

Topological Defects and Symmetry

Nordic Network Meeting on Branes, Fields, and Strings

University of Stavanger – December 5, 2023

Michele Del Zotto



UPPSALA
UNIVERSITET



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Based on joint projects with

- Christian **Copetti**, Kantaro **Ohmori**, Yifan **Wang**
- Kantaro **Ohmori**
- Robert **Moscrop** and Shani Nadir **Meynet**
- Vladimir **Bashmakov**, **Azeem Hasan**, and Justin **Kaidi**
- Matteo **Dell'Acqua**, Shani Nadir **Meynet**, and Elias **Riedel Gårding**

 **annonse** 

- **GCS24:** workshop and school on symmetry categories (June 2024)
- **KITP Program on Symmetries** (in spring 2025 - deadline now: 15/12/2023)

scgcs.berkeley.edu

for details and links!

Executive Summary

Symmetries are a topological subsector of the spectrum of operators of a given QFT.

Today I will explain some features and some first applications of symmetries that arise from this perspective.

Generalized Symmetries: Lightning Review

Well-known **fact**: quantum fields can have **extended operators**.

Wilson 1974
't Hooft 1975
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**Generalized
global
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≡

Framework to express corresponding
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- $\mathcal{O}(p)$

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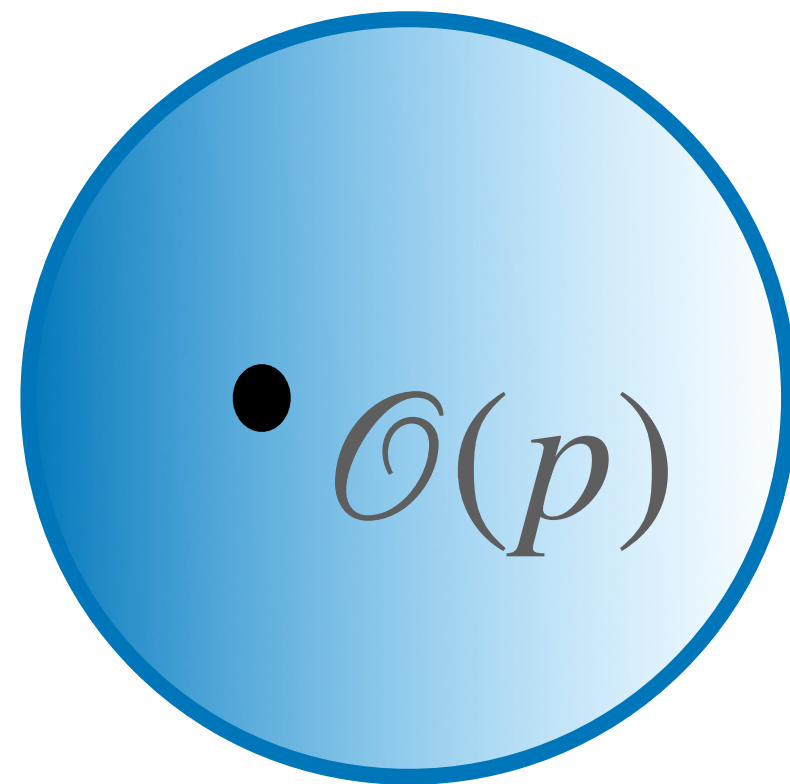
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$$\langle \mathcal{U}_g(S^D) \mathcal{O}(p) \dots \rangle$$

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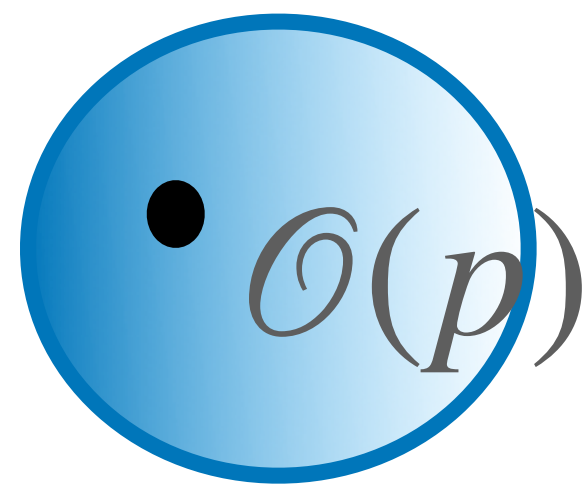
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$$\bullet R_g \mathcal{O}(p) \quad \langle \mathcal{U}_g(S^D) \mathcal{O}(p) \cdots \rangle = \langle R_g \mathcal{O}(p) \cdots \rangle$$

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$\mathcal{O}(\Sigma^k)$

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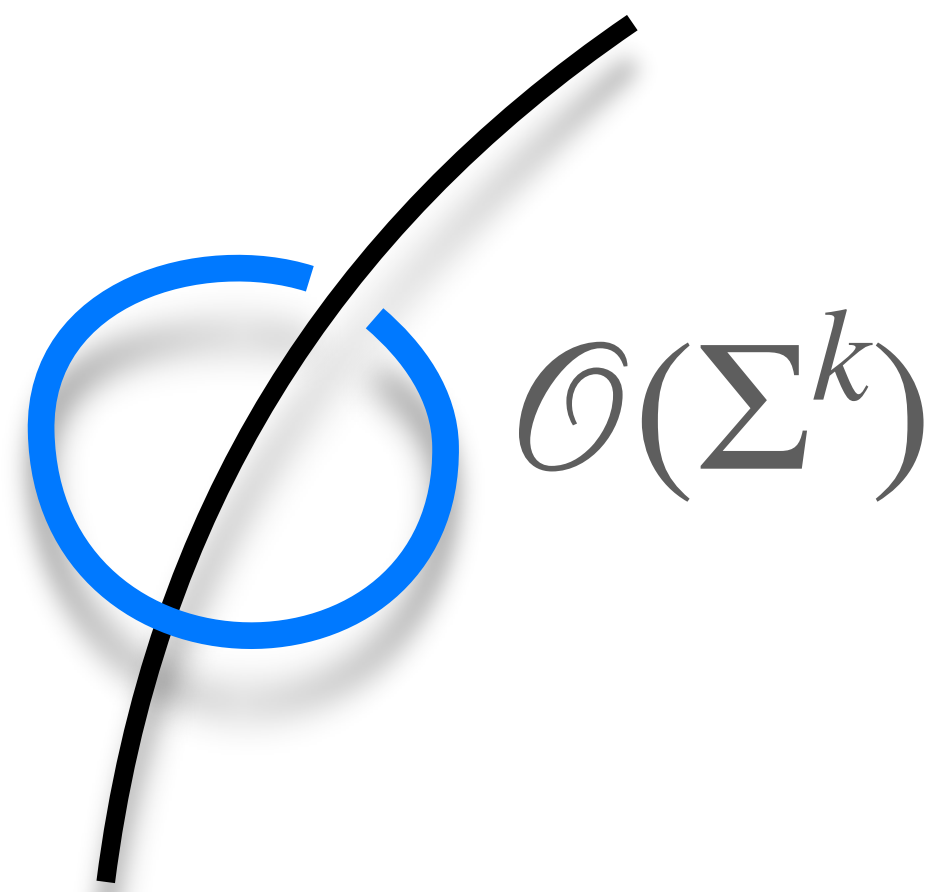
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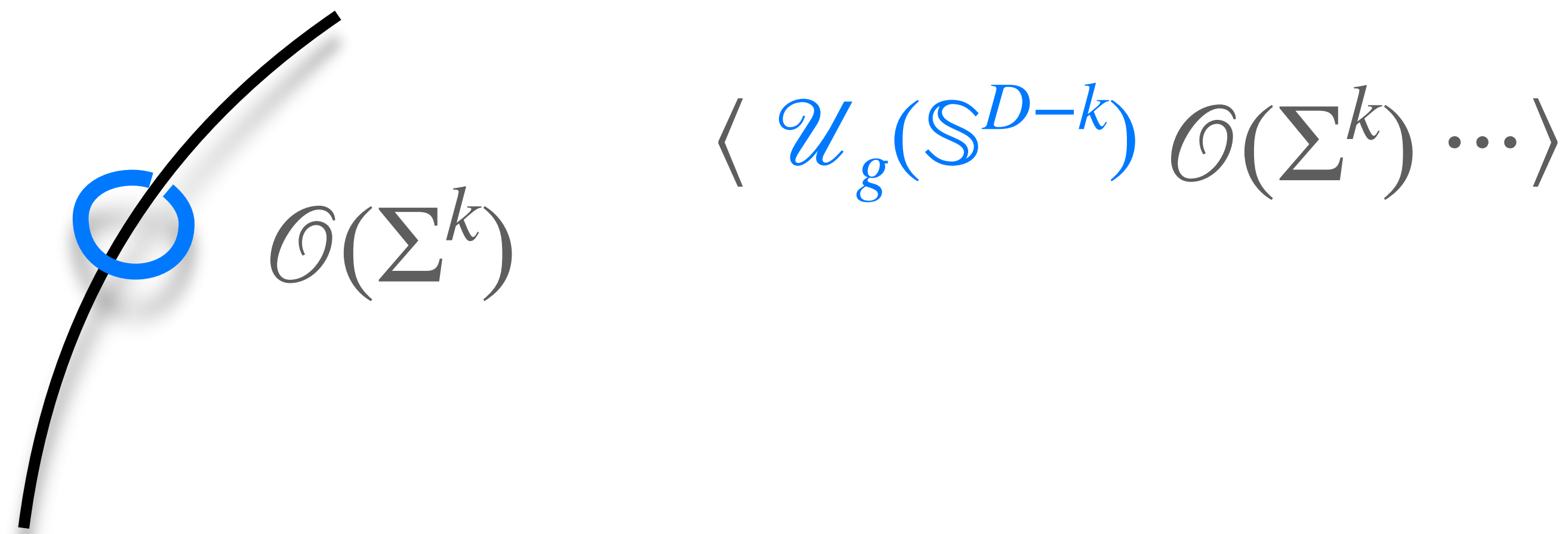
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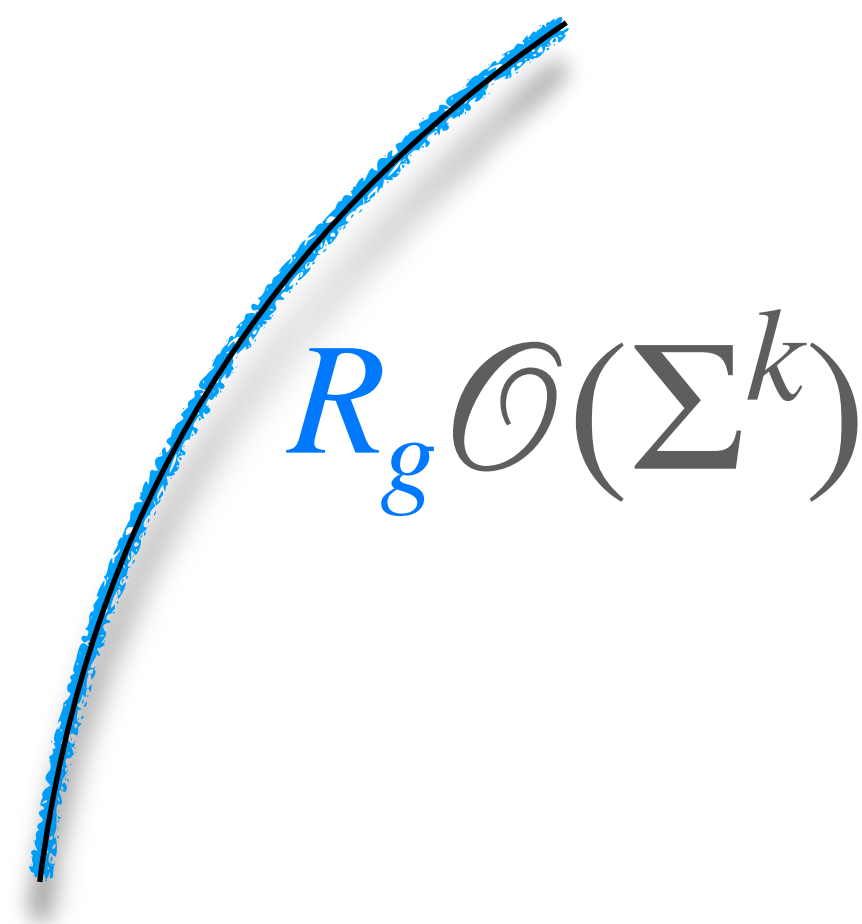
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$$\langle \mathcal{U}_g(S^{D-k}) \mathcal{O}(\Sigma^k) \cdots \rangle = \langle R_g \mathcal{O}(\Sigma^k) \cdots \rangle$$

Example: Maxwell theory

Electric and magnetic 1-form symmetries

$$\mathcal{U}_\theta^{(e)}(\mathbb{S}^2) = e^{i\theta \int_{\mathbb{S}^2} *J_e^{(2)}}$$

$$\mathcal{U}_\theta^{(m)}(\mathbb{S}^2) = e^{i\theta \int_{\mathbb{S}^2} *J_m^{(2)}}$$

$$J_e^{(2)} = \frac{f}{e^2}$$

$$J_m^{(2)} = * \frac{f}{2\pi}$$

$$\mathcal{O}_{q_e}(\Sigma^1) = e^{iq_e \int_{\Sigma^1} a^{(1)}}$$

$$\mathcal{O}_{q_m}(\Sigma^1) = e^{iq_m \int_{\Sigma^1} a_D^{(1)}}$$

Wilson lines

't Hooft lines

$$\mathcal{U}_\theta^{(\bullet)}(\mathbb{S}^2) \mathcal{O}_{q_\bullet}(\Sigma^1) = e^{i\theta q_\bullet} \mathcal{O}_{q_\bullet}(\Sigma^1) \quad \bullet = e, m$$

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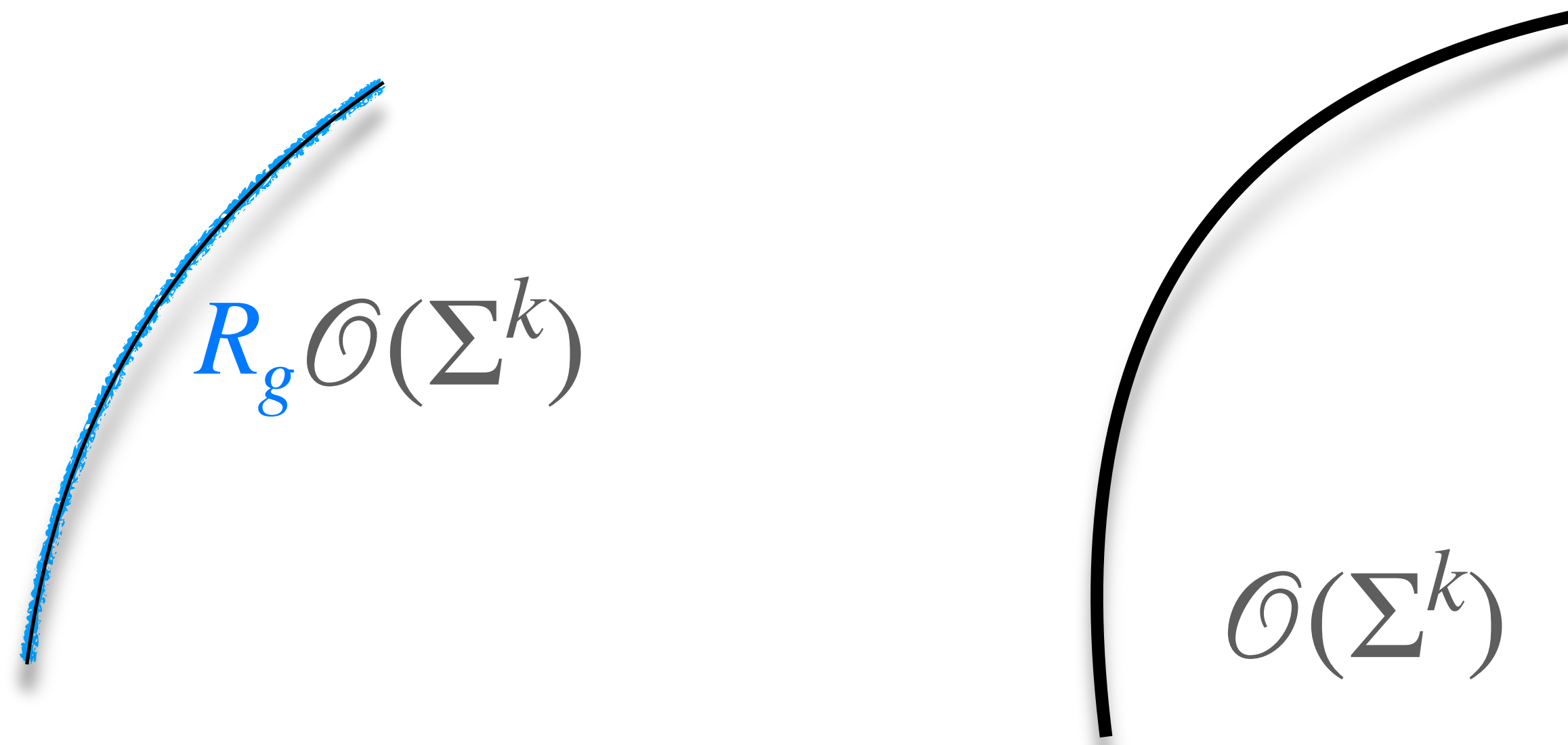
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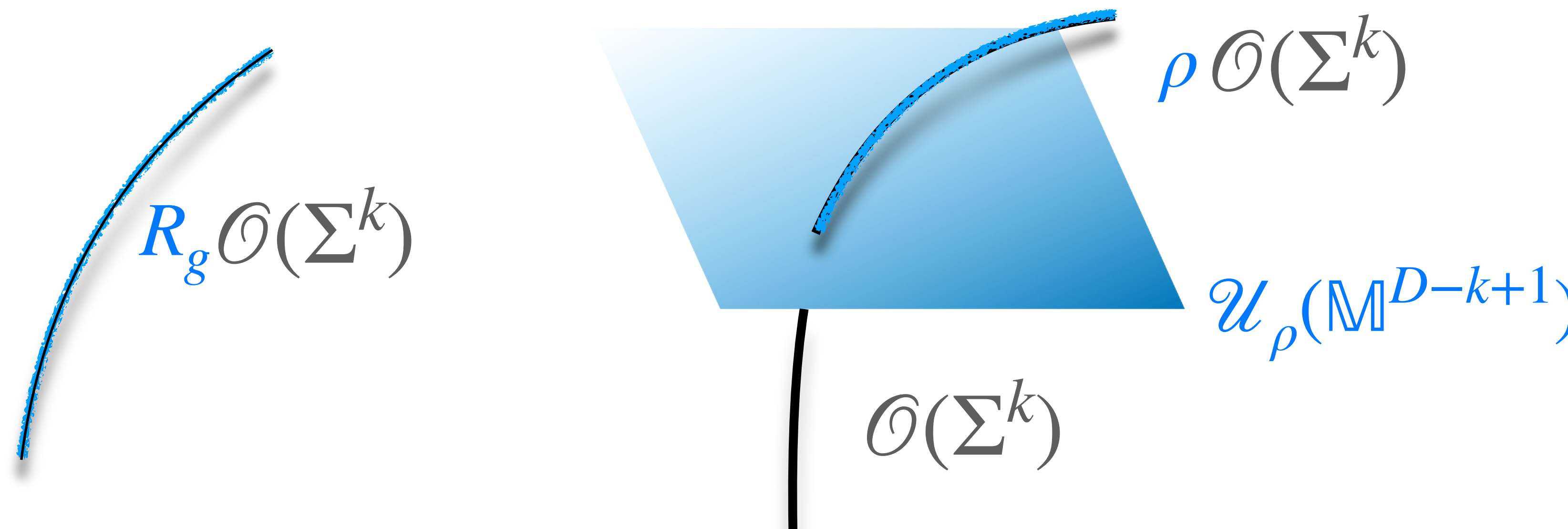
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More charges

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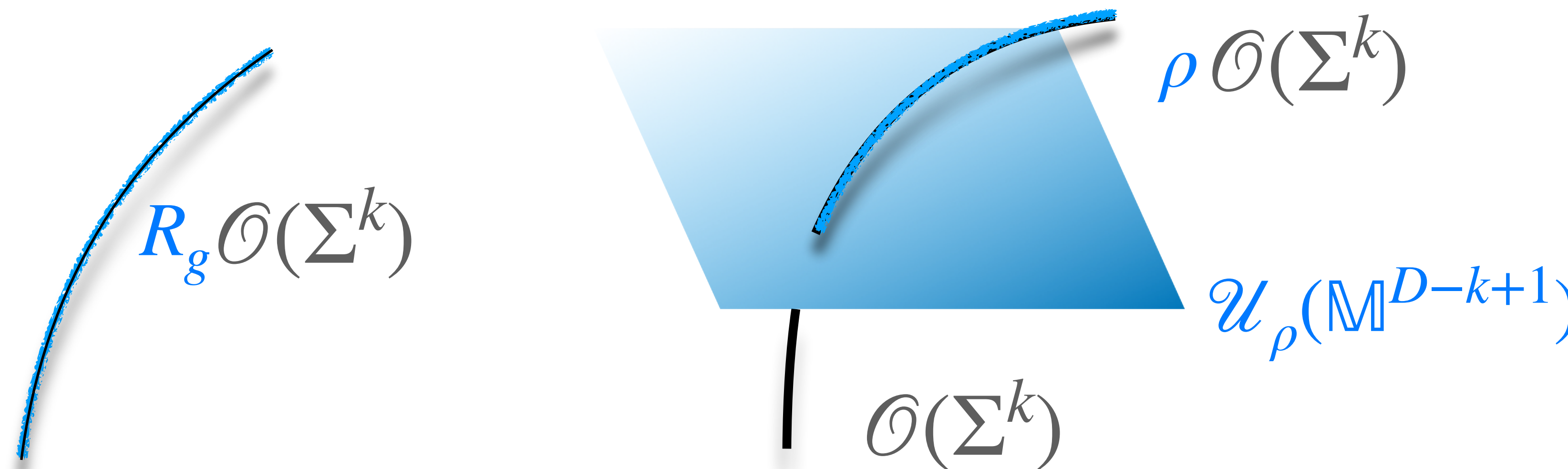
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- Ward identities

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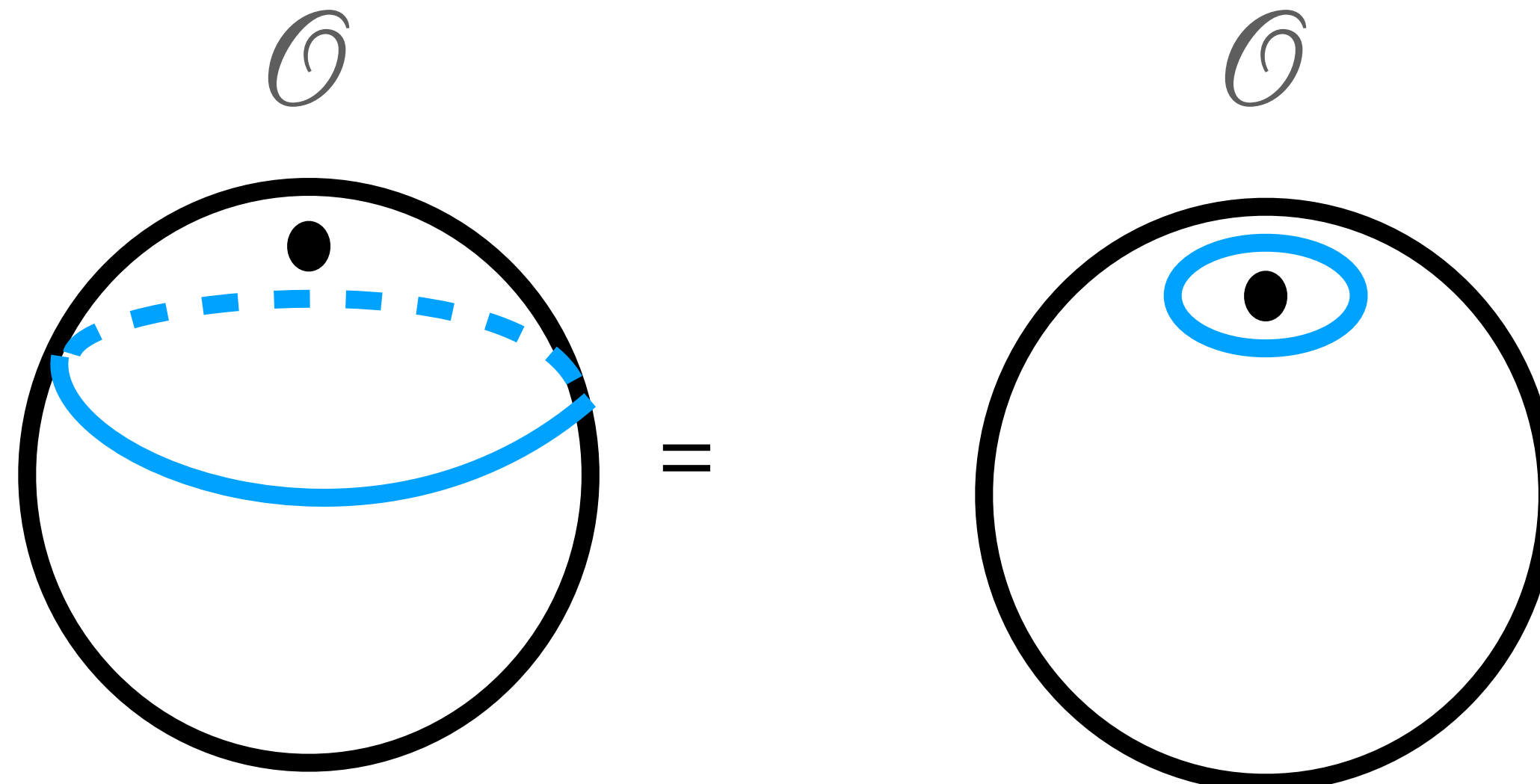
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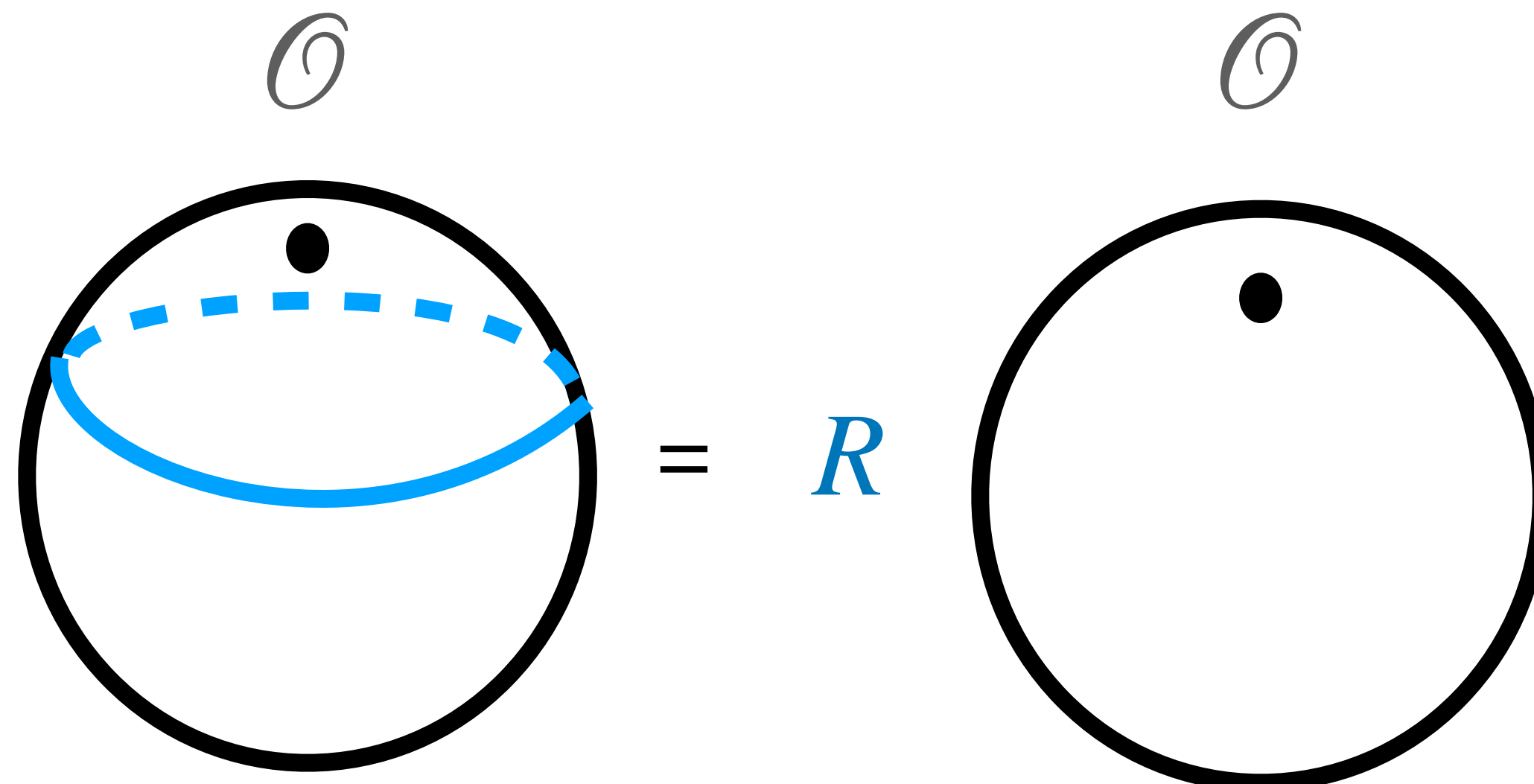
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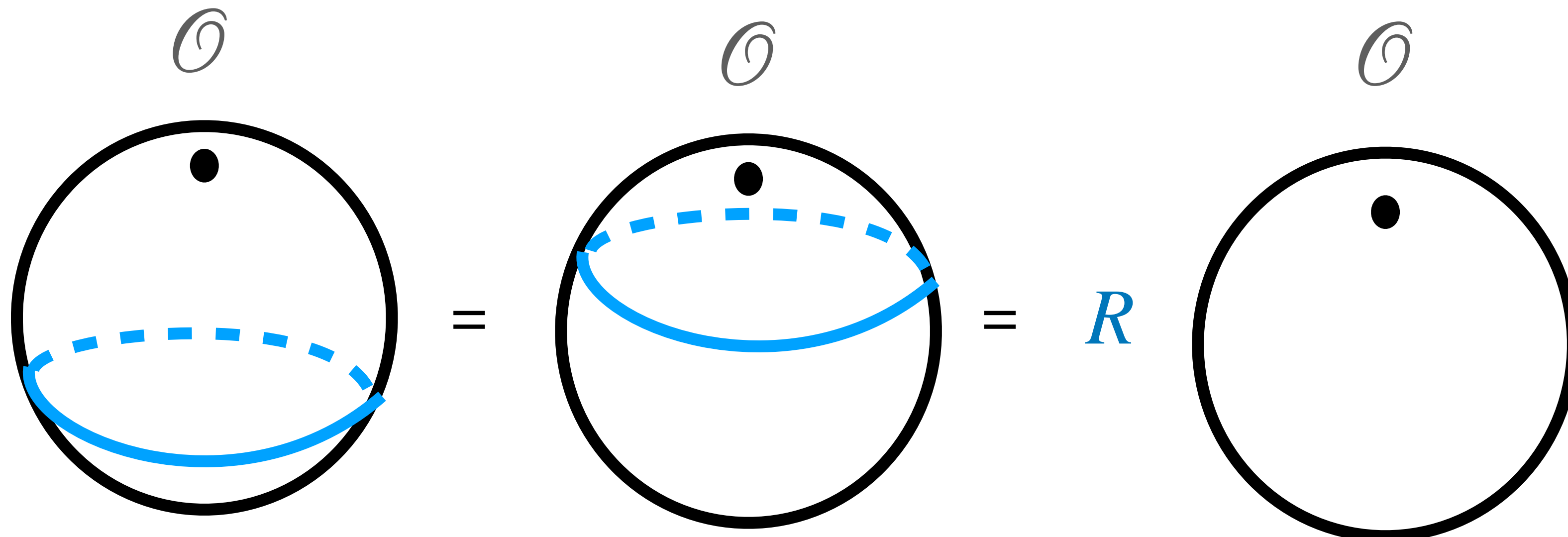
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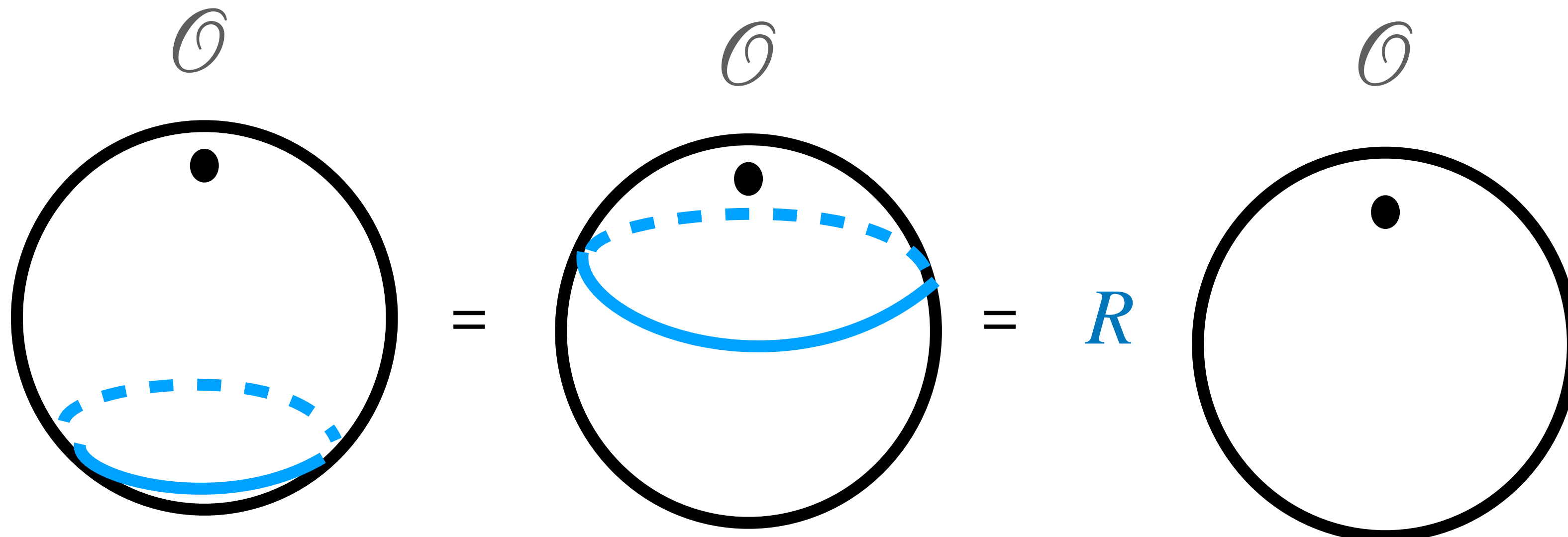
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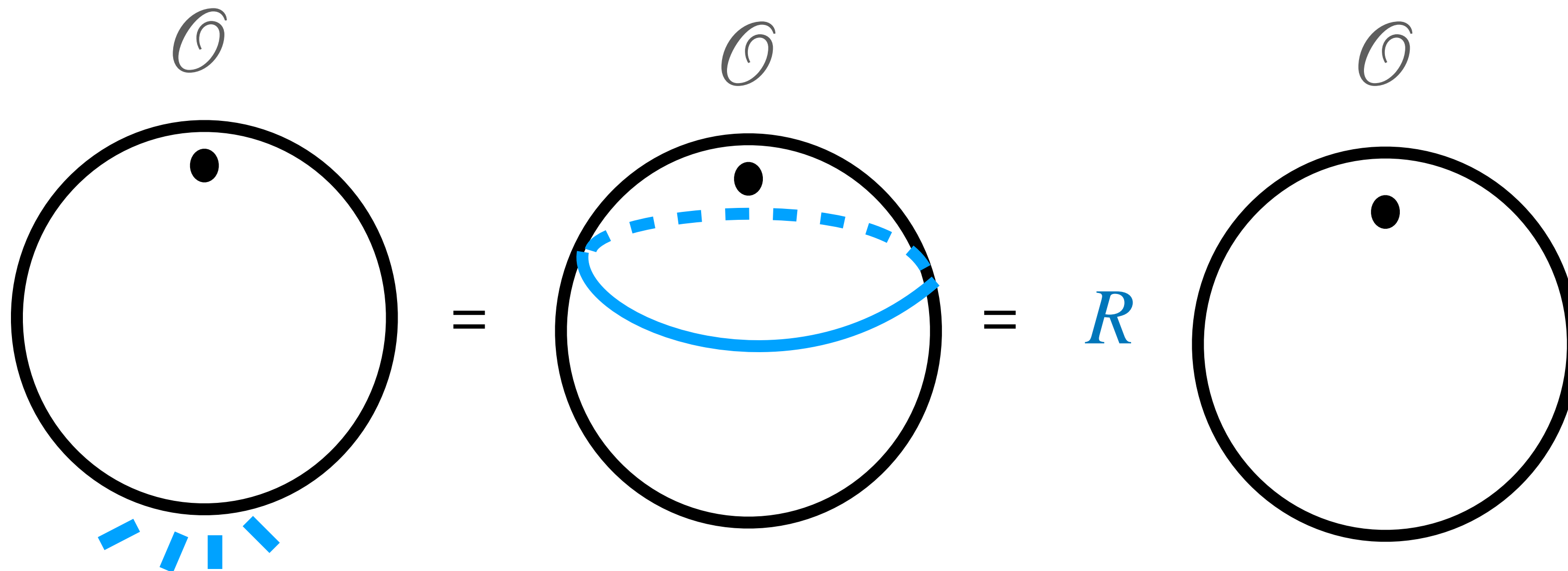
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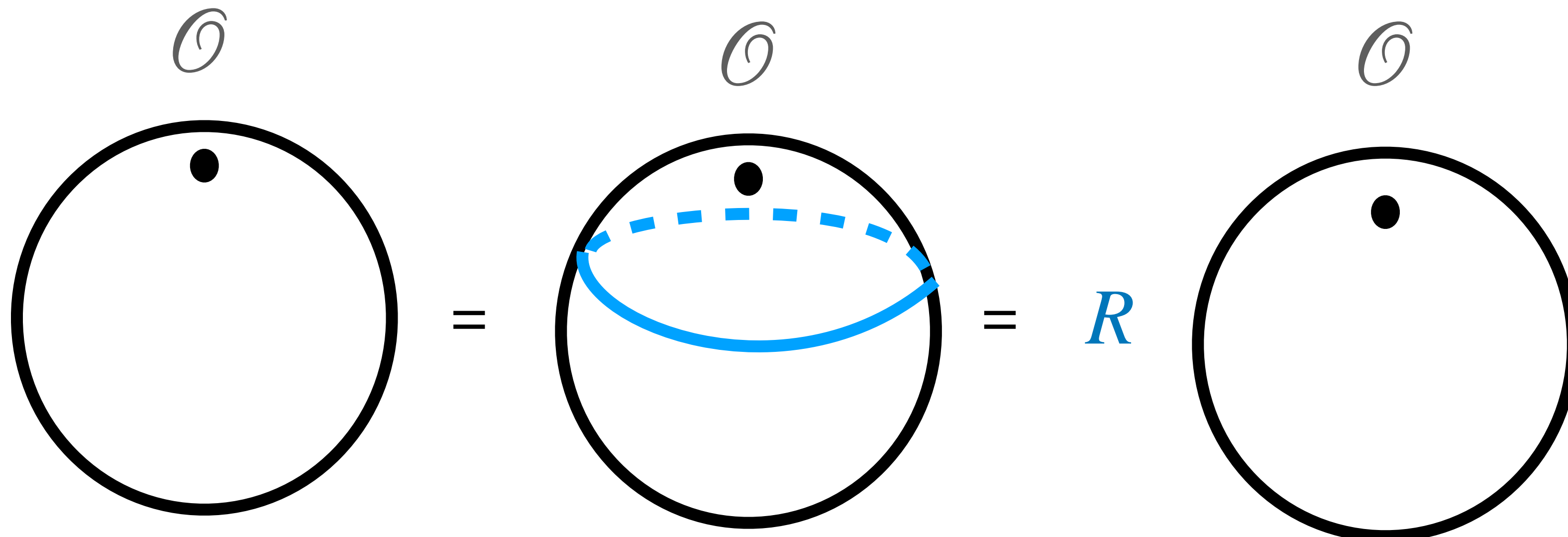
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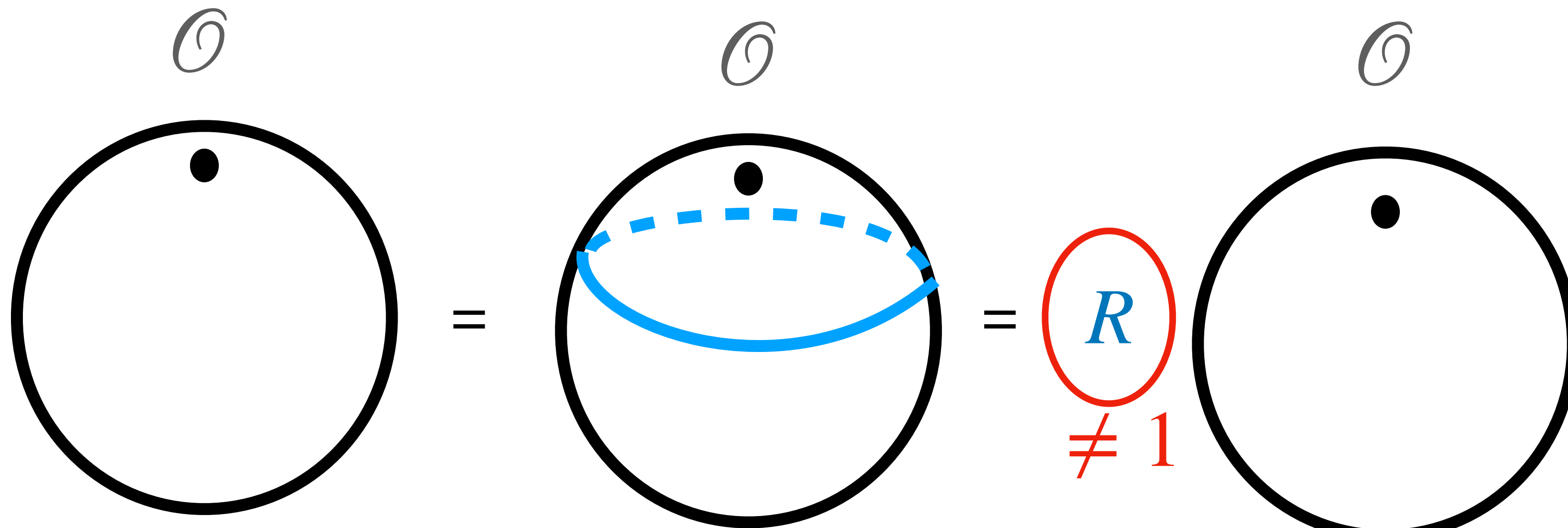
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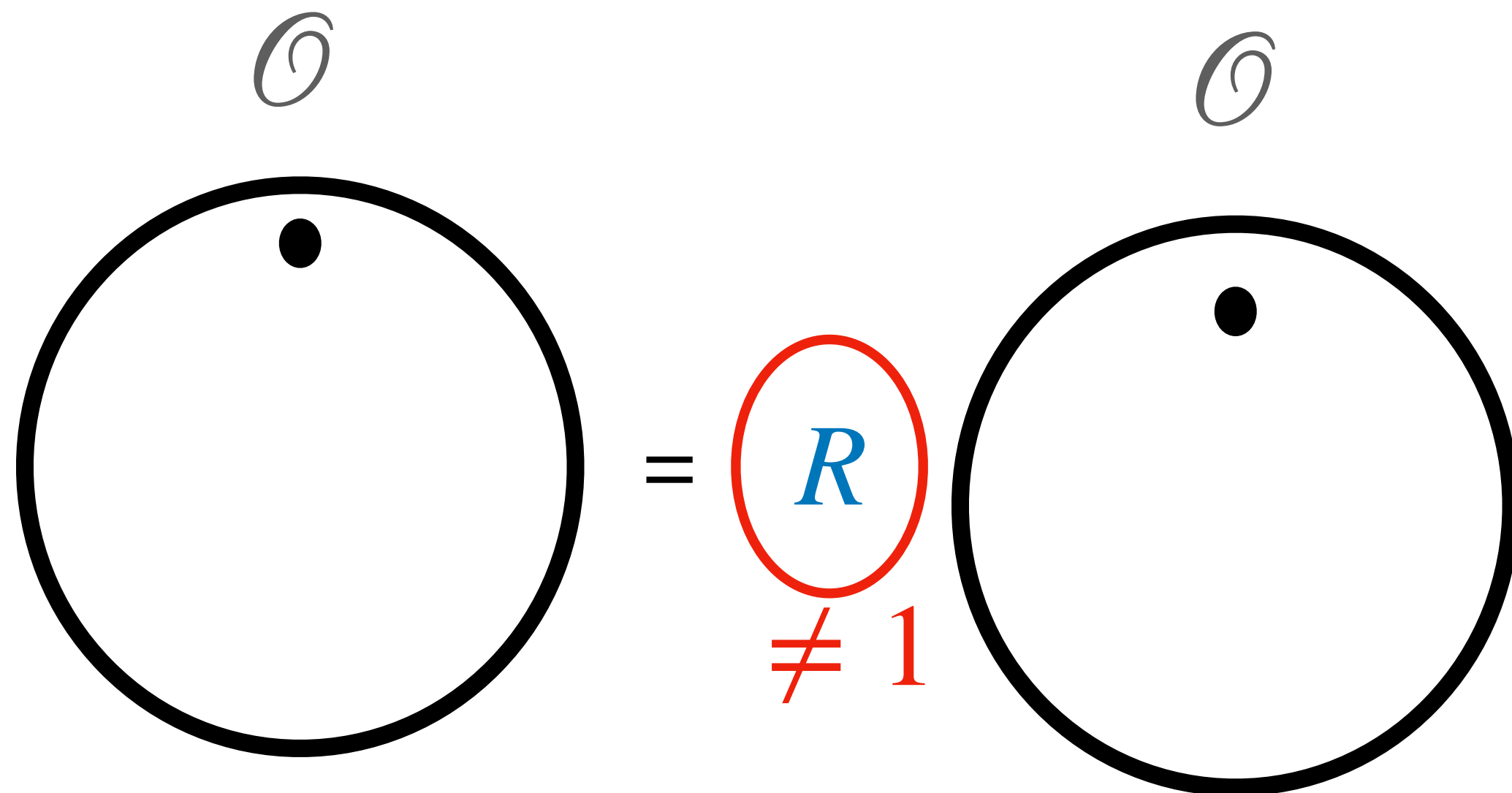
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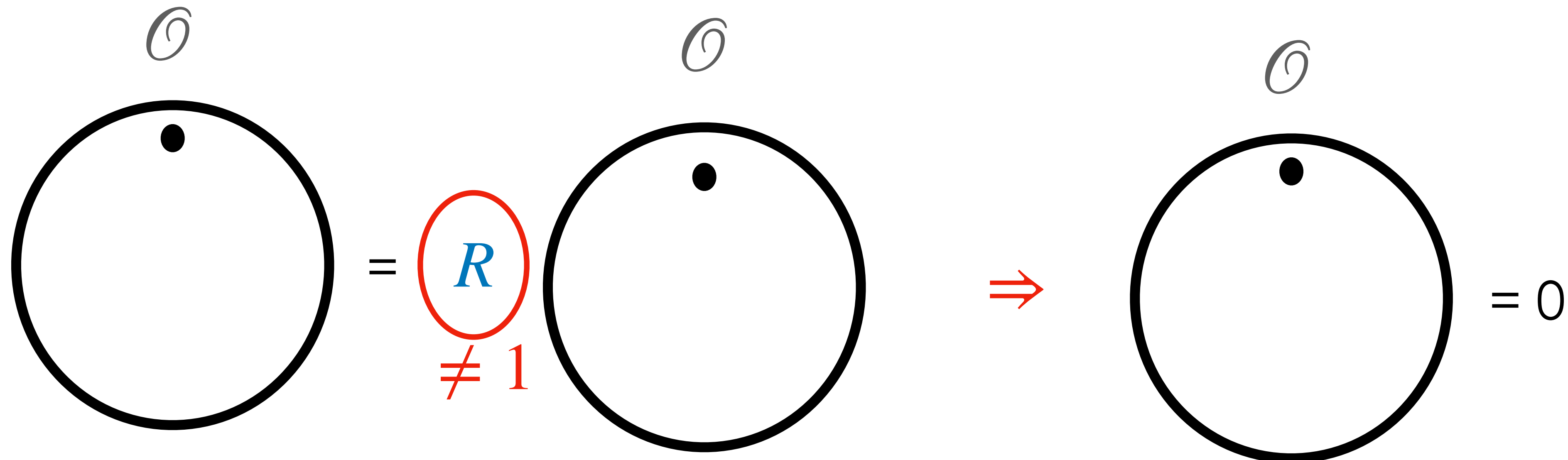
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$$D_1 \otimes D_2$$

