Using Feynman diagrams in orbit dynamics including extra fields

Vegard Undheim

UiS

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Norwegian Particle, Astroparticle & Cosmology Theory network

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Outline

1. Outline
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   - Why the interest in binary dynamics
   - Feynman diagrams in gravity
3. Gravity from gravitons - Using Feynman diagrams in GR
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   - References

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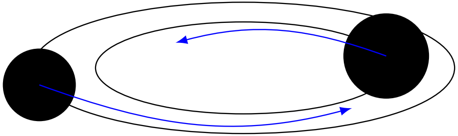
What we have done



$$L = \overbrace{\frac{1}{2}\mu \boldsymbol{v}^2\left(1+\frac{1-3\eta}{4}\frac{\boldsymbol{v}^2}{c^2}\right)}^{\text{kin. terms}} + \overbrace{\frac{GM\mu}{r}\left(1+\frac{3}{2}\frac{\boldsymbol{v}^2}{c^2}+\eta\frac{\boldsymbol{v}^2+(\boldsymbol{v}\cdot\hat{\boldsymbol{r}})^2}{2c^2}-\frac{GM}{2rc^2}\right)}^{\text{Gravity}} + \mathcal{O}\left(c^{-3}\right)$$

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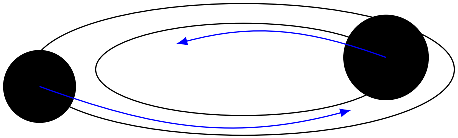

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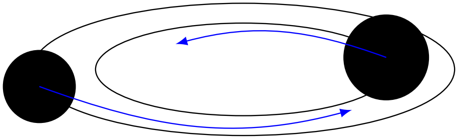

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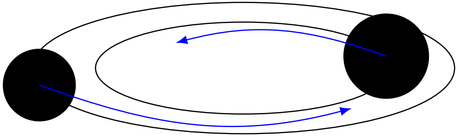

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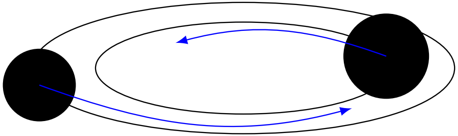

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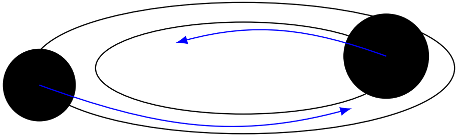

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Why should we care?

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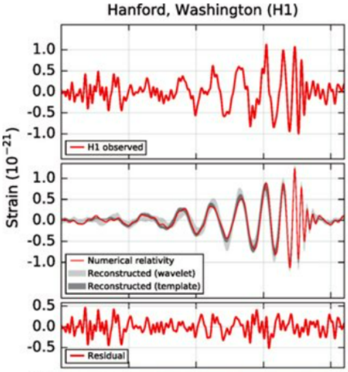
Binary dynamics - the Rosetta Stone of GW astronomy



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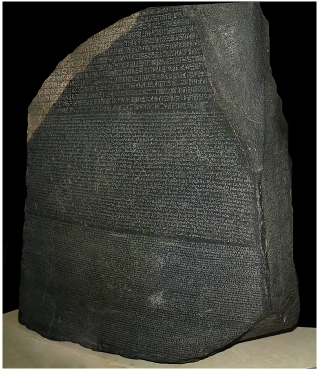

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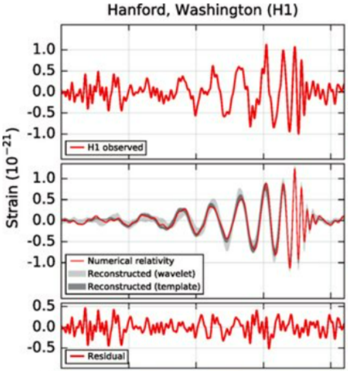

