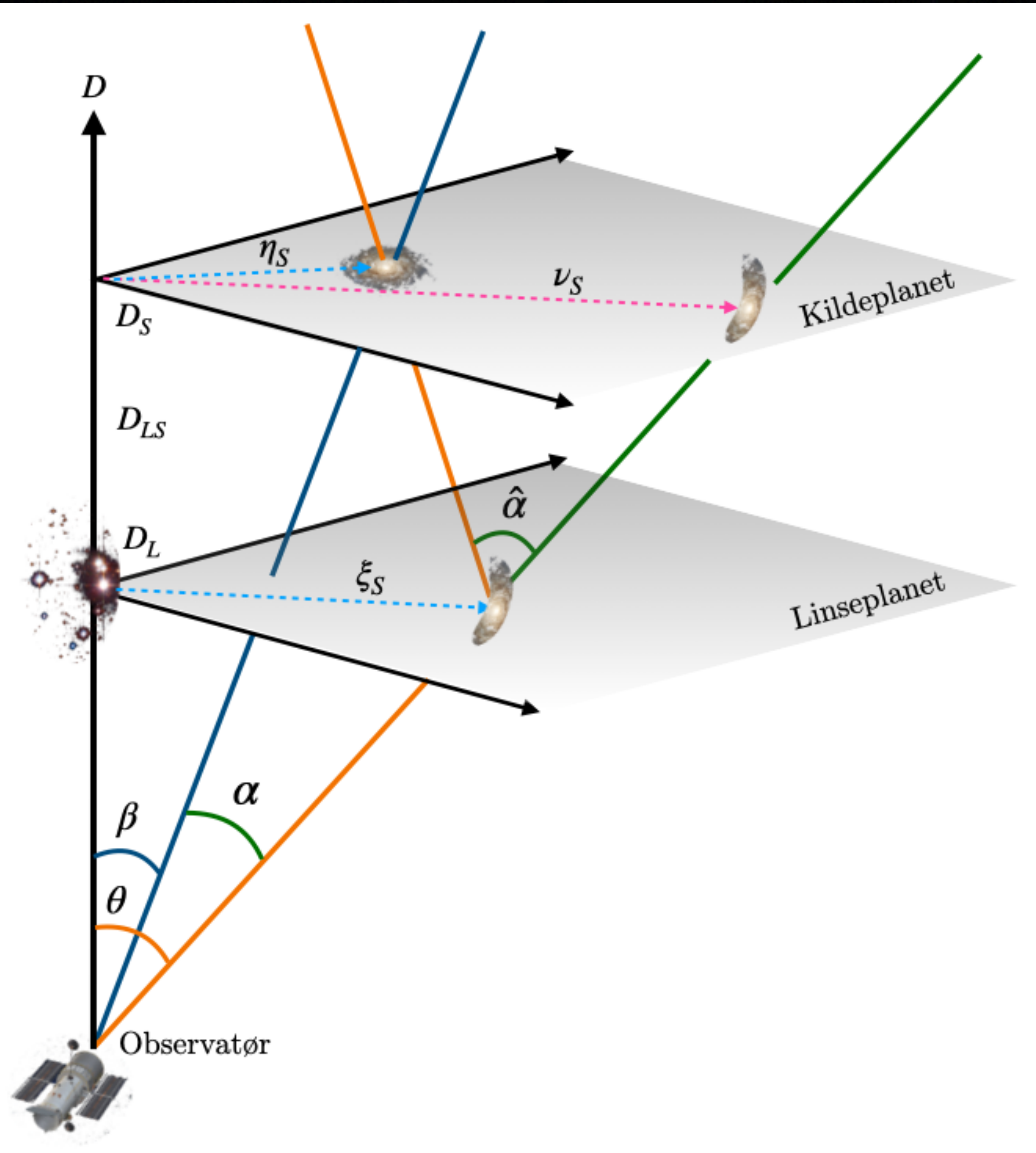


POTENTIAL

Real-data phase



Formalism



$$d\beta = A^{-1}d\theta$$

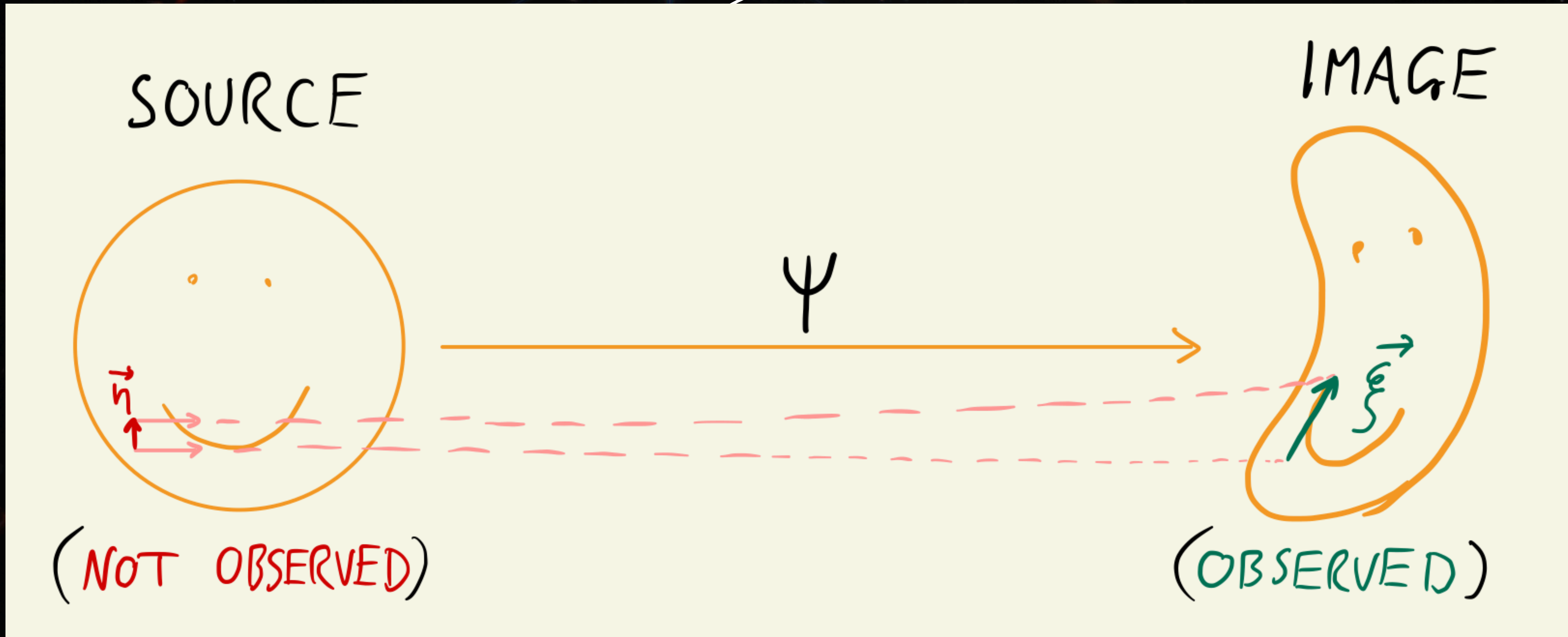
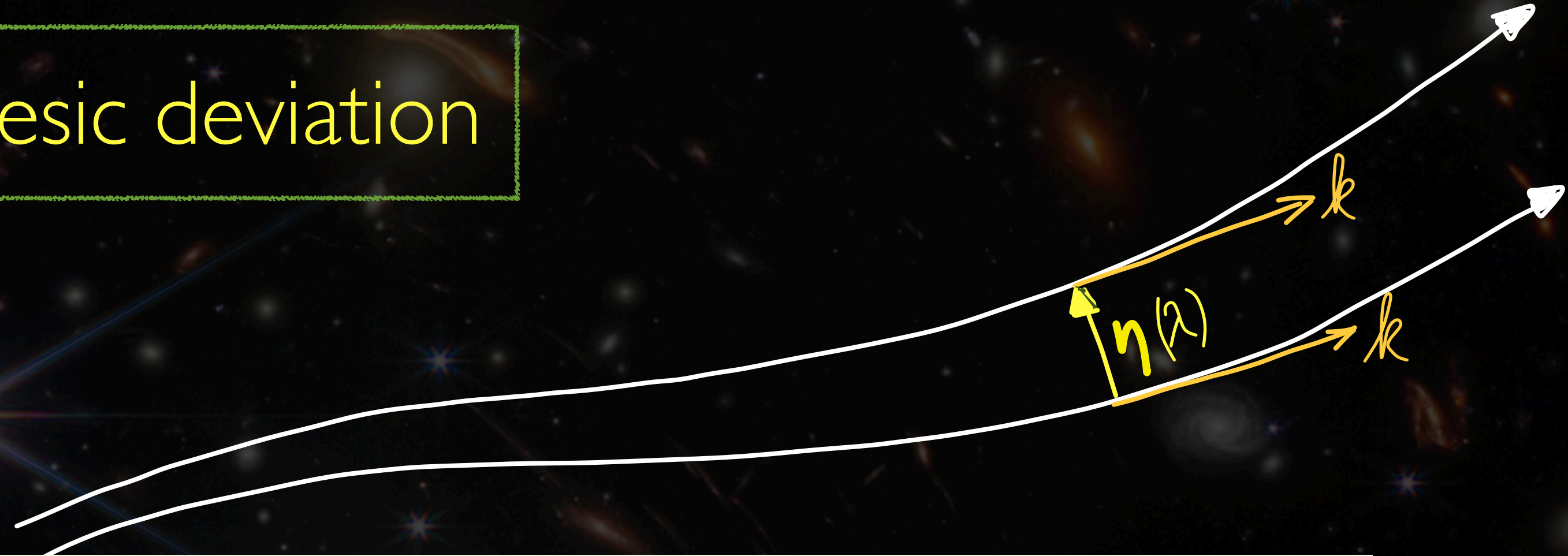
$$A^{-1} = (1 - \kappa)I + \gamma_+ \mathbf{R}_- + \gamma_\times \mathbf{R}_/$$