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$$D_1 \otimes D_2 = D_3$$

Group Multiplication

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• **Fusion product**


$$D_1 \otimes D_2 = D_3 \oplus D_4 \cdots \oplus D_k$$

Group Multiplication \rightarrow Fusion Product

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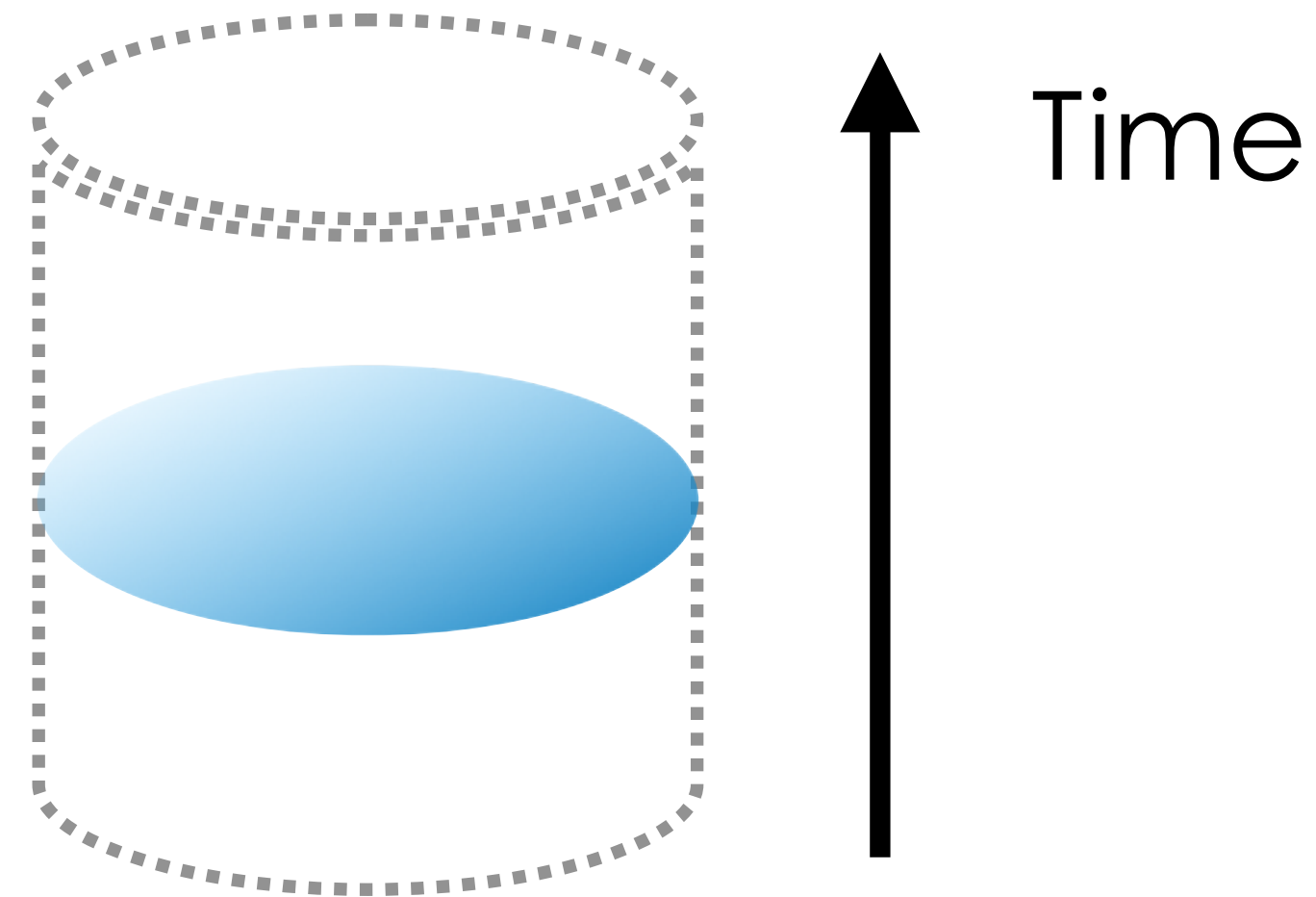
Group Multiplication \rightarrow Fusion Product

only positive integers on the RHS

Locality principle

So far we have not made a distinction among defects and operators (spacetime is Euclidean), but for RQFT there is:

- **Operators** : inserted along space



$$D_1 \otimes D_2 = D_3 \oplus D_4 \cdots \oplus D_k$$

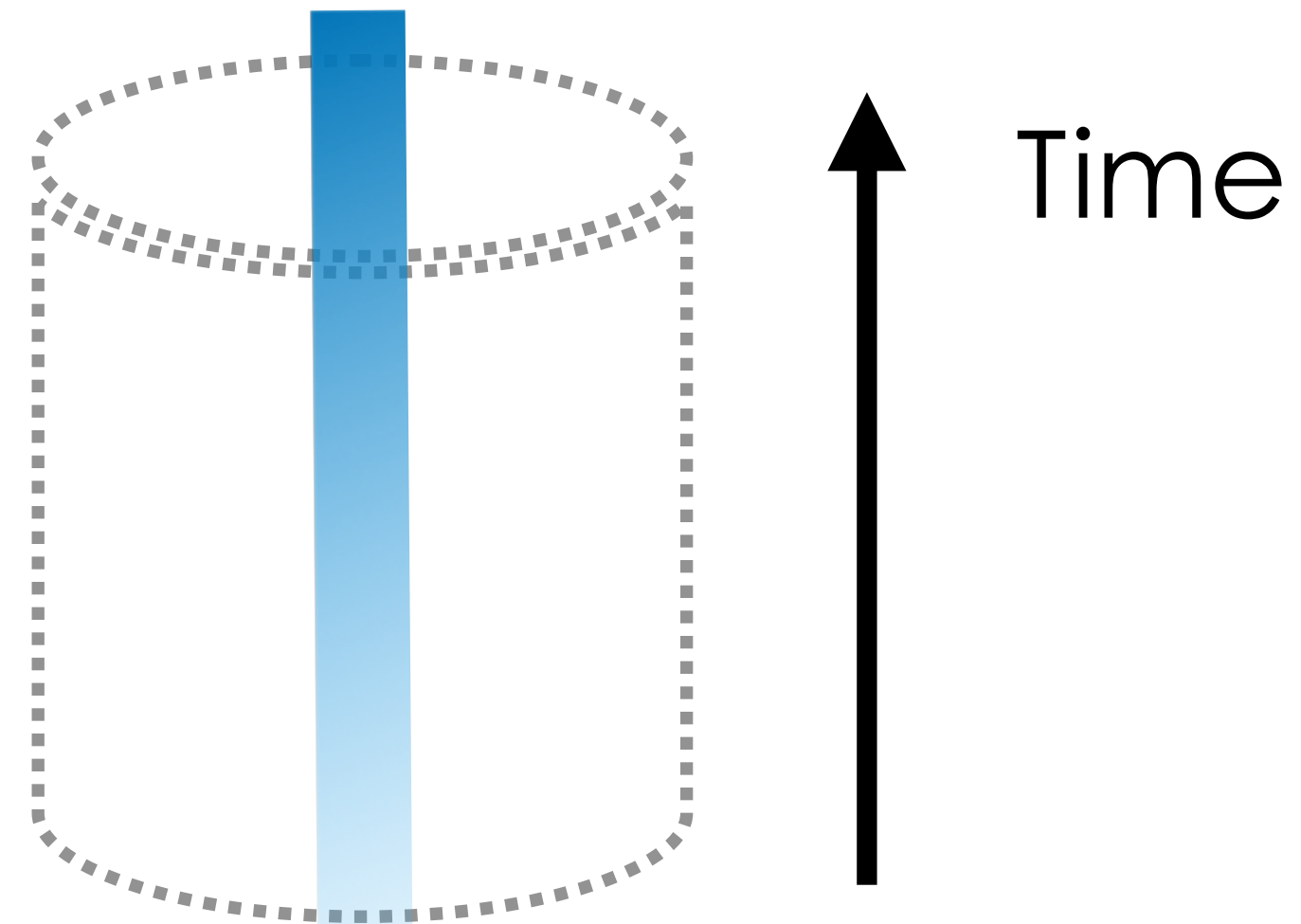
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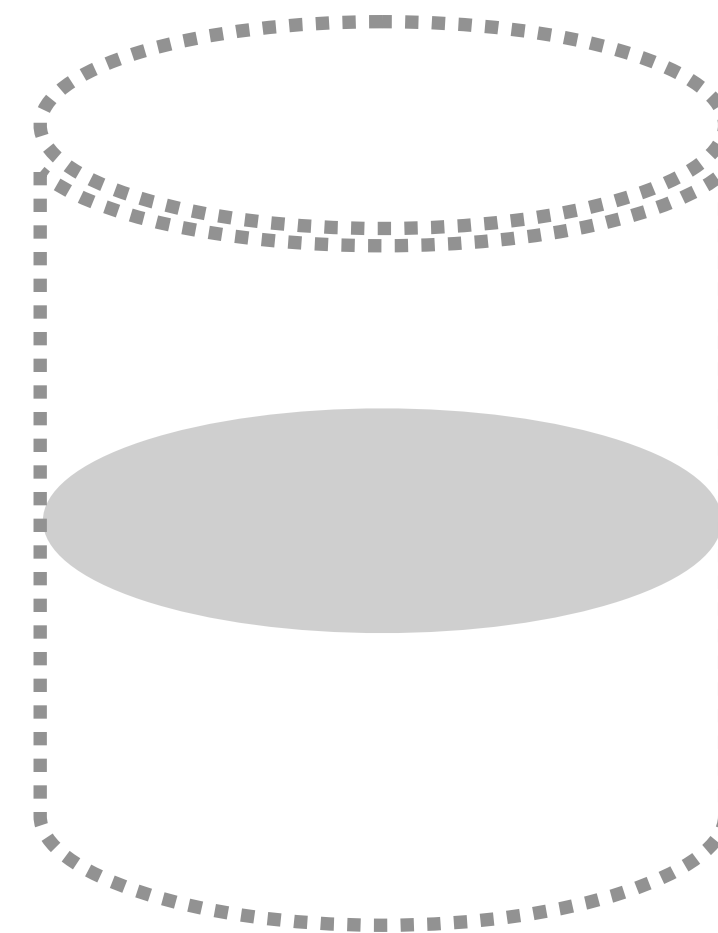
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As defects: **twisted Hilbert spaces**



Time

eg. Free scalar in 2d

$$\phi(x + 2\pi) = \phi(x)$$

S^1 Hilbert space



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Group Multiplication \rightarrow Fusion Product

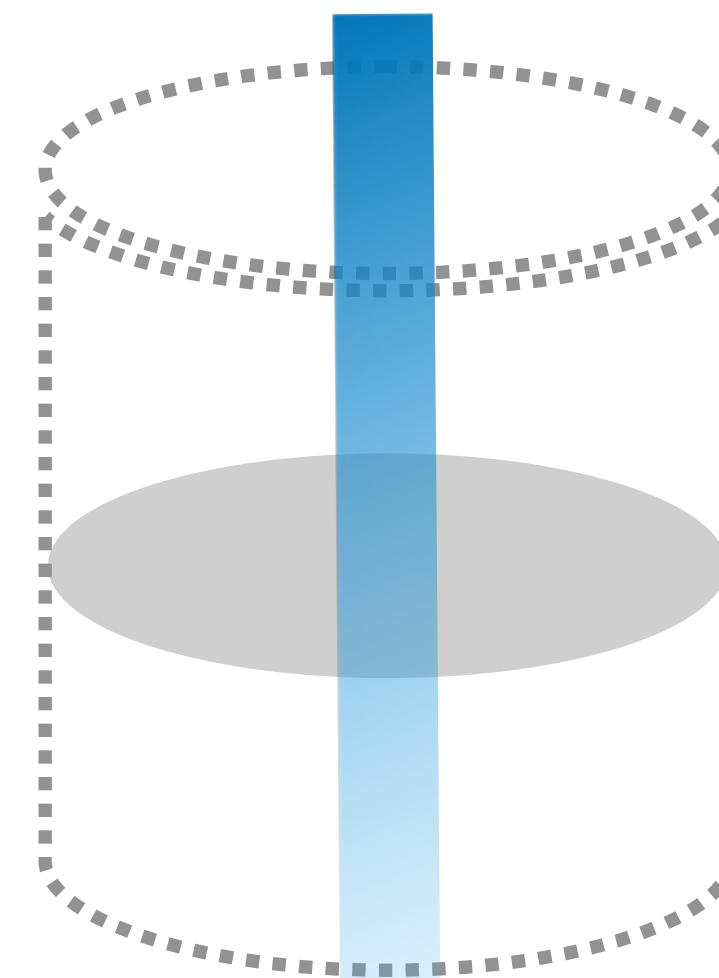
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S^1 Hilbert space

twisted

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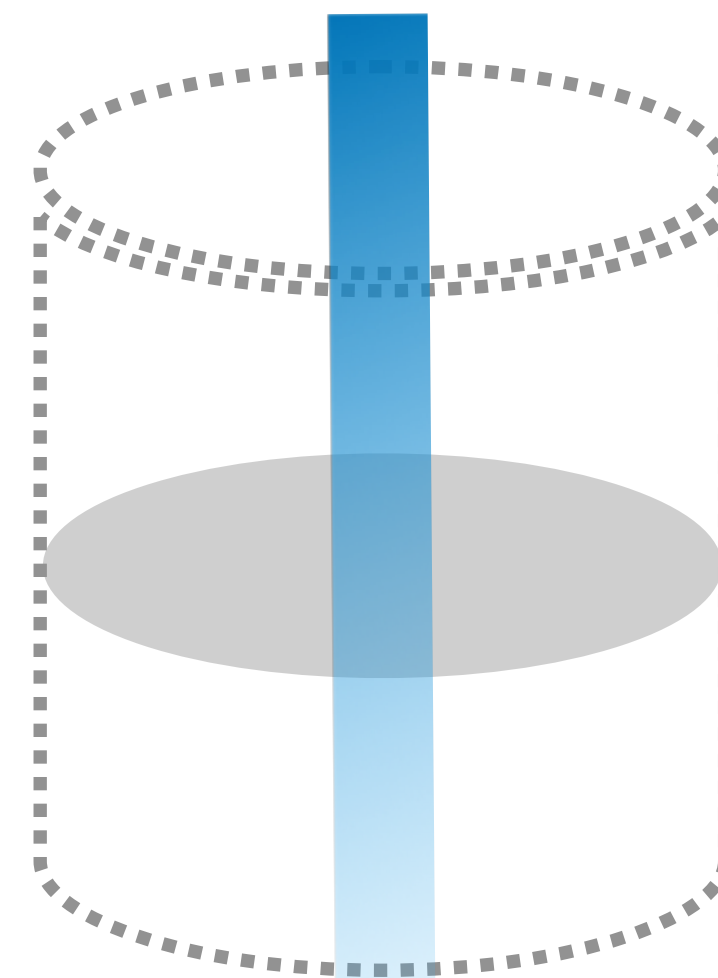
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⇒ can only take direct sums



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S^1 Hilbert space

twisted

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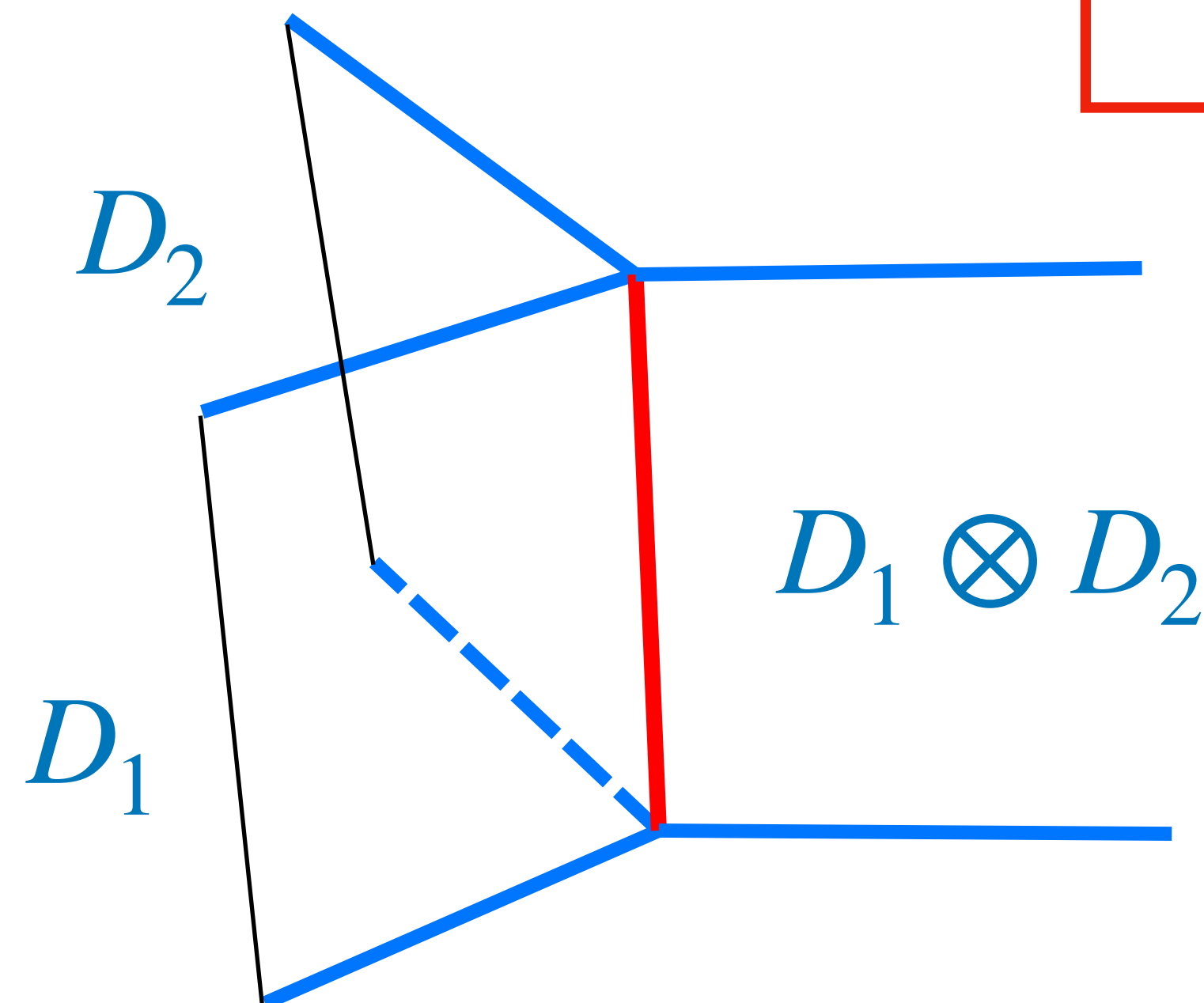
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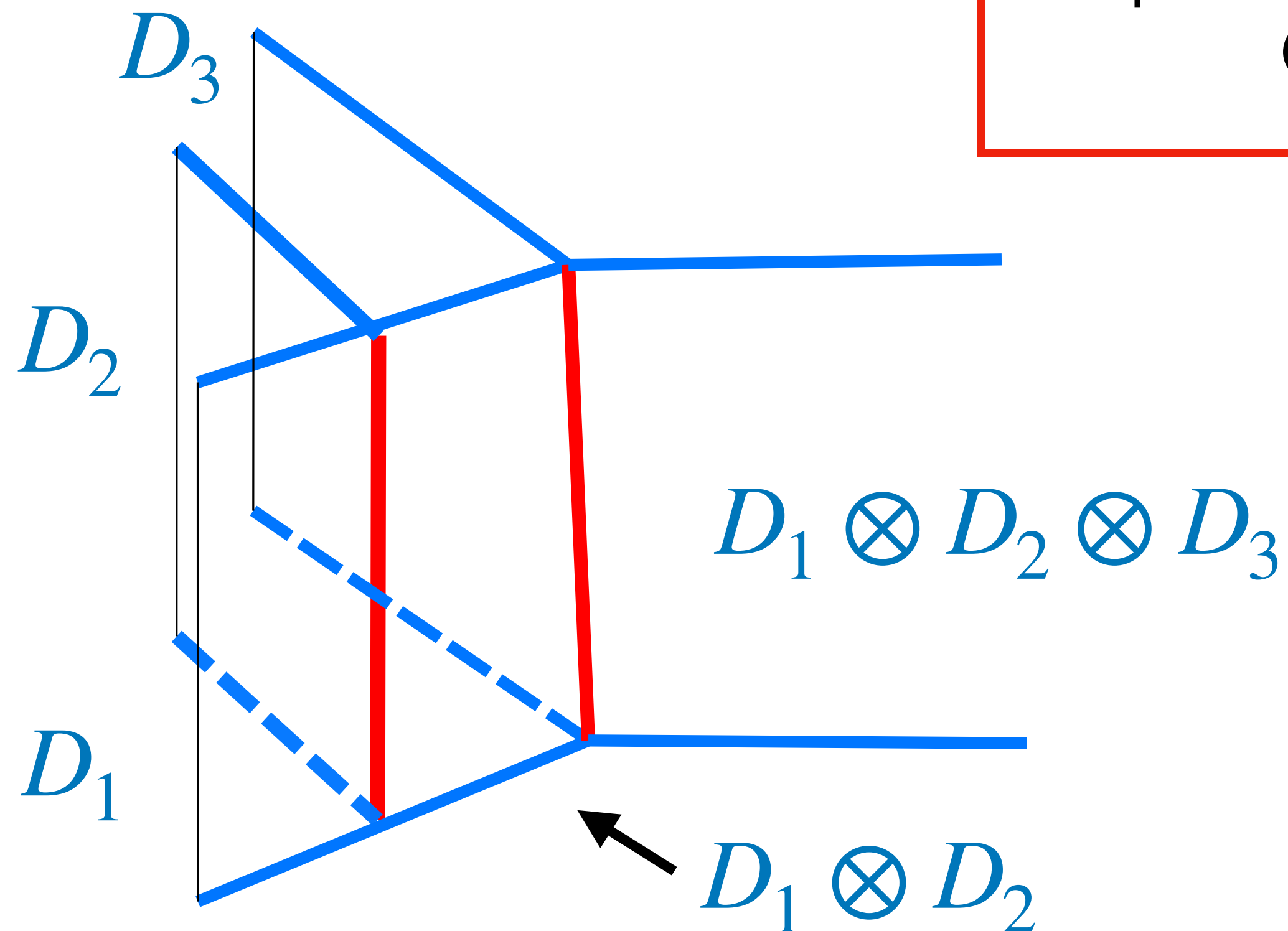


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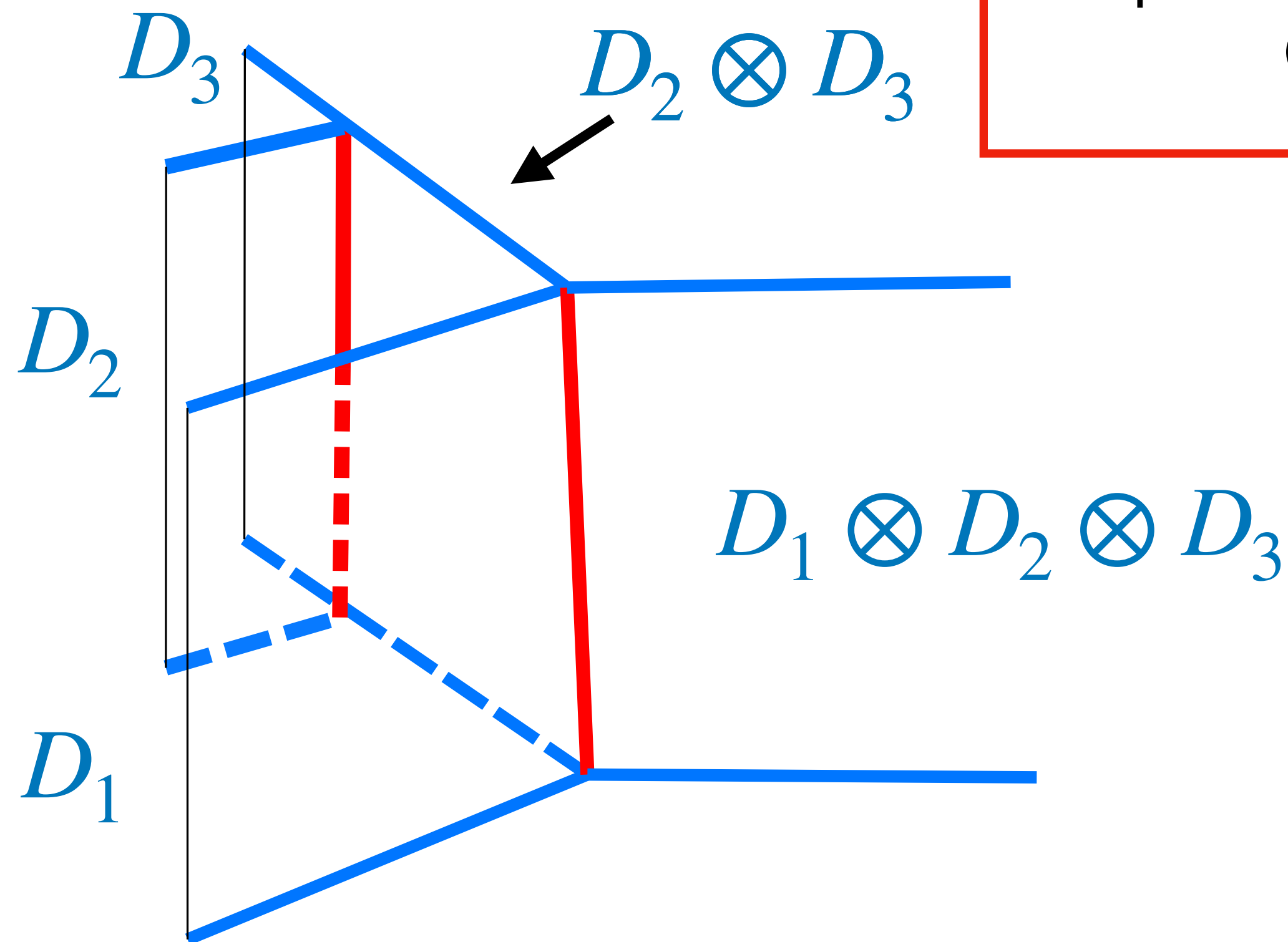
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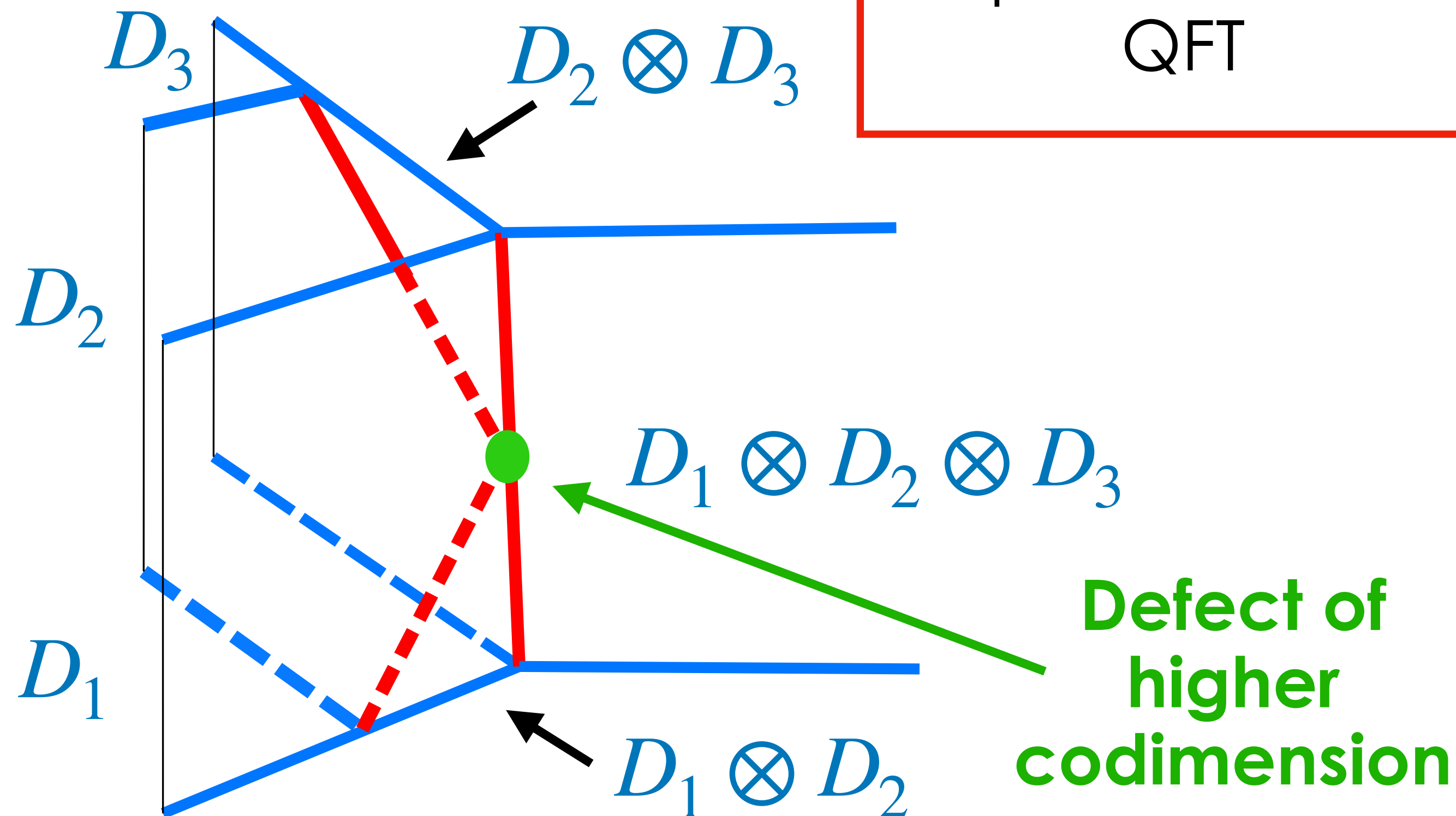
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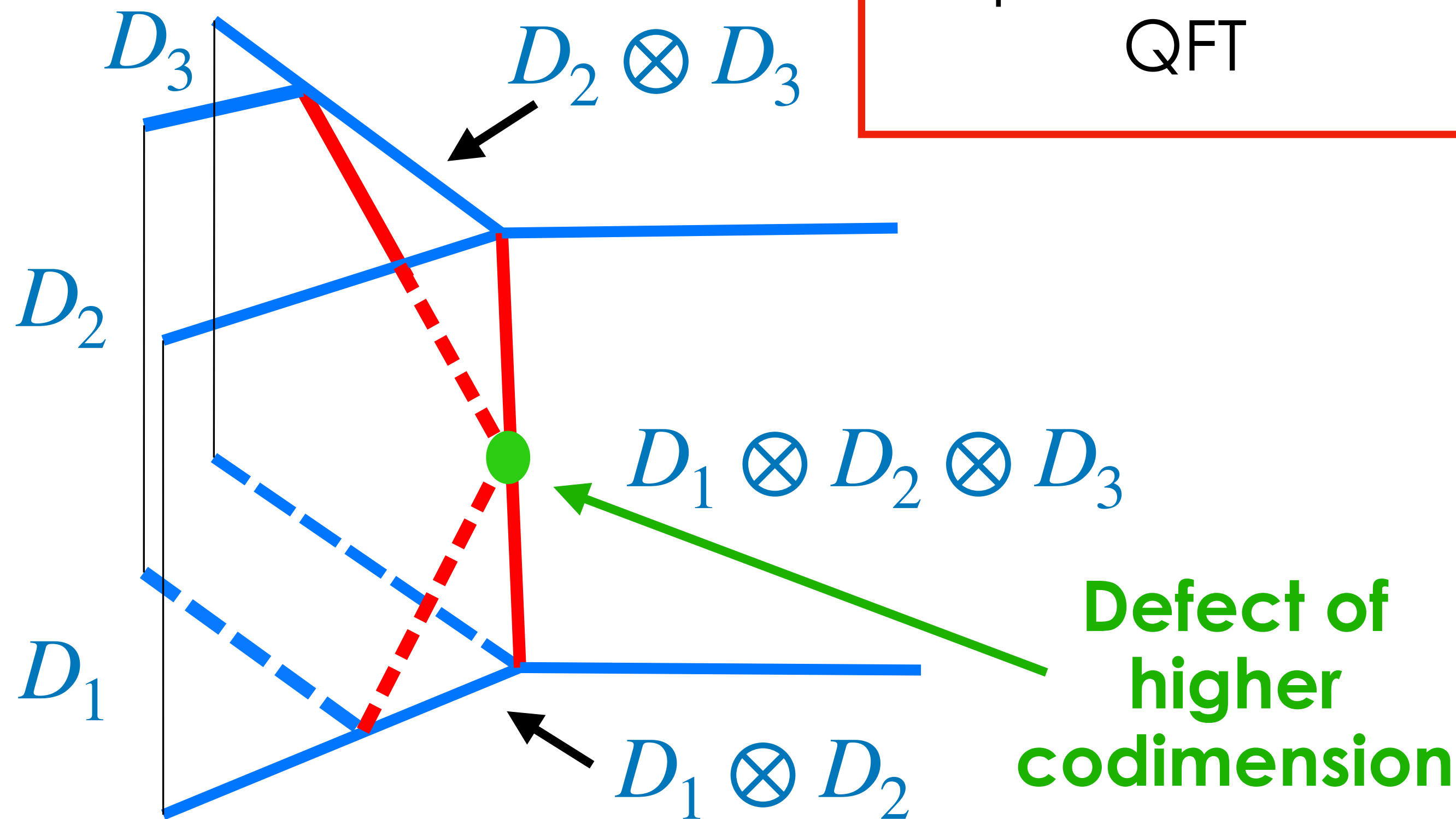
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“Topological” subsector of the spectrum of operators of a QFT



Defect of higher codimension

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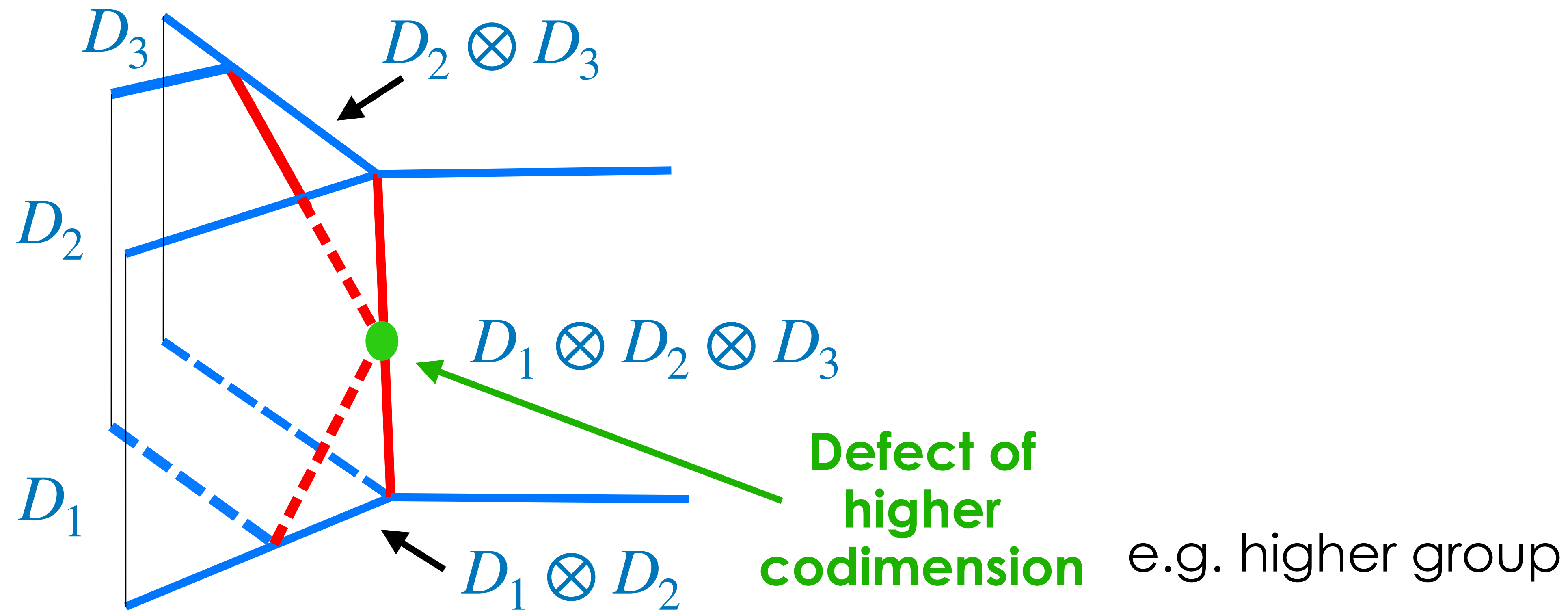
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Extended defects can form **junctions**

e.g. higher group

Cordóva, Dumitrescu, Intriligator 16; Benini, Cordóva, Hsin 18; Bhardwaj, Shafer-Nameki 23; Bullimore, Barscht, Grigoletto 23