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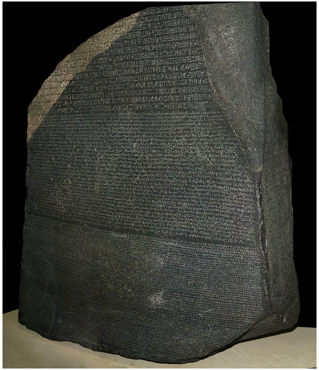

Accurate understanding of dynamics = accurate GW astronomy

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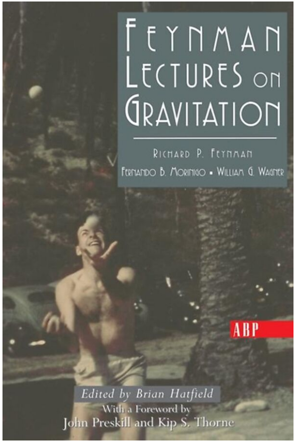
Using Feynman diagrams



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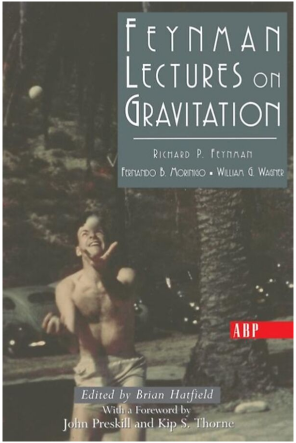

arXiv:hep-th/0409156v2 11 Mar 2005

An Effective Field Theory of Gravity for Extended Objects

Walter D. Goldberger[1] and Ira Z. Rothstein[2]
[1]Department of Physics, Yale University, New Haven, CT 06511
[2]Department of Physics, Carnegie Mellon University, Pittsburgh, PA 15213
(Dated: March 2005)

Using Effective Field Theory (EFT) methods we present a Lagrangian formalism which describes the dynamics of non-relativistic extended objects coupled to gravity. The formalism is relevant to understanding the gravitational radiation power spectra emitted by binary star systems, an important class of candidate signals for gravitational wave observatories such as LIGO or VIRGO. The EFT allows for a clean separation of the three relevant scales: r_s, the size of the compact objects, r the orbital radius and r/v, the wavelength of the physical radiation (where the velocity v is the expansion parameter). In the EFT radiation is systematically included in the v expansion without need to separate integrals into near zones and radiation zones. Using the EFT, we show that the renormalization of ultraviolet divergences which arise at v^6 in post-Newtonian (PN) calculations requires the presence of two non-minimal worldline gravitational couplings linear in the Ricci curvature. However, these operators can be removed by a redefinition of the metric tensor, so that the divergences at arising at v^6 have no physically observable effect. Because in the EFT finite size features are encoded in the coefficients of non-minimal couplings, this implies a simple proof of the decoupling of internal structure for spinless objects to at least order v^6. Neglecting absorptive effects, we find that the power counting rules of the EFT indicate that the next set of short distance operators, which are quadratic in the curvature and are associated with tidal deformations, do not play a role until order v^{10}. These operators, which encapsulate finite size properties of the sources, have coefficients that can be fixed by a matching calculation. By including the most general set of such operators, the EFT allows one to work within a point particle theory to arbitrary orders in v.

I. INTRODUCTION

The experimental progam in gravitational wave detection that is currently under way [1, 2] has brought renewed attention to the problem of obtaining high accuracy predictions for the evolution and gravitational radiation power spectra of binary star systems with neutron star or black hole constituents. The conventional approach to calculating the initial inspiral of the system as it slowly loses energy to gravitational radiation employs the post-Newtonian (PN) approximation to general relativity. Essentially, this formalism consists of systematically solving the Einstein equations with non-relativistic (NR) sources as a power series expansion in $v < 1$, where v is a typical three-velocity of the system under consideration [3, 4]. For binary systems that can be detected by LIGO, which have binary constituents with masses in the range $\sim (1-10)\, m_\odot$, the inspiral phase spans roughly given by

$$\Delta\phi(t) = \int_{t_0}^{t} d\tau\omega(\tau) = \frac{2}{G_N M}\int_{v(t)}^{v_0} dv' v'^3 \frac{dE/dv'}{P(v')}, \quad (1)$$