$$\kappa(\theta) = \frac{1}{2}(\psi_{yy} + \psi_{xx}),$$

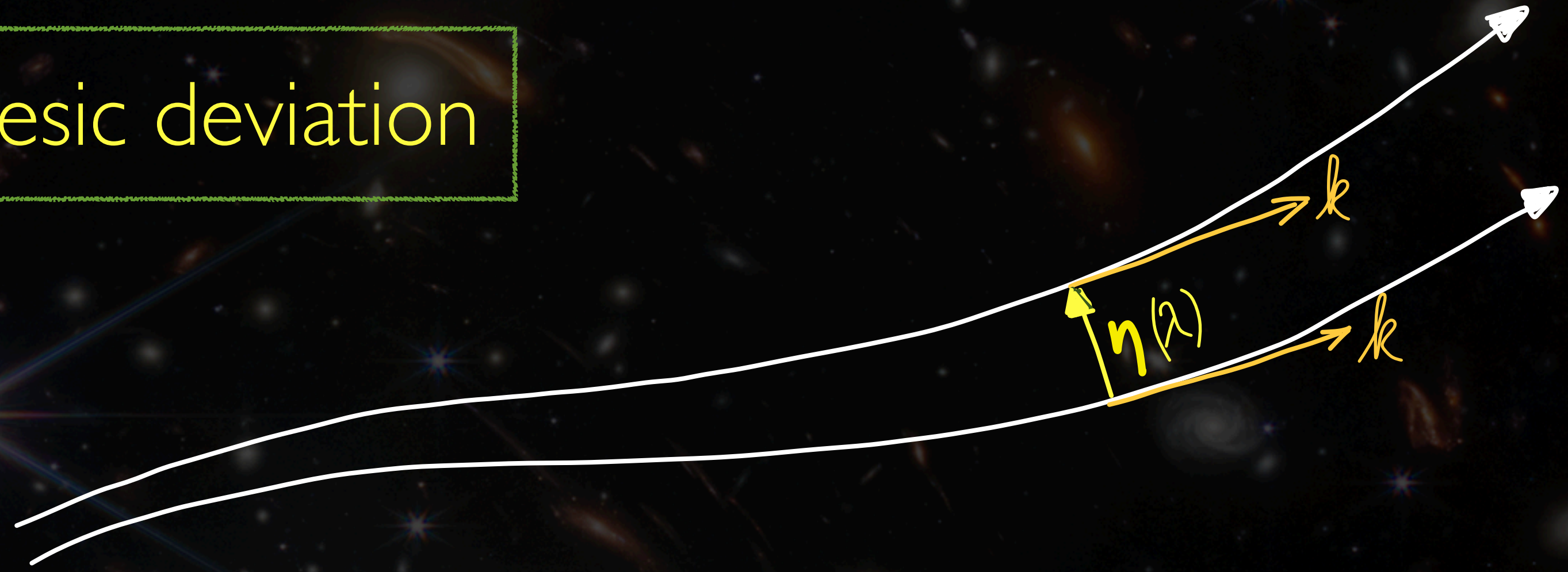
$$\gamma_+ = \frac{1}{2}(\psi_{yy} - \psi_{xx}),$$

$$\gamma_\times = \psi_{xy}.$$

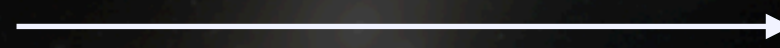
Geodesic deviation



Geodesic deviation



$$\ddot{\eta} - \mathcal{R}(\psi)\eta = \mathcal{O}^1(\eta, \dot{\eta})$$



convergence + shear

$$\ddot{\eta} - \mathcal{R}(\psi)\eta = \mathcal{F}(\psi)$$



Shear +
flexion + 2.flexion + ... inf

leading-order screen-space
derivatives

Geodesic deviation



Roulette formalism

$$\ddot{\eta} - \mathcal{R}(\psi)\eta = \mathcal{F}(\psi)$$

leading-order screen-space
derivatives

Solution via series expansion

$$\eta(\xi) = \sum_{m=1}^{\infty} \frac{\epsilon^m}{m!} \hat{\eta}^{(m)}(\xi)$$

$$\eta_{(m)}^A = \mathcal{M}_{B_1 \dots B_m}^A \xi^{B_1} \dots \xi^{B_m}$$

$$\begin{aligned} \mathcal{M}_{AB_1 \dots B_m} \hat{\xi}^{B_1} \dots \hat{\xi}^{B_m} = & \sum_{s=0}^{m+1} \frac{[1 - (-1)^{m+s}]}{4} \times \\ & \left(\left[[C^+ \alpha_s^m + \bar{\beta}_s^m] \mathbf{R}_- + [C^+ \beta_s^m - \bar{\alpha}_s^m] \mathbf{R}_/ \right] \mathbf{p}_{s-1} \right. \\ & \left. + \left[[C^- \alpha_s^m - \bar{\beta}_s^m] \mathbf{I} + [C^- \beta_s^m + \bar{\alpha}_s^m] \boldsymbol{\varepsilon} \right] \mathbf{p}_{s+1} \right), \end{aligned}$$

$$C^\pm = 1 \pm \frac{s}{m+1}.$$

$$\mathbf{p}_{(s)} = \cos s\theta \mathbf{e}_x + \sin s\theta \mathbf{e}_y.$$

$$\boldsymbol{\varepsilon}_- = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \mathbf{R}_- = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \mathbf{R}_/ = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Our job is therefore to determine $\alpha_s^m, \beta_s^m, \bar{\alpha}_s^m, \bar{\beta}_s^m$

$$\alpha_s^m = -2^{-\delta_{0s}} \chi^{m+1} \sum_{k=0}^m \binom{m}{k} (\mathcal{C}_s^{m(k)} \partial_X + \mathcal{C}_s^{m(k+1)} \partial_Y) \partial_X^{m-k} \partial_Y^k \psi,$$

$$\beta_s^m = -\chi^{m+1} \sum_{k=0}^m \binom{m}{k} (\mathcal{S}_s^{m(k)} \partial_X + \mathcal{S}_s^{m(k+1)} \partial_Y) \partial_X^{m-k} \partial_Y^k \psi,$$

$$\mathcal{C}_s^{m(k)} = \frac{1}{\pi} \int_{-\pi}^{\pi} d\theta \sin^k \theta \cos^{m-k+1} \theta \cos s\theta,$$

$$\mathcal{S}_s^{m(k)} = \frac{1}{\pi} \int_{-\pi}^{\pi} d\theta \sin^k \theta \cos^{m-k+1} \theta \sin s\theta.$$

Determine these by machine learning!


$$\alpha_s^m = -2^{-\delta_{0s}} \chi^{m+1} \sum_{k=0}^m \binom{m}{k} (\mathcal{C}_s^{m(k)} \partial_X + \mathcal{C}_s^{m(k+1)} \partial_Y) \partial_X^{m-k} \partial_Y^k \psi$$

$$\beta_s^m = -\chi^{m+1} \sum_{k=0}^m \binom{m}{k} (\mathcal{S}_s^{m(k)} \partial_X + \mathcal{S}_s^{m(k+1)} \partial_Y) \partial_X^{m-k} \partial_Y^k \psi$$

$$\kappa = \alpha_0^1, \quad \gamma_+ = \alpha_2^1, \quad \gamma_\times = \beta_2^1$$

$$\mathcal{C}_s^{m(k)} = \frac{1}{\pi} \int_{-\pi}^{\pi} d\theta \sin^k \theta \cos^{m-k+1} \theta \cos s\theta,$$

$$\mathcal{S}_s^{m(k)} = \frac{1}{\pi} \int_{-\pi}^{\pi} d\theta \sin^k \theta \cos^{m-k+1} \theta \sin s\theta.$$

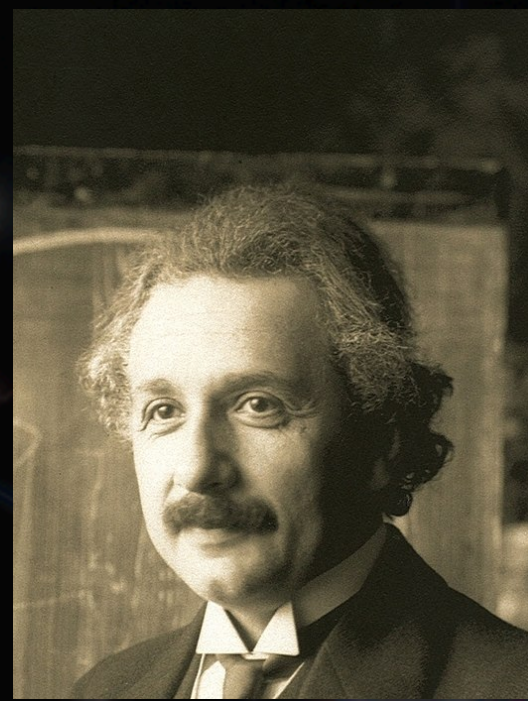


A red circle highlights a point in a star field. The background is a dark space filled with numerous stars of varying colors and sizes. A faint coordinate grid is overlaid on the image, with a central point where the axes intersect. The red circle is positioned in the lower right quadrant of the grid.