Example: 2-group symmetry

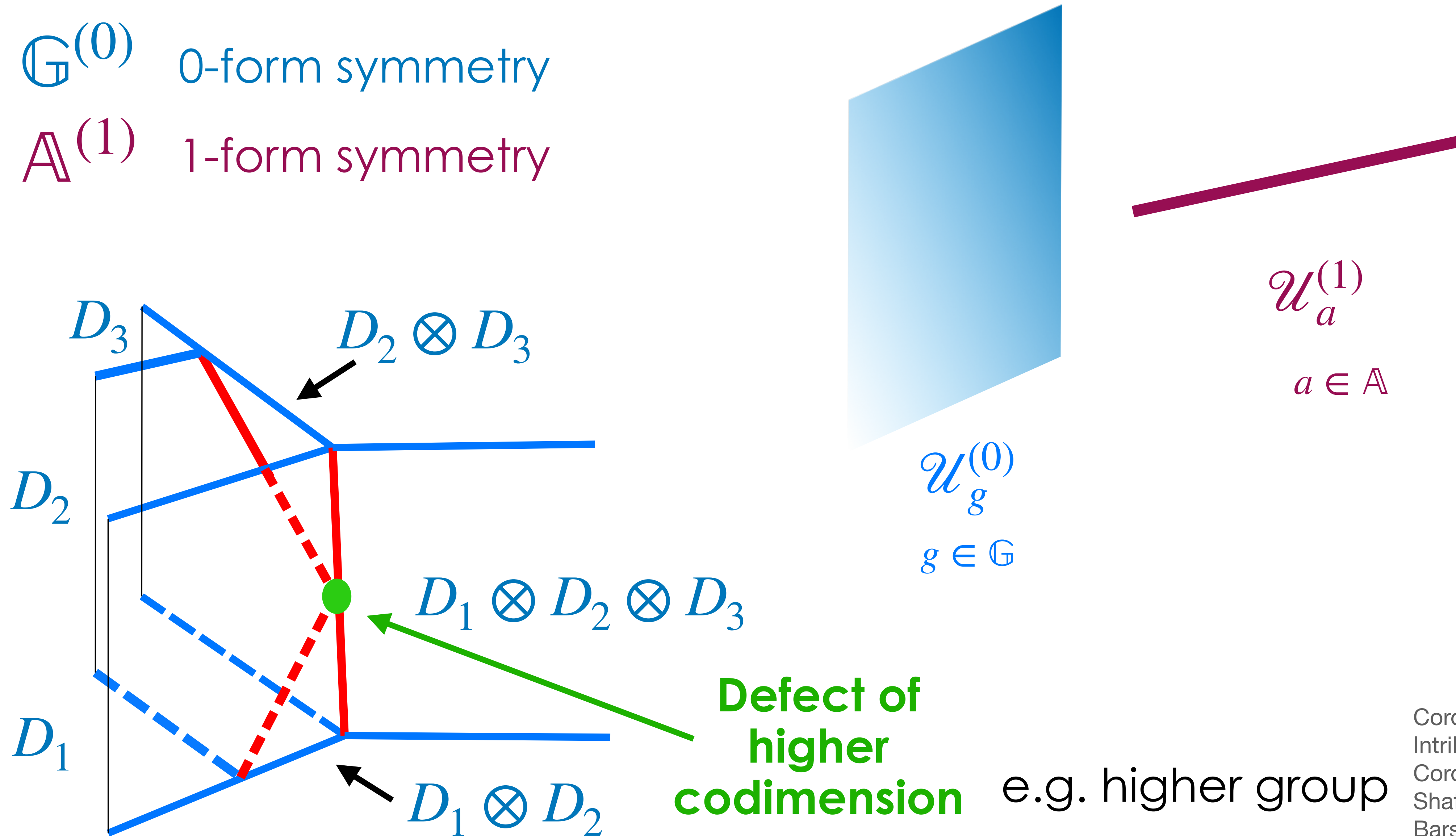


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Example: 2-group symmetry

$\mathbb{G}^{(0)}$ 0-form symmetry

$\mathbb{A}^{(1)}$ 1-form symmetry

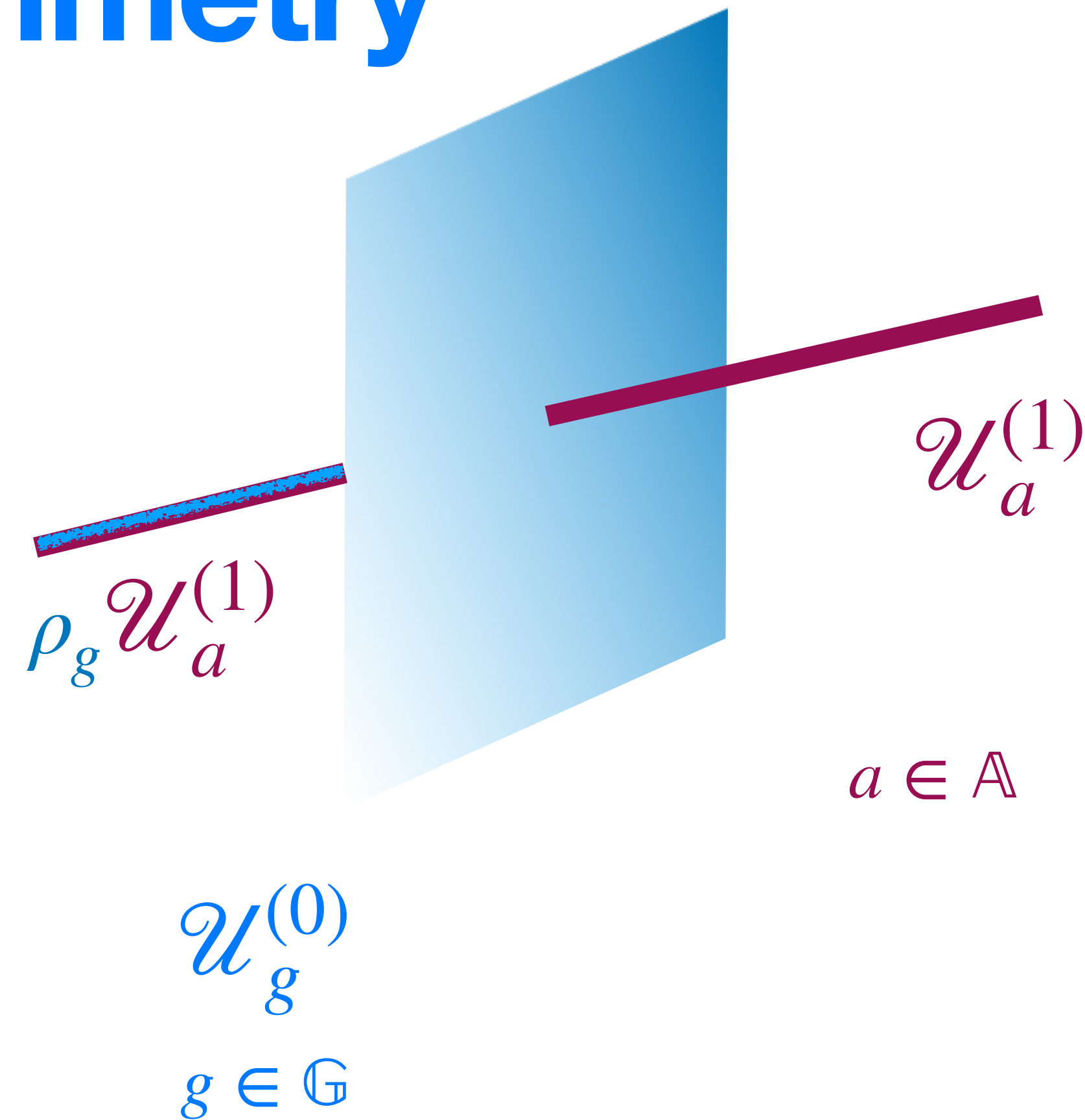
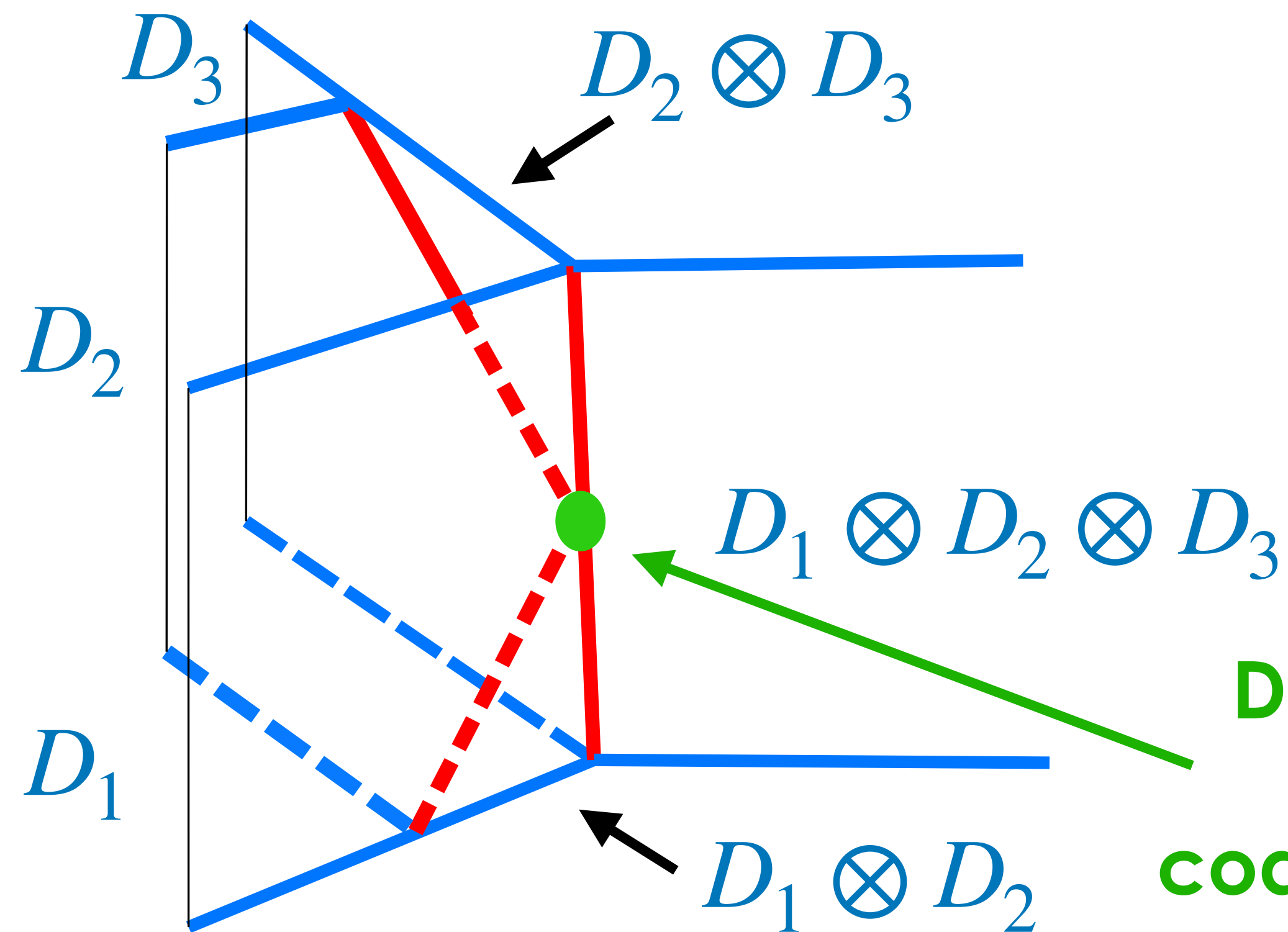


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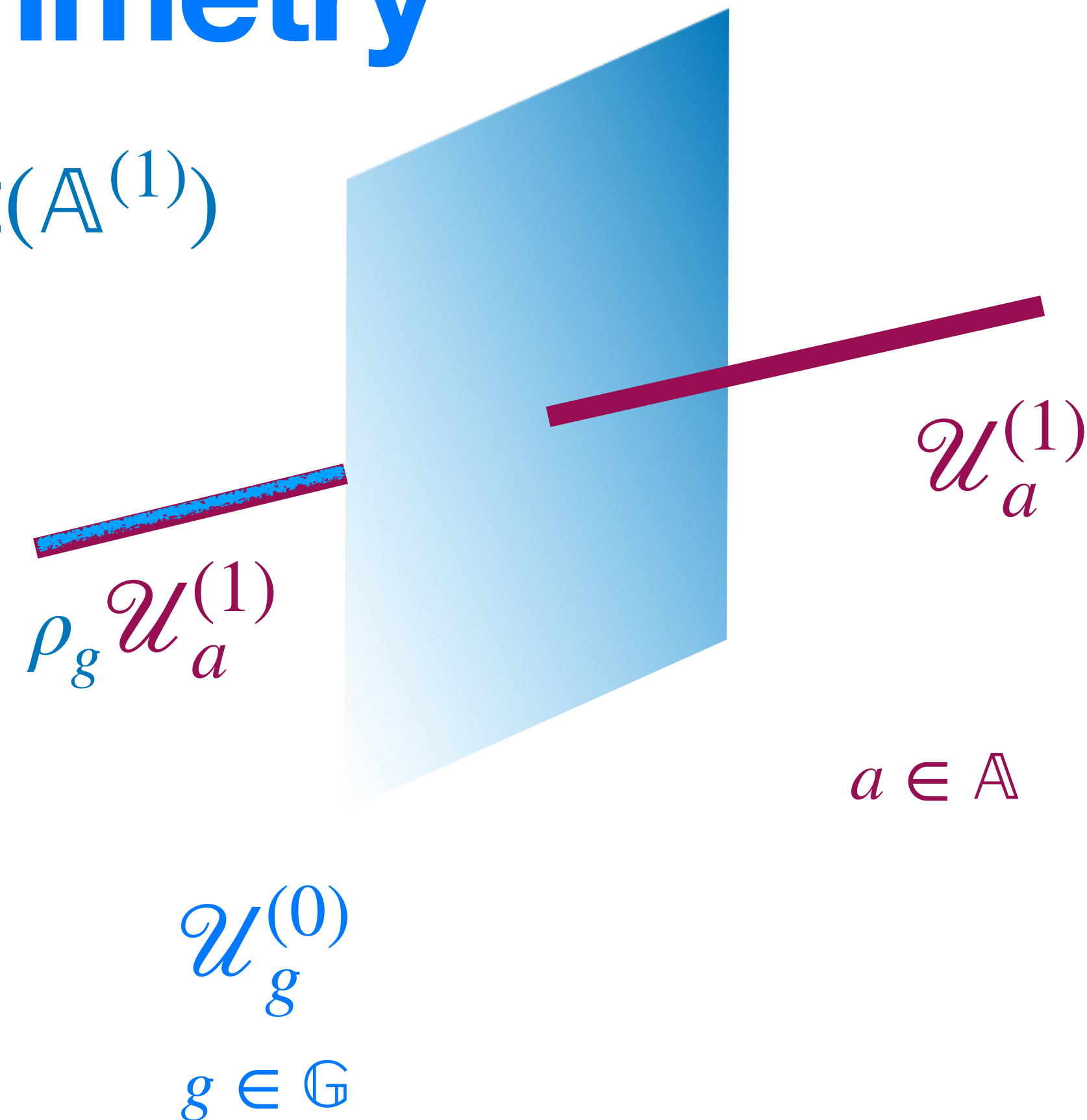
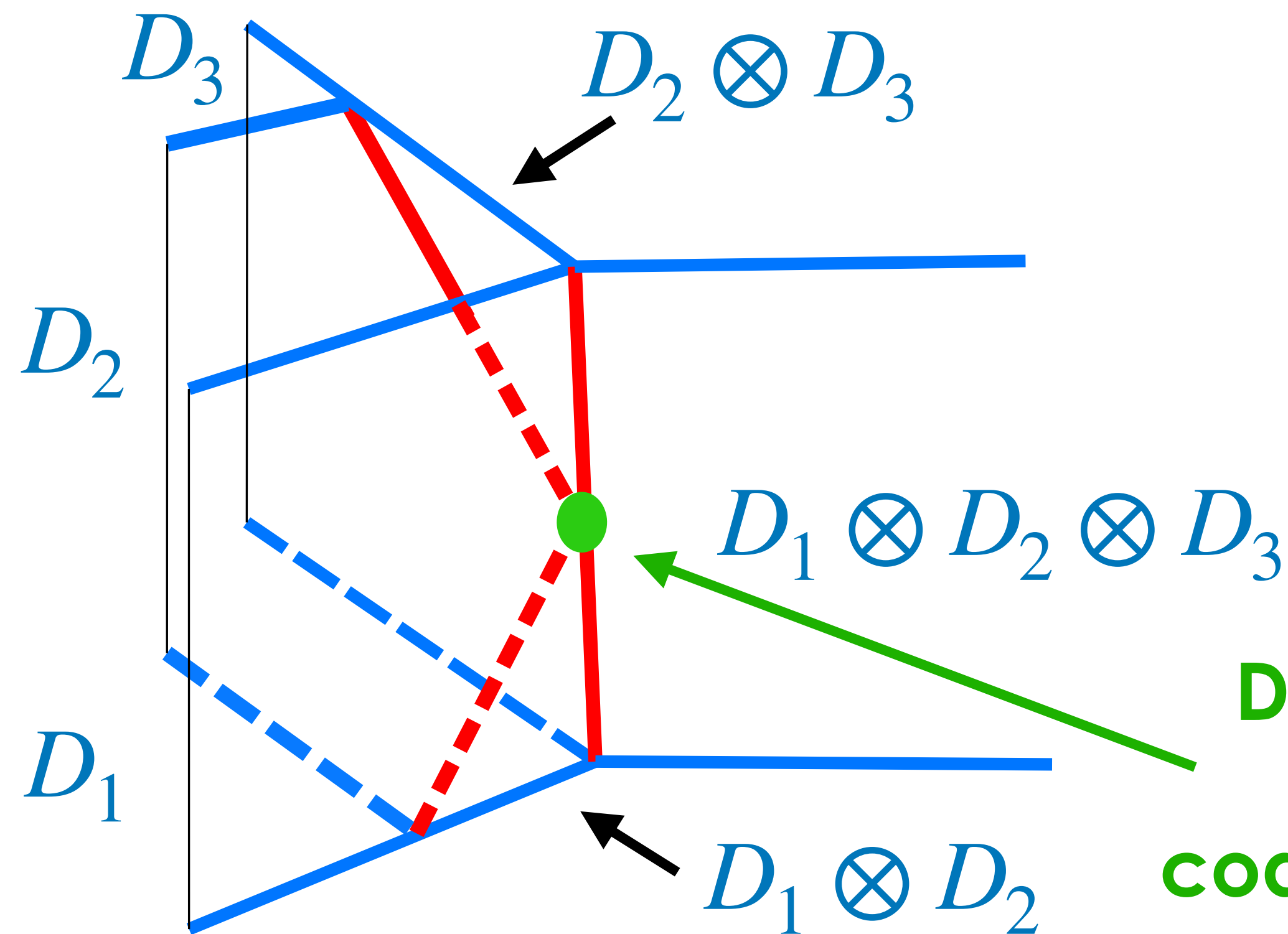


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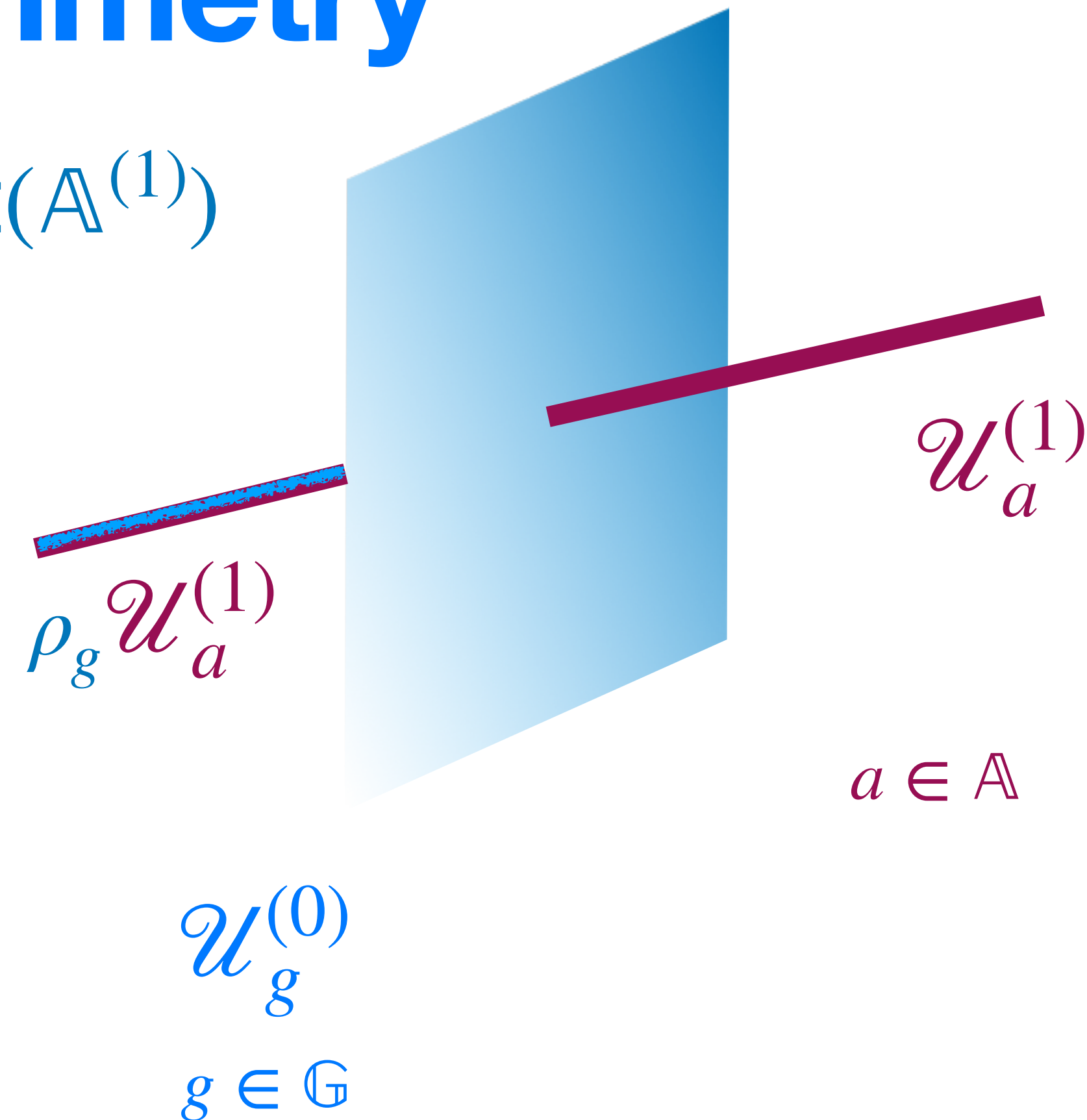
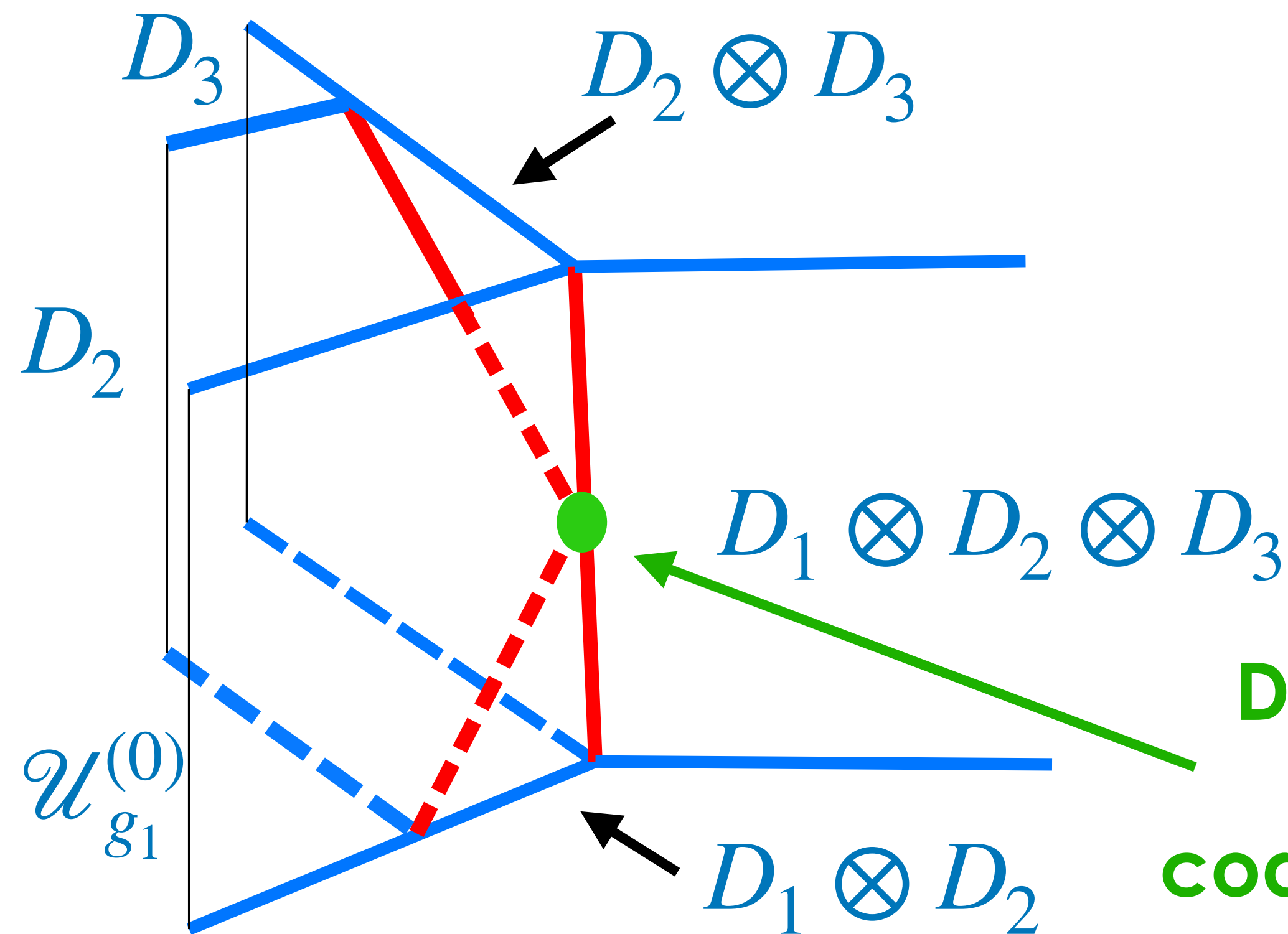


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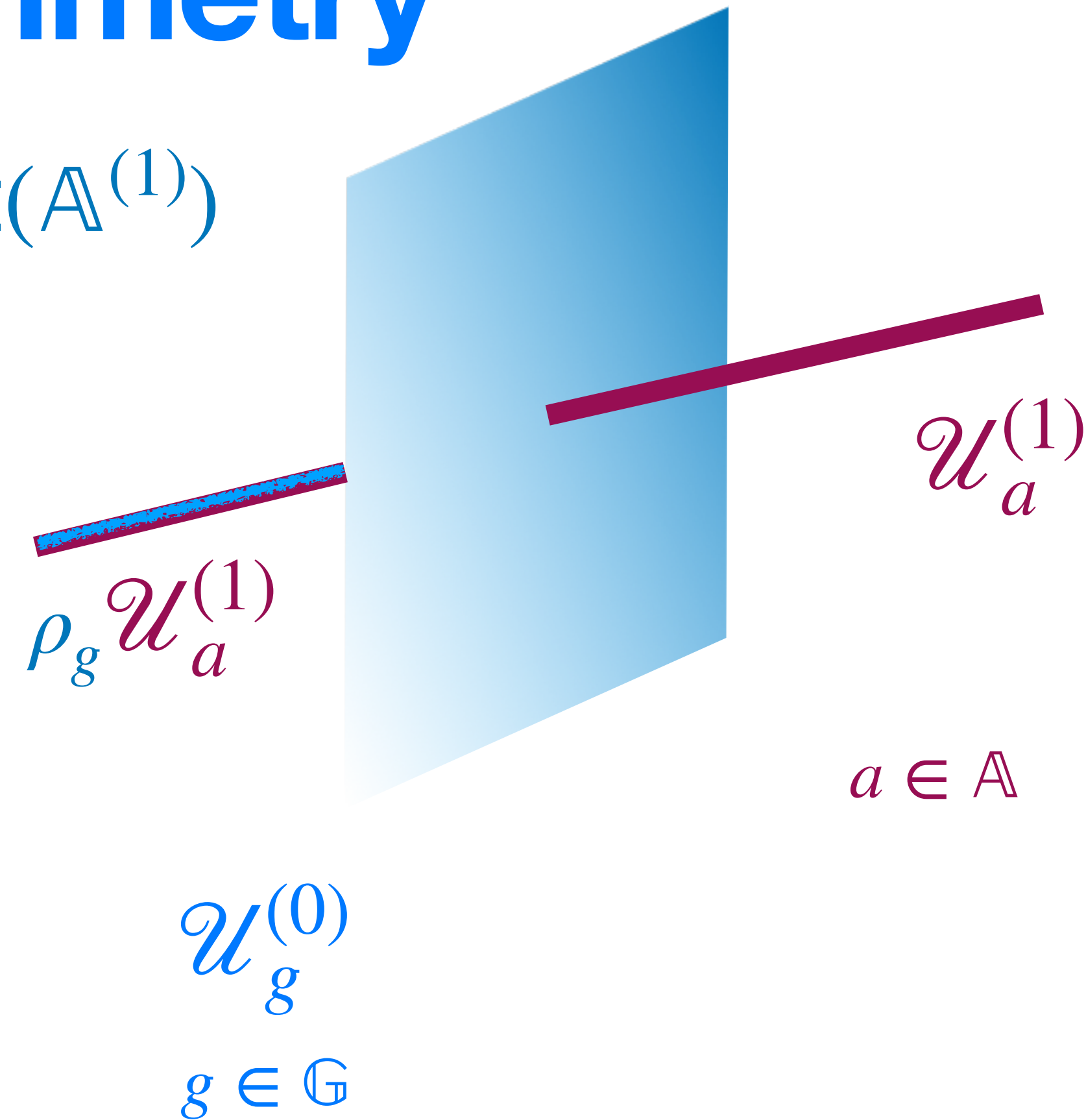
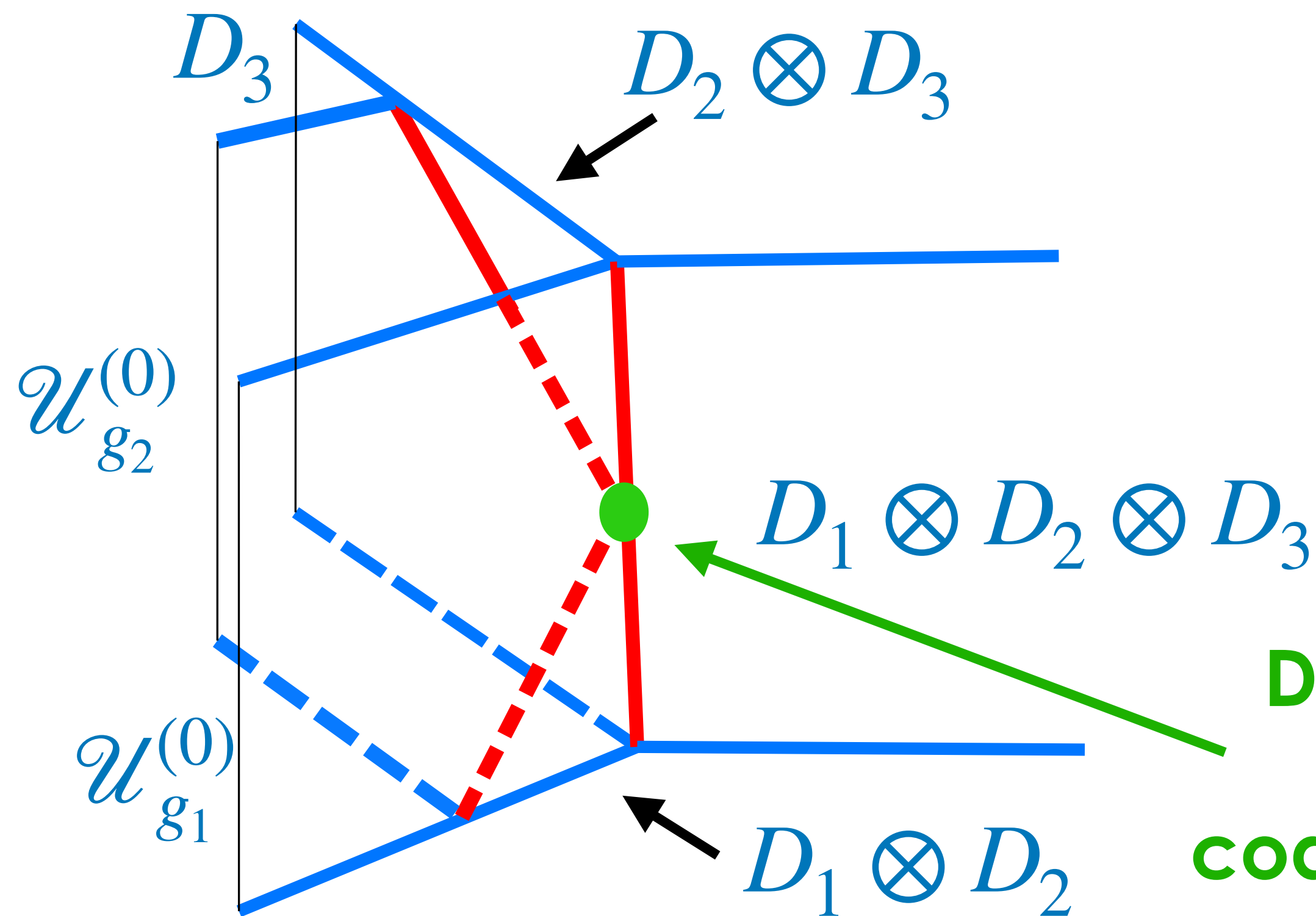