where $\omega(\tau)$ is the wave's frequency and M the mass of the heavy component star. Because a typical inspiral event will sweep a large number of radians of phase as it scans the detector frequency band (e.g, for LIGO, with a frequency range $1-10^4$ Hz, the number of cycles for a neutron star/neutron star binary is of order 10^4), to compare the predictions of GR with the experimental data requires computing the observable $\Delta\phi(t)$ to extremely high order in the velocity expansion. For instance, for binary neutron star inspirals in the LIGO band, one must know $\Delta\phi(t)$ to $\mathcal{O}(v^6)$ (or "3PN") beyond the leading order quadrupole radiation results, and higher order terms in v may be necessary for tracking the phase variation of the gravitational waves emitted by more massive objects [5].

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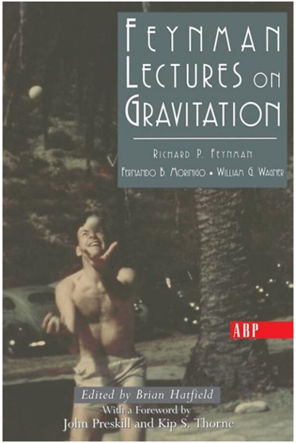

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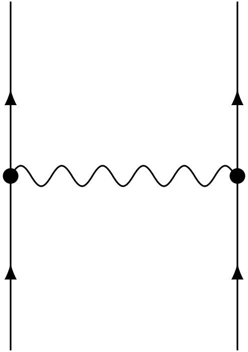

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Advantages of using Feynman diagrams for relativistic binaries

Action → Diagrams → Effective Potential

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E.g. the additional *Kalb-Ramond field* $cS = -\int \mathrm{d}^4x \sqrt{-g} \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho}$

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How to use Feynman diagrams in GR

In the limit of weak gravity ($\frac{GM}{rc^2} \ll 1$) the dynamics should reduce to the Kepler problem:

$$\omega^2 = \frac{GM}{r^3} \quad \leftrightarrow \quad r^2\omega^2 = \frac{GM}{r}.$$

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In order to treat GR as a *typical field theory* we expand the metric

$$g_{\mu\nu} = \eta_{\mu\nu} + \lambda h_{\mu\nu}$$

and treat $h_{\mu\nu}$ as a typical field on a flat space-time. $\lambda \propto \sqrt{G}$.

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To leading order in $\lambda h_{\mu\nu}$ the EoM of the Einstein-Hilbert action is

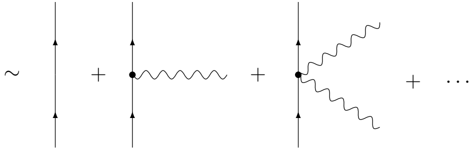
$$\Box\, h_{\mu\nu} = -\lambda P_{\mu\nu\alpha\beta} T^{\alpha\beta}.$$

3 types of expansions to obtain the binary Lagrangian

Interaction term expansion

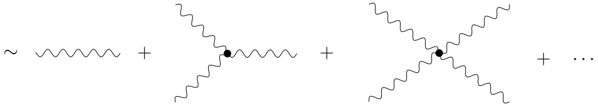
$$L_{pp} = -mc^2\gamma^{-1}\sqrt{1-\lambda h_{\mu\nu}\frac{\dot{x}^\mu}{c}\frac{\dot{x}^\nu}{c}}$$

$$= m\gamma^{-1}\left[-c^2 + \frac{\lambda}{2}h_{\mu\nu}\dot{x}^\mu\dot{x}^\nu + \frac{\lambda^2}{8}\left(h_{\mu\nu}\dot{x}^\mu\dot{x}^\nu\right)^2 + \dots\right]$$



Graviton expansion

$$h_{\mu\nu} = -\lambda D_{\mu\nu\alpha\beta}\left(T^{\alpha\beta} + t^{\alpha\beta}_{(2)} + \lambda t^{\alpha\beta}_{(3)} + \dots\right)$$