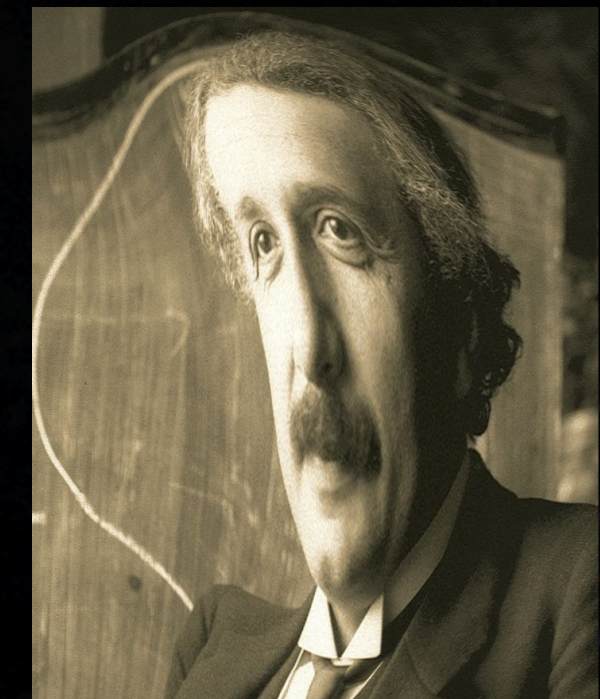
$$\varepsilon_- = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \mathbf{R}_- = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \mathbf{R}_/ = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\mathbf{p}_{(s)} = \cos s\theta \mathbf{e}_x + \sin s\theta \mathbf{e}_y.$$

$$\boldsymbol{\eta}_o \longrightarrow \boldsymbol{\eta}_o + \alpha \mathbf{R} \cdot \mathbf{p}_s$$



	$s = 1$	$s = 2$	$s = 3$	$s = 4$	$s = 5$
$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$					
$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$					
$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$					
$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$					



$$\eta_o \longrightarrow \eta_o + \alpha R \cdot p_s$$



Results

Results

Theory

Simulator

Visualisation tool

Machine learning

bachelor thesis
(2022)

Schaatun, Normann, et al. (2023):
Simulation

Clarkson (2016): Roulette theory

Normann & Clarkson (2020): Recursion relations

Normann, Schaatun, Solevåg-Hoti (2022):
diversity of theoretical results

bachelor thesis
(2023)