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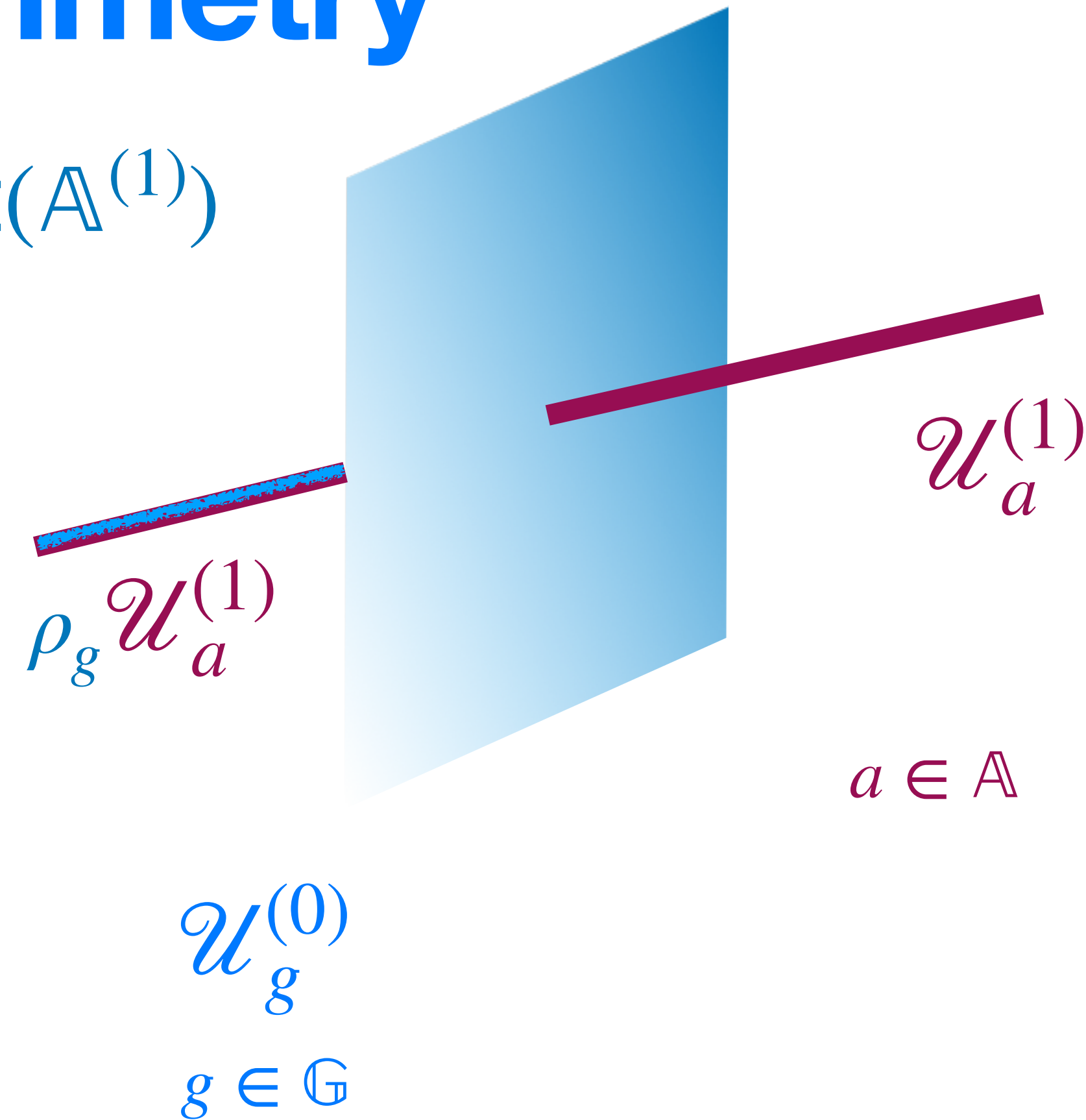
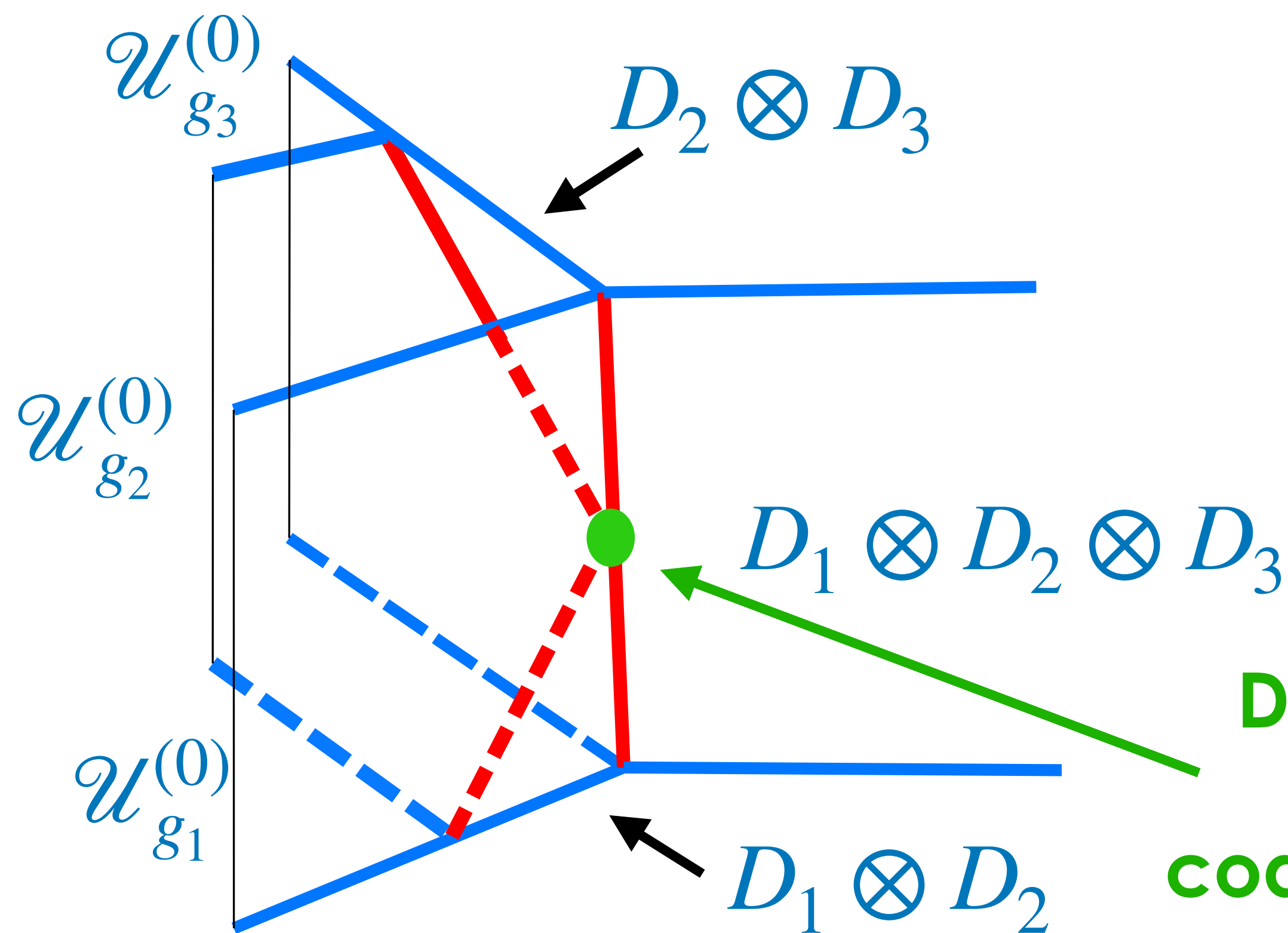
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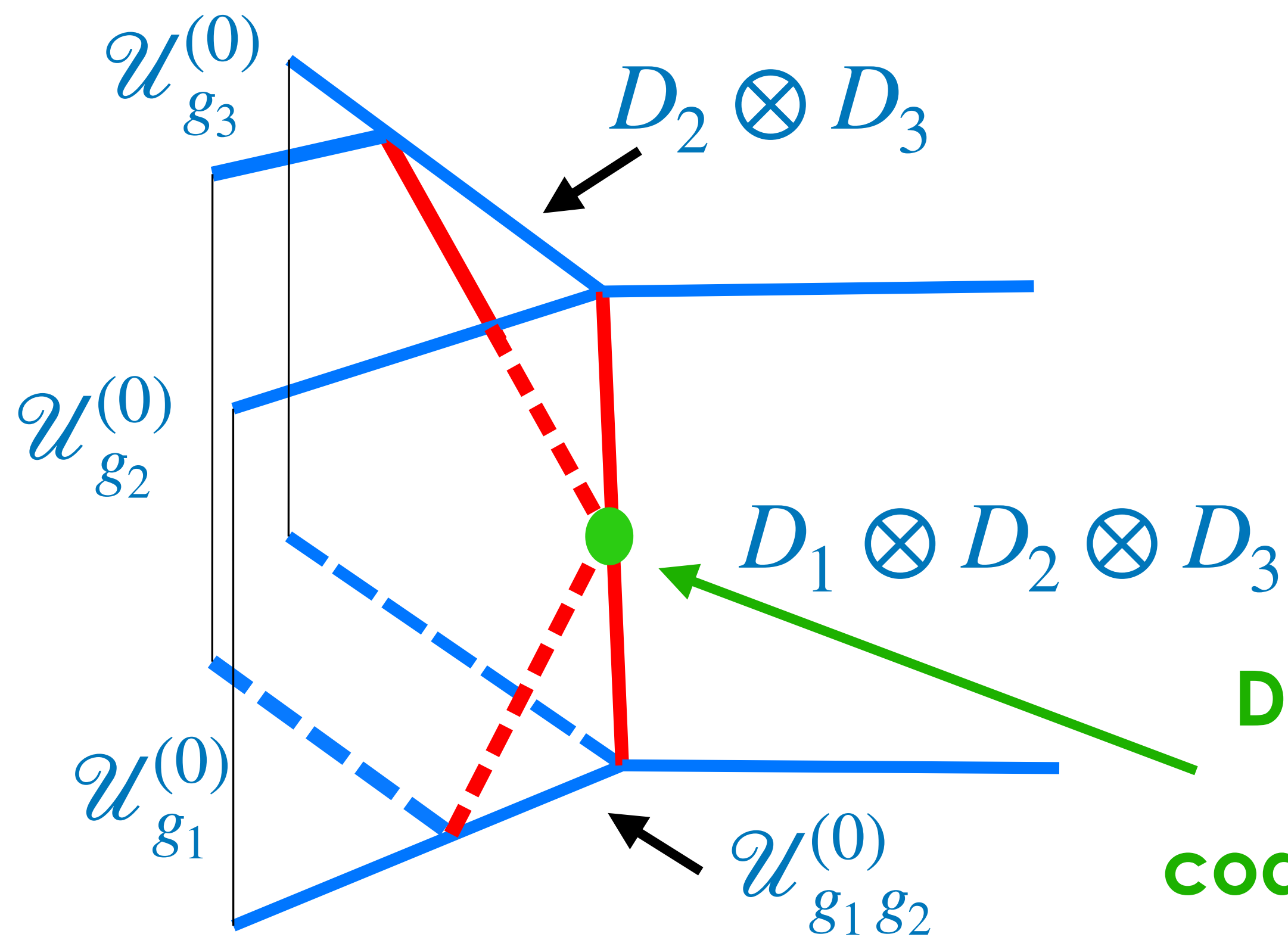
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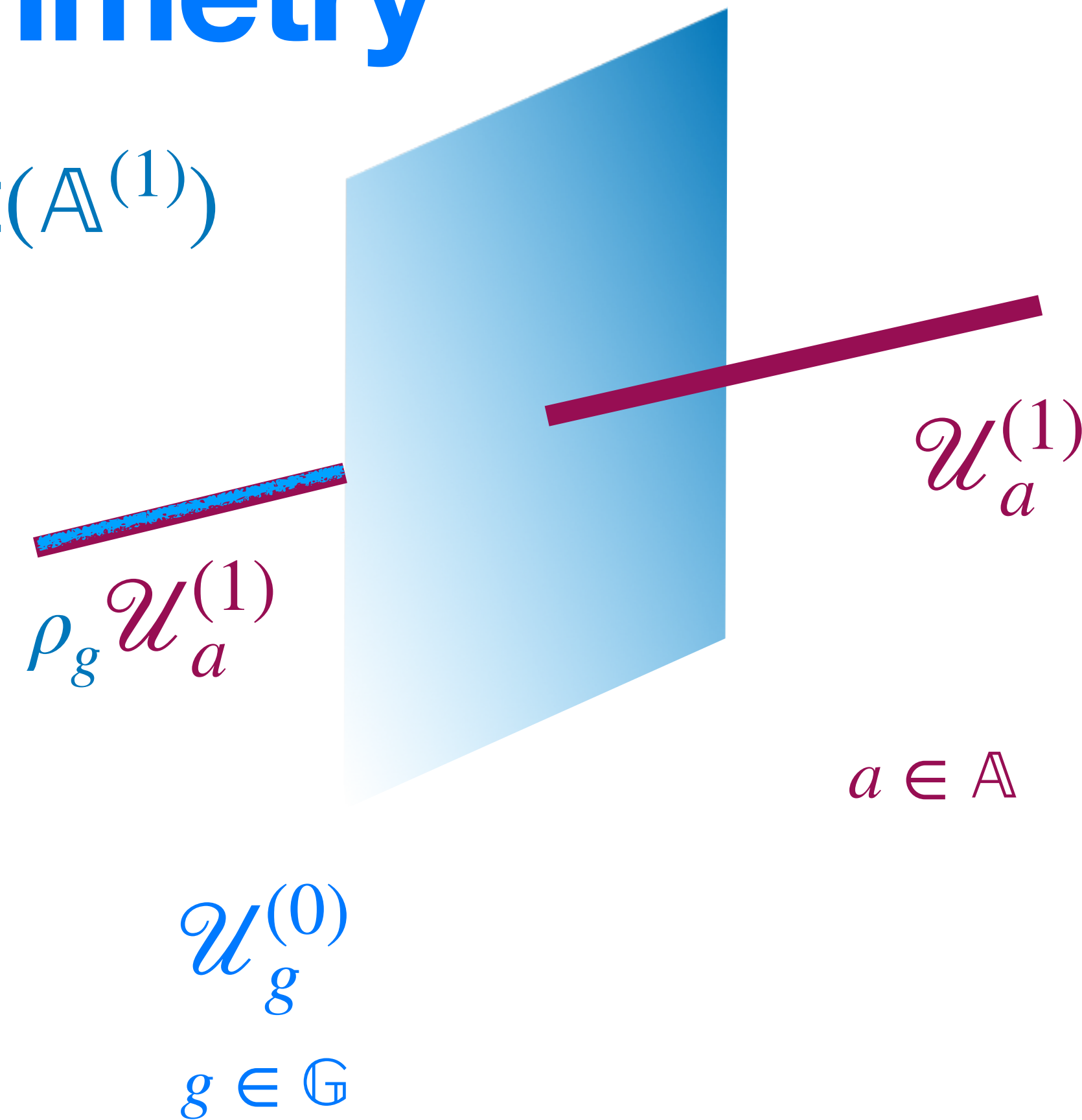
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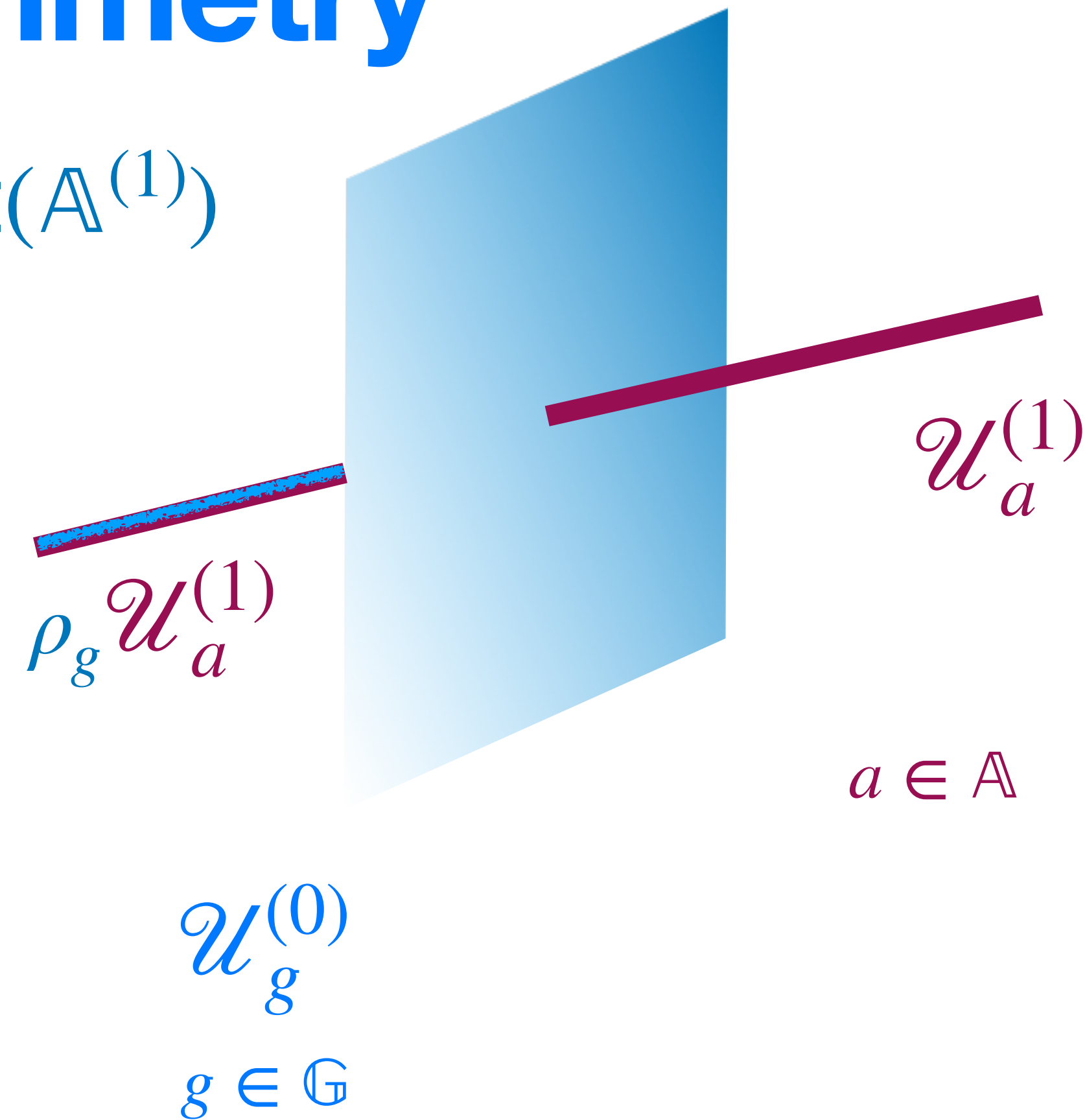
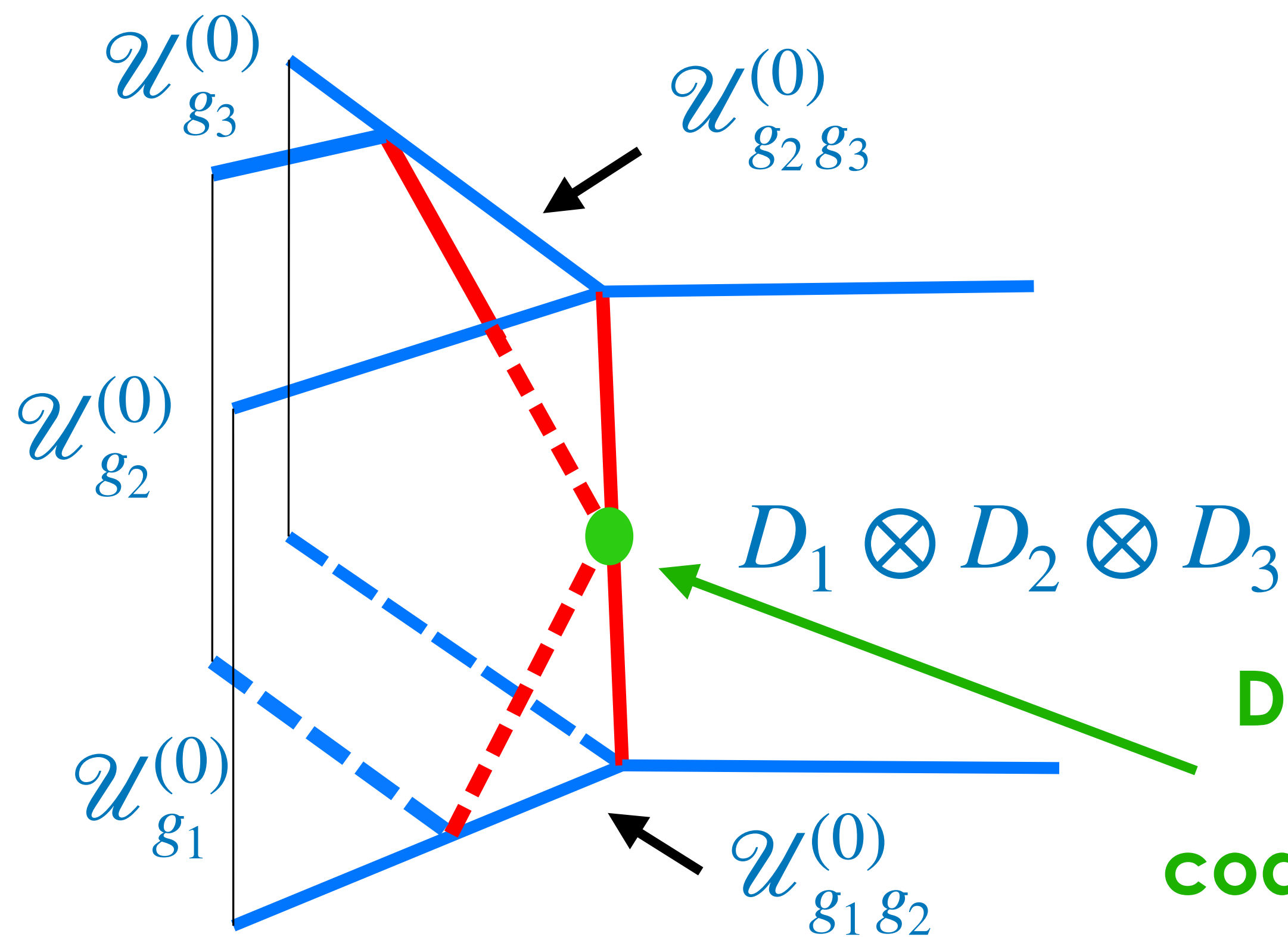


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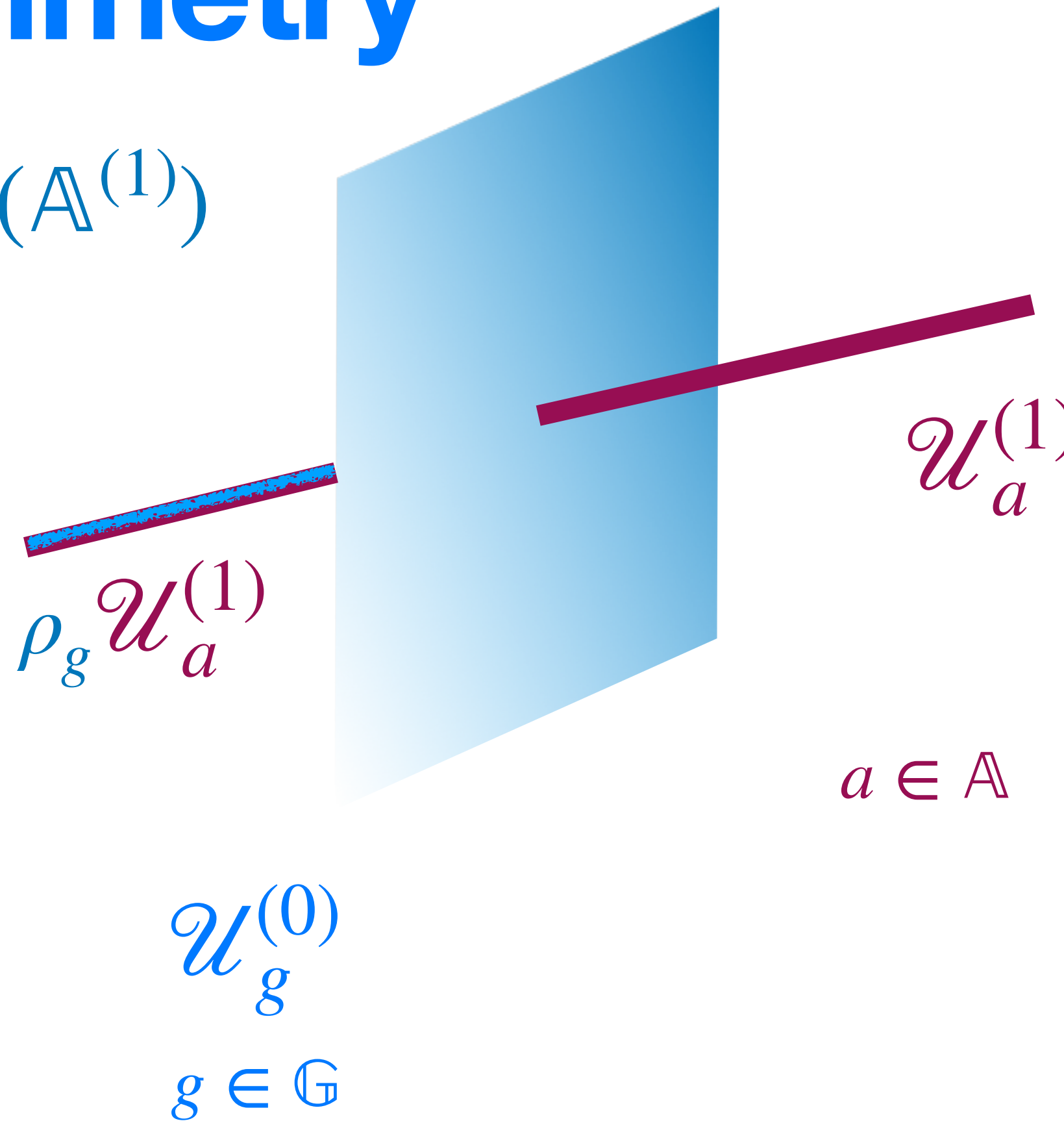
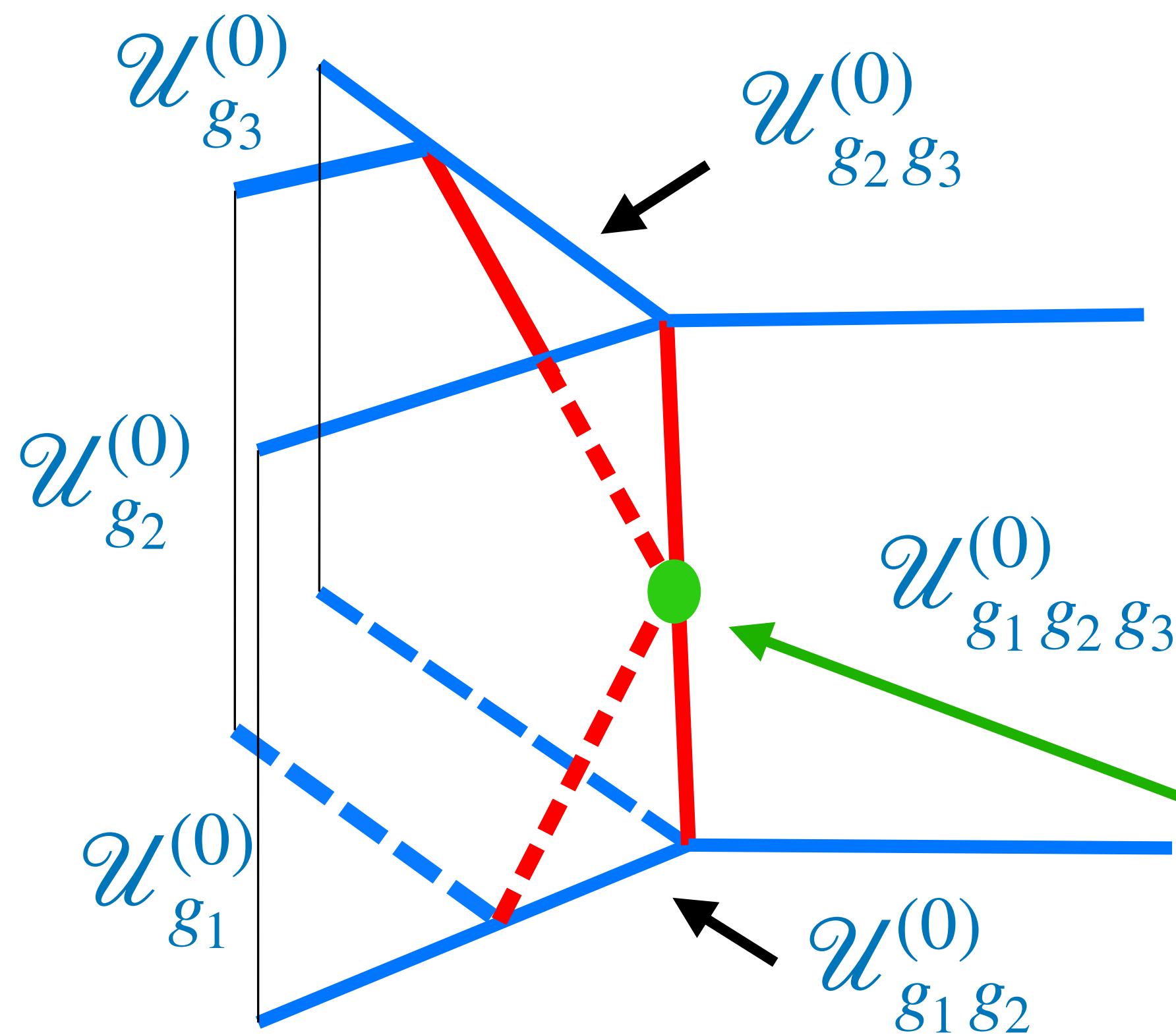
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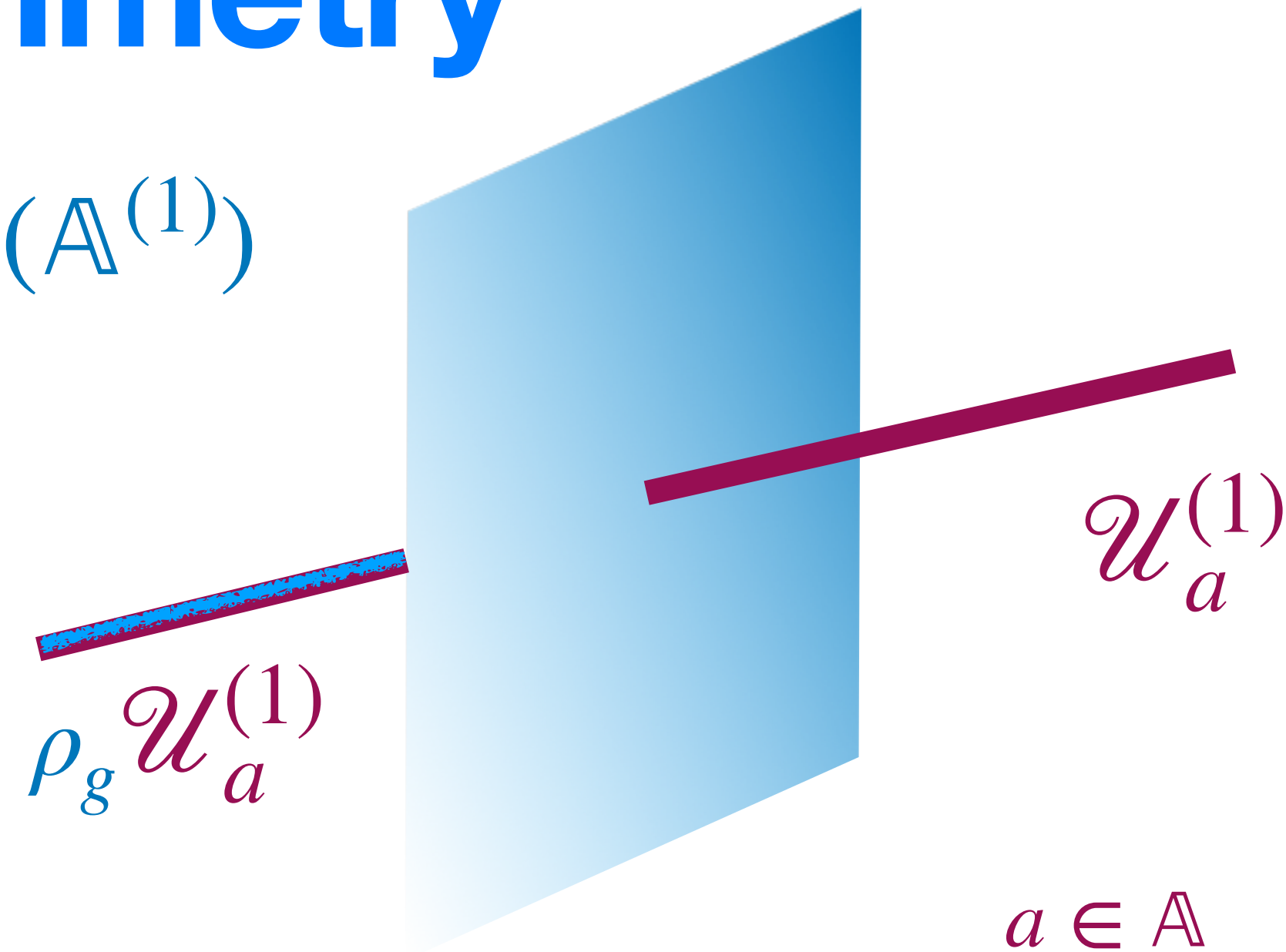
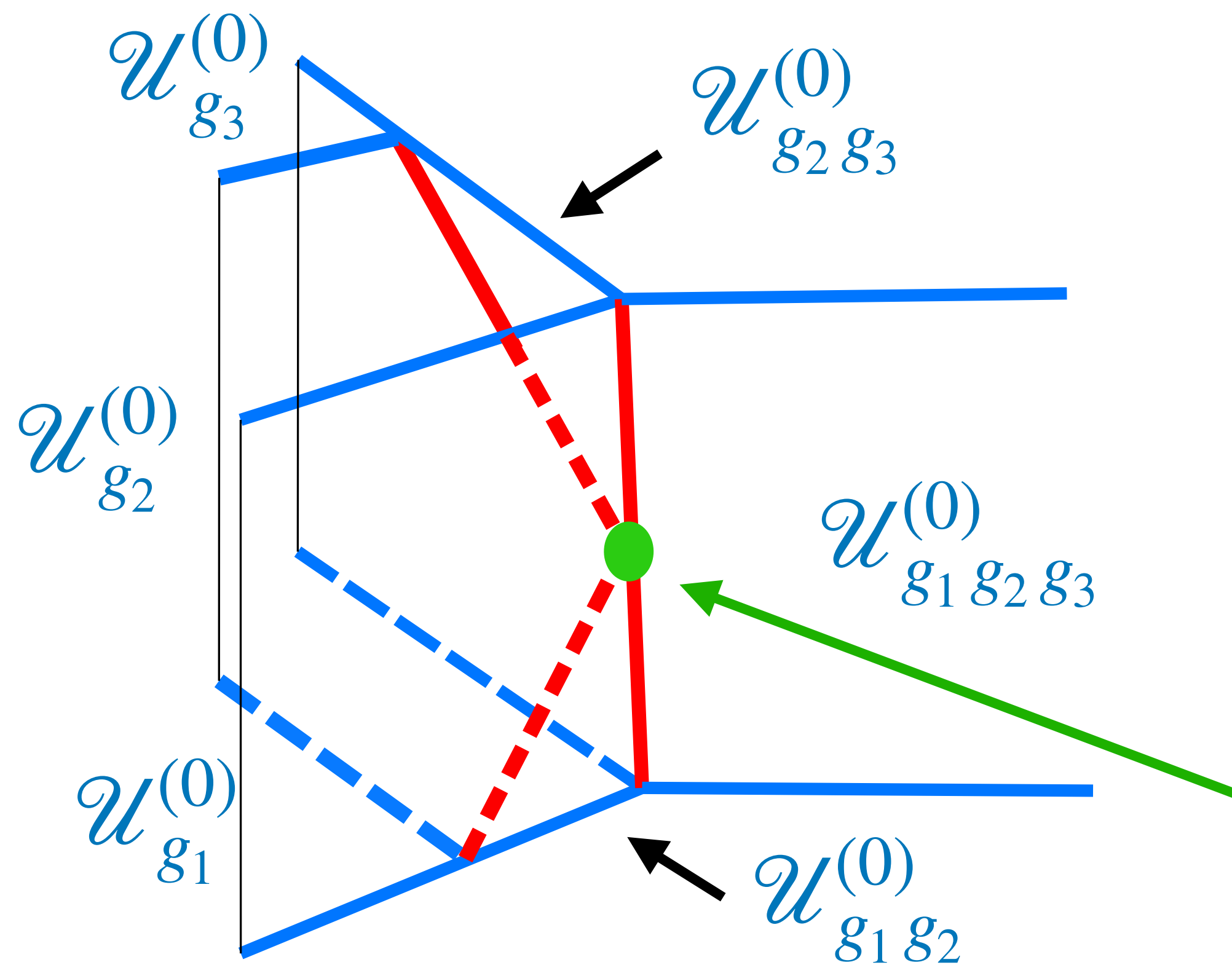
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$\beta^{(2)}$

e.g. higher group

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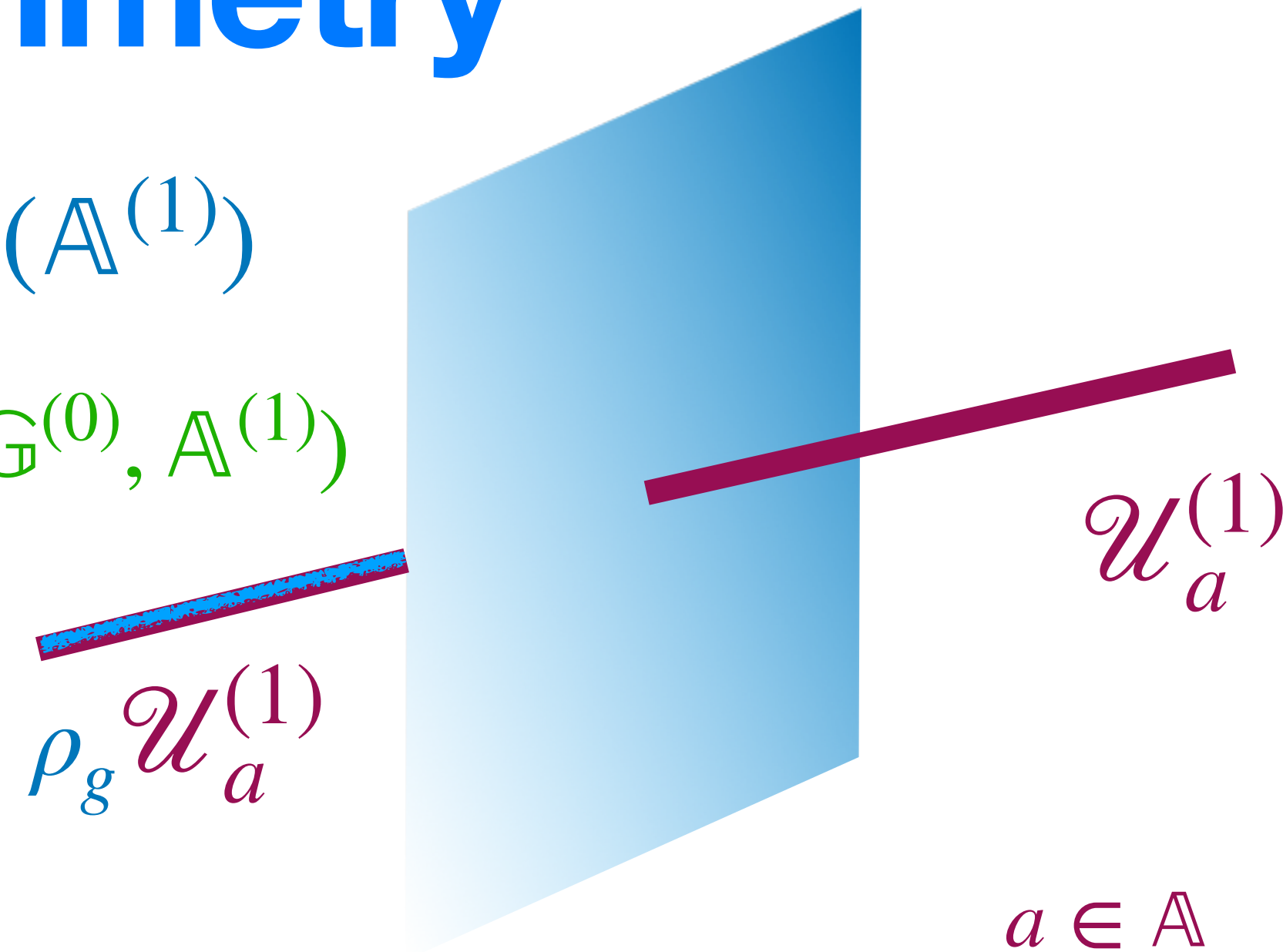
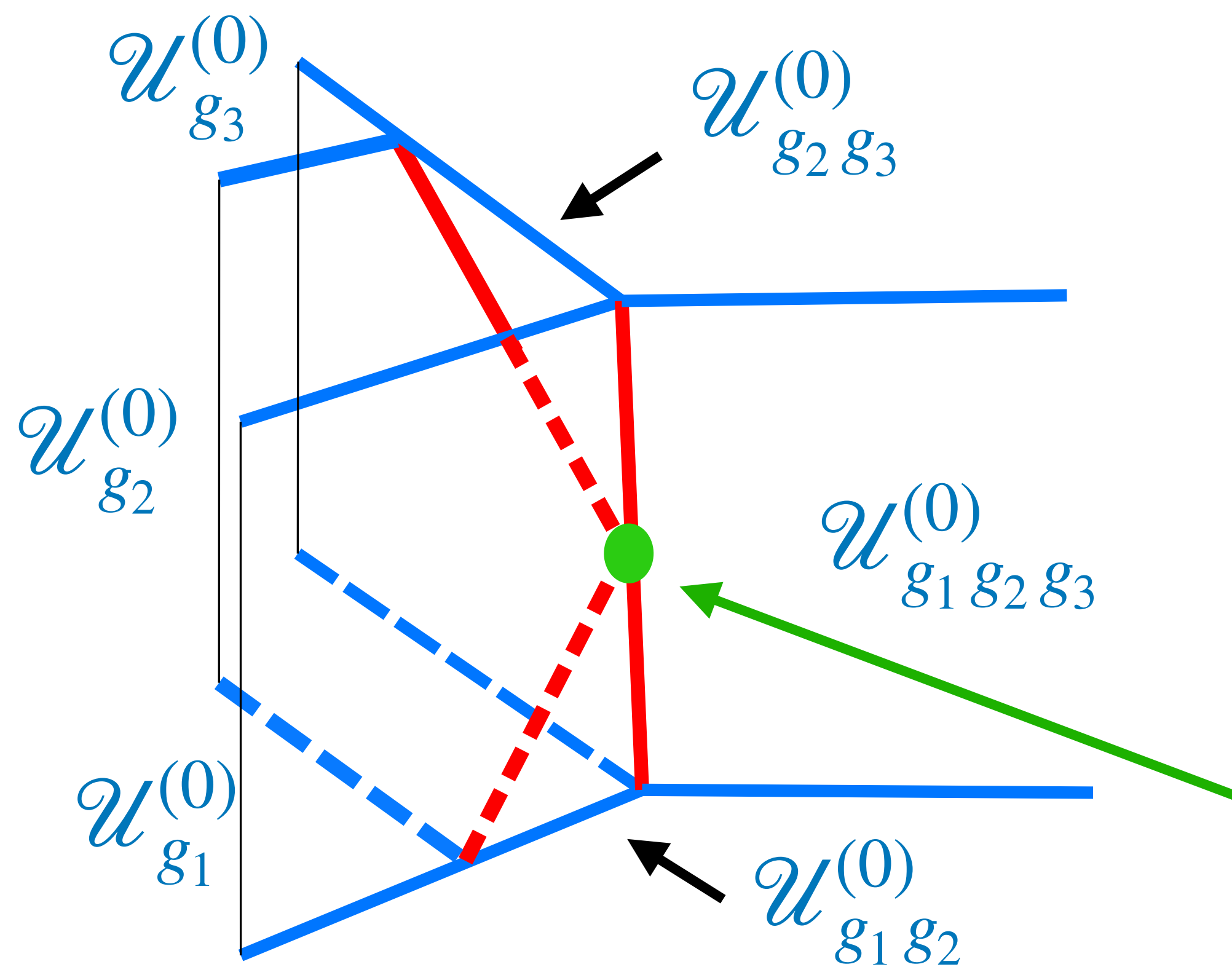
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$$\mathcal{U}_g^{(0)}$$

$$g \in \mathbb{G}$$

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e.g. higher group

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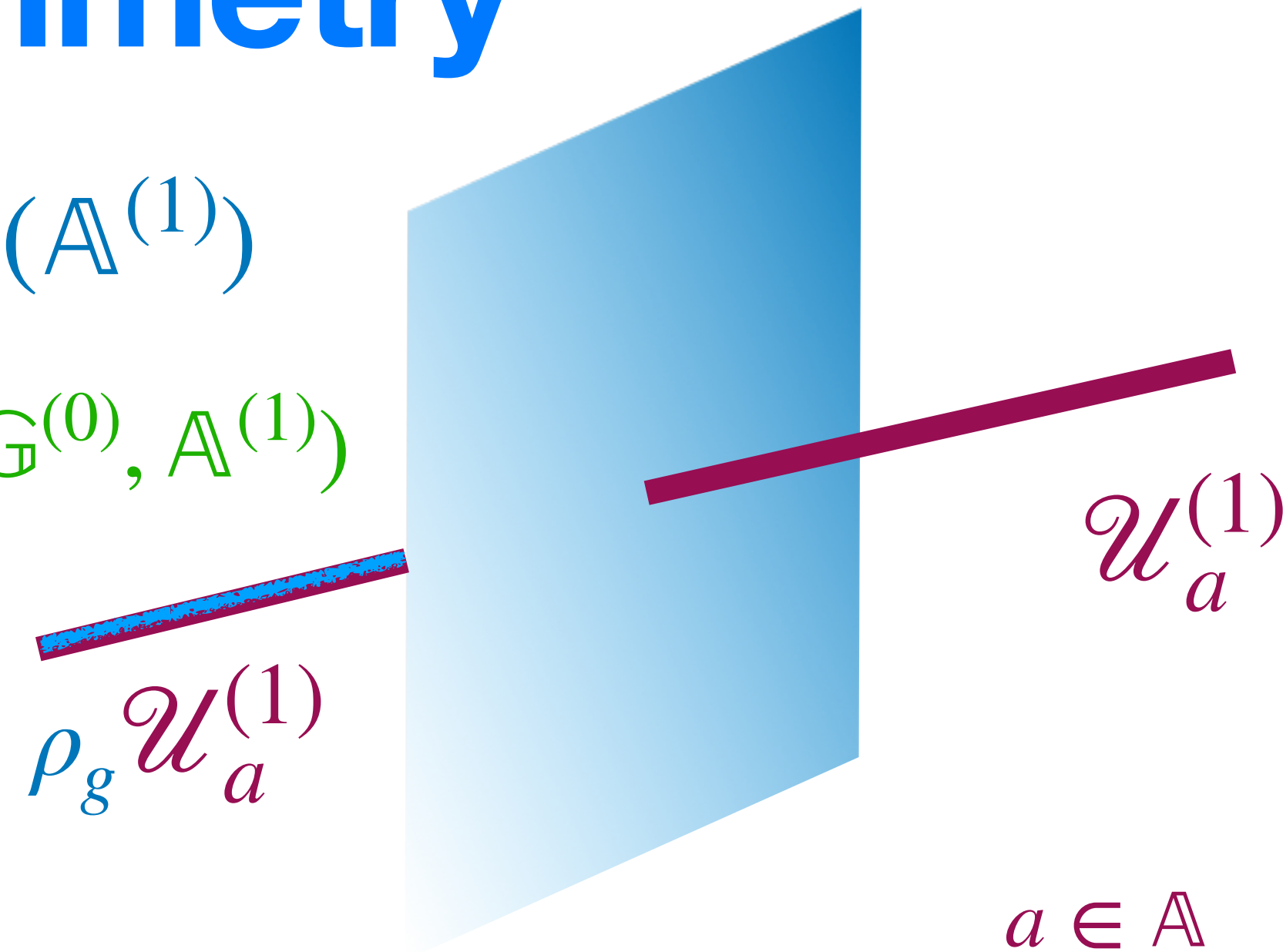
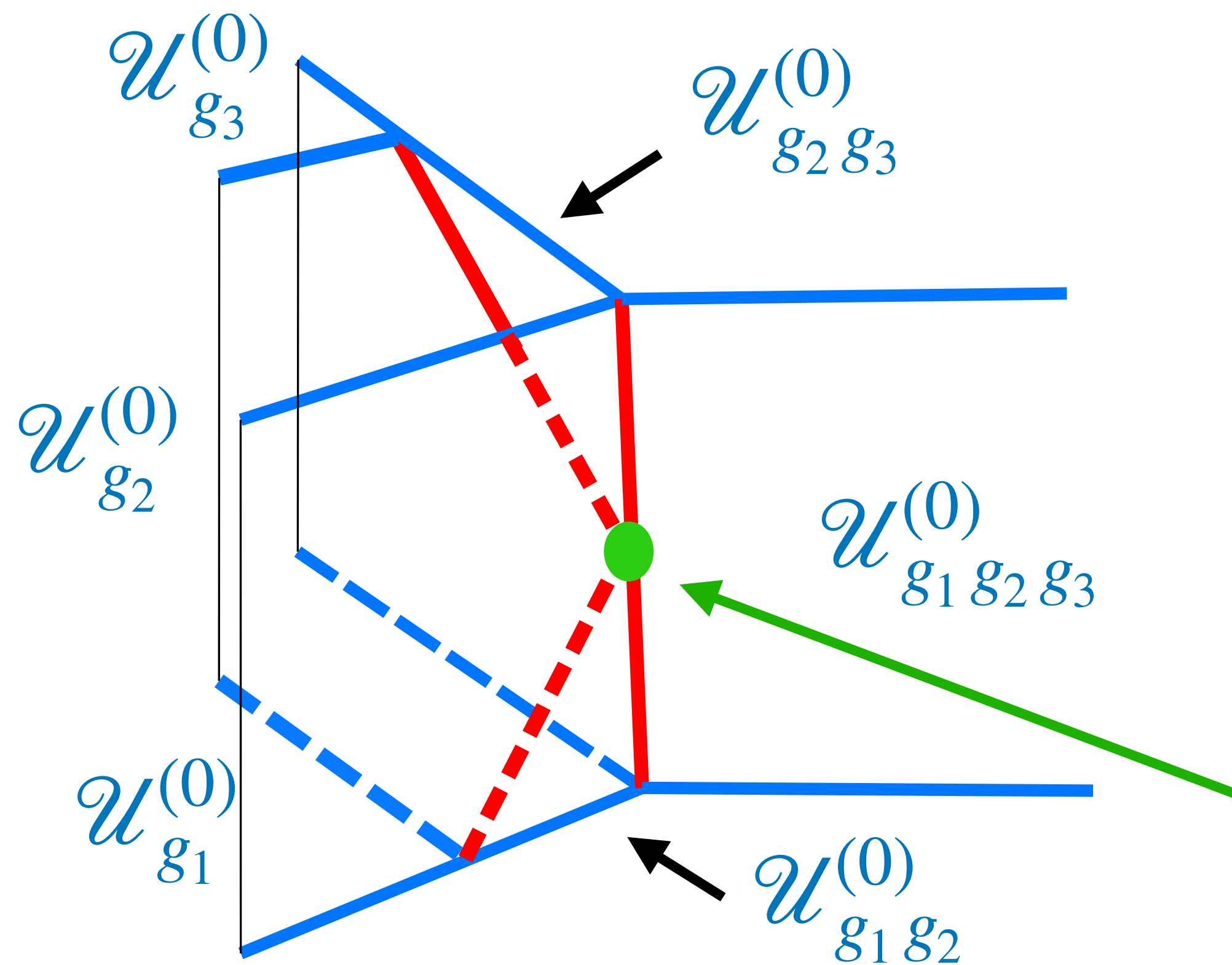
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Useful: all these data
match across dualities

Del Zotto, Ohmori 20
Lee, Ohmori, Tachikawa 21

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How to compute it in practice?