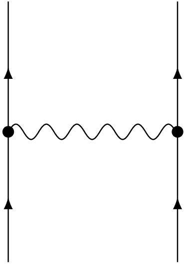


Velocity expansion

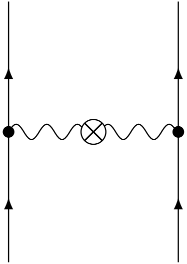
$$\gamma^{-1} = 1 - \frac{v^2}{2c^2} - \frac{v^4}{8c^4} - \dots,$$

$$D_{\mu\nu\alpha\beta}(k) = \frac{-1}{k_\sigma k^\sigma} = \frac{-1}{\boldsymbol{k}^2}\left(1 + \frac{k_0^2}{\boldsymbol{k}^2} + \frac{k_0^4}{\boldsymbol{k}^4} + \dots\right)$$

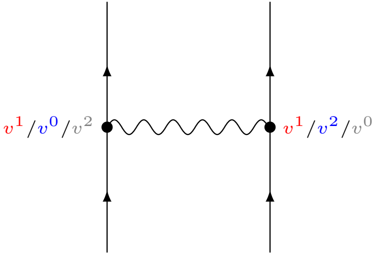
The resulting potentials



$$V = -\frac{Gm_1m_2}{r}$$



$$V = -\frac{Gm_1m_2}{r}\,\frac{\boldsymbol{v}_1\cdot\boldsymbol{v}_2 - (\boldsymbol{v}_1\cdot\hat{\boldsymbol{r}})(\boldsymbol{v}_2\cdot\hat{\boldsymbol{r}})}{2c^2}$$



$$V = \begin{cases} \frac{4Gm_1m_2}{r}\,\frac{\boldsymbol{v}_1\cdot\boldsymbol{v}_2}{c^2} \\ -\frac{3Gm_1m_2}{4r}\,\frac{\boldsymbol{v}_1^2+\boldsymbol{v}_2^2}{c^2} \\ -\frac{3Gm_1m_2}{4r}\,\frac{\boldsymbol{v}_1^2+\boldsymbol{v}_2^2}{c^2} \end{cases}$$

For details see [1, 2]

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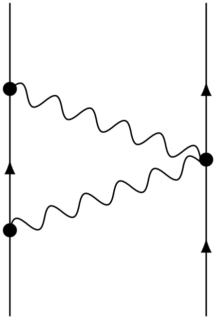
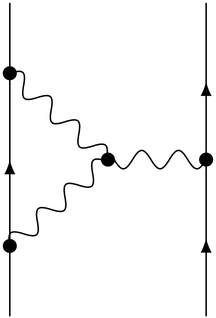


$$V = -\frac{G^2 M m_1 m_2}{2r^2 c^2}$$



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The modification of additional fields

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$$S_{KR} = -\int \sqrt{-g}\left\{\frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} + \mathcal{L}_{\text{int}}\right\}\mathrm{d}^4x$$

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$$\begin{aligned} H_{\alpha\beta\gamma} &= \epsilon_{\alpha\beta\gamma\delta} K^{;\delta}, \\ \mathcal{L}_{\text{int}} &= \sum_a \left[q_a K(x) + p_a K^2(x) + \dots \right] \delta^3(\boldsymbol{x} - \boldsymbol{x}_a(t)) \end{aligned}$$

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We use dashed lines to represent the K-scalar in diagrams.

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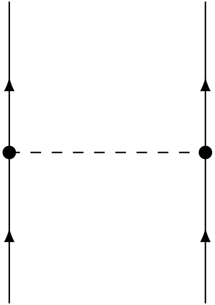
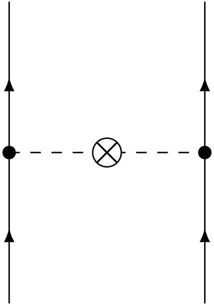
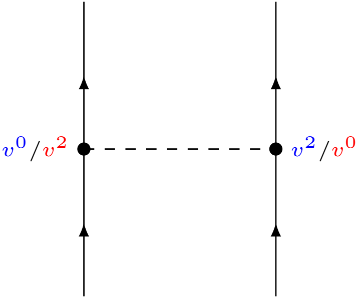
The resulting potentials