Schaatun, Normann, Solevåg-Hoti (2023):
On mapping DM



Visualisation tool

Lens Model Source Model

Einstein Radius Source Size

Distance Ratio (chi) Secondary Size

Number of Terms (Roulettes only) Source Rotation

x

y

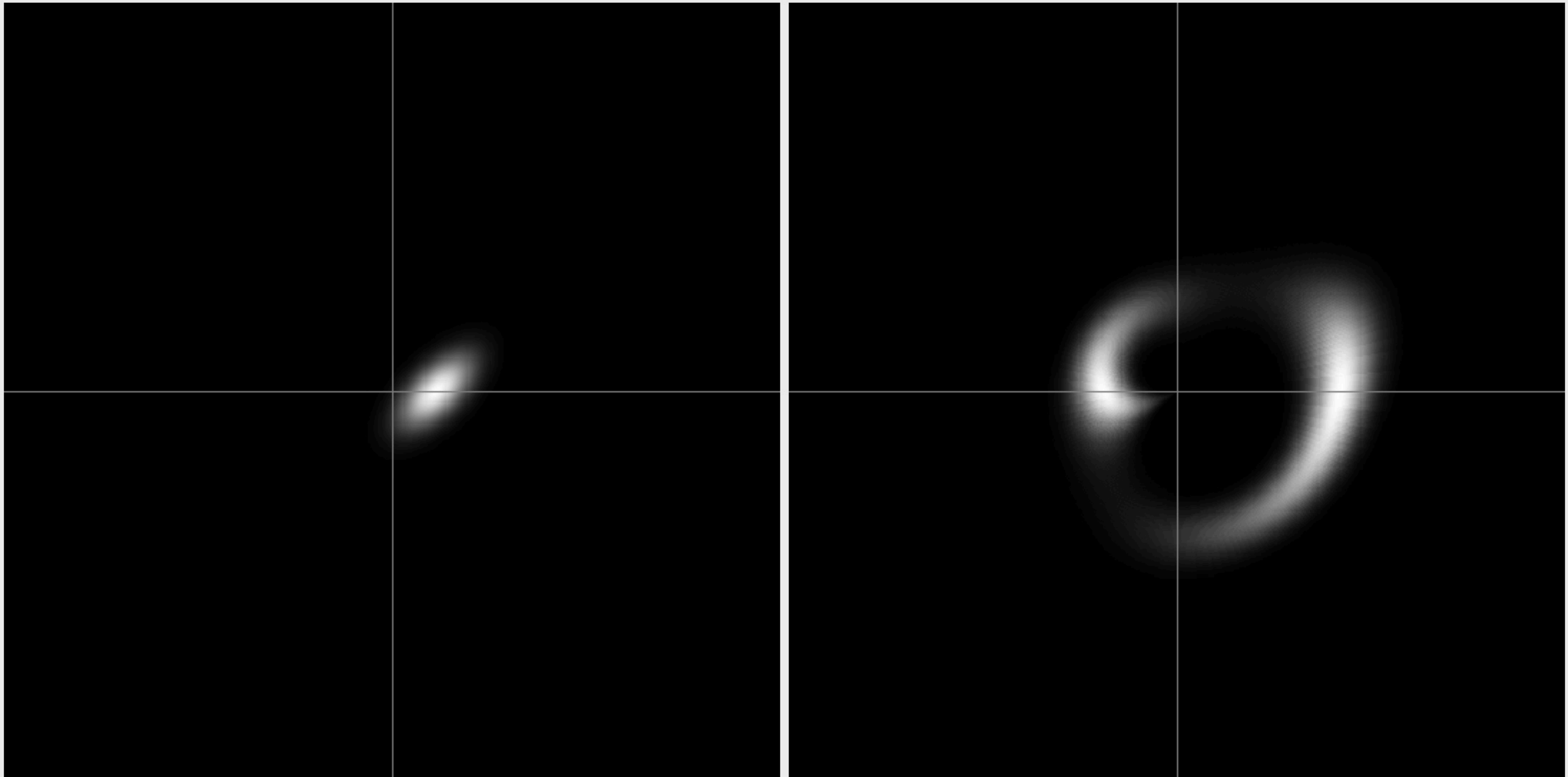
r

theta

Image Size Show Reference Lines Mask Mode

Image Resolution

Background Colour Show Masks Postprocessing Mask



Lens Model **Roulette SIS** Source Model **Ellipsoid**

Einstein Radius Source Size

Distance Ratio (chi) Secondary Size

Number of Terms (Roulettes only) Source Rotation

x

y

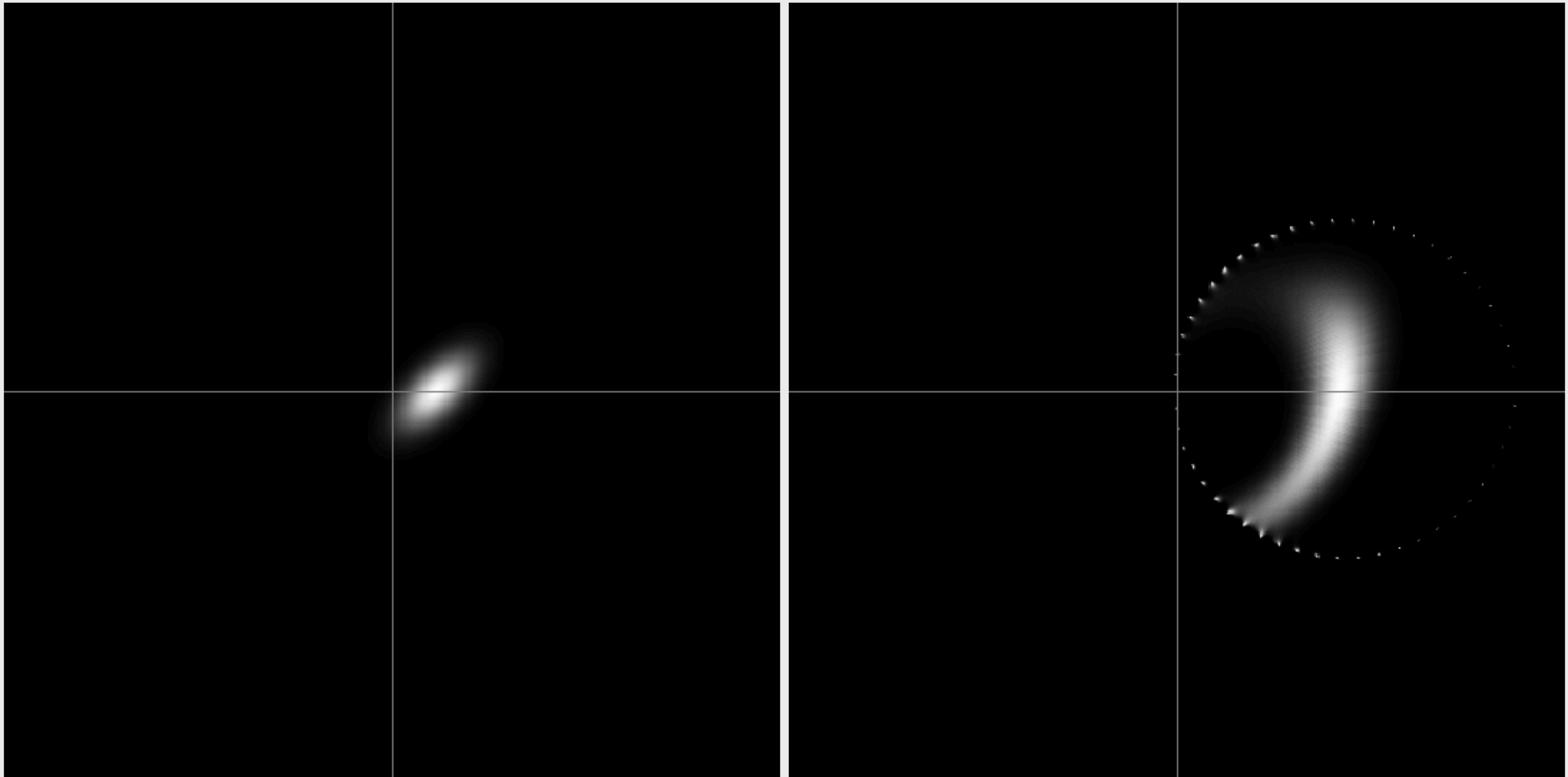
r

theta

Image Size Show Reference Lines Mask Mode

Image Resolution

Background Colour Show Masks Postprocessing Mask



Lens Model Source Model

Einstein Radius Source Size

Distance Ratio (chi) Secondary Size

Number of Terms (Roulettes only) Source Rotation

x

y

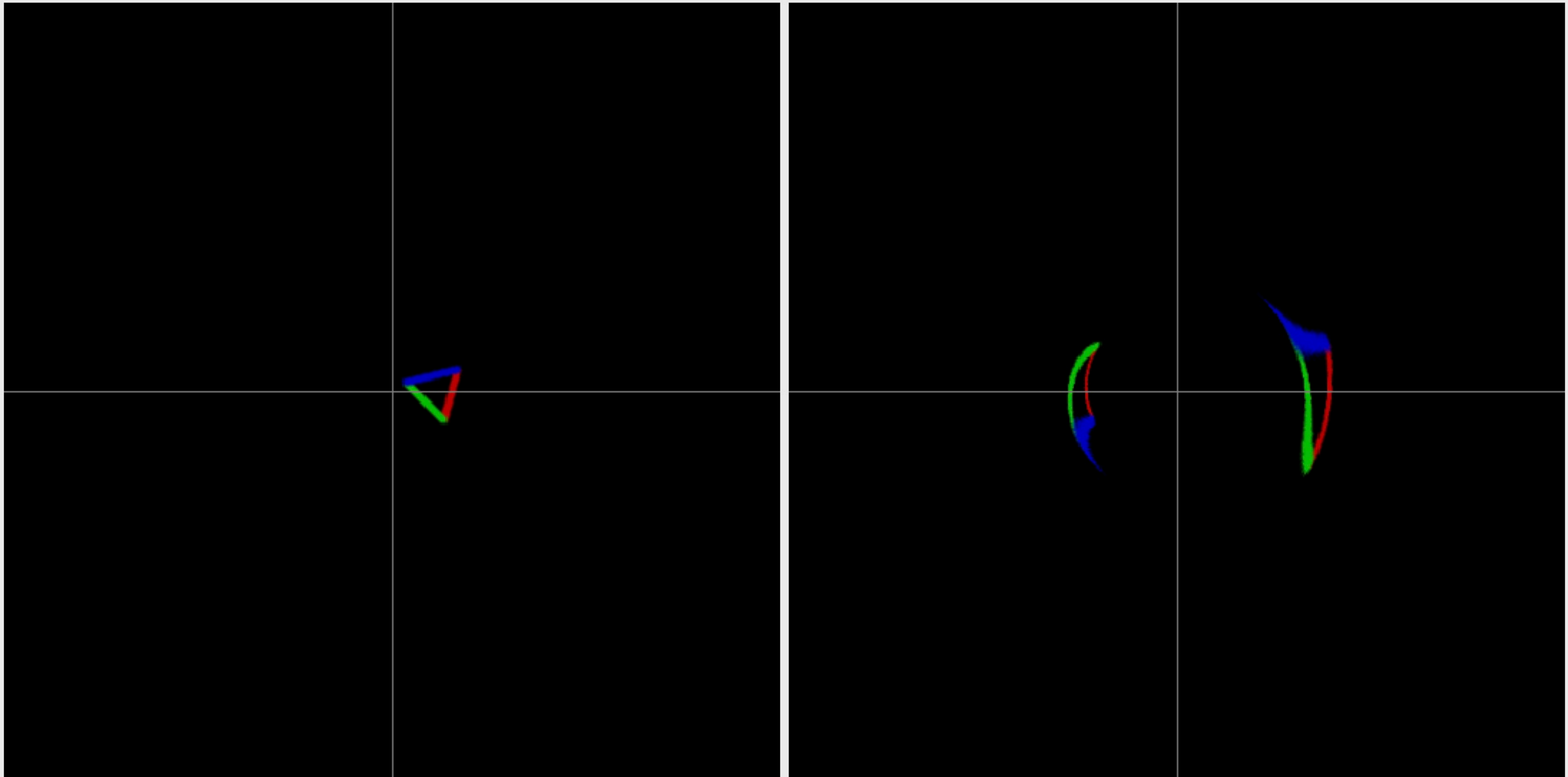
r

theta

Image Size Show Reference Lines Mask Mode

Image Resolution

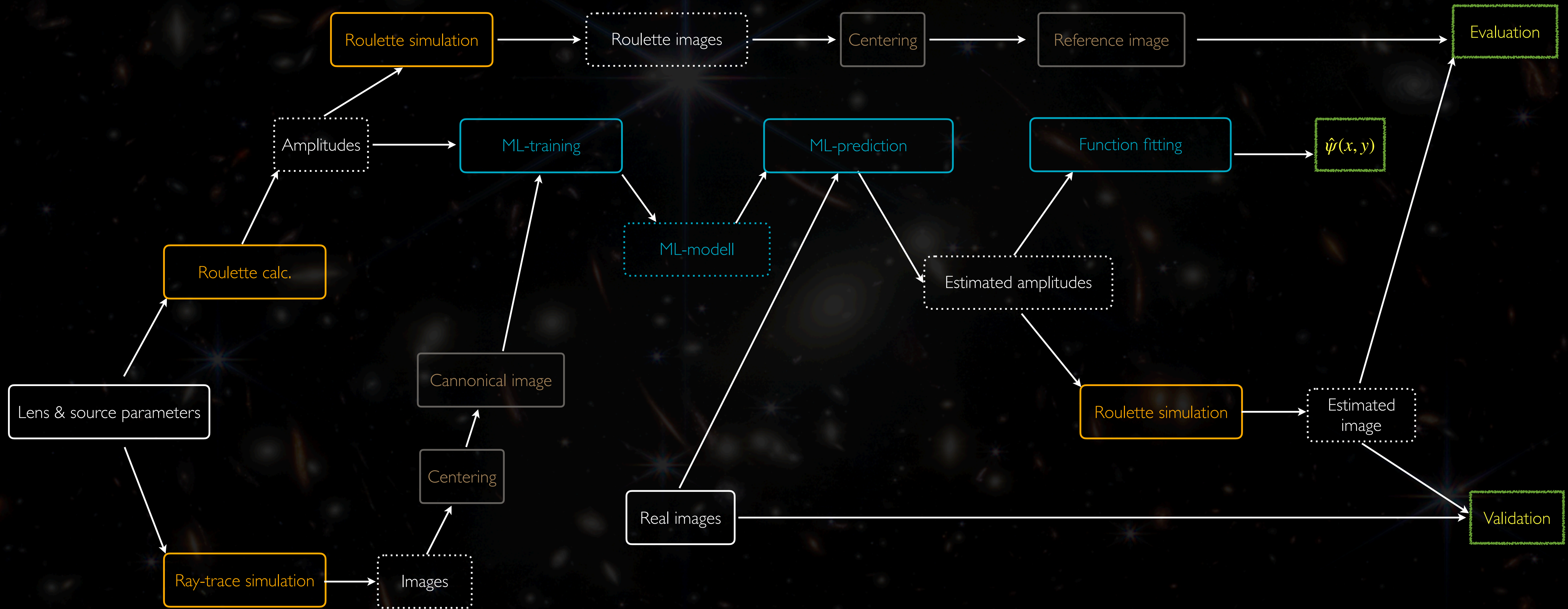
Background Colour Show Masks Postprocessing Mask



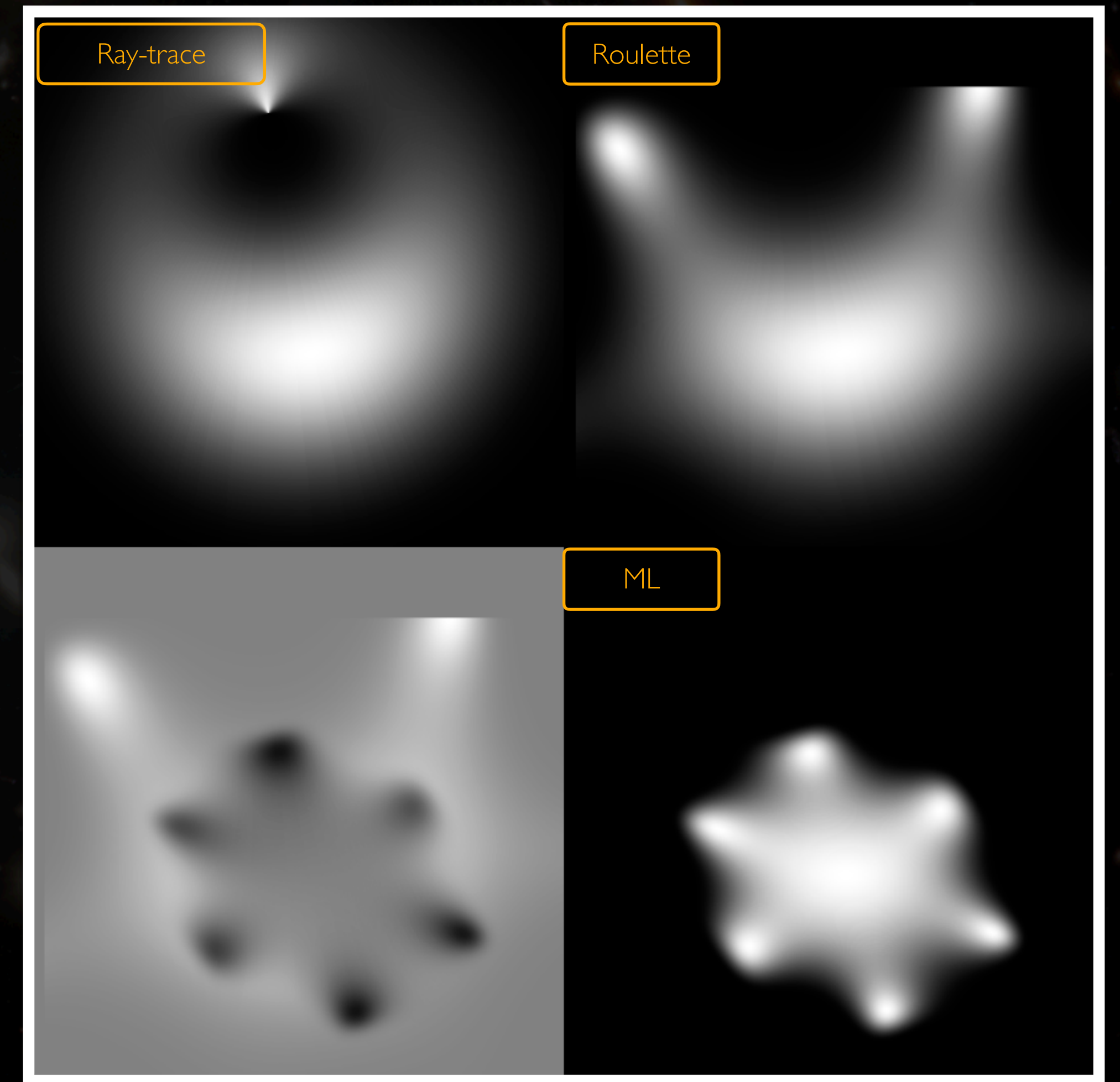
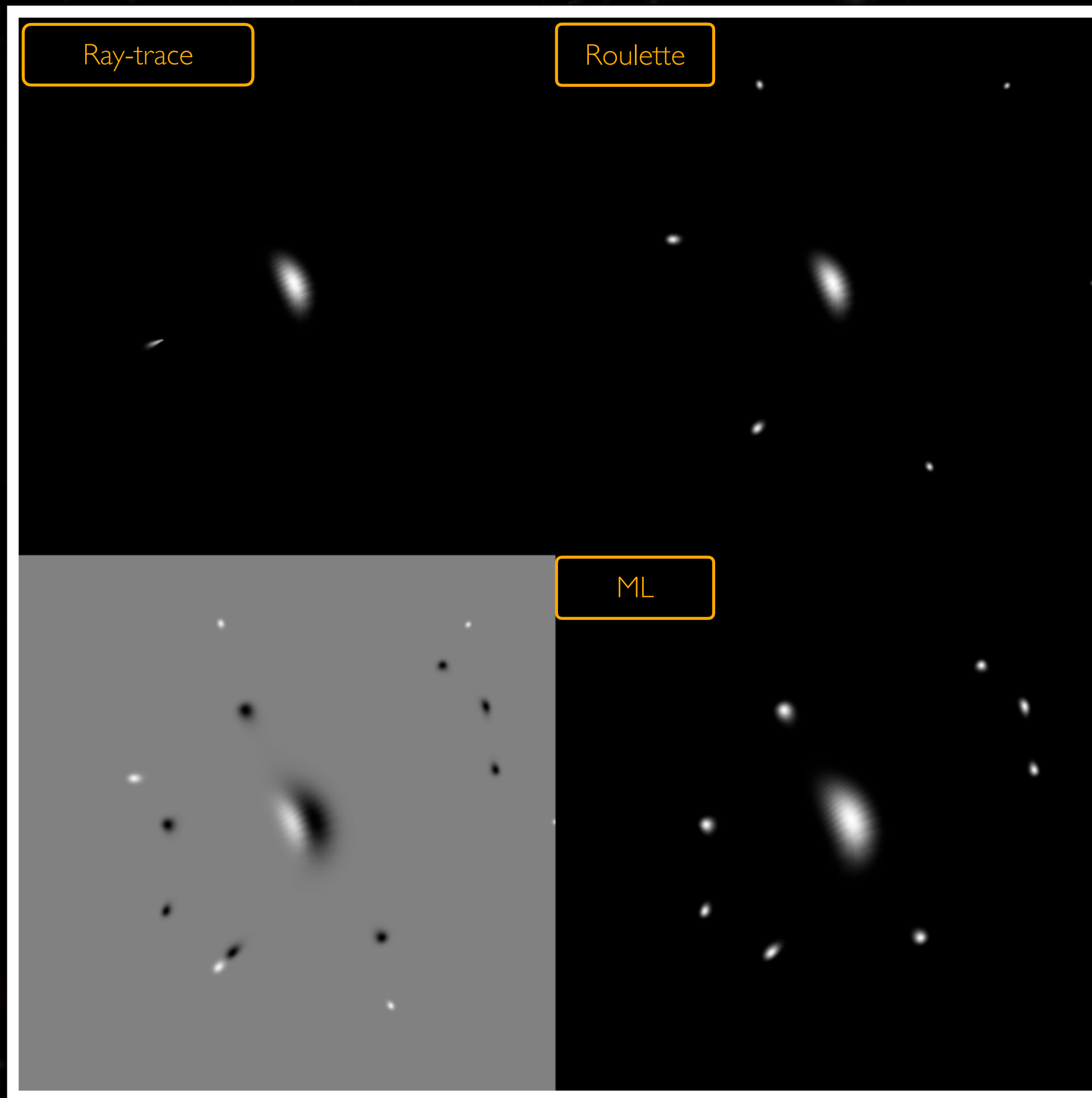
The image shows a Cosmic Microwave Background (CMB) fluctuation map. The background is dark with a complex pattern of light and dark spots, representing temperature variations in the early universe. Several bright, multi-pointed star-like patterns are visible, likely representing foreground stars or galaxies. A yellow rectangular box is drawn over the central portion of the map, containing the letters 'ML' in white. The box is positioned roughly in the middle of the image, both horizontally and vertically.

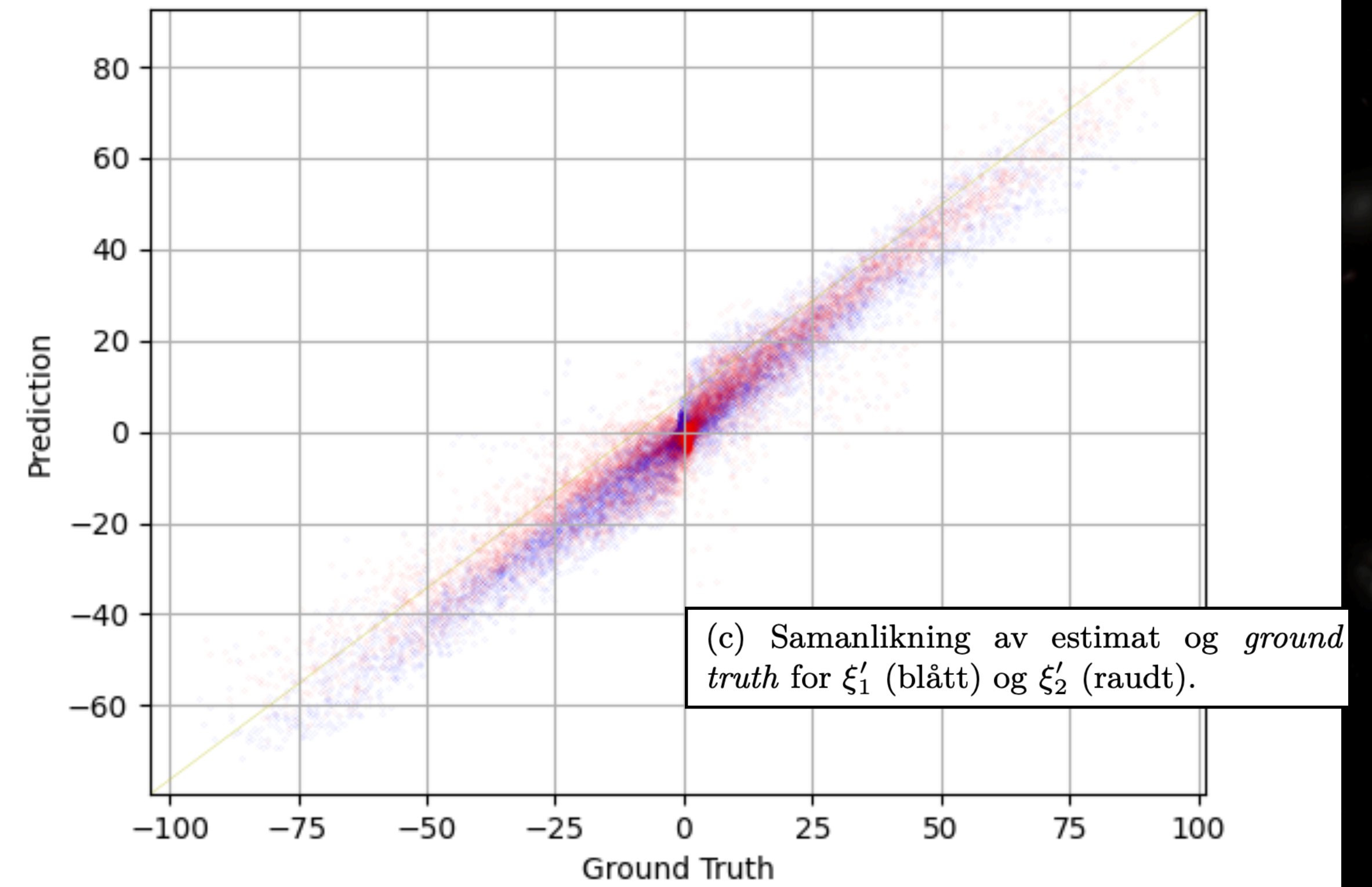
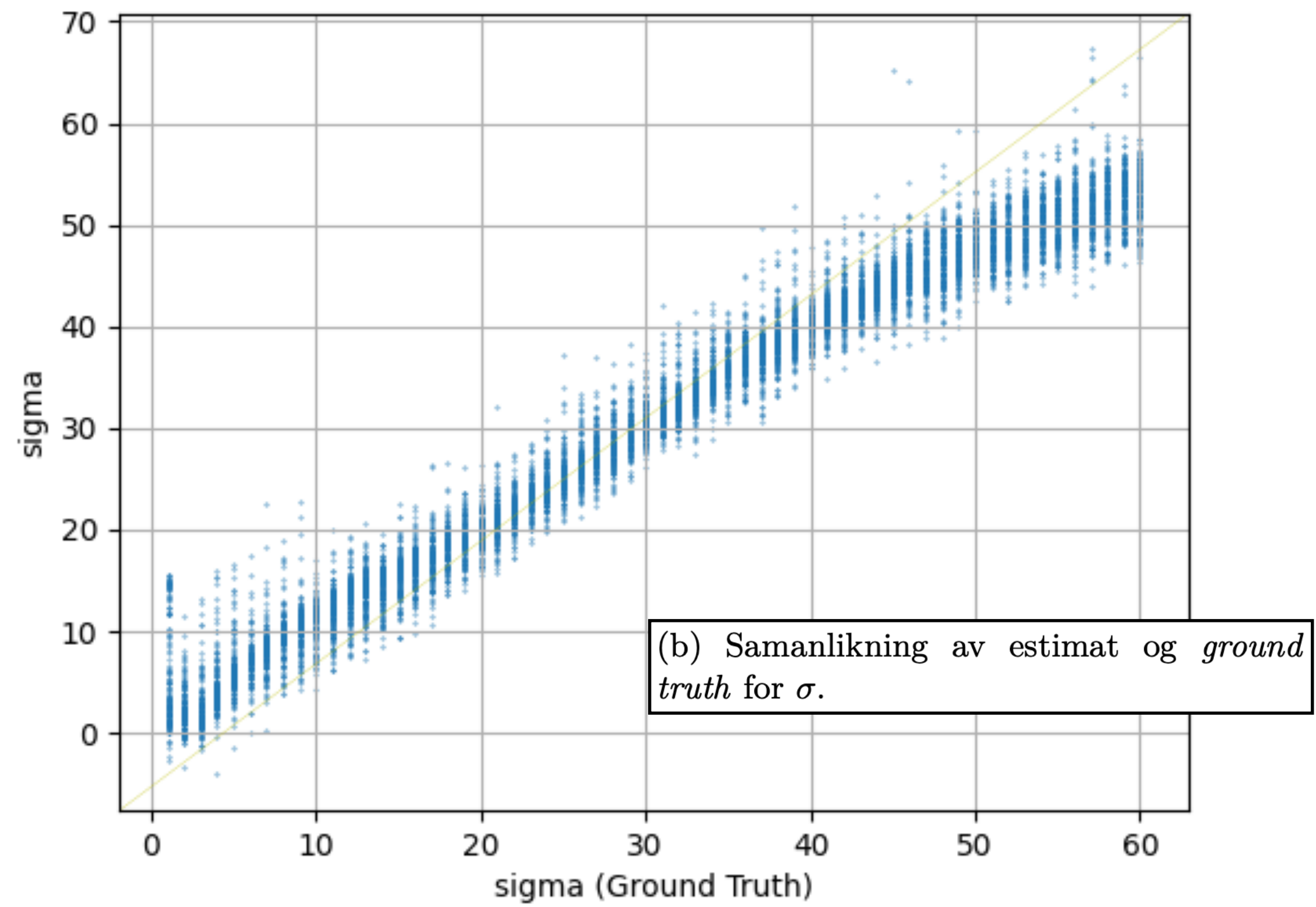
ML

MACHINE LEARNING

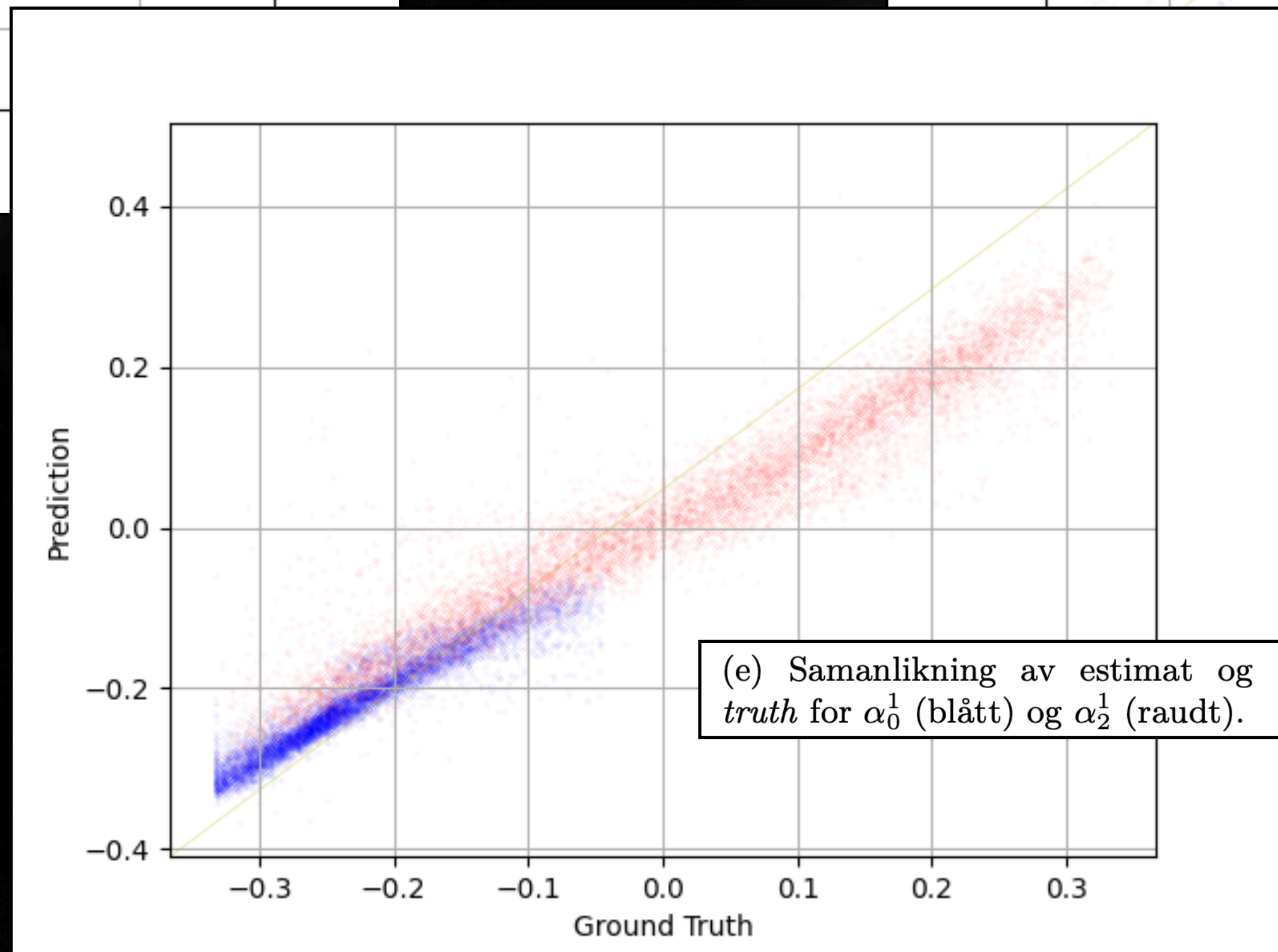
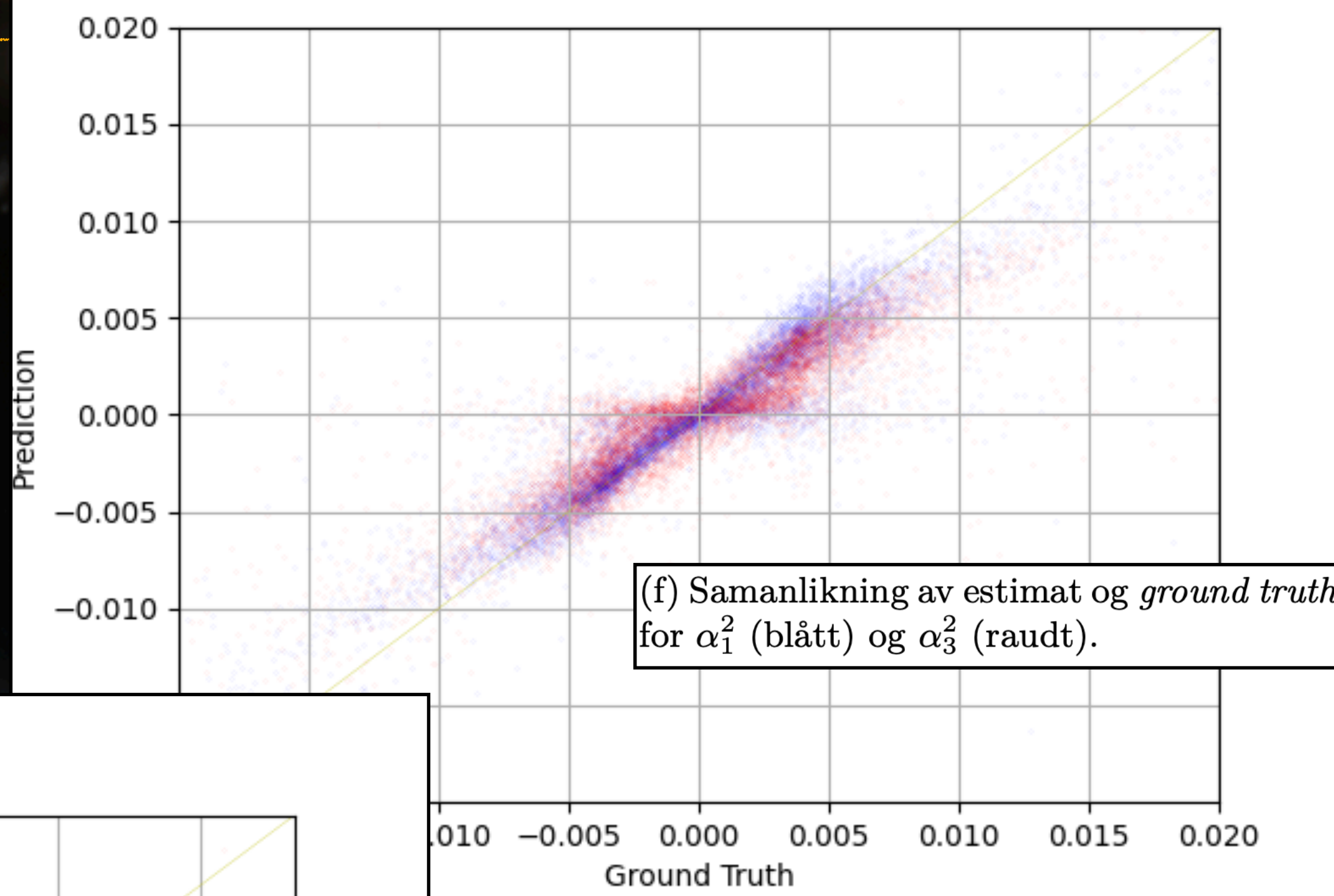
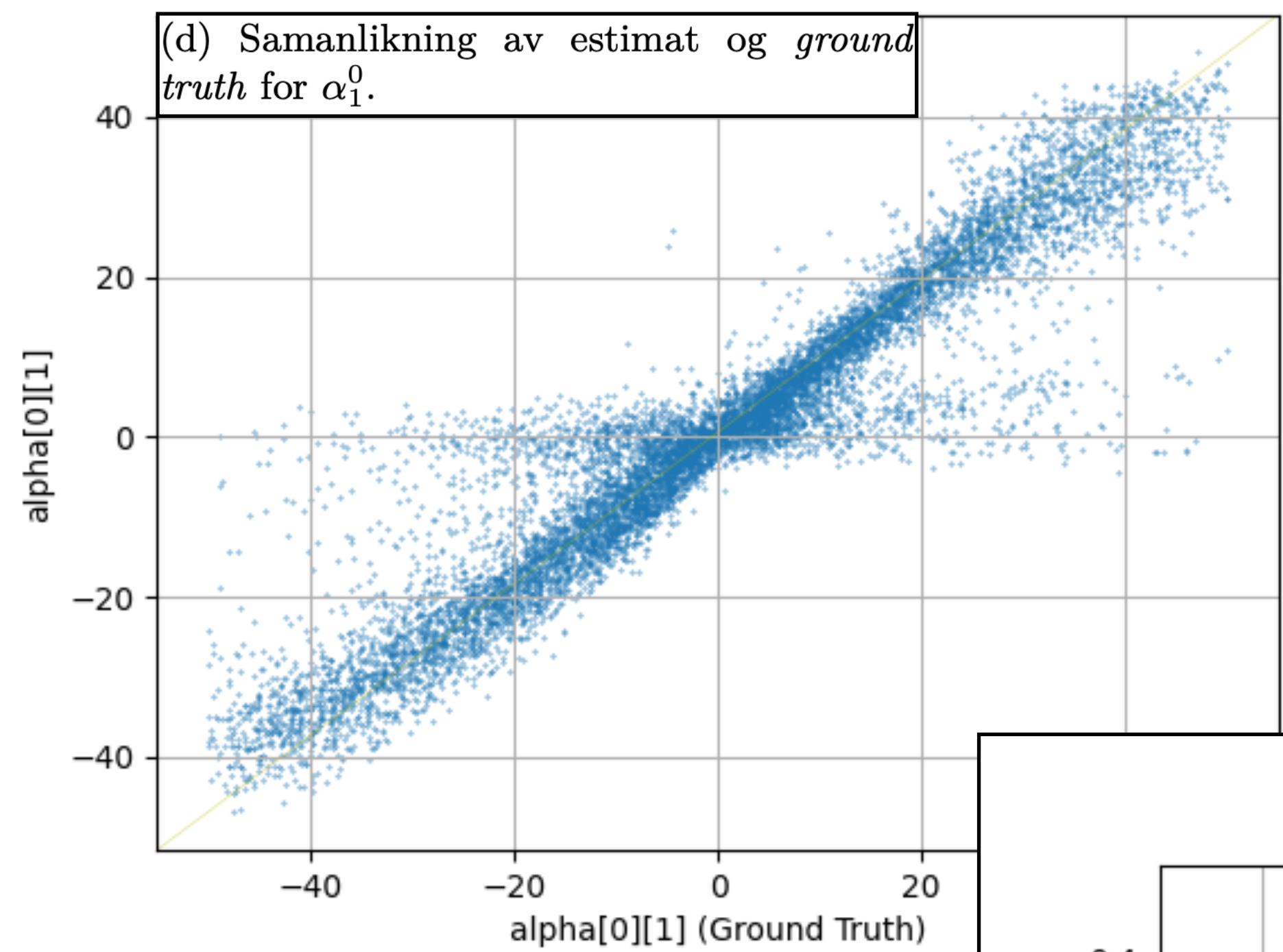


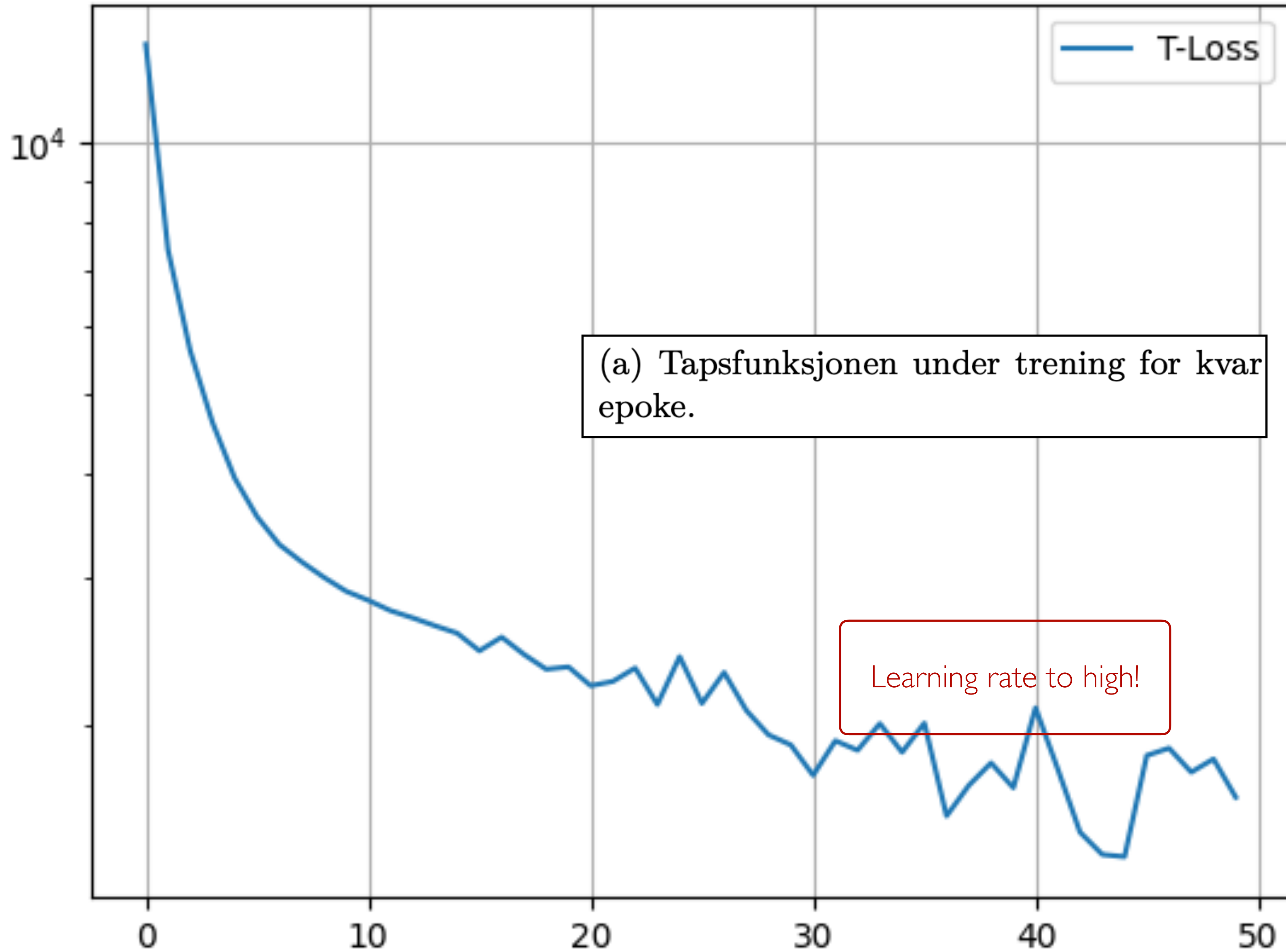
MACHINE LEARNING





MACHINE LEARNING





(a) Tapsfunksjonen under trening for kvar epoke.

Learning rate to high!

Future work

Theory

Roulette 2.0

Cluster lenses

Visualisation

Critical curves / caustics

ML

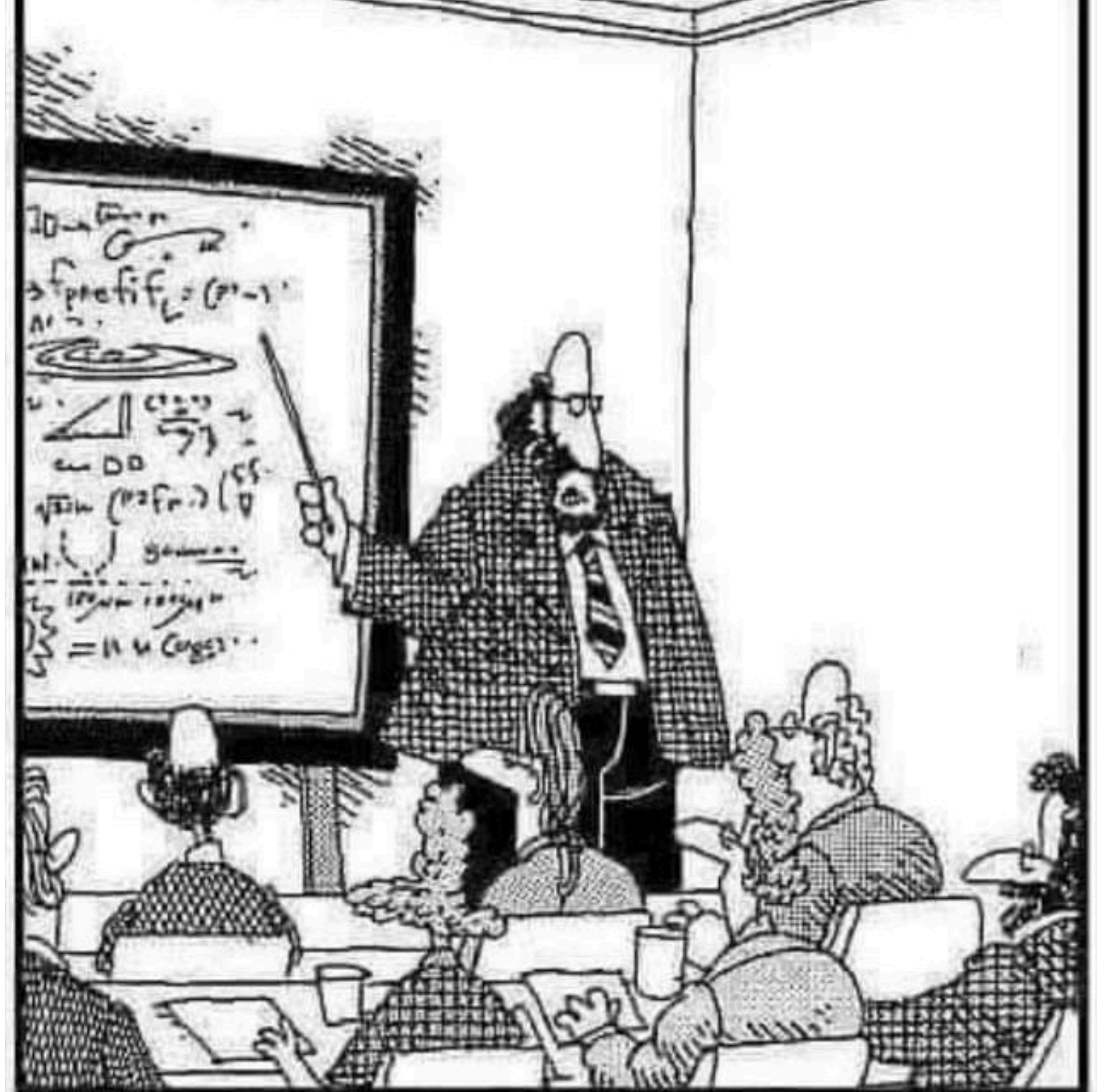
systematics

Real data

Thank you for your attention!

References

- [GitHub: CosmoAI-AES](#)
- [Conference paper for ECMS \(arXiv:2303.11824\)](#)



"Along with 'Antimatter,' and 'Dark Matter,' we've recently discovered the existence of 'Doesn't Matter,' which appears to have no effect on the universe whatsoever."