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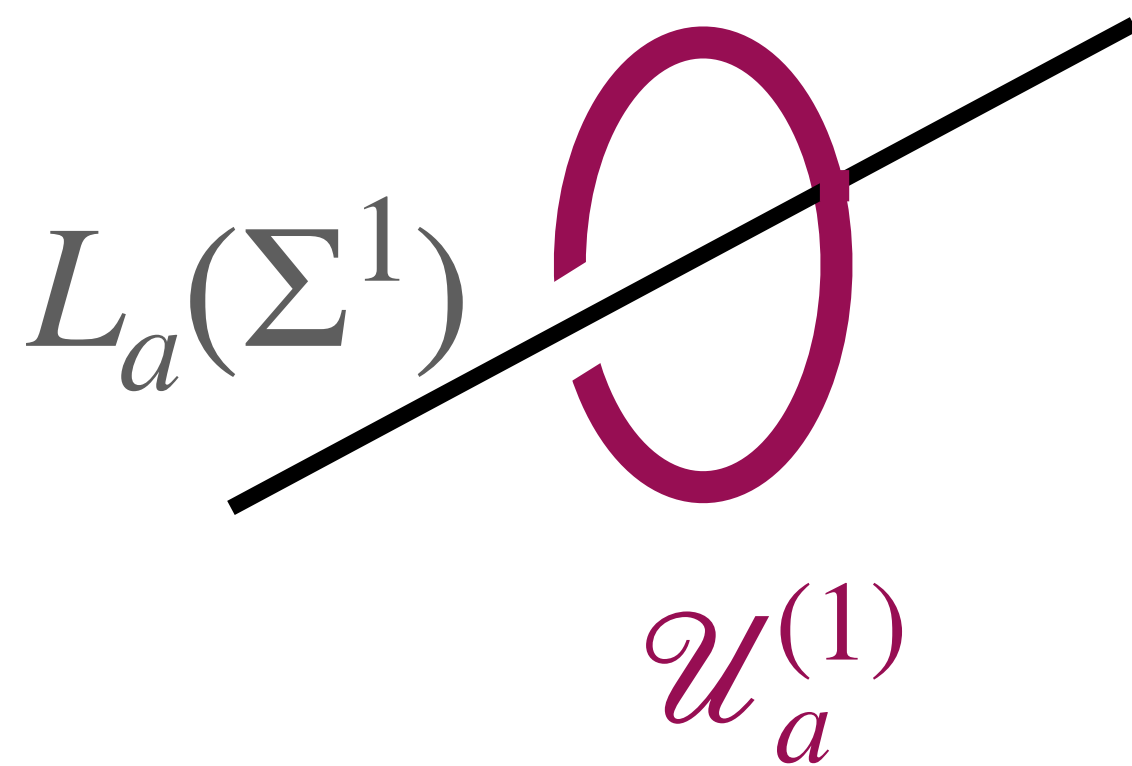
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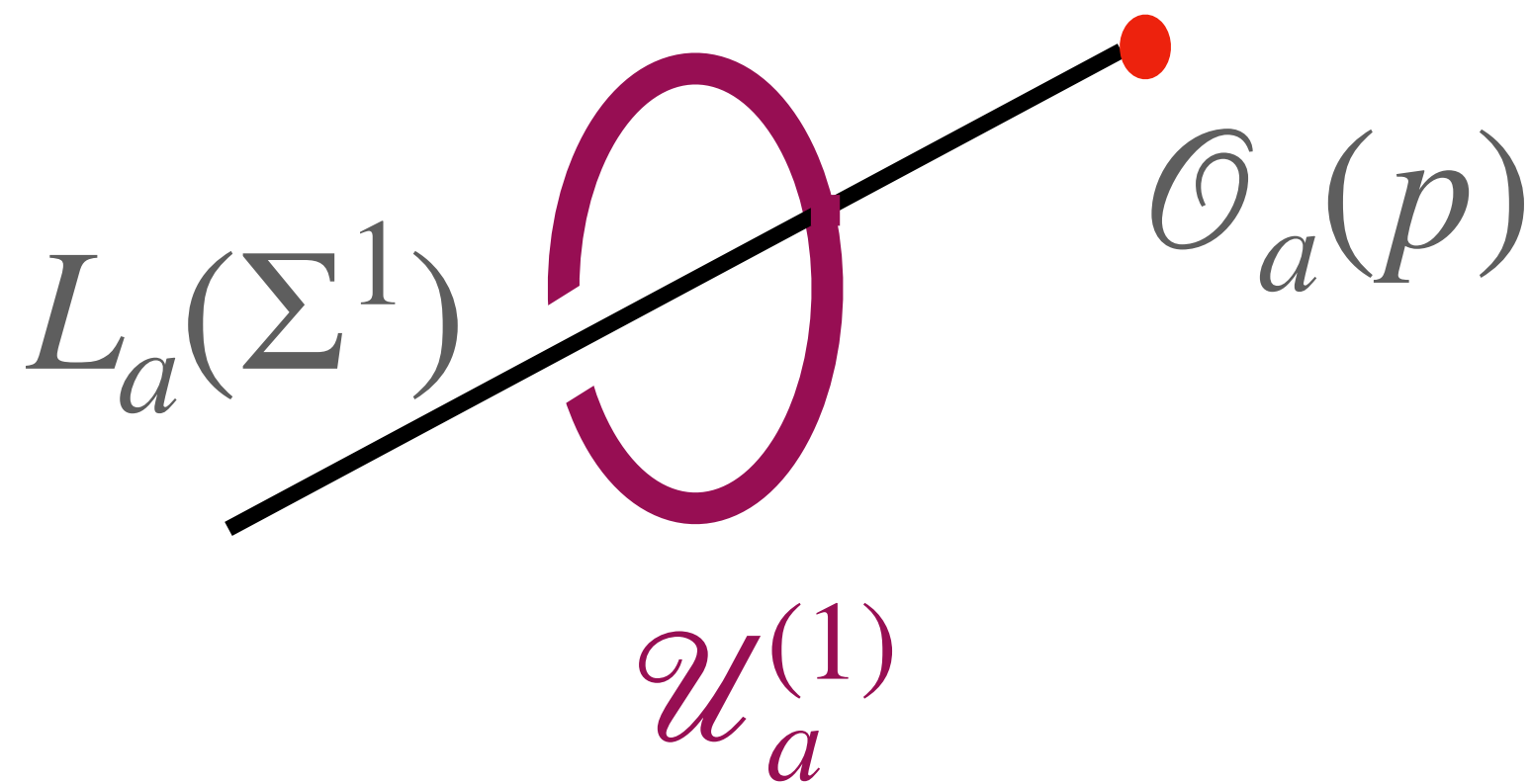
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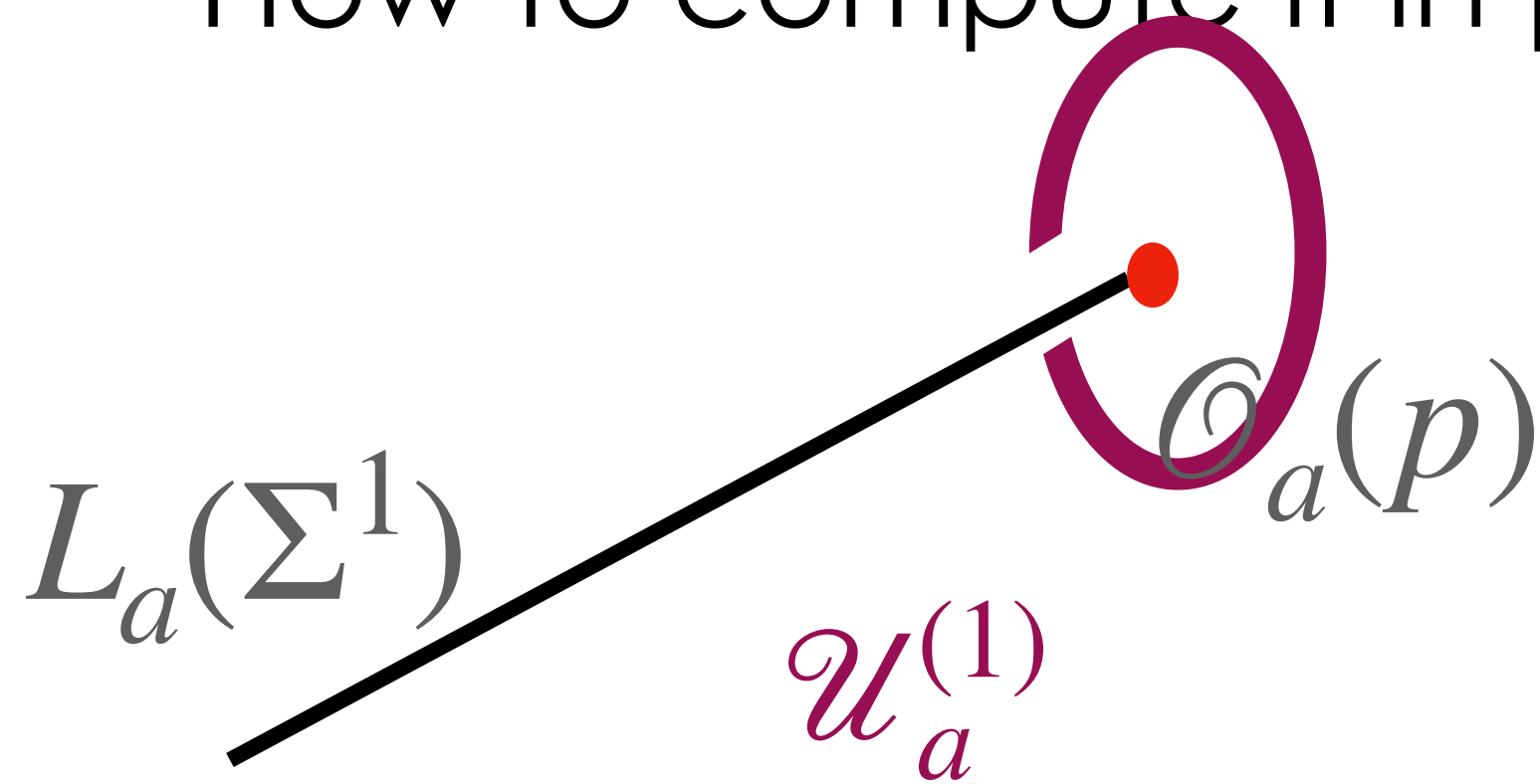
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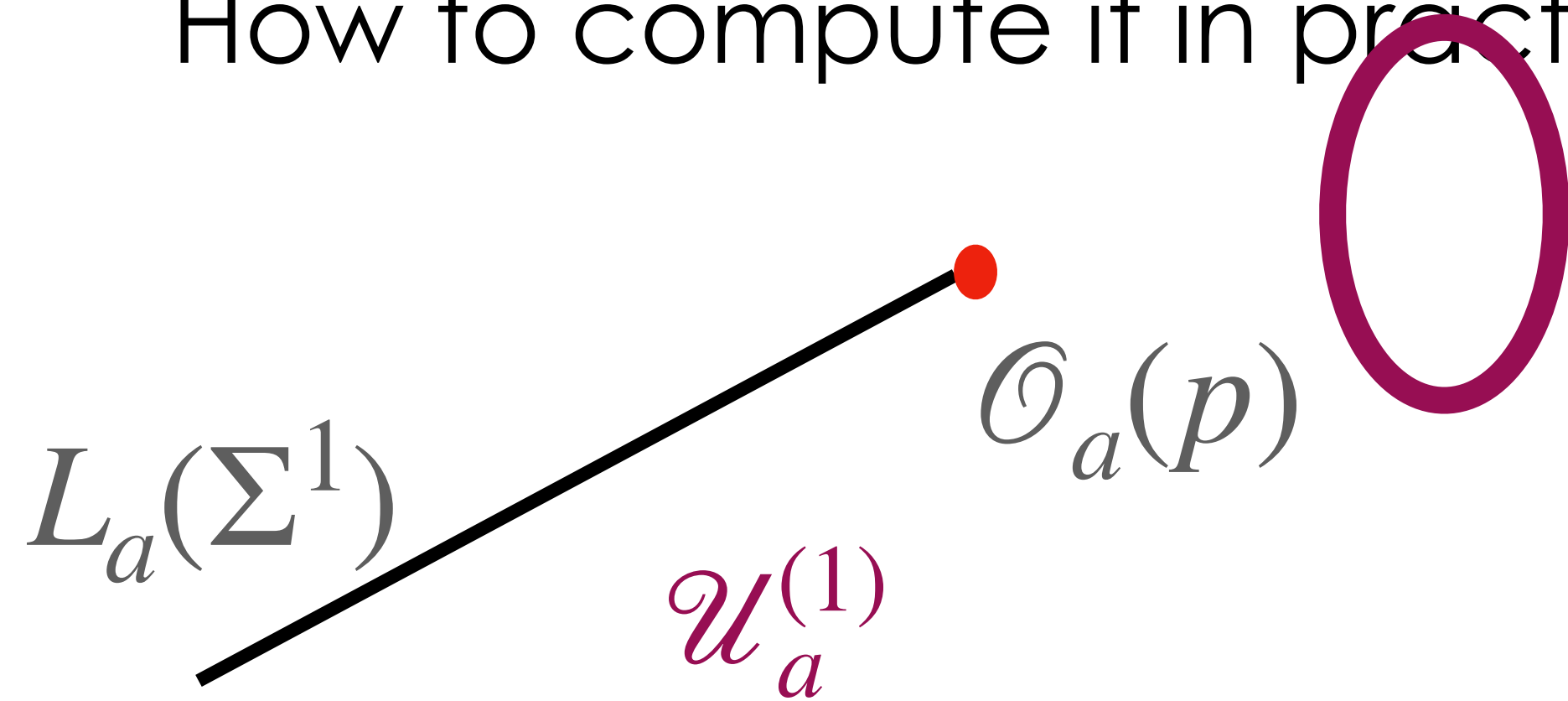
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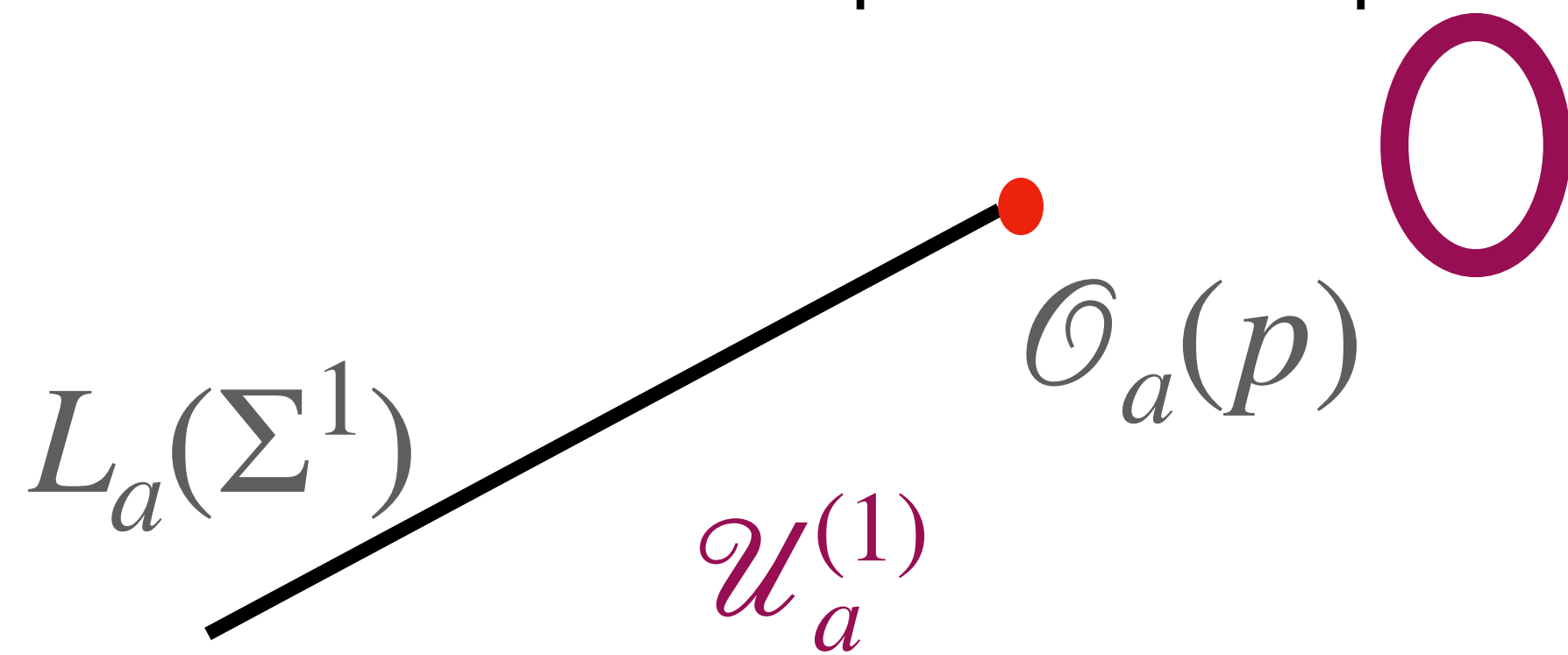
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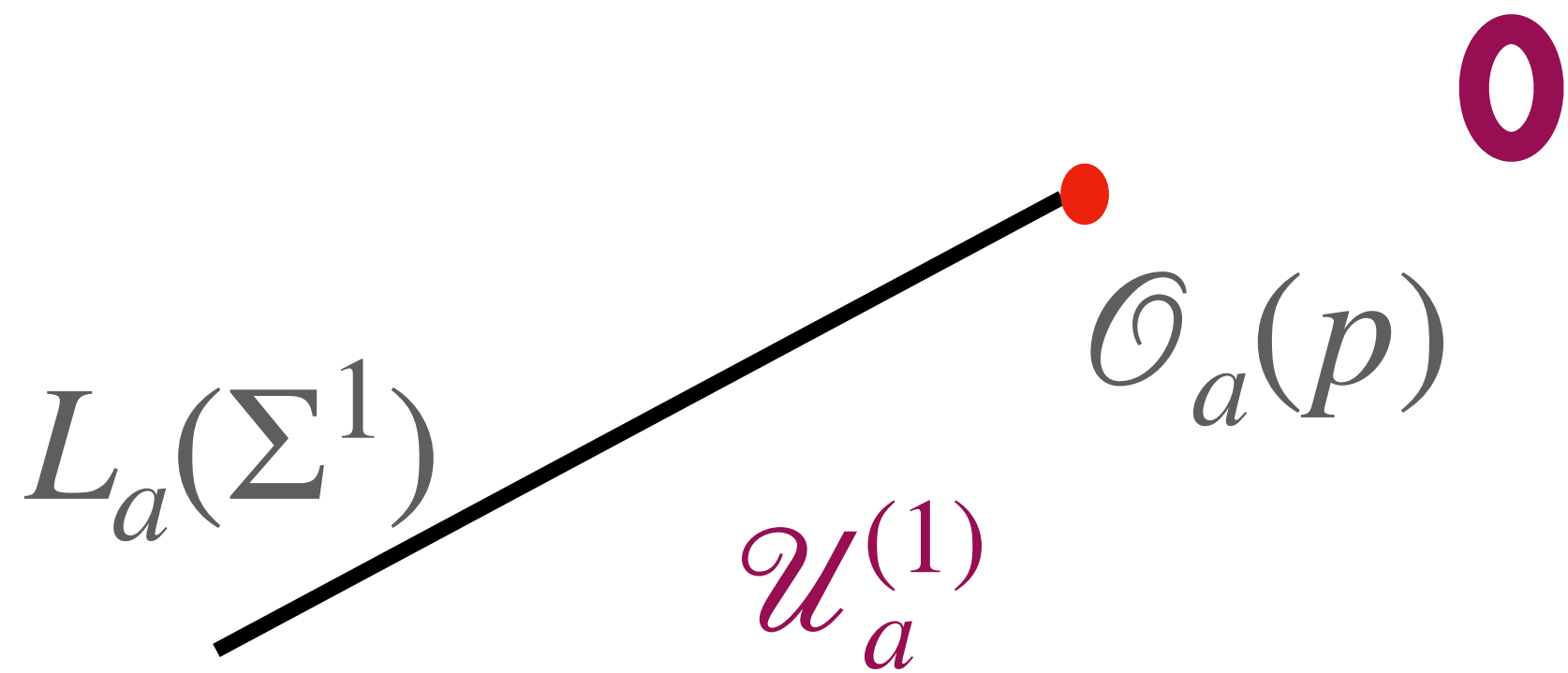
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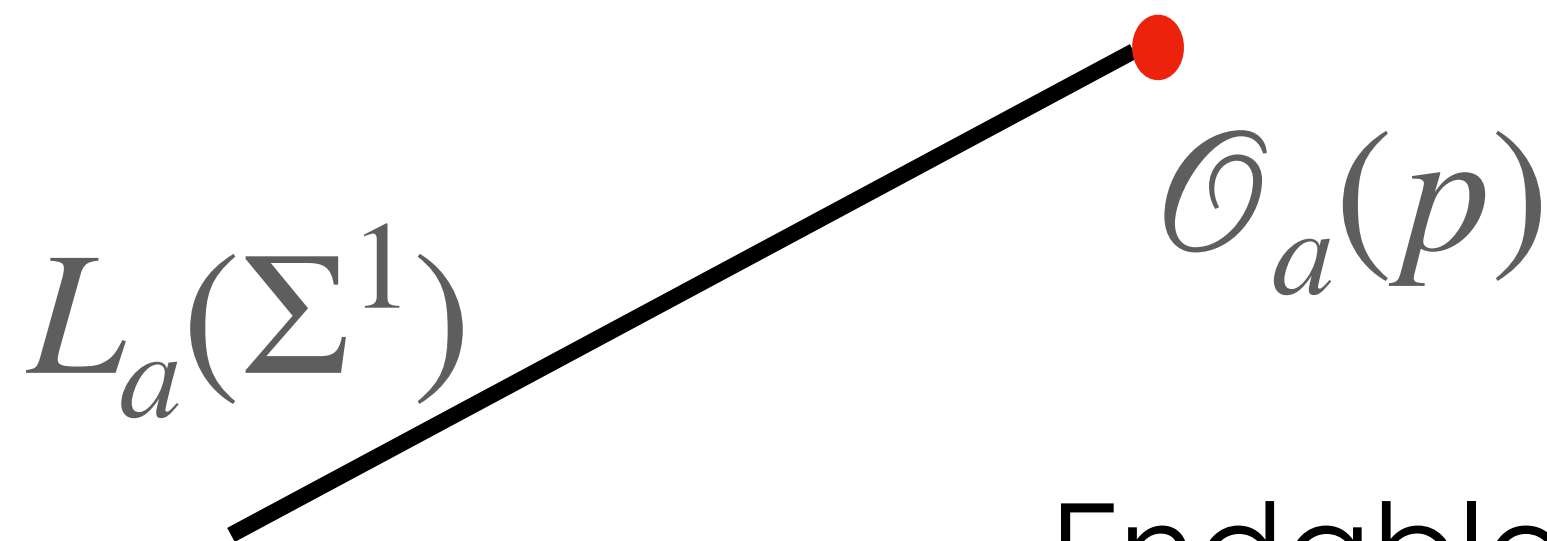
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Endable lines are screened

$\mathbb{A}^{(1)}$ = charges of non-endable lines

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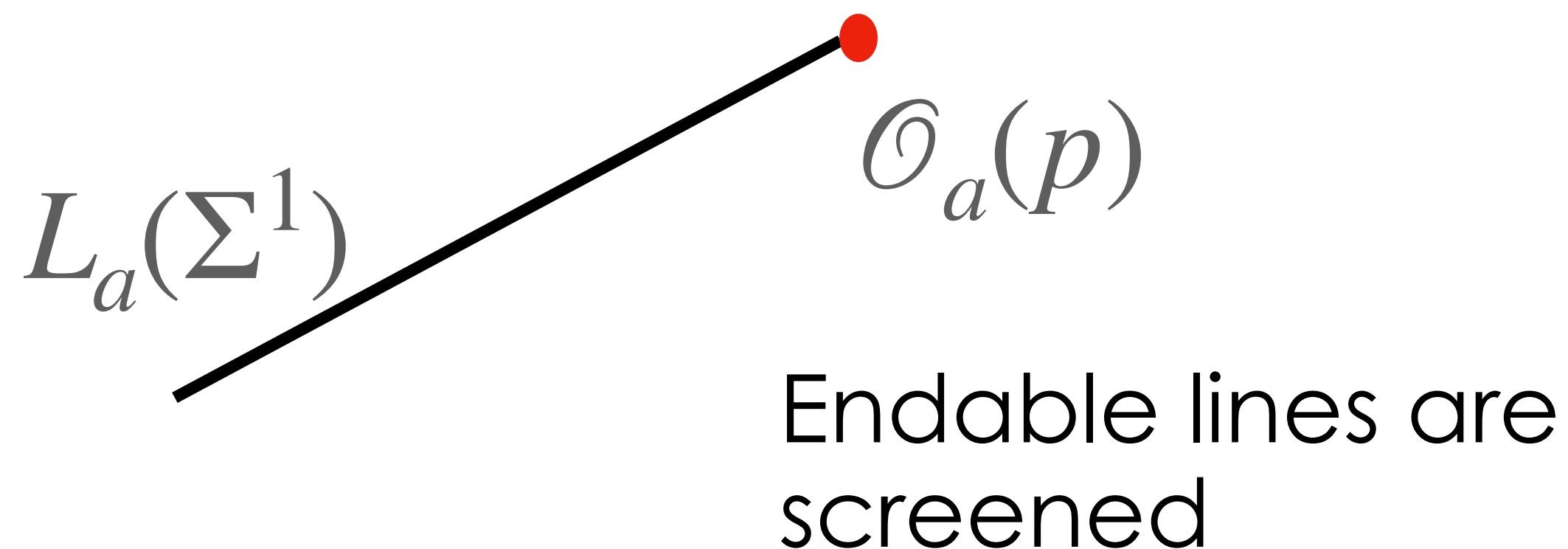
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Consider screening only with operators in definite representations of $\mathbb{G}^{(0)}$

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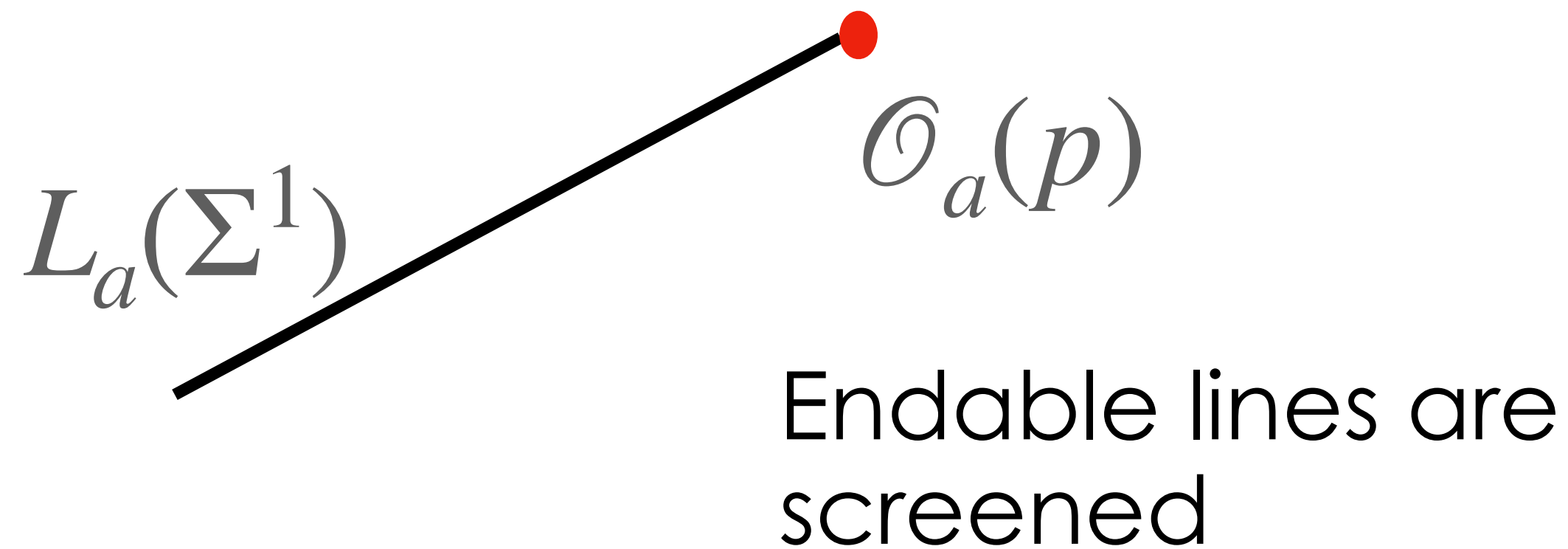
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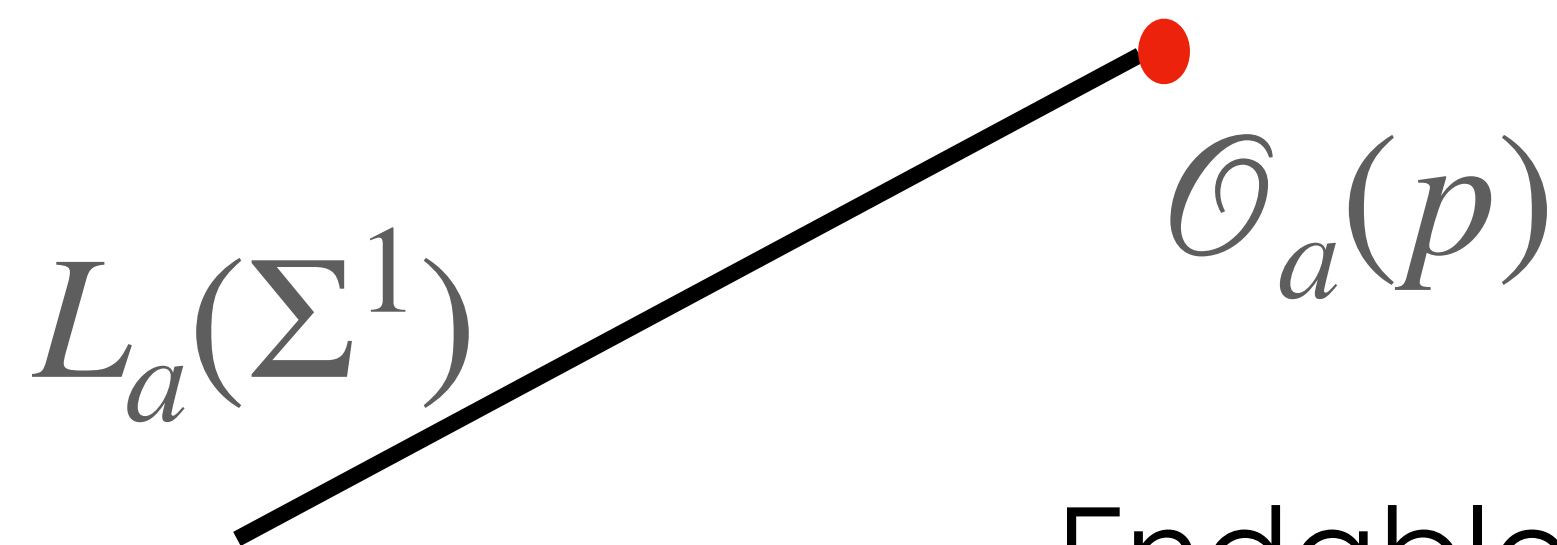
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How to compute it in practice?

$$1 \rightarrow \mathbb{A}^{(1)} \rightarrow \Gamma^{(1)} \rightarrow \mathbb{C} \rightarrow 1$$



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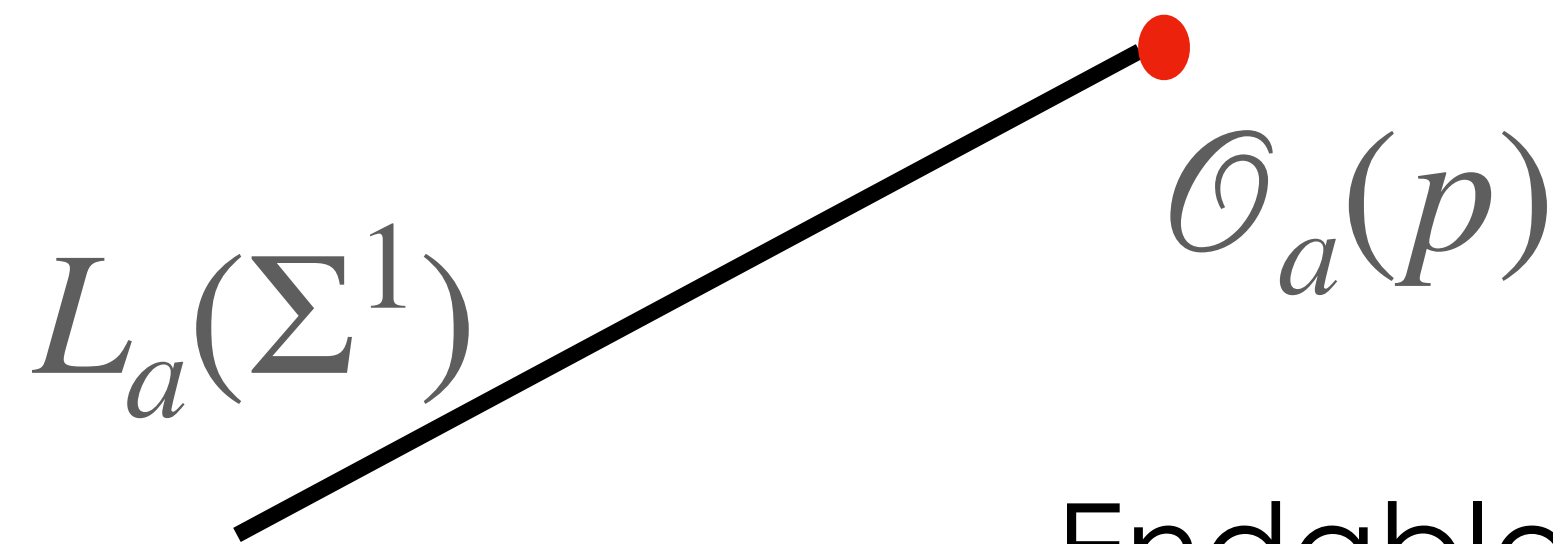
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$$w \in H^2(\mathbb{G}^{(0)}, \mathbb{C})$$

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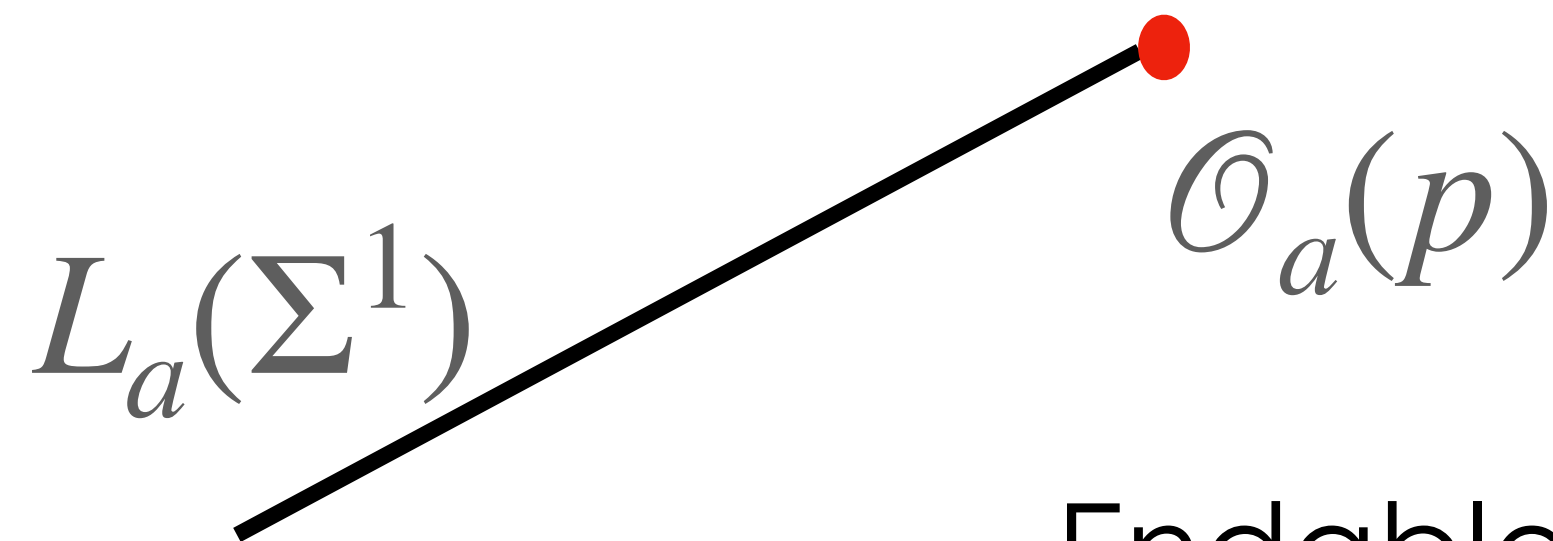
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$$\beta^{(2)} = \delta_\rho w$$

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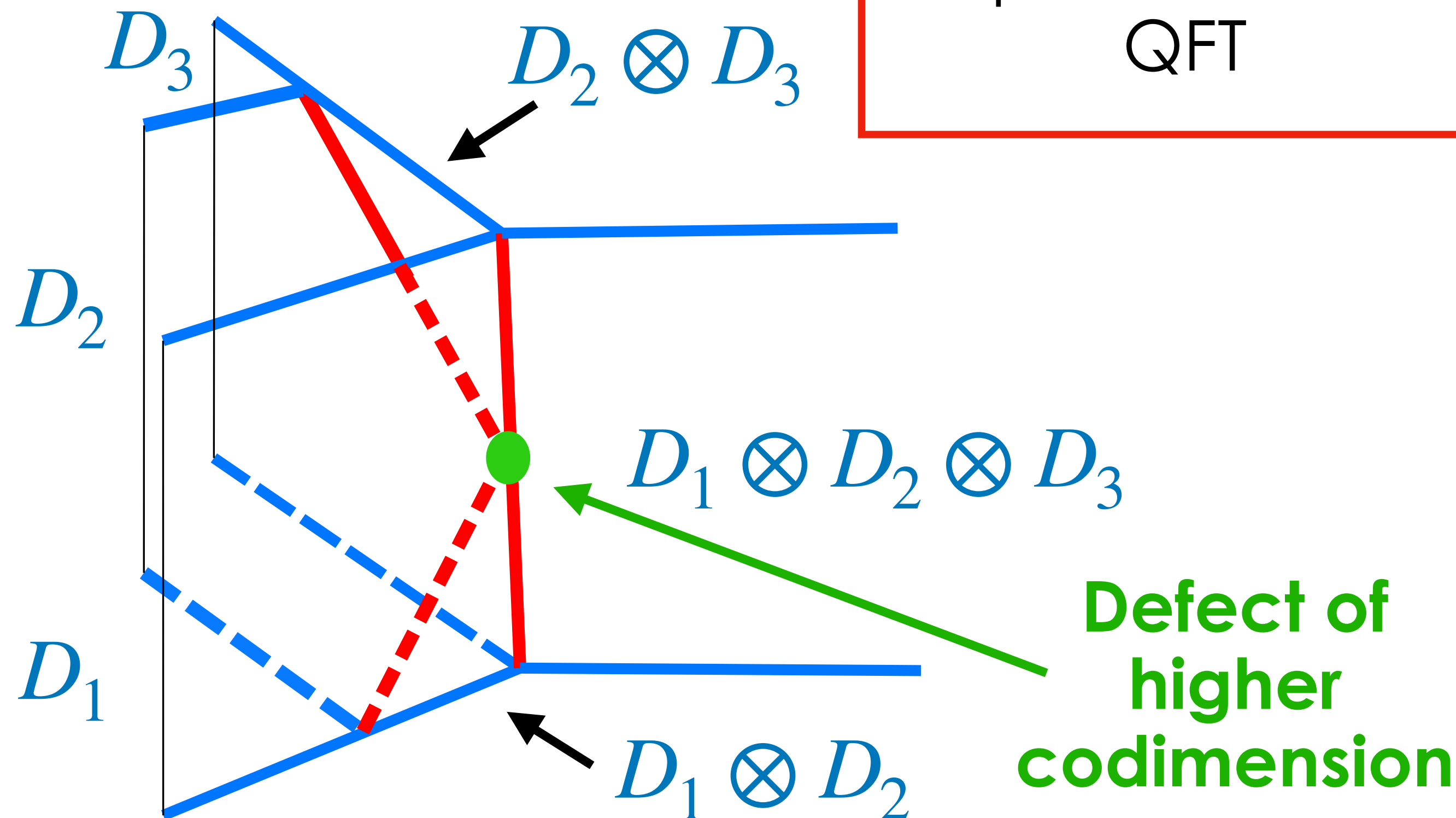
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Generalized Symmetries: Lightning Review

Generalized global symmetries

\equiv

“Topological” subsector of the spectrum of operators of a QFT



- Ward identities
 - Selection Rules
 - Anomalies
 - Spontaneous breaking
- ... but also **other features!**

- **Fusion product**
- **Higher structure**

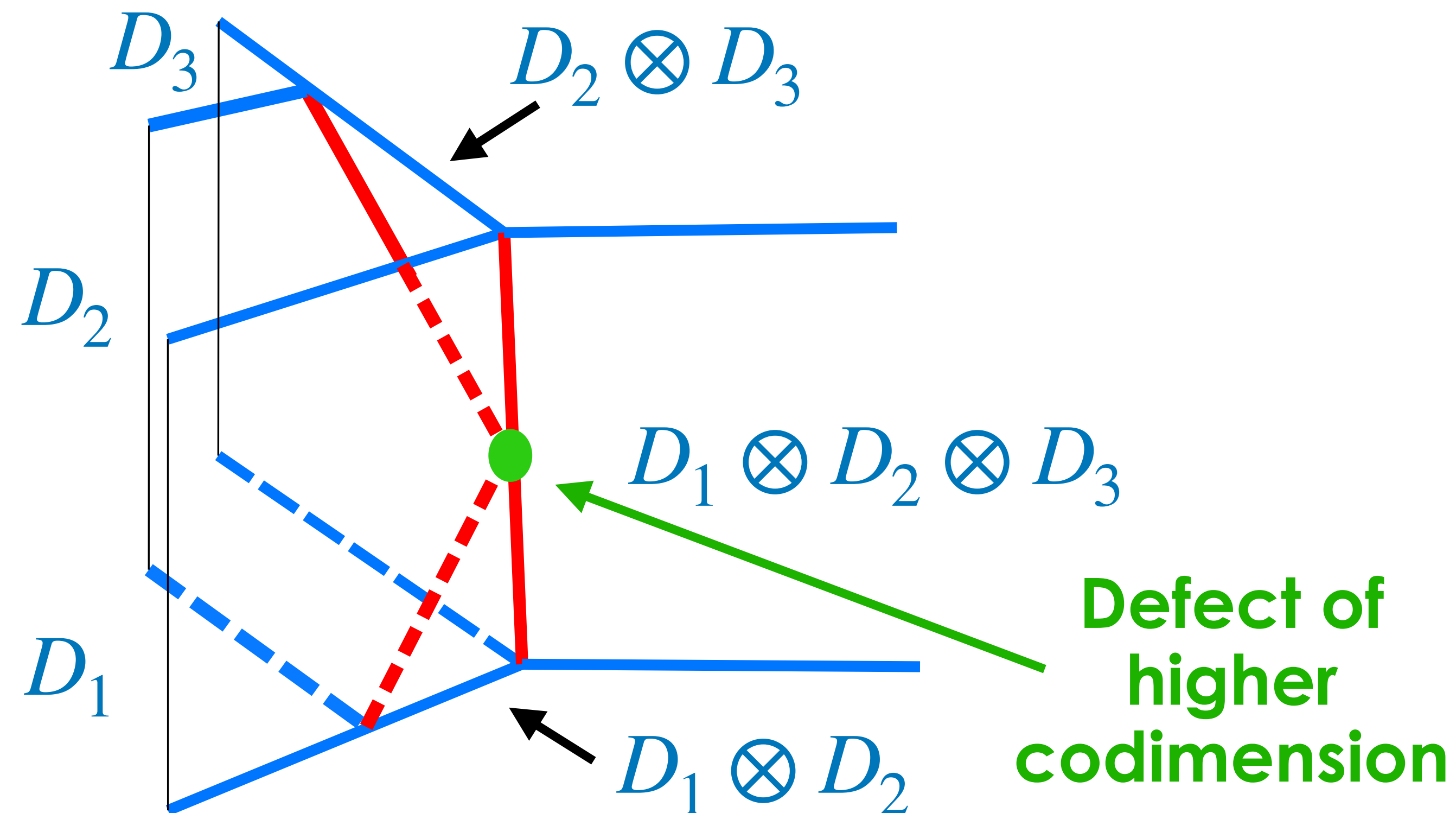
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Symmetry Groups \rightarrow Symmetry Categories

- Ward identities
- Selection Rules
- Anomalies
- Spontaneous breaking



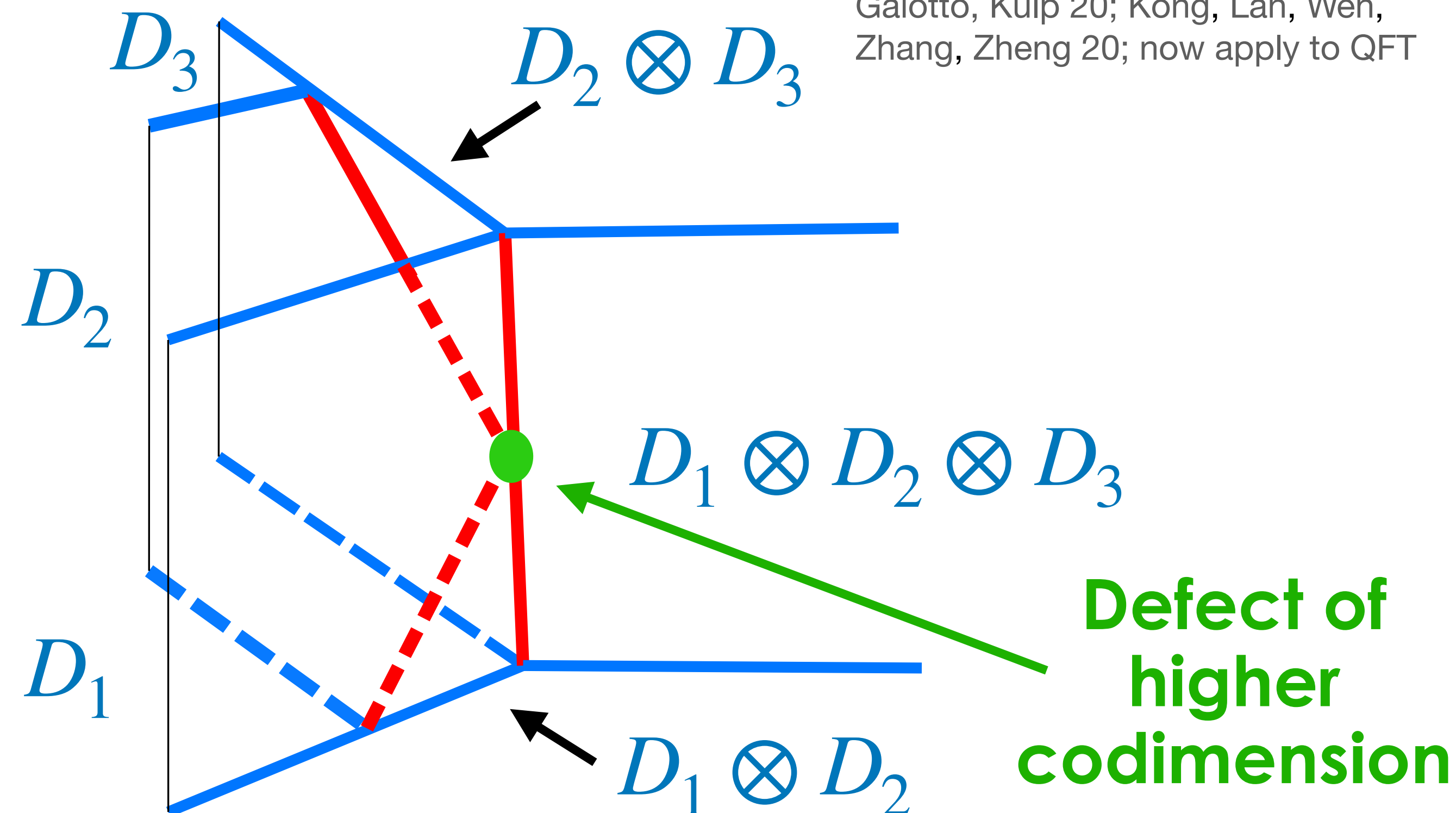
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The structure of symmetries is better characterized exploiting **higher categories** than groups

Well-known in the TQFT community
Finds applications in cond-mat
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Gaiotto 17,19; Johnson-Freyd 20;
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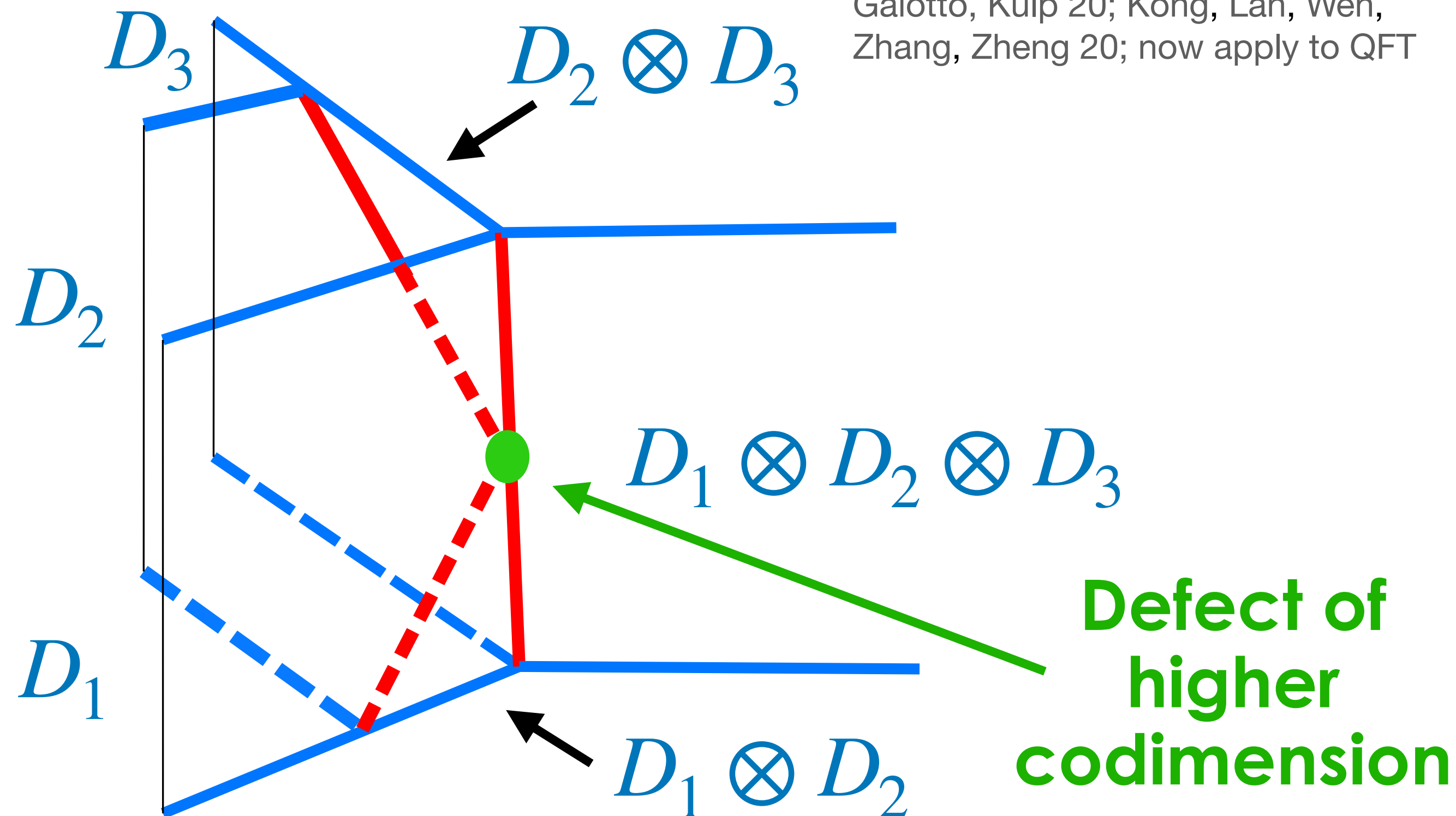
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- **Fusion product**
- **Higher structure**

See also: Córdova, Ohmori 21; Choi, (Córdova), Lam, (Hsin), Shao 21,22; Roumpedakis, Seifnashri, Shao 22; Kaidi, Ohmori, Zheng 21,22; Kaidi, Nardoni, Zafrir, Zheng 23; Oxford group (Apruzzi, Bhardwaj, Bonetti, Bottini, ..., Schäfer-Nameki); Durham group (Bartsch, Bullimore, ... + García Etxebarria, Hosseini),...

Symmetry Categories oversimplified

Symmetry category graded by **charged operator** dimensions:

$$\mathcal{C} = (\mathcal{C}^{(0)}, \mathcal{C}^{(1)}, \dots, \mathcal{C}^{(D-1)})$$

$$\mathcal{C}^{(k)} = \text{codim } k+1 \text{ top-ops}$$

slogan: **everything is a morphism and every morphism is an interface**

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Remark: From this perspective it makes sense to consider a category of (D+1)-dimensional QFTs with morphisms topological interfaces

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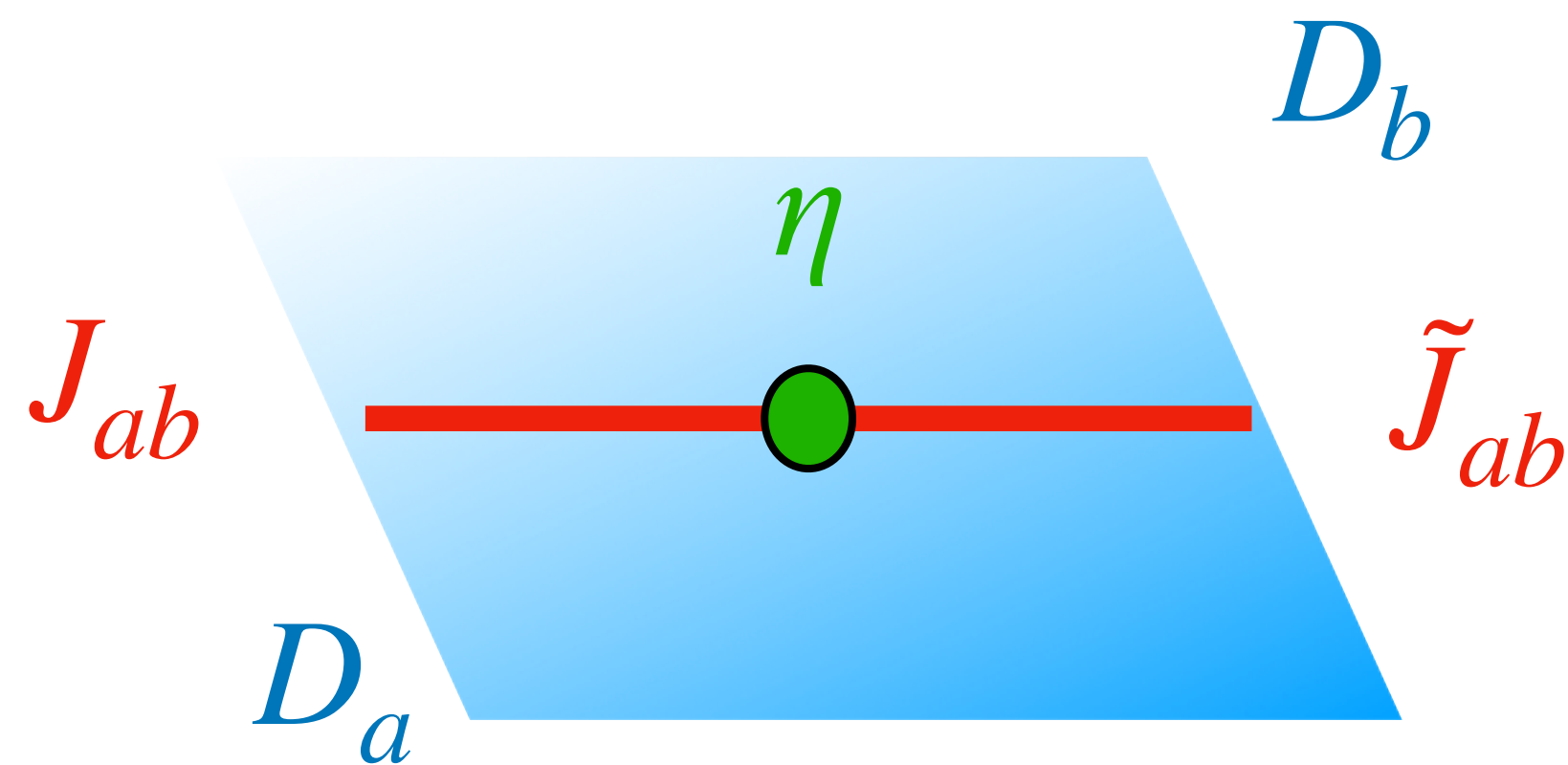
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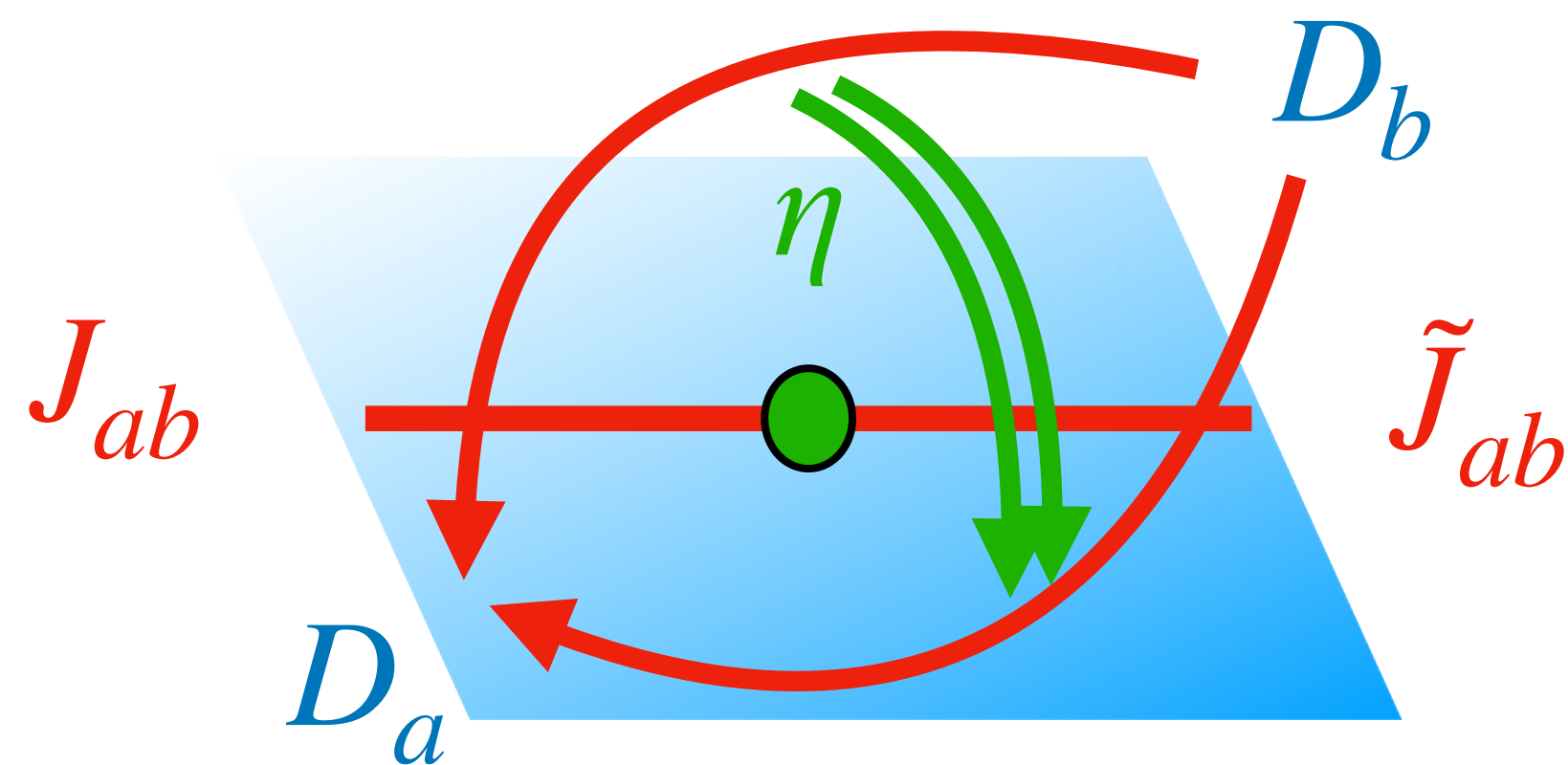
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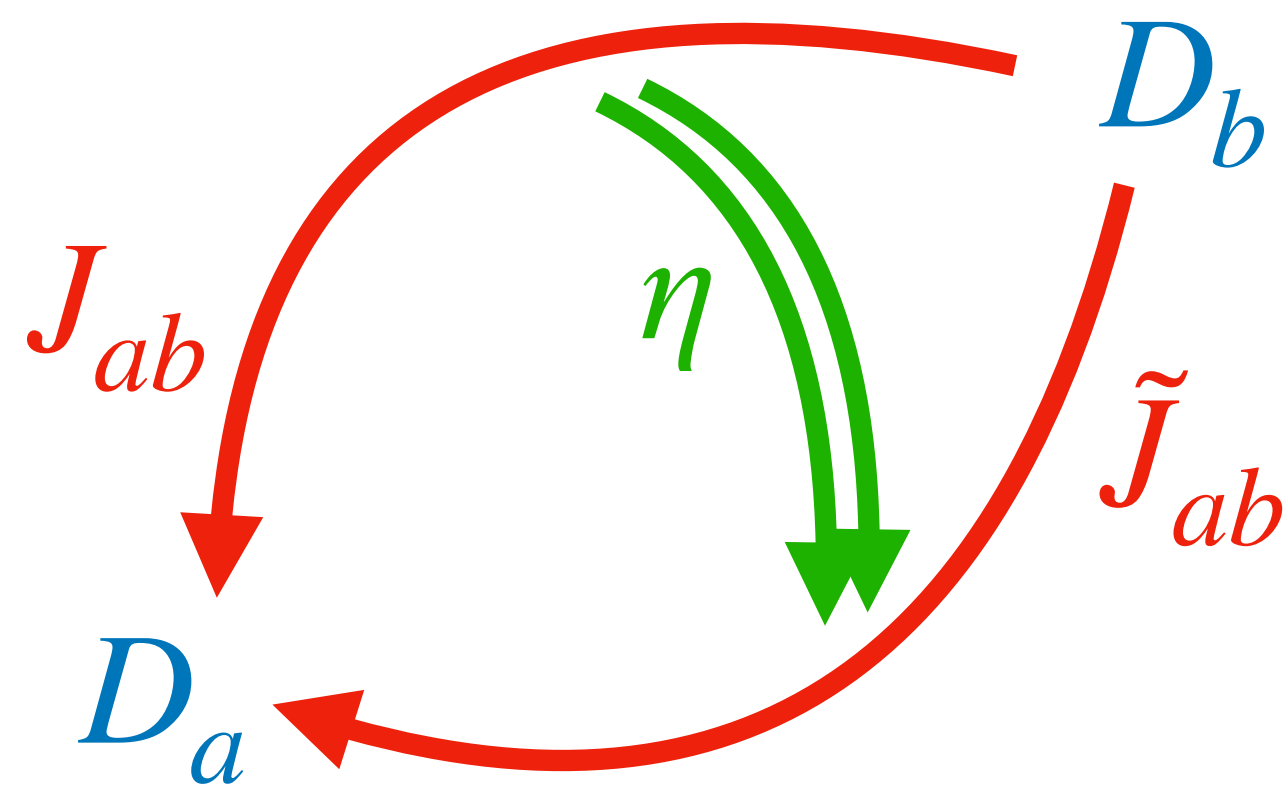
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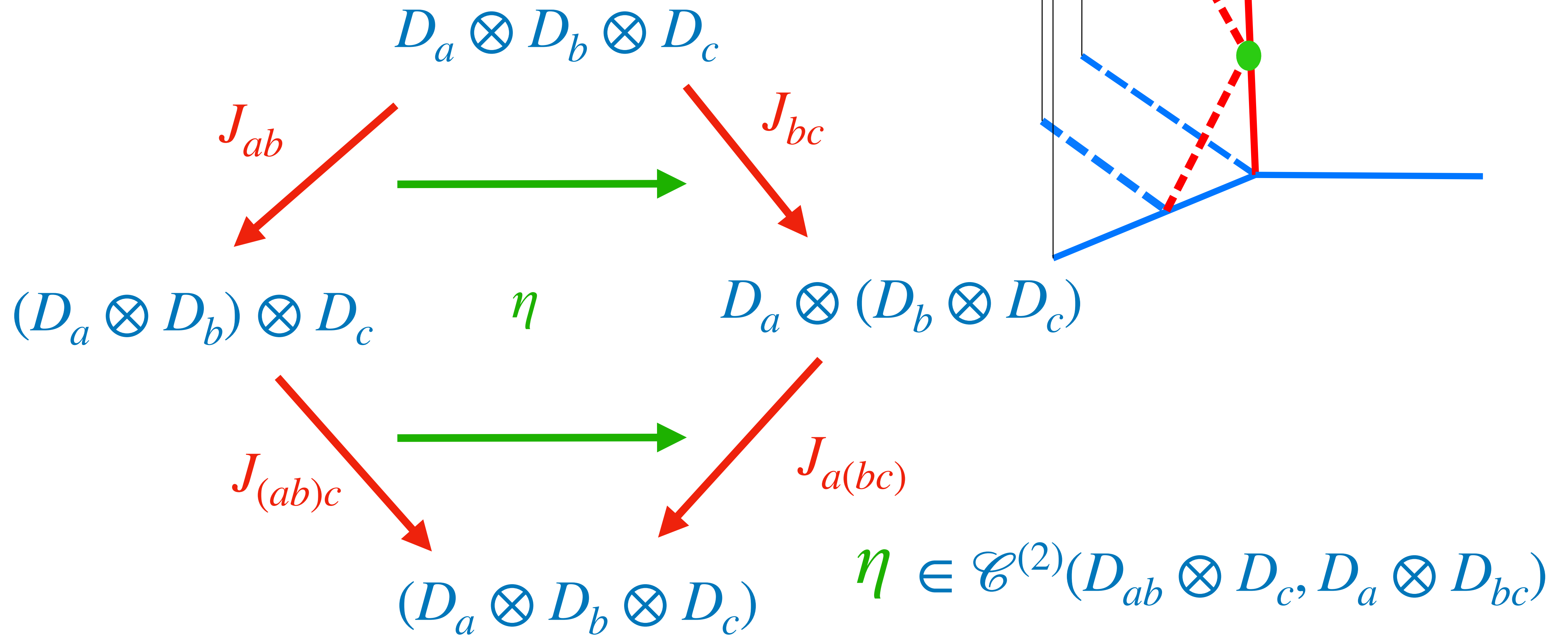
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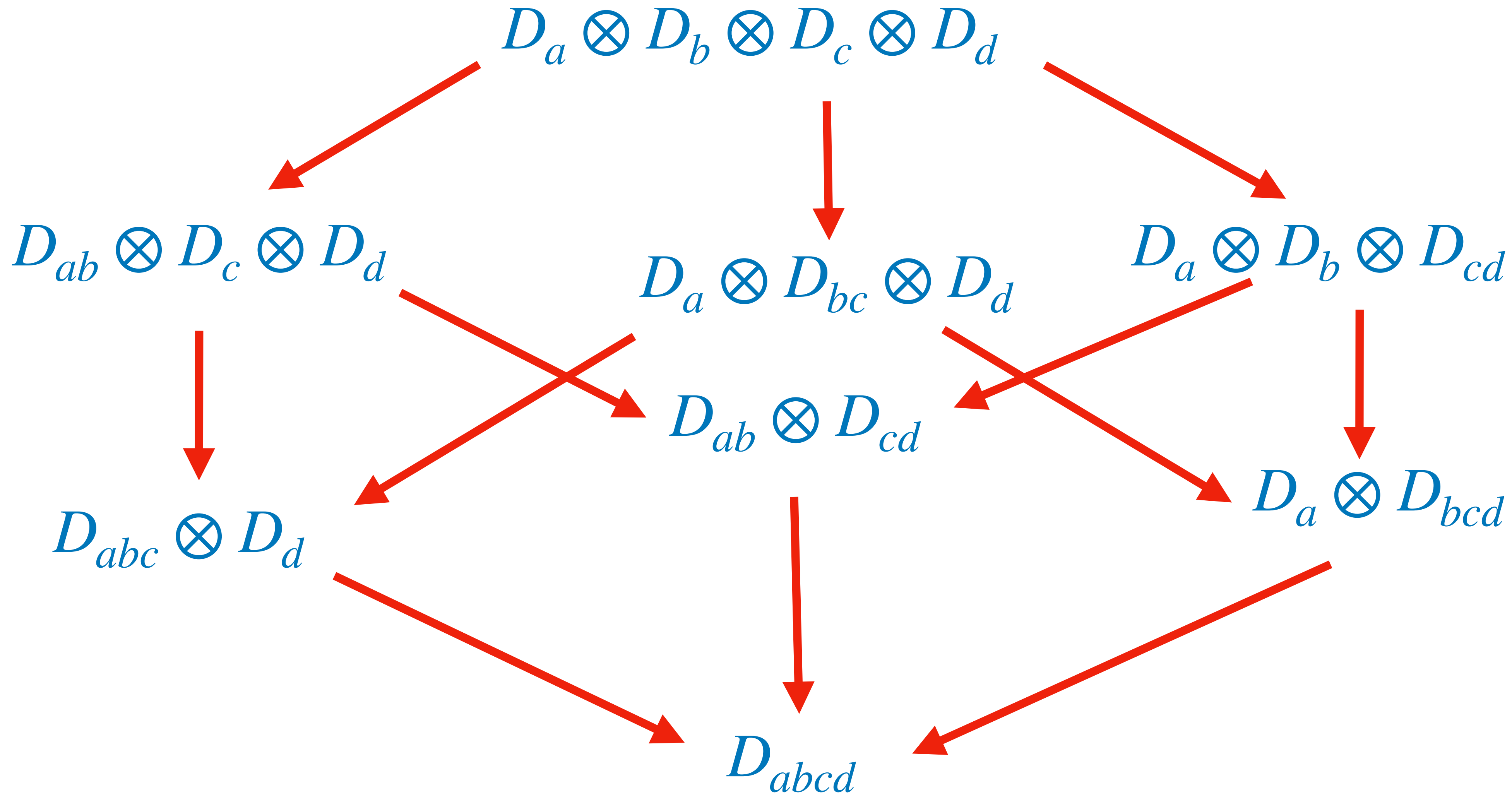
Example



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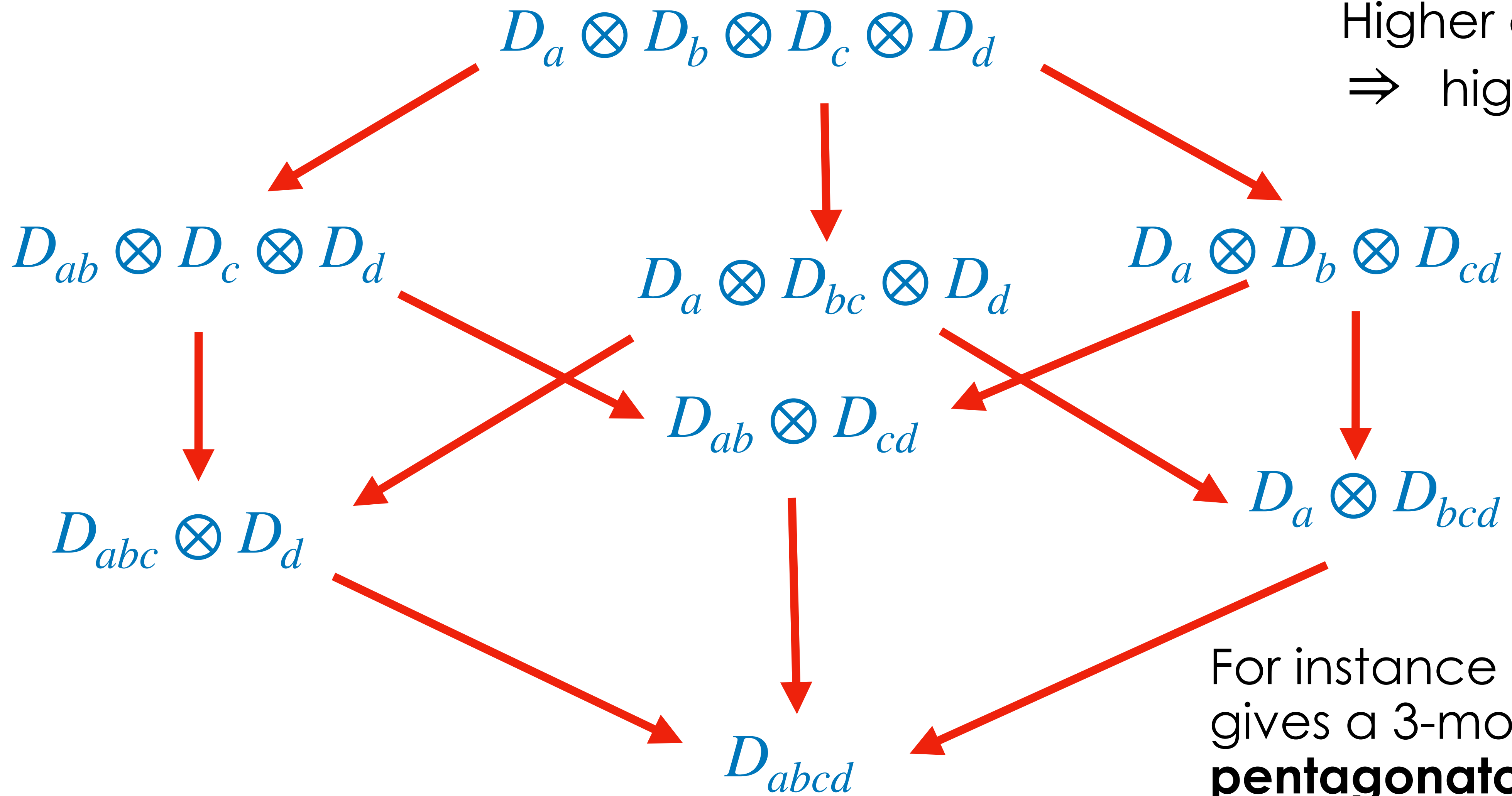
Higher associators

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Higher associativity
 \Rightarrow higher codimension

For instance this diagram gives a 3-morphism (the **pentagonator**).

Gauging and Condensates

Feature of N-fusion categories for $N > 1$: can form **condensates**

Gaiotto, Johnson-Freyd 19
Roumpedakis, Seifnashri, Shao 22

i.e. one can build lower codimension defects from higher codimension ones via the **higher gauging procedure**

For each **p-gaugeable** $\mathbb{A} \subseteq \mathcal{C}^{(k)}$ consistent higher structure requires

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$$\Sigma^{D+1-p}$$

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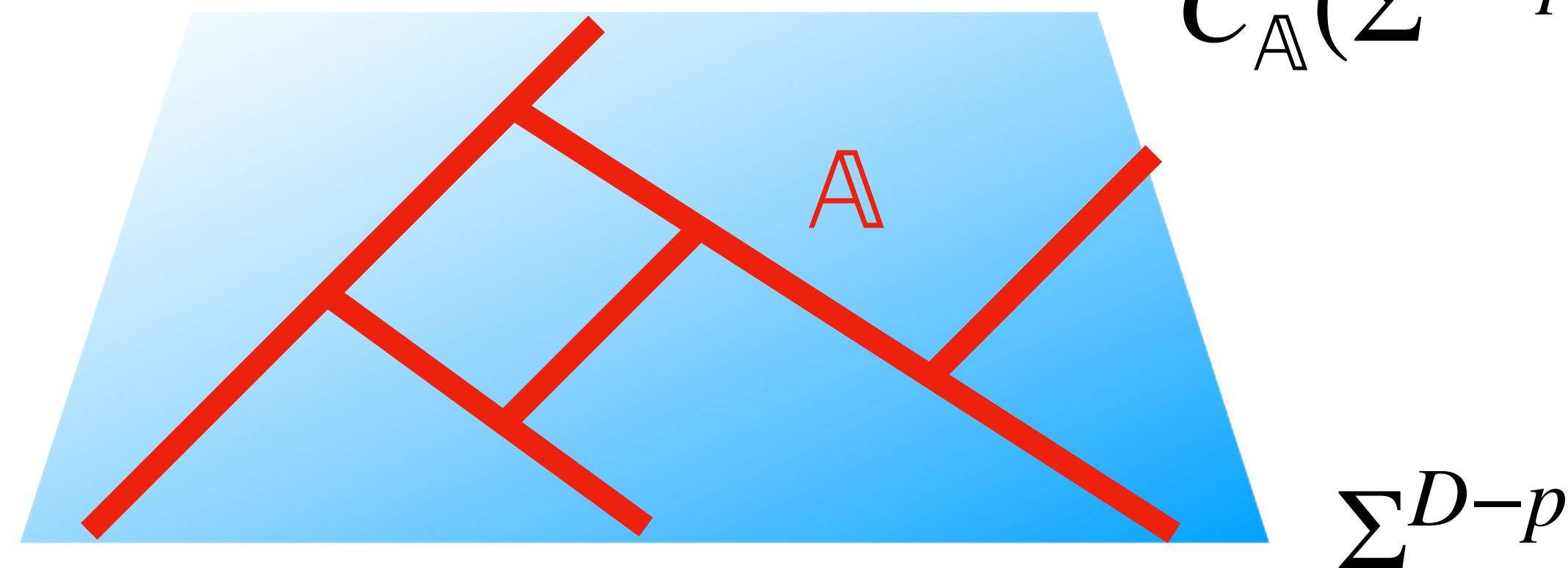
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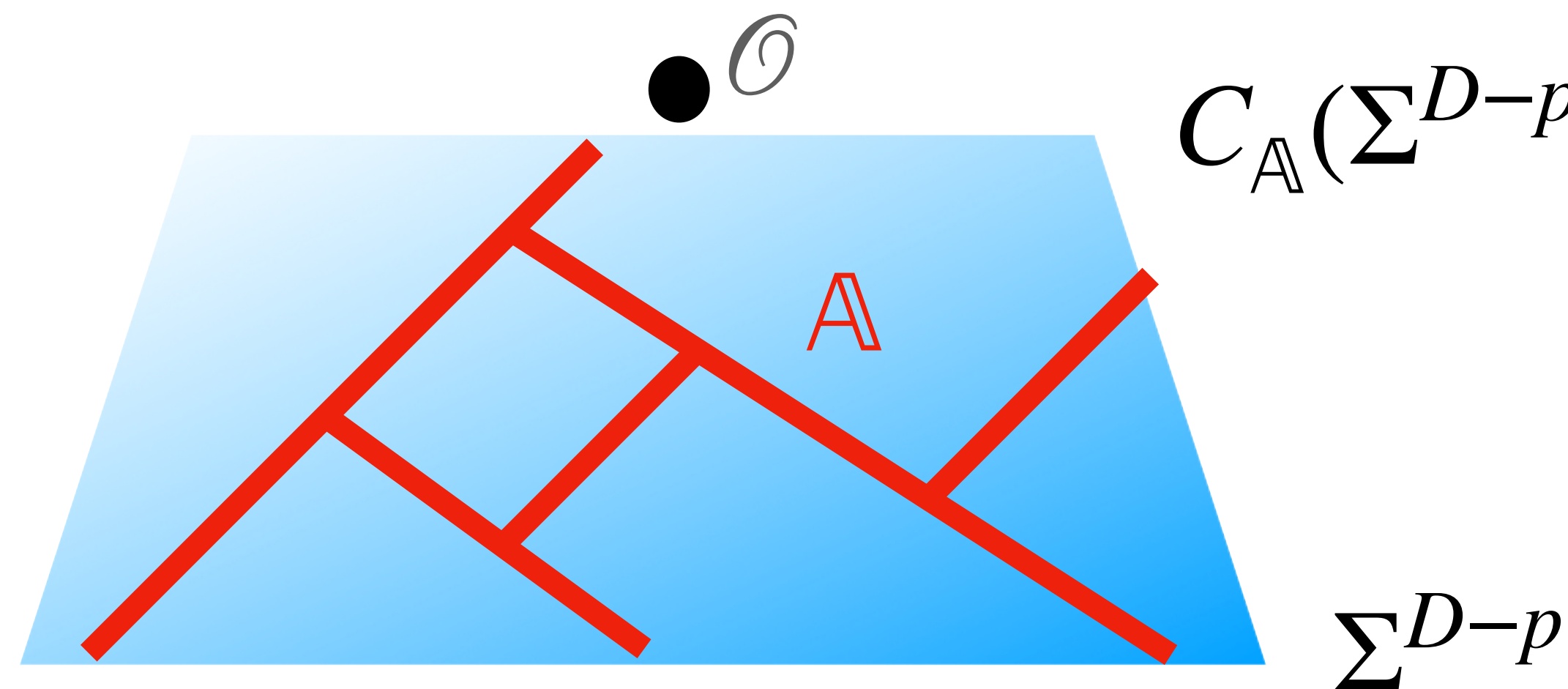
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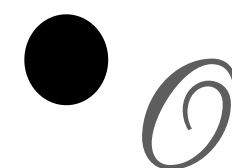
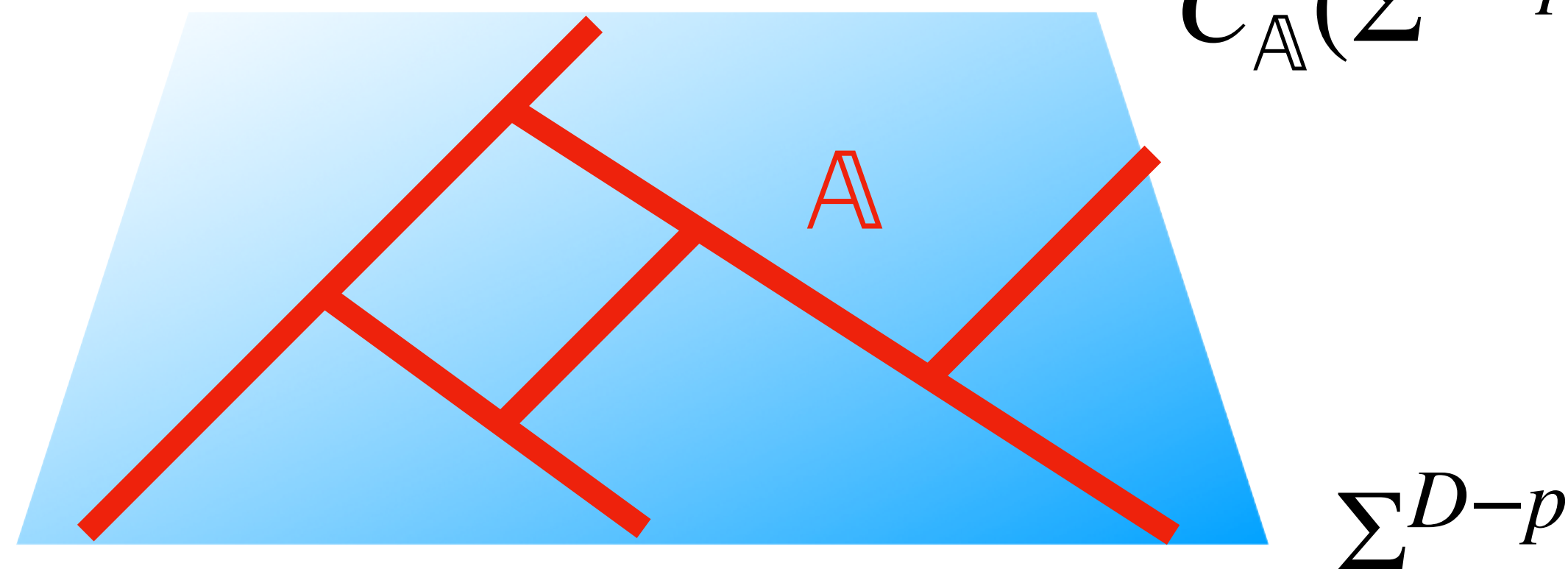
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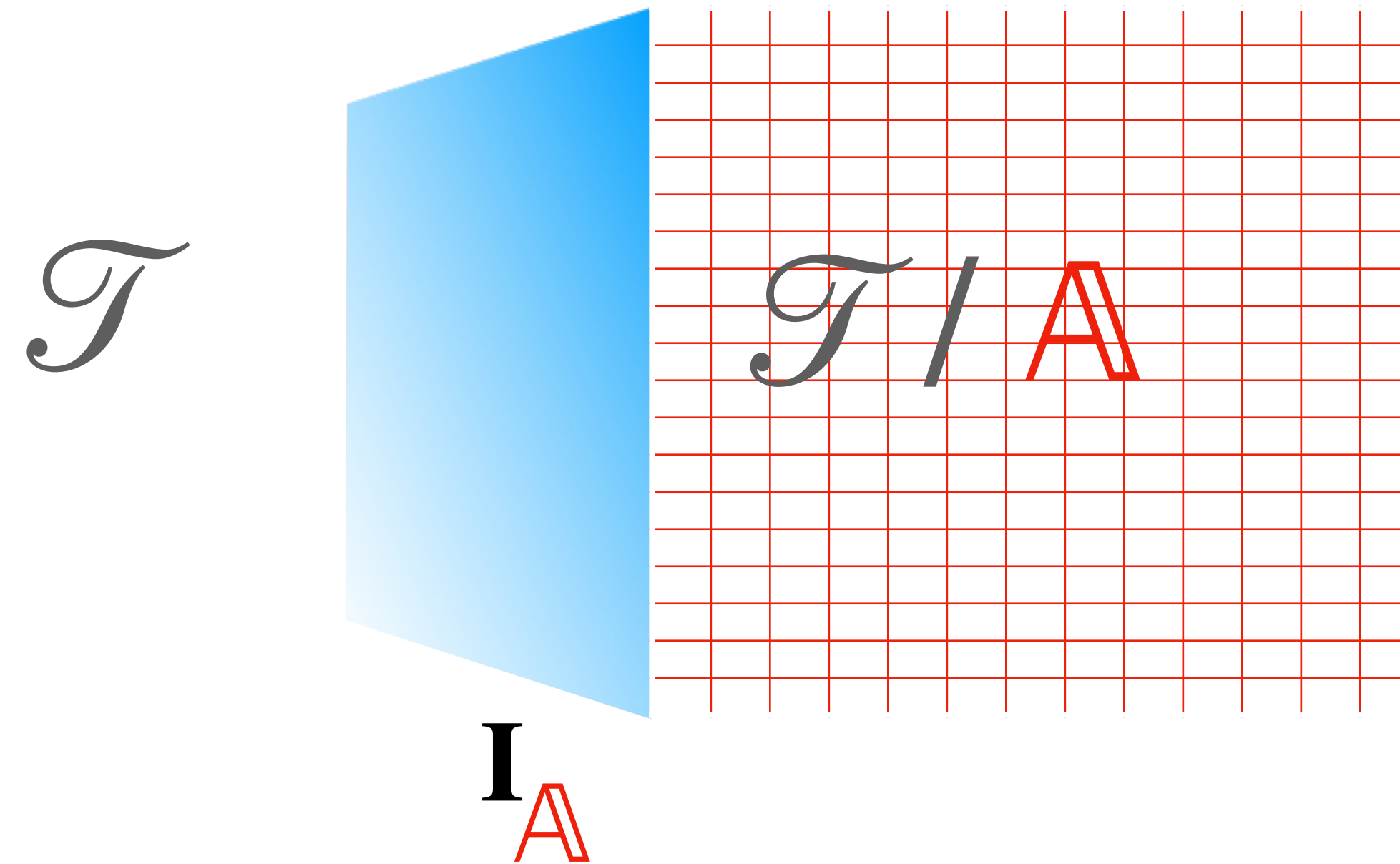


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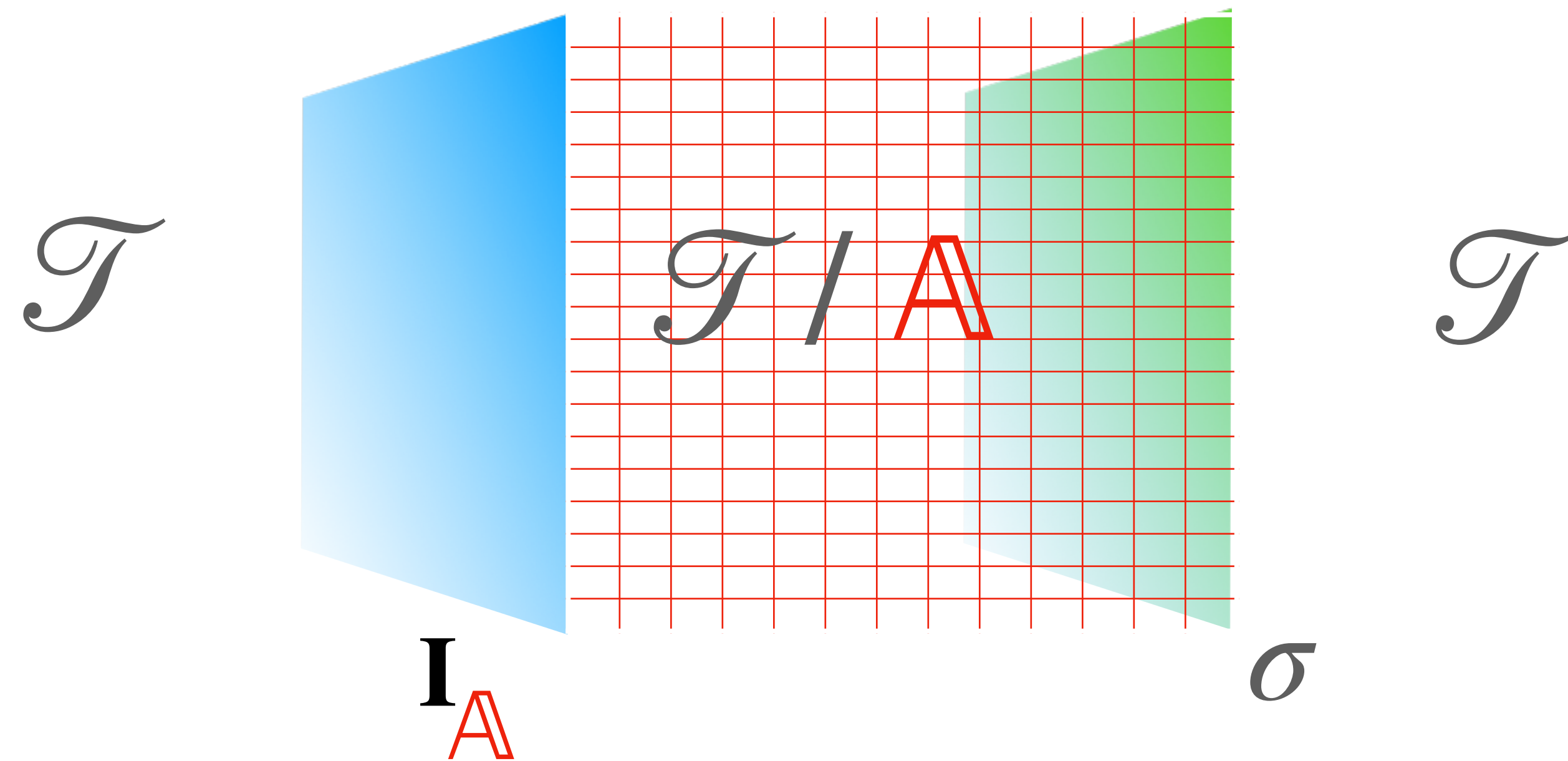
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One obtains a **duality defect** from composition

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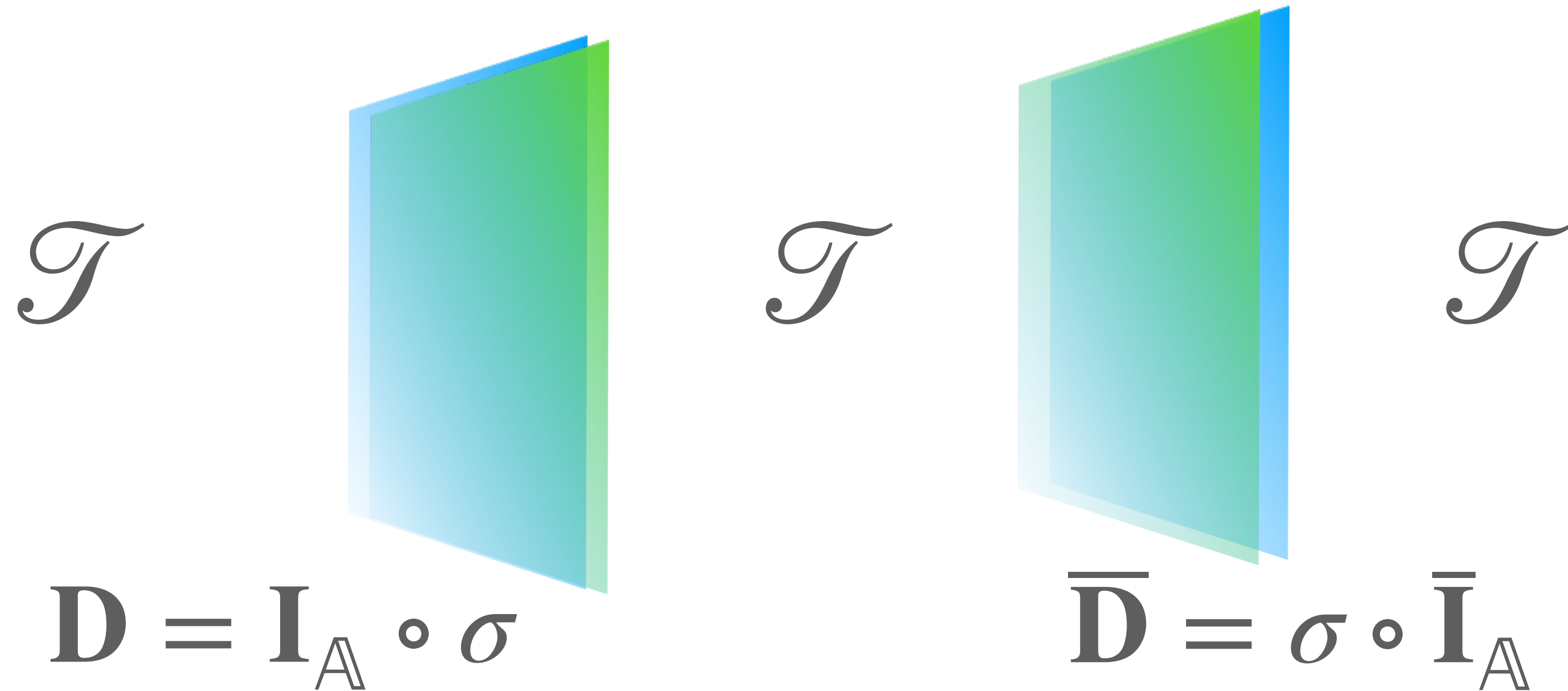


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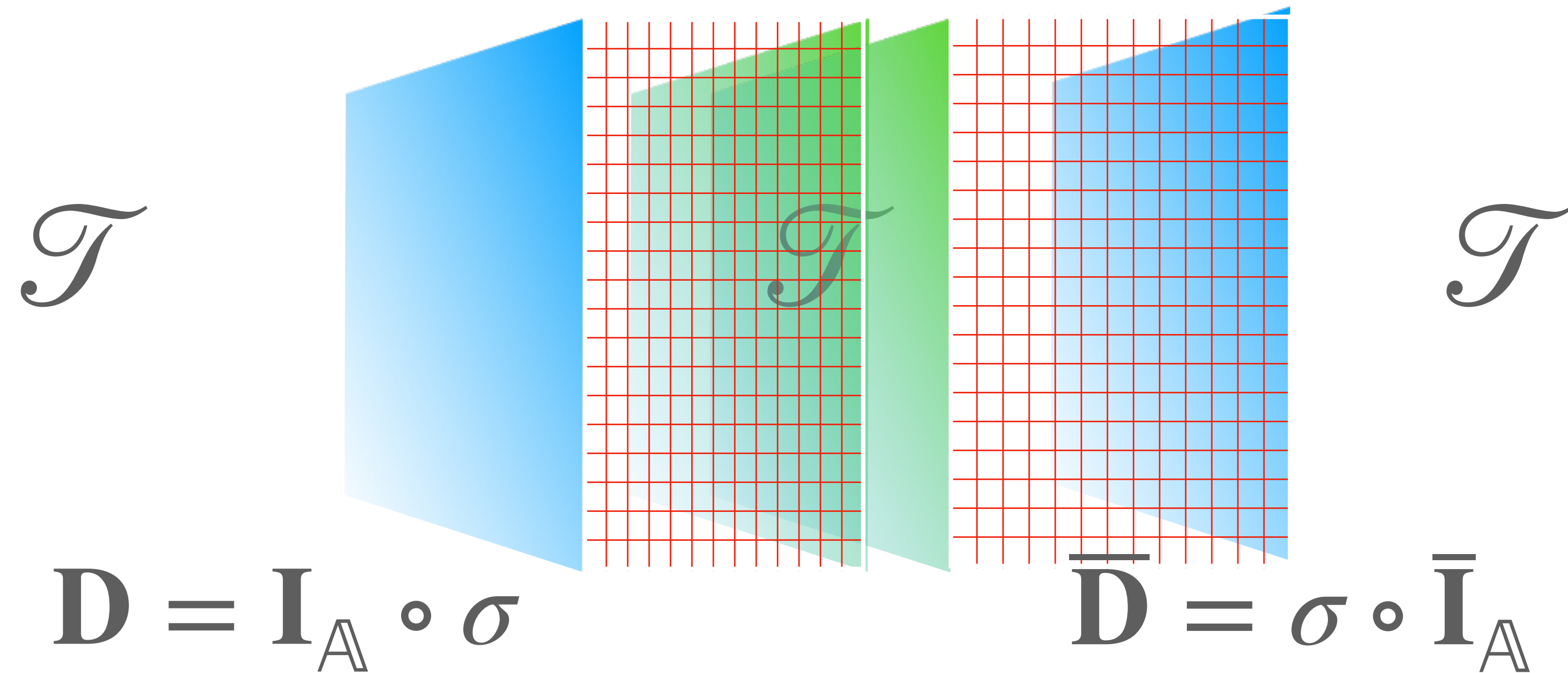


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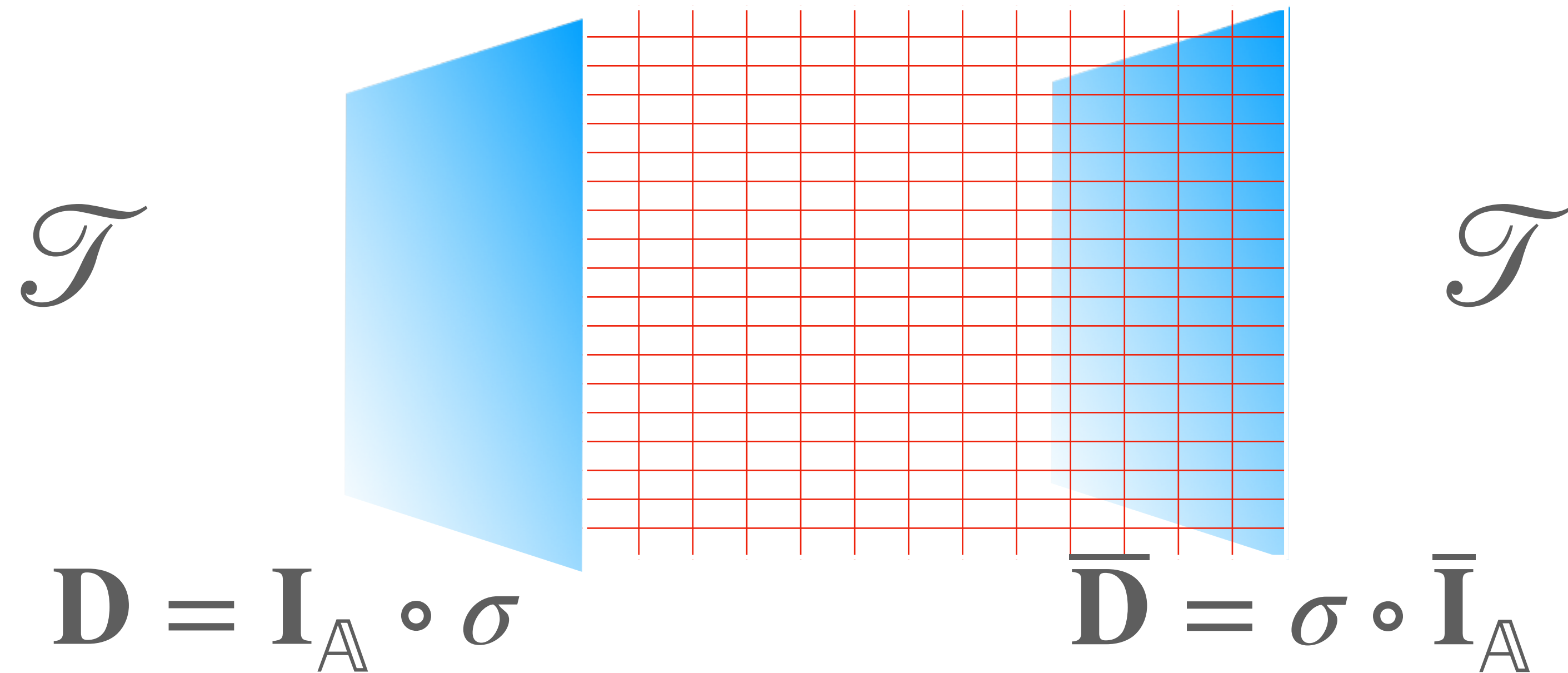


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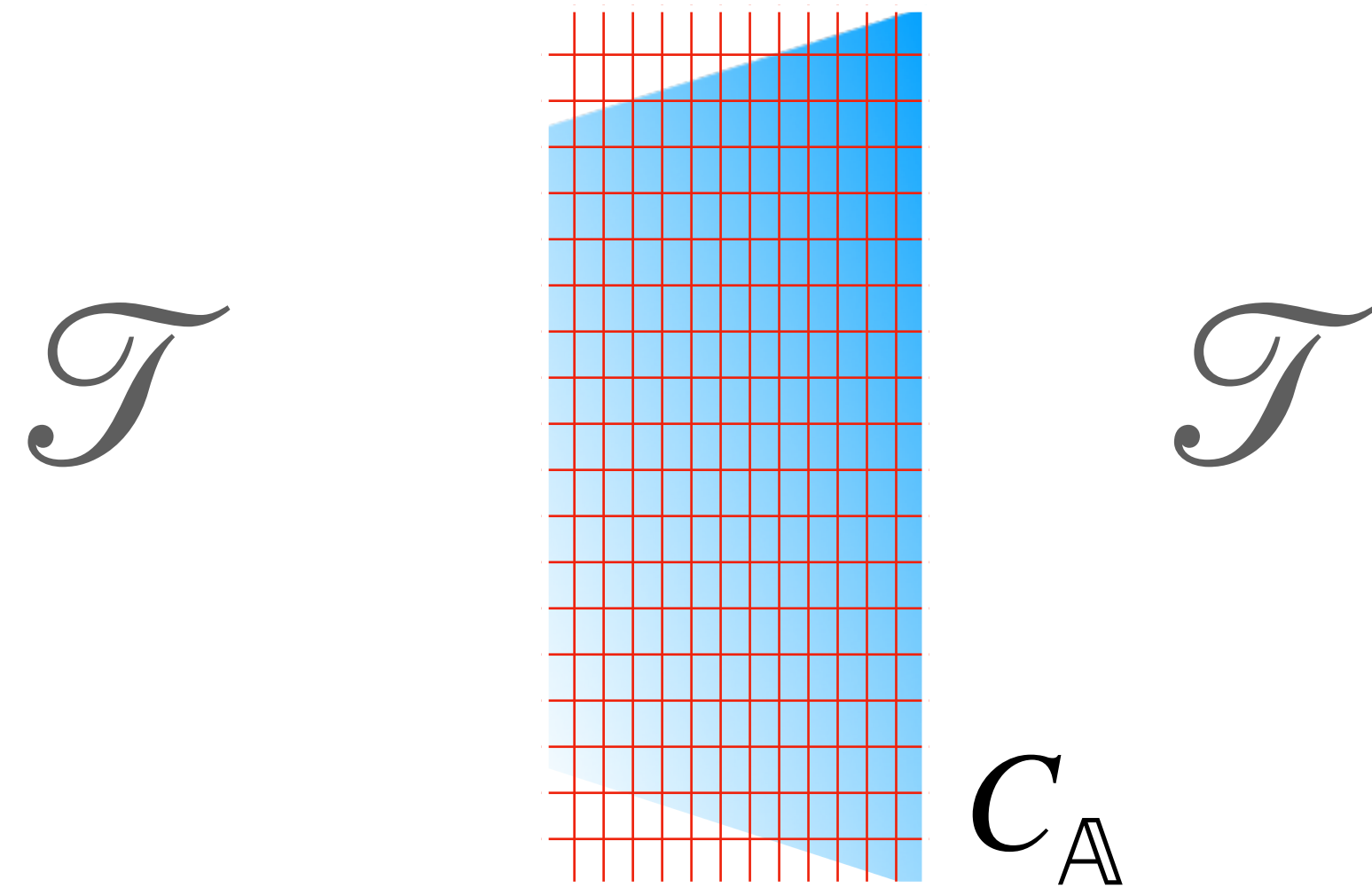


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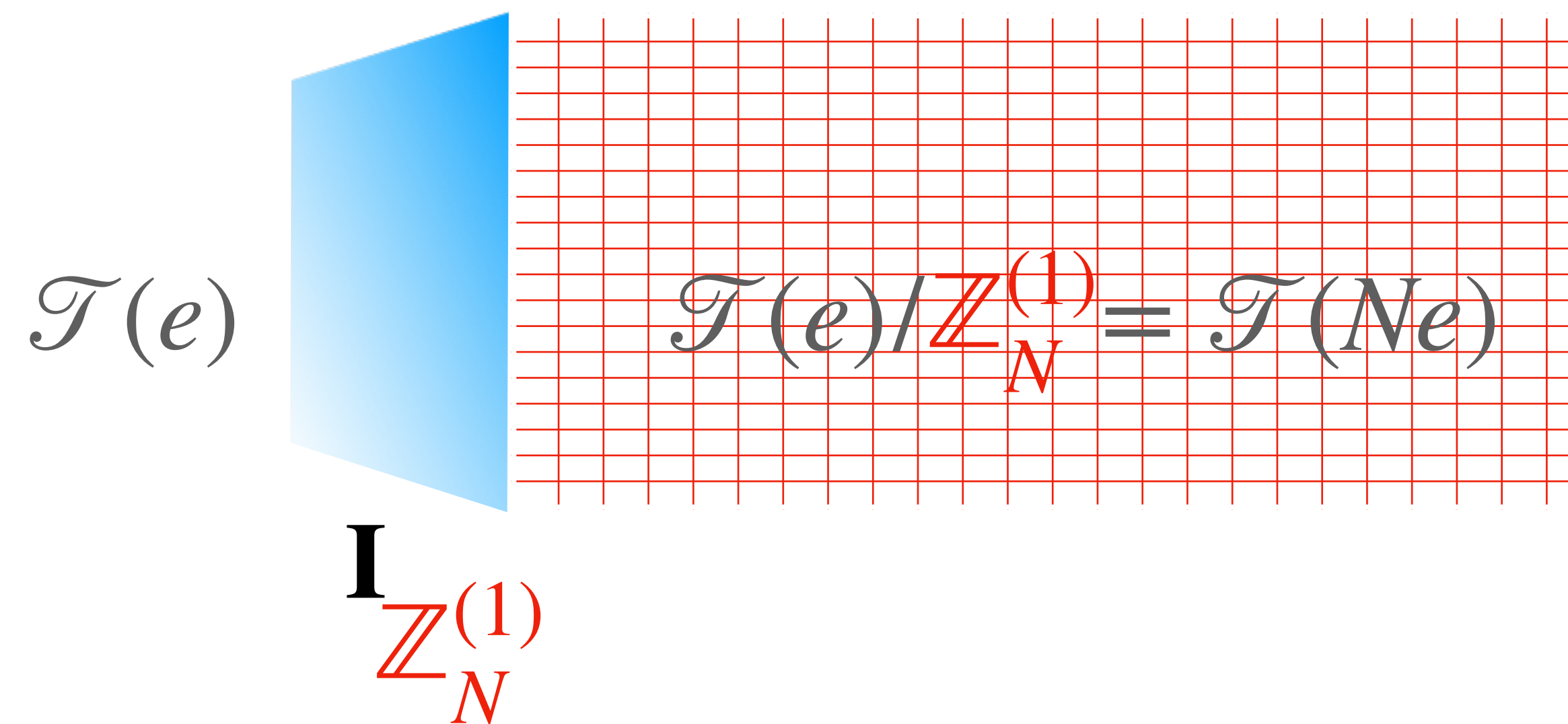
$$\mathbf{D} \otimes \bar{\mathbf{D}} = C_A$$

Example: Maxwell Theory

Consider gauging a $\mathbb{Z}_N^{(1)}$ subgroup of $U(1)_e^{(1)}$

This has the same effect as shifting the gauge potentials

$$a \rightarrow \frac{1}{N}a \quad a_D \rightarrow Na_D \quad e \rightarrow Ne$$



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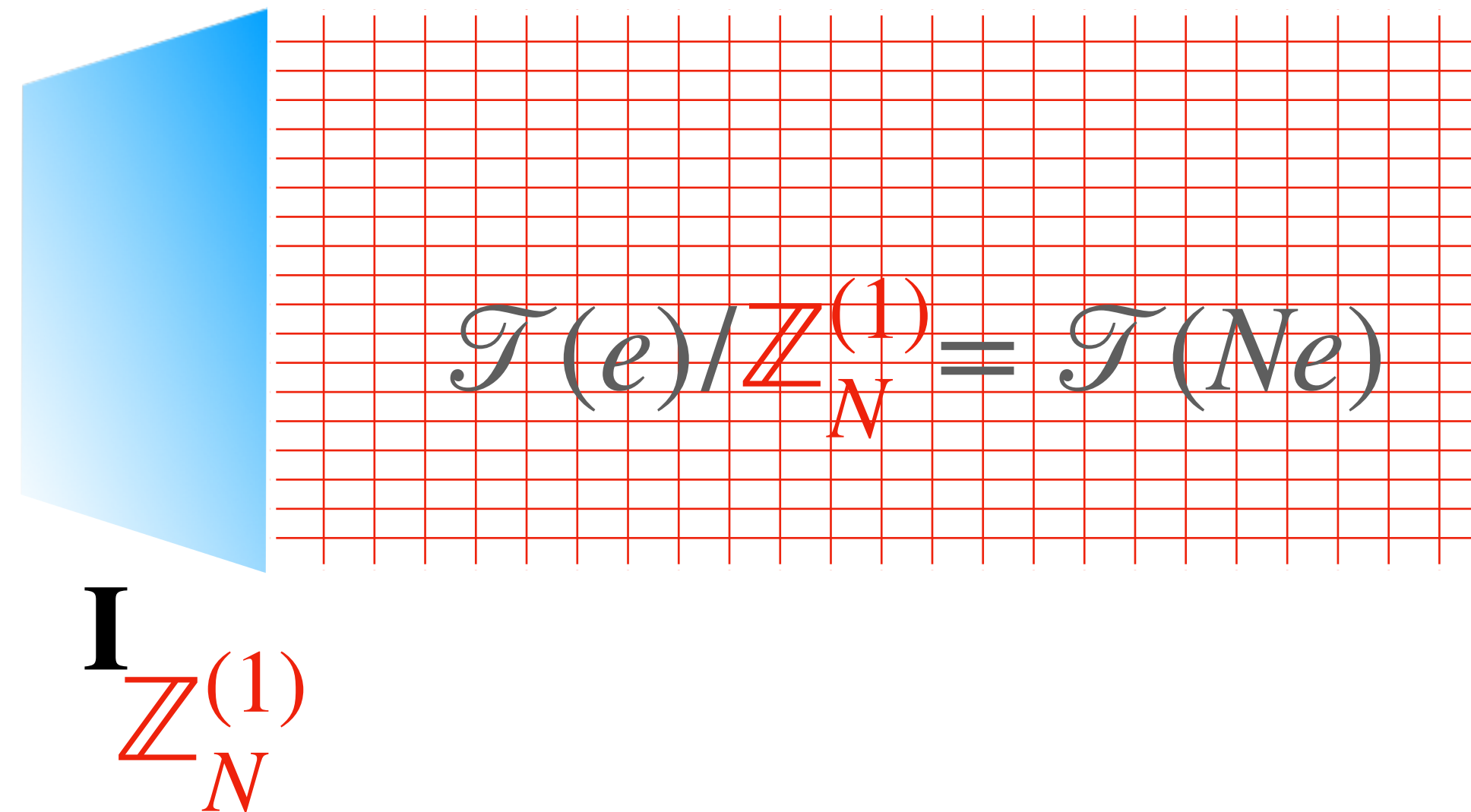
EM duality

$$\mathcal{T}(Ne) \cong \mathcal{T}(2\pi/Ne)$$

$$\Rightarrow e_* = \sqrt{2\pi/N} \quad \mathcal{T}(e)$$

Gives an equivalence

$\Rightarrow \mathcal{T}(e_*)$ has duality defects



Many more examples can be realized

- Using SYM at the self dual coupling $\tau = i$ one has the equivalence

$$SU(N) \cong PSU(N) = SU(N)/\mathbb{Z}_N$$

Cordova, Choi, Shao 21
Kaidi, Ohmori, Zheng 21

- Many more examples can be constructed exploiting class S theories at special points of their moduli spaces

$$\mathcal{X}_{(2,0)}^{6D}/\Sigma_{g,p}$$

Bashmakov, Del Zotto, Hasan 22

In particular one finds examples of n-ality defects, and generalized duality defects labeled by non-abelian finite groups in this way

Bashmakov, Del Zotto, Hasan, Kaidi 22
Antinucci, Copetti, Galati, Rizi 22

Symmetry theory

Idea: topological operators are encoded in a $D+2$ dimensional TFT



Kapustin Seiberg 14

Gaiotto, Kulp 20

Apruzzi, Bonetti, Garcia-Etxebarria,

Schafer-Nameki, Hosseini 21

Freed, Moore, Teleman 22

Bhardwaj, Shafer-Nameki 23 (many)

- Separates the topological symmetry data from the theory
- Allows to import techniques from TFT (cobordism hypothesis)
- Gives generalization of 't Hooft anomaly matching
- Streamlines construction of duality defects:



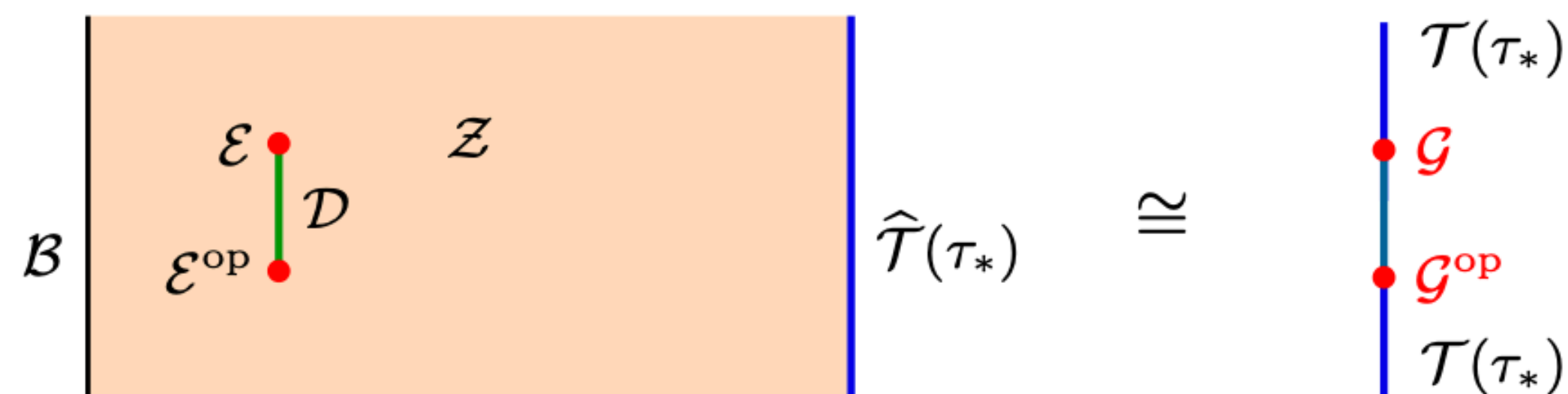
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Higher associators - continued

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Associativity condition with codimension higher than the defect worldvolume itself \rightarrow If non trivializable becomes **obstruction to gauging**

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Examples:

- 1. Quantum mechanics:** by Wigner \mathcal{H} can be in a projective representation: symmetry operators are inserted at points and associativity itself measures the anomaly
- 2. QFT in 1+1:** symmetries are lines, associativity is encoded by F-symbol, consistency of F-symbol is encoded by pentagonator. When symmetry is a group, pentagonators are parametrized by class in $H^3(\mathbb{G}, U(1))$ which is the standard anomaly

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Other notion: theory cannot have **trivial gapped symmetric phase**.

For non-invertible symmetries the two notions don't coincide and the second is more restrictive

Choi, Rayhuan, Sanghavi, Shao 23

Cordova, Hsin, Zheng 23

Antinucci, Benini, Copetti, Rizi 23

Chiral Symmetry

Choi, Lam, Shao 22
Córdova, Ohmori 22

Consider 3+1 dimensional QFT with $U(1)^{(1)}$ symmetry and a 1-form that satisfies an anomalous conservation equation of the ABJ type

$$d \star j_{\chi}^{(1)} = \star j^{(2)} \wedge \star j^{(2)}$$

Then **there is** a symmetry for wannabe $U(1)_{\chi}^{(0)}$ quantum numbers

Generators:

$$D_{p/N}^{(0)}(\Sigma^3) = \mathcal{U}_{p/N}^{\chi} \otimes \mathcal{A}^{N,p}[b] \quad b = \frac{2\pi}{N} \star j^{(2)}$$

chiral rotation
generator

Hsin-Lan-Seiberg
minimal 3d \mathbb{Z}_N TFT

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Choi, Lam, Shao 22
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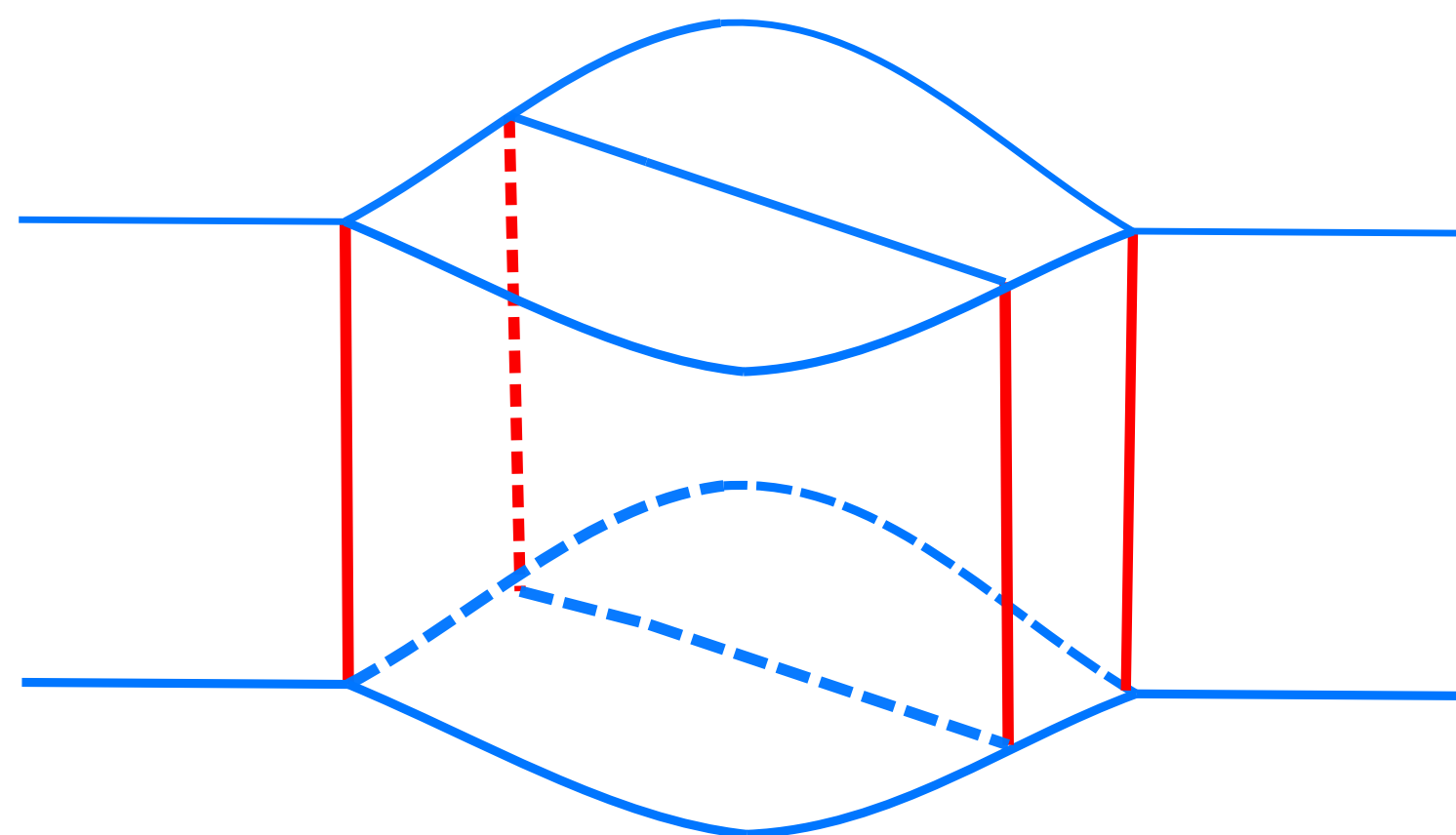
$\mathbb{Z}_L^{(1)}$ subgroup
condensates

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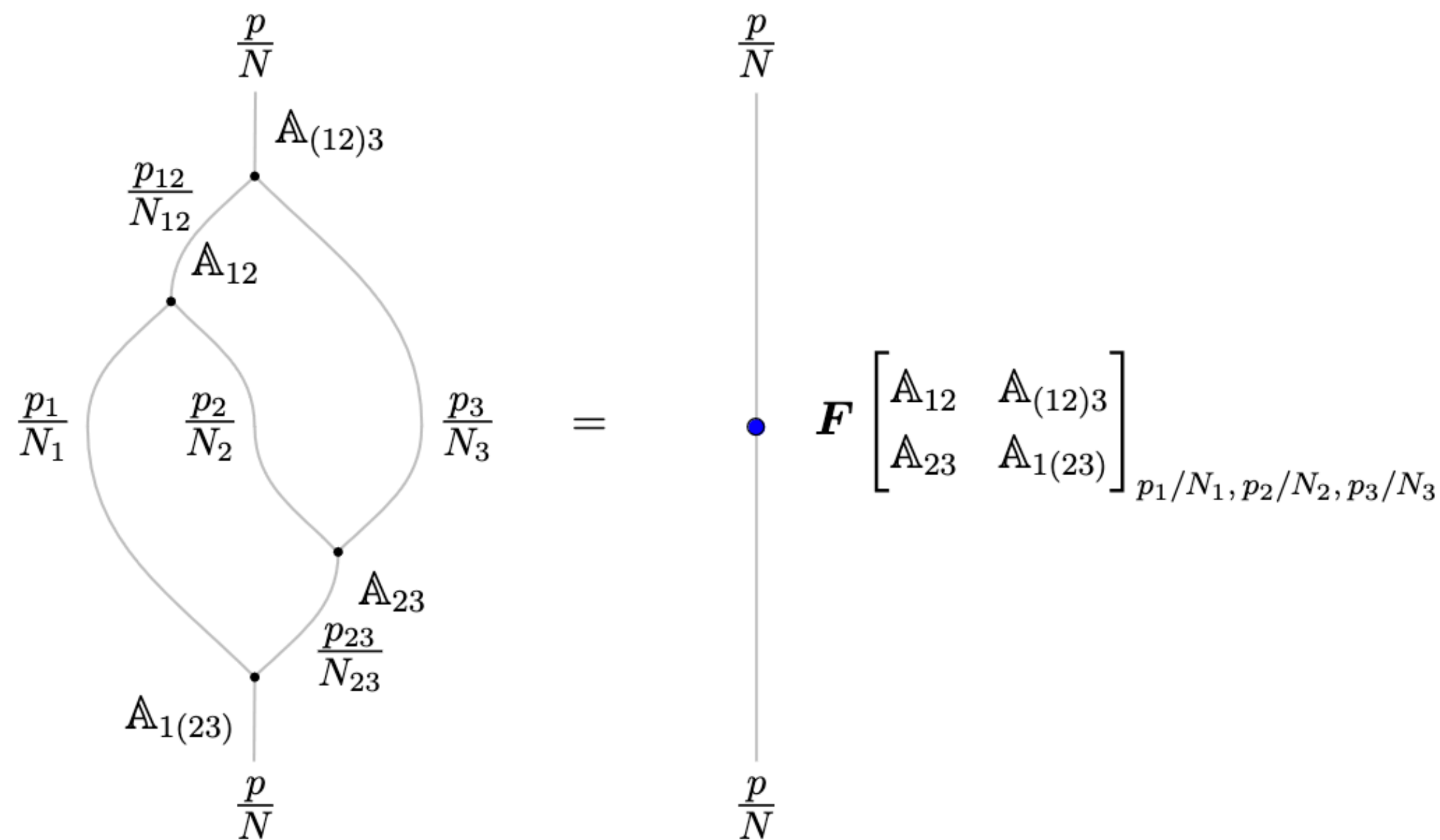
Discrete torsion

$$\alpha \in H^3(B\mathbb{Z}_L, U(1))$$

Chiral Symmetry Associator



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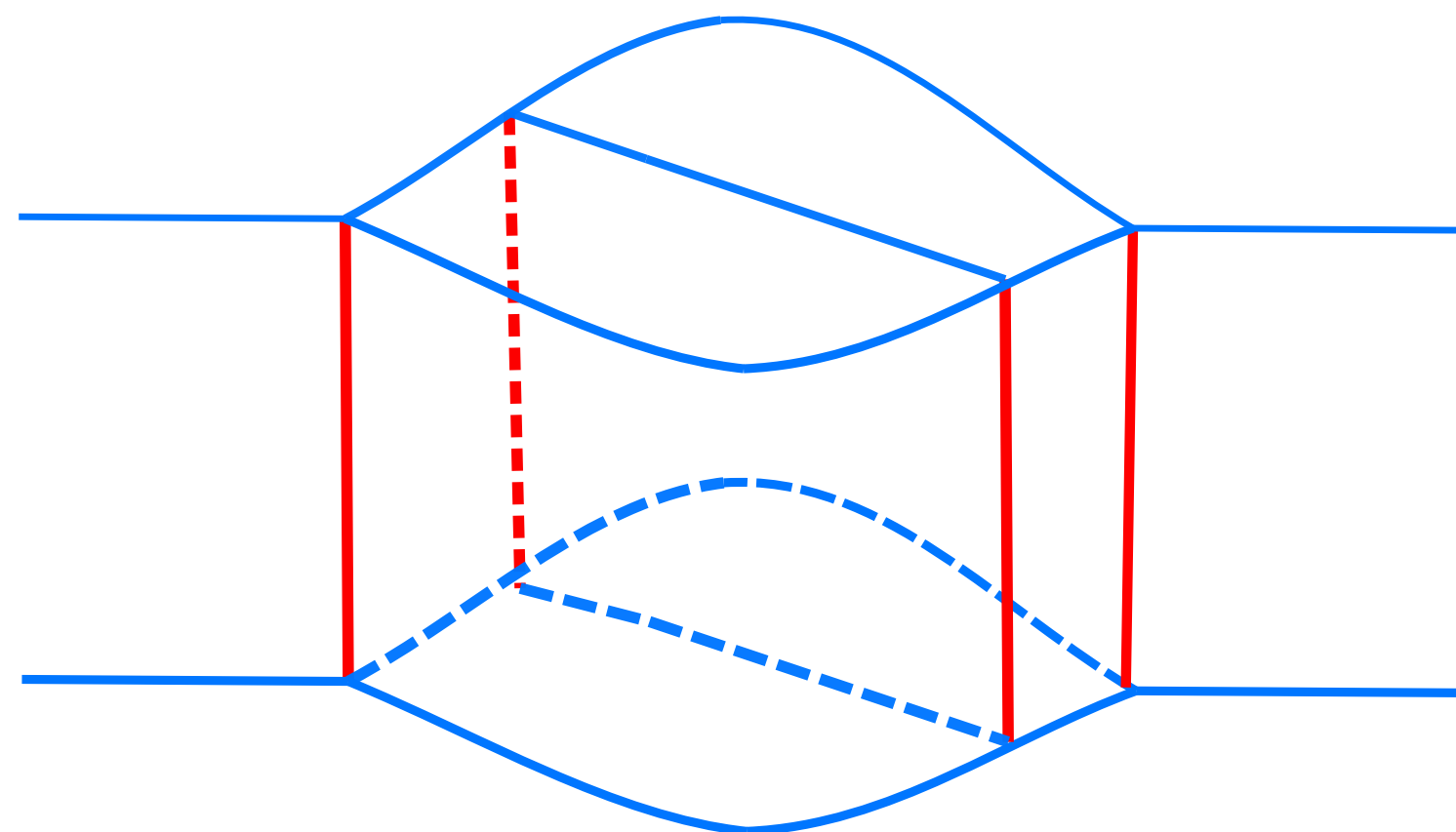
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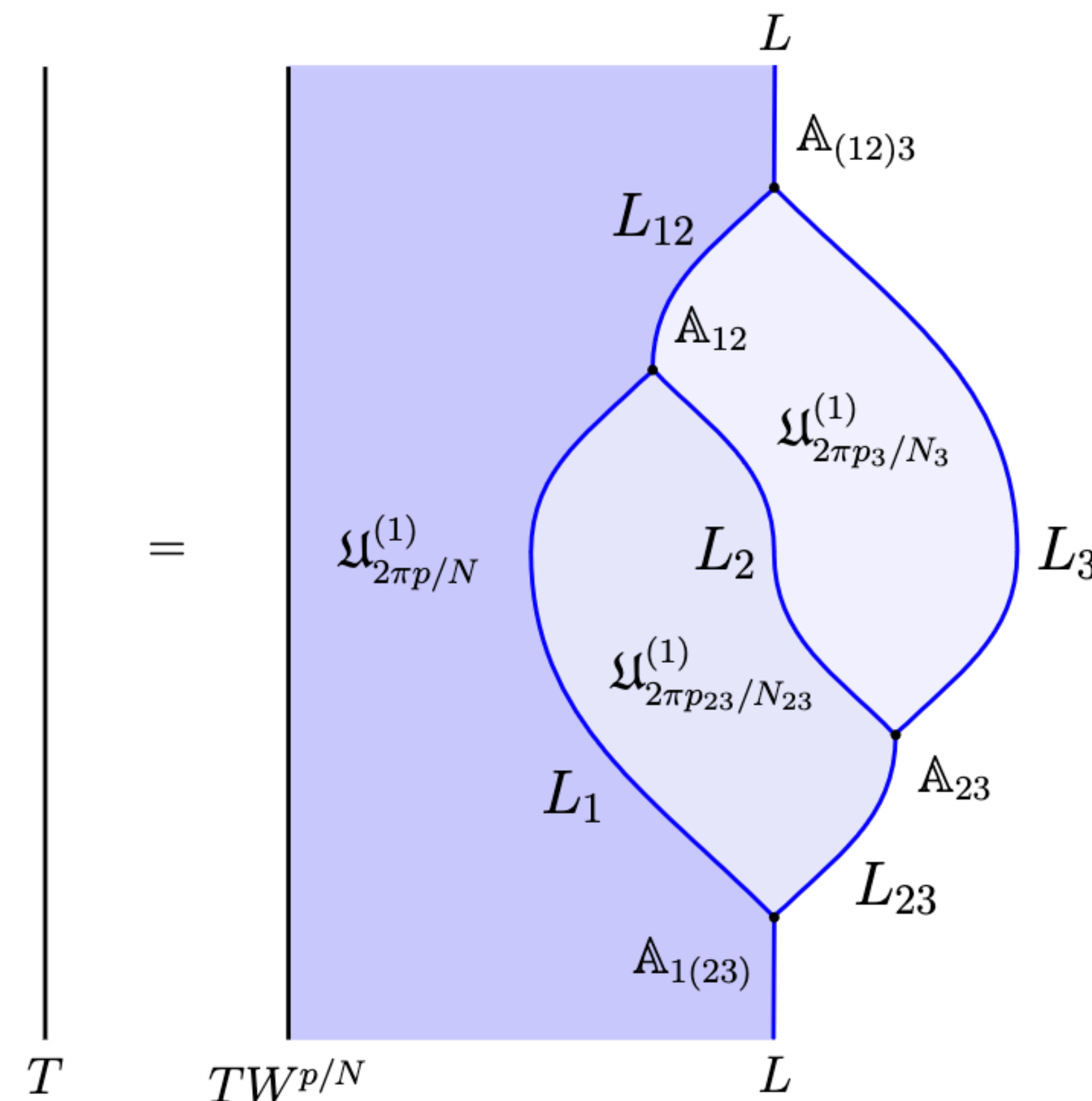
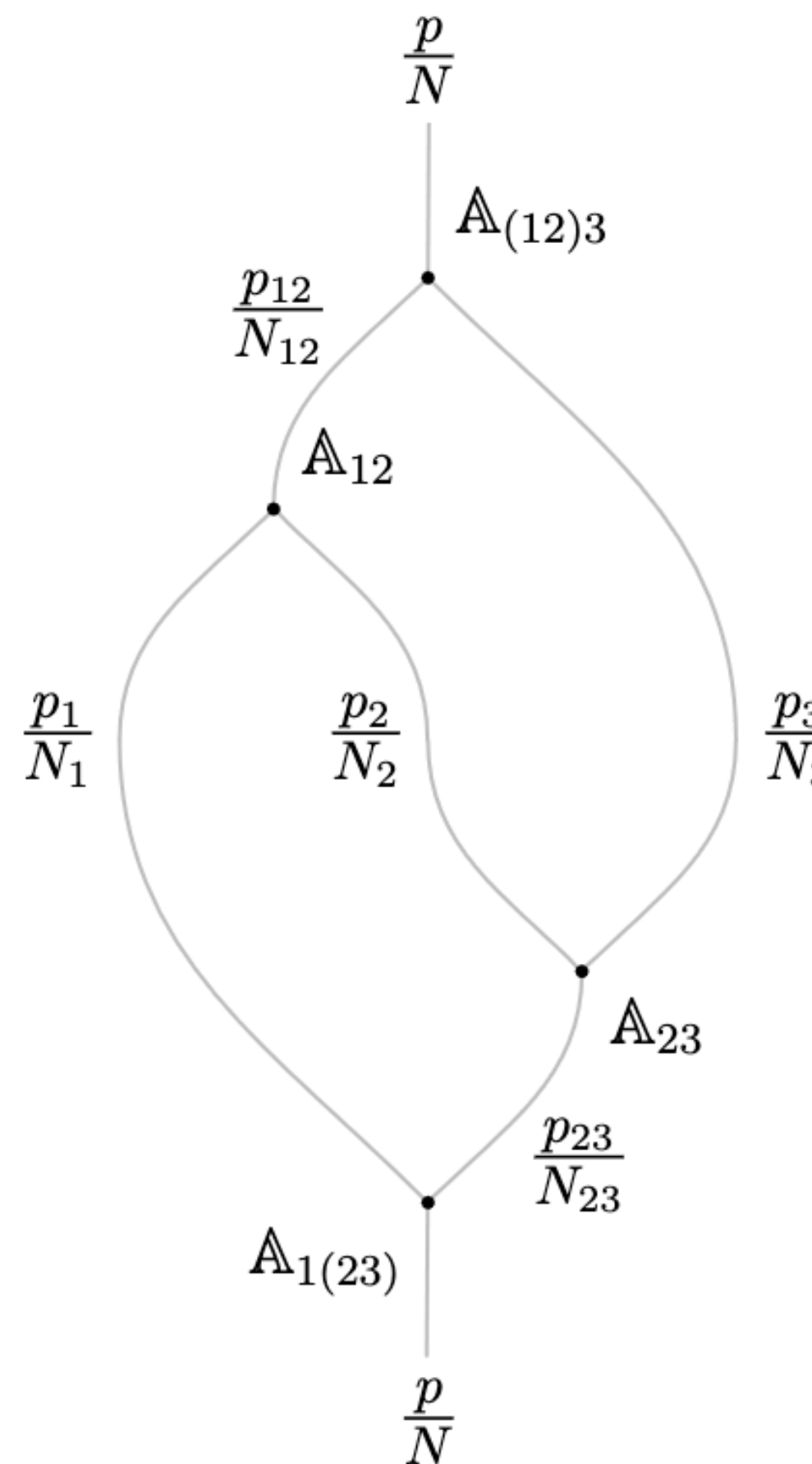
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Chiral Symmetry Associator



To **detect** the associator:
 we throw it against a 't
 Hooft line, which gets
 dressed by Wilson lines
 because of the Witten
 effect



Chiral Symmetry Ward Identity

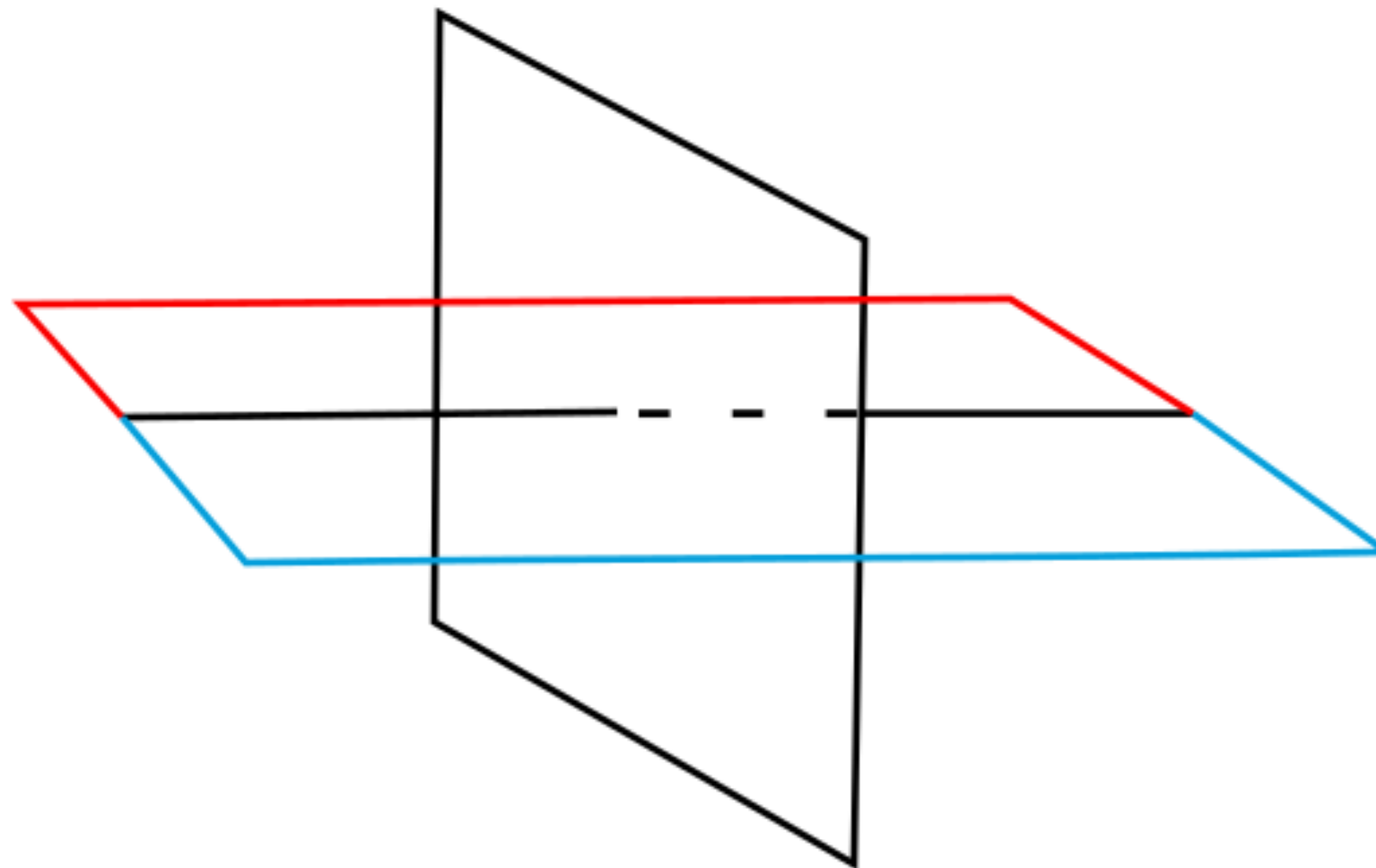
Consider correlators on $S^2 \times S^2$

Chiral Symmetry Ward Identity

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A bit hard to visualize 4-manifolds. **IDEA:** draw fourth direction as time.

- **Solid black** we draw \mathbb{R}^3
- **Blue** we draw past
- **Red** we draw future



These are two planes intersecting transversally at a point in \mathbb{R}^4

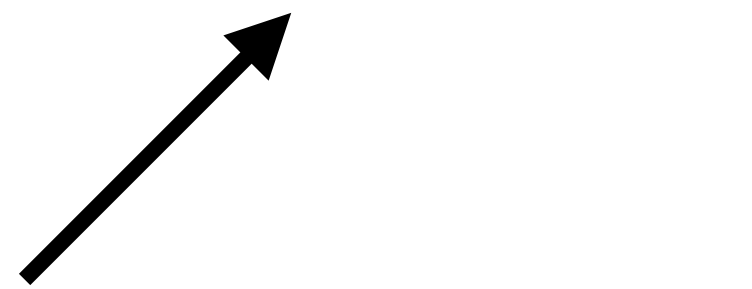
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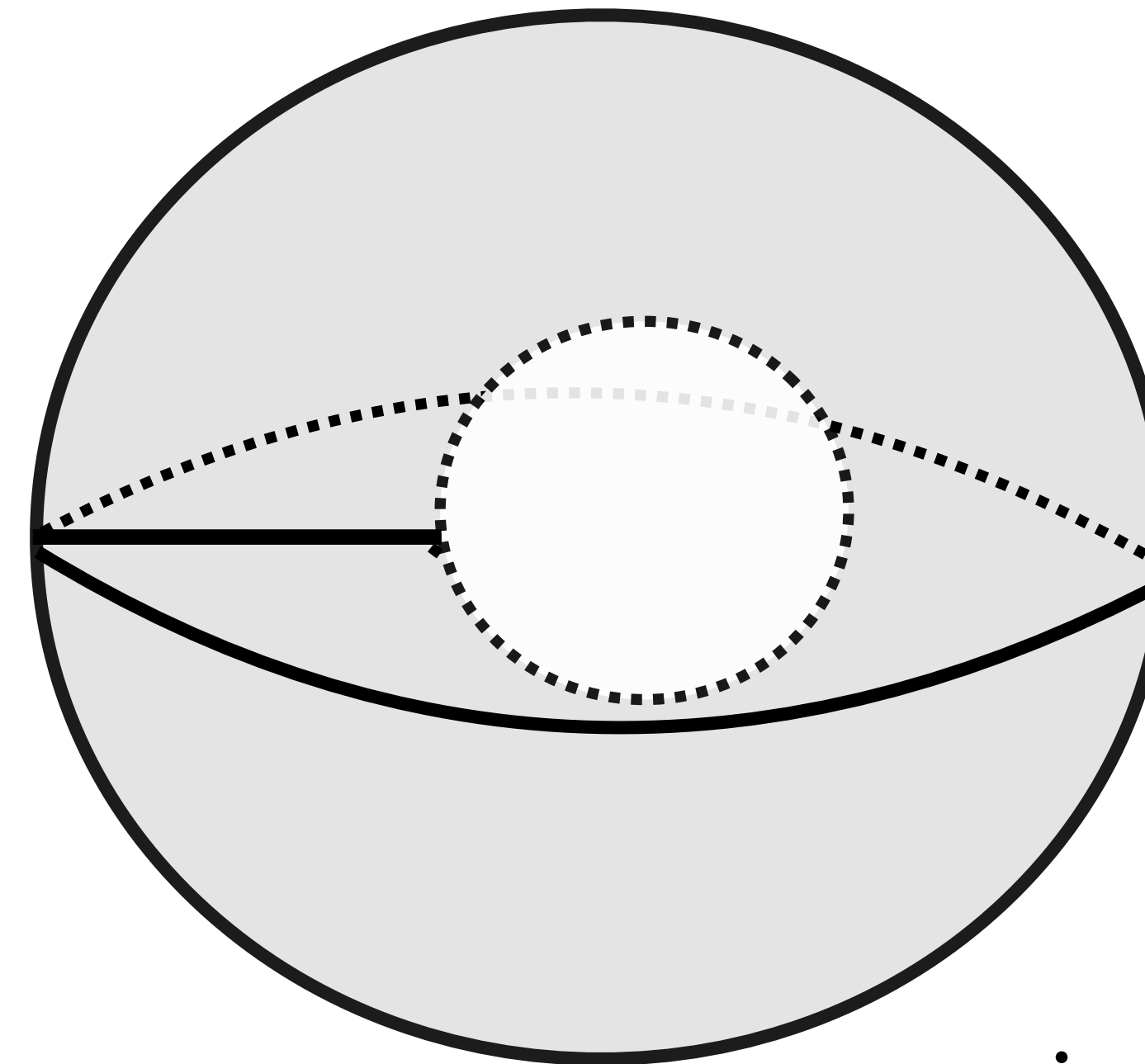
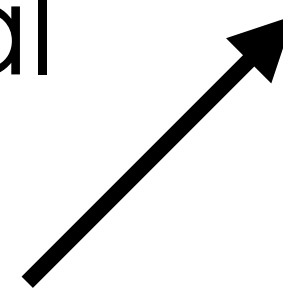
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inner S^2 and outer
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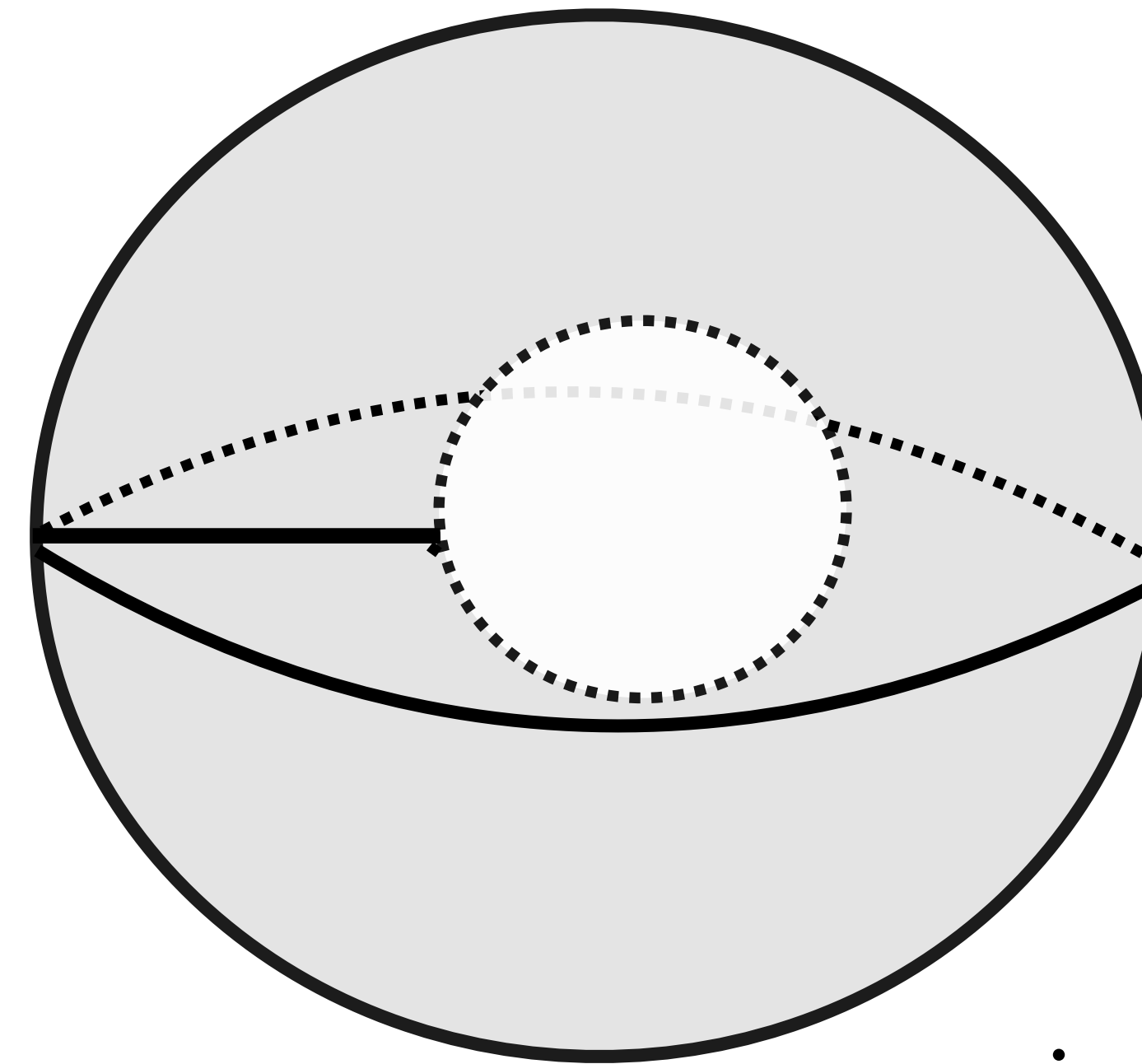
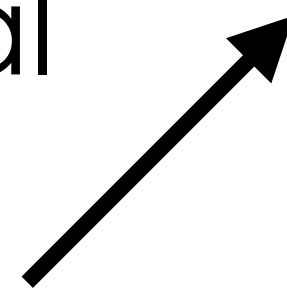
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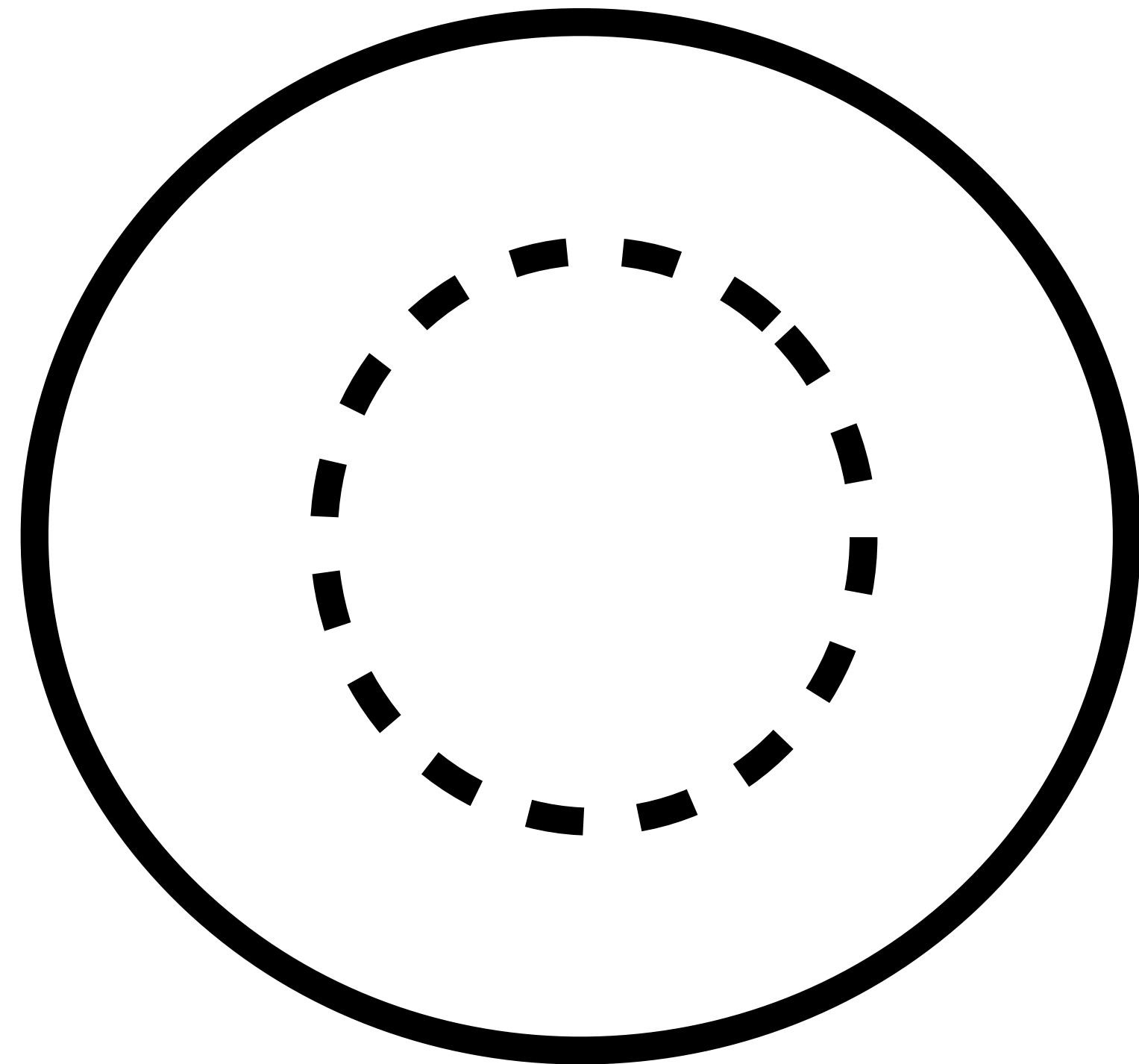
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Simplify the drawing



Chiral Symmetry Ward Identity

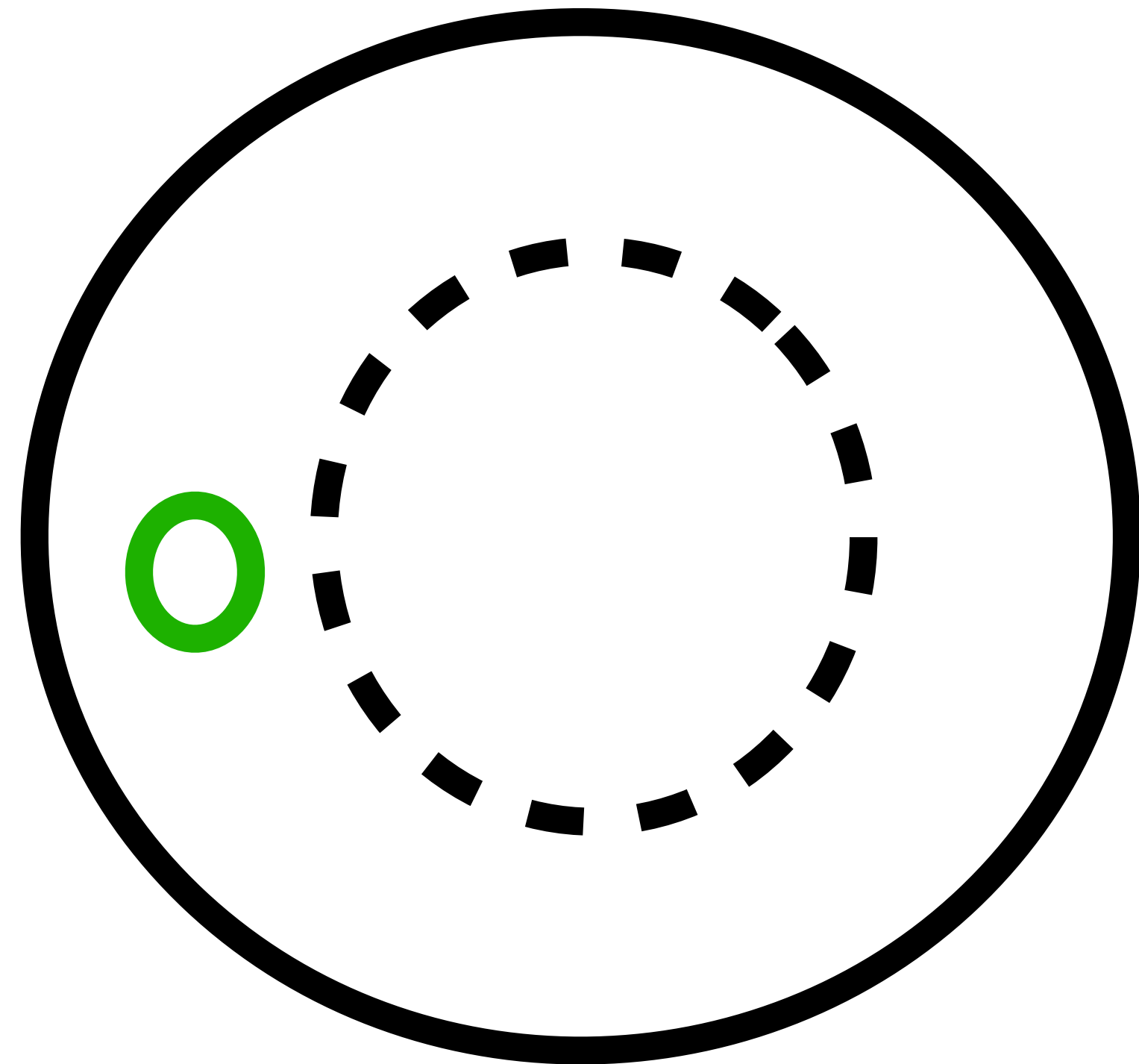
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Simplify the drawing

Nucleate a bubble of
chiral symmetry: S^3
obtained by S^2 that shrinks
in time.



Chiral Symmetry Ward Identity

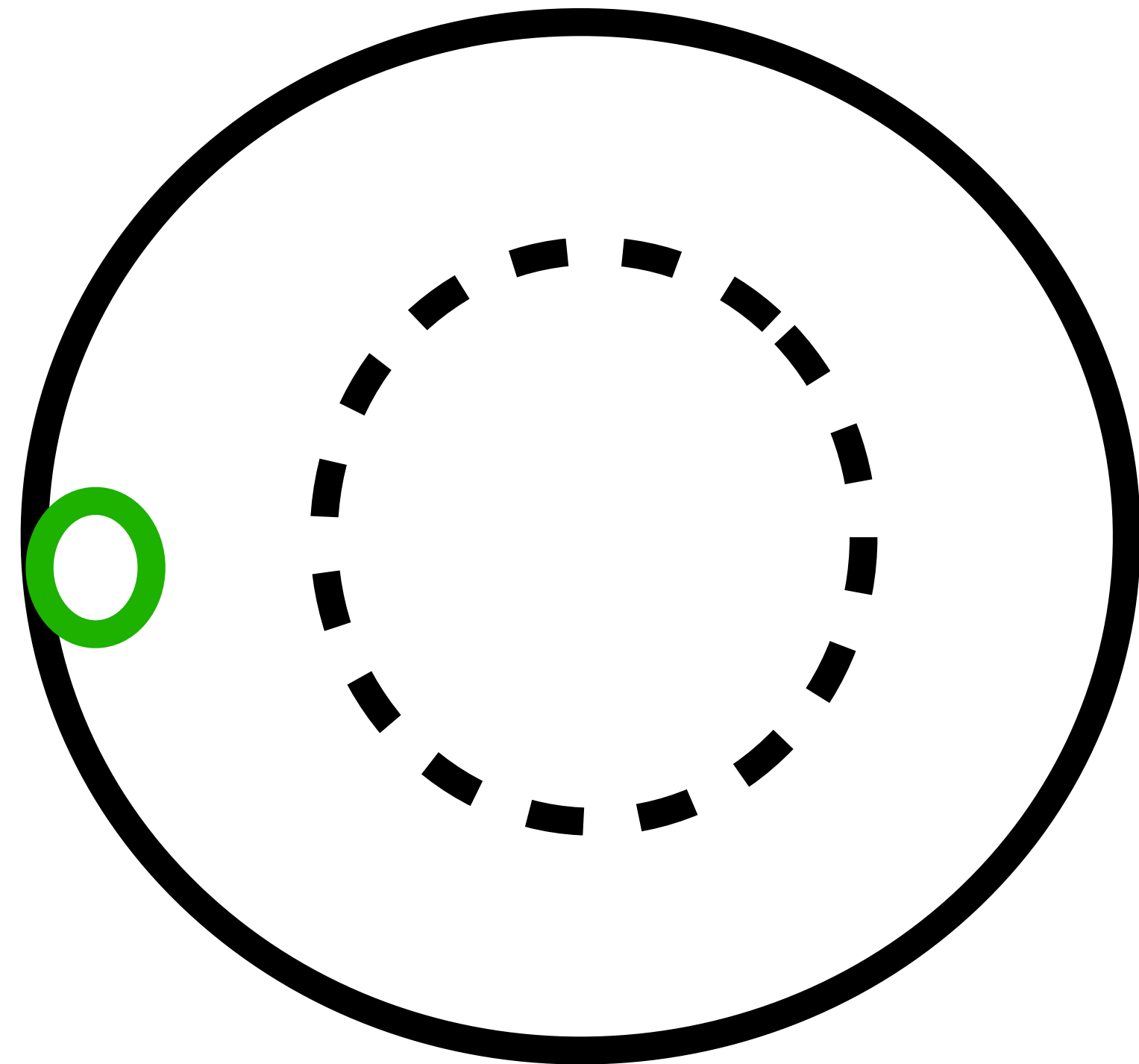
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- **Solid black** we draw $S^1 \times S^2$
- **Blue** we draw past S^1 shrinks
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Simplify the drawing

Nucleate a bubble of
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Chiral Symmetry Ward Identity

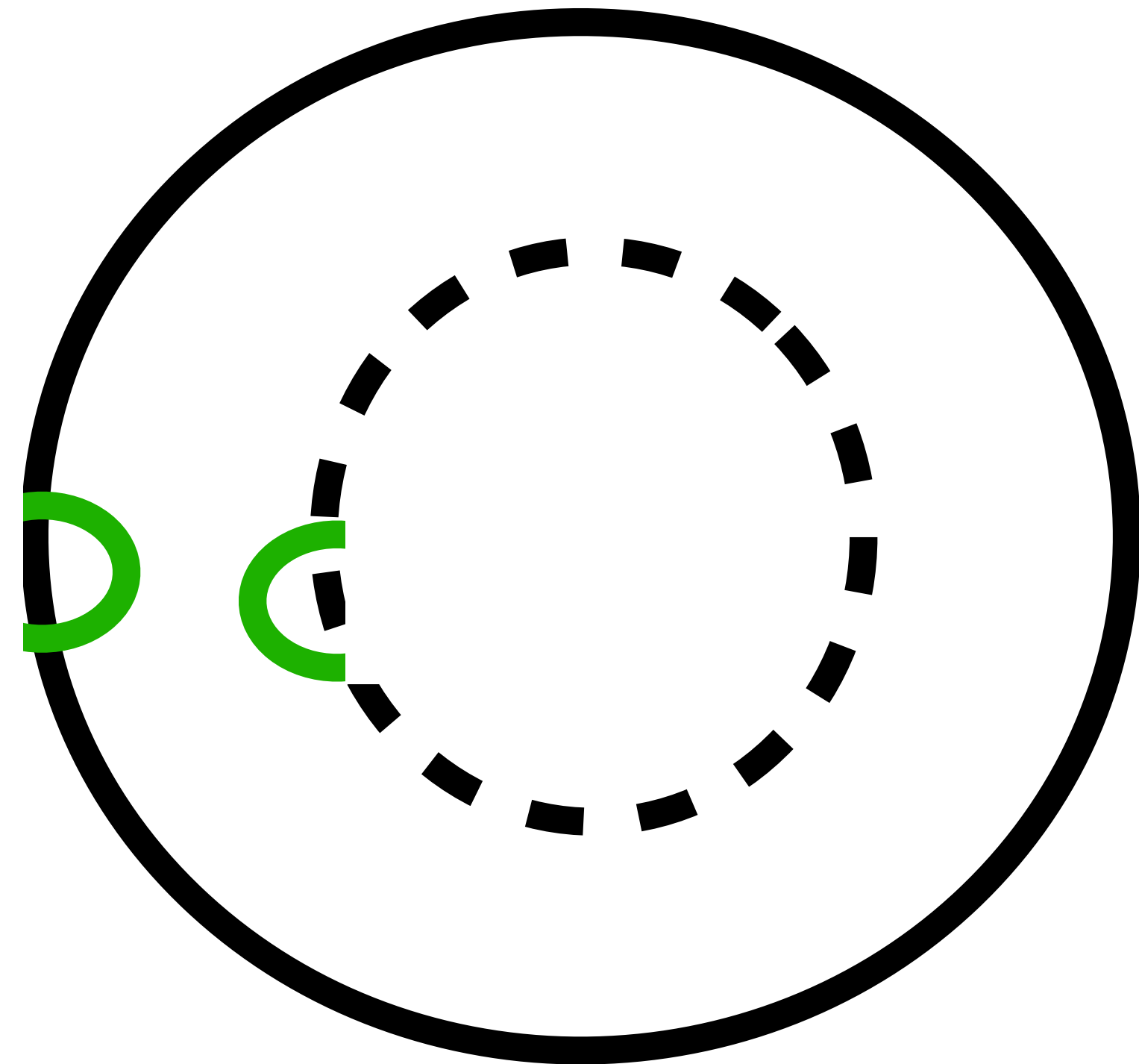
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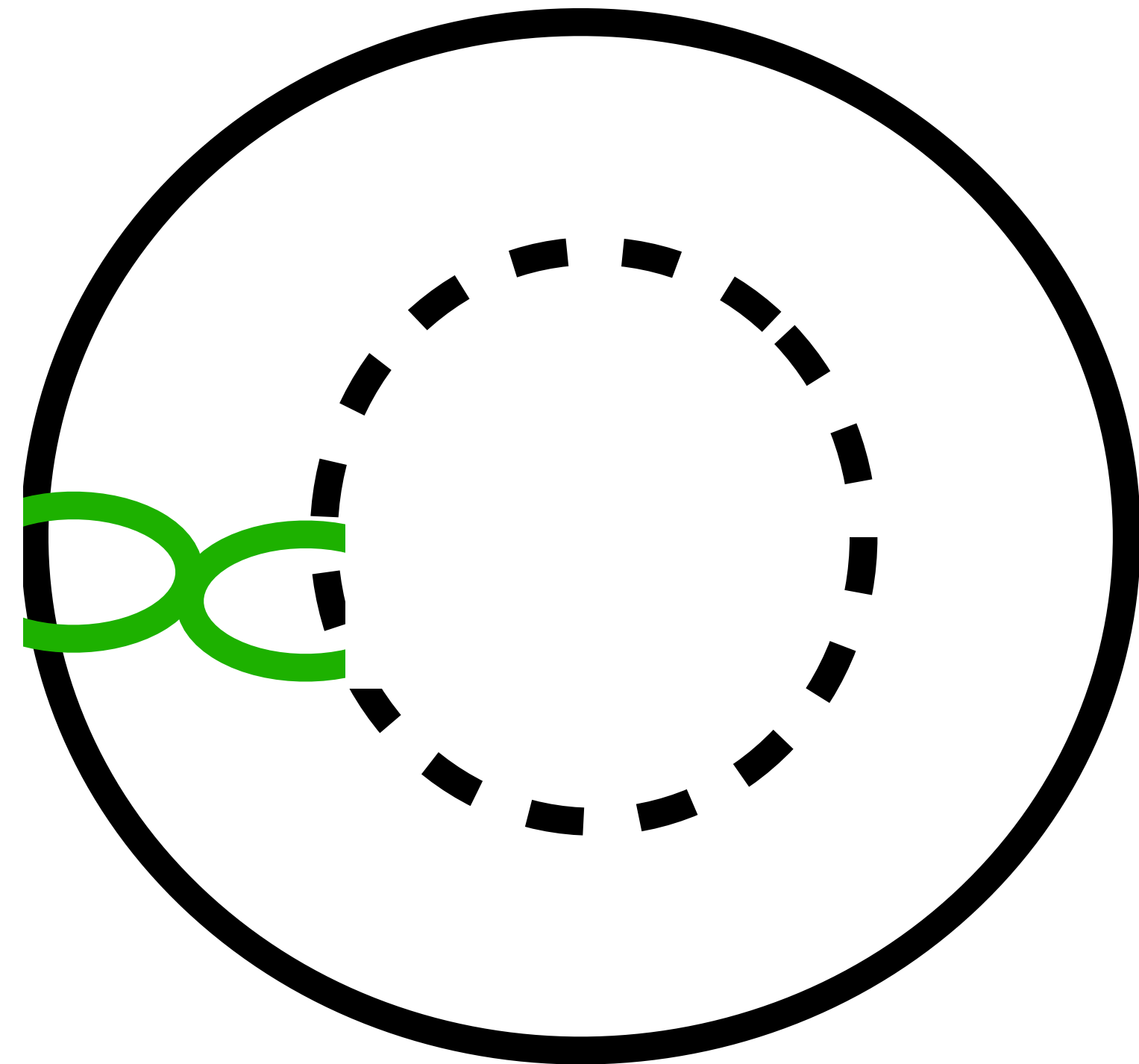
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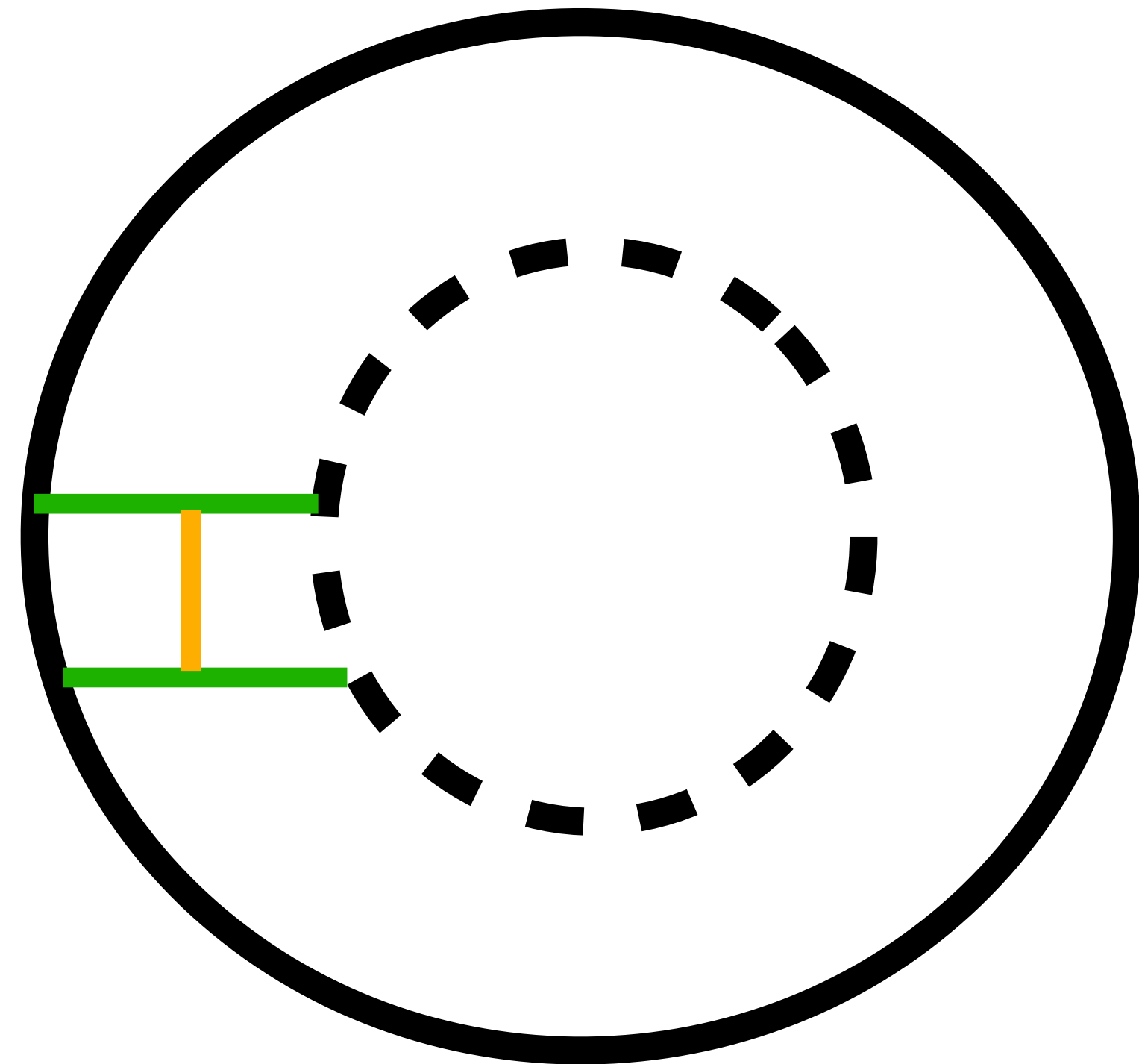
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