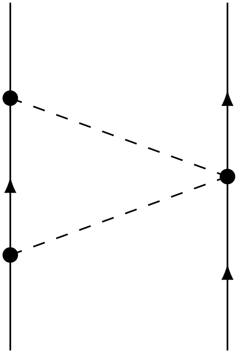


$$V = -\frac{q_1 q_2}{4\pi r}$$

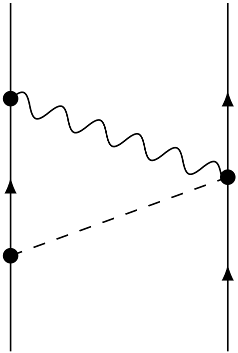
$$V = \frac{q_1 q_2 \eta}{4\pi r} \frac{\boldsymbol{v}^2 - (\boldsymbol{v} \cdot \hat{\boldsymbol{r}})^2}{2c^2}$$

$$V = \frac{q_1 q_2}{4\pi r} \frac{(1 - 2\eta)\boldsymbol{v}^2}{2c^2}$$

The resulting potentials



$$V = \frac{-q_1 q_2}{(4\pi r)^2}\left(\frac{p_1 q_2}{q_1} + \frac{p_2 q_1}{q_2}\right)$$



$$V = \frac{GM q_1 q_2}{4\pi r^2 c^2}$$

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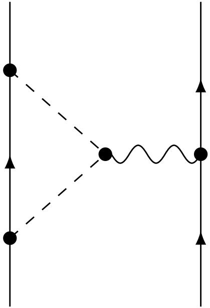
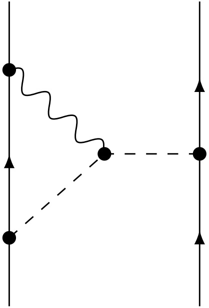
The resulting potentials



$$V = -\frac{G(m_1 q_2^2 + m_2 q_1^2)}{6\pi r^2 c^2}$$



$$V = -\frac{G M q_1 q_2}{6\pi r^2 c^2}$$

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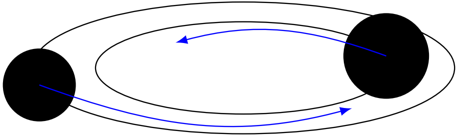
Total Lagrangian



$$L = \overbrace{\frac{1}{2}\mu\boldsymbol{v}^2\left(1+\frac{1-3\eta}{4}\frac{\boldsymbol{v}^2}{c^2}\right)}^{\text{kin. terms}} + \overbrace{\frac{GM\mu}{r}\left(1+\frac{3}{2}\frac{\boldsymbol{v}^2}{c^2}+\eta\frac{\boldsymbol{v}^2+(\boldsymbol{v}\cdot\hat{\boldsymbol{r}})^2}{2c^2}-\frac{GM}{2rc^2}\right)}^{\text{Gravity}} + \mathcal{O}\left(c^{-3}\right)$$

$$+\frac{q_1q_2}{4\pi r}\left(1-\frac{1-2\eta}{2}\frac{\boldsymbol{v}^2}{c^2}-\eta\frac{\boldsymbol{v}^2-(\boldsymbol{v}\cdot\hat{\boldsymbol{r}})^2}{2c^2}-\frac{GM}{3rc^2}+\left(\frac{p_1q_2}{q_1}+\frac{p_2q_1}{q_2}\right)\frac{1}{4\pi r}\right)+\frac{G\left(m_1q_2^2+m_2q_1^2\right)}{6\pi c^2r^2}$$

Thank you for your attention!

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2. Undheim, V. *First post-Newtonian correction to gravitational waves produced by compact binaries: How to compute relativistic corrections to gravitational waves using Feynman diagrams.* MA thesis (NTNU, 2021). https://arxiv.org/abs/2110.15643.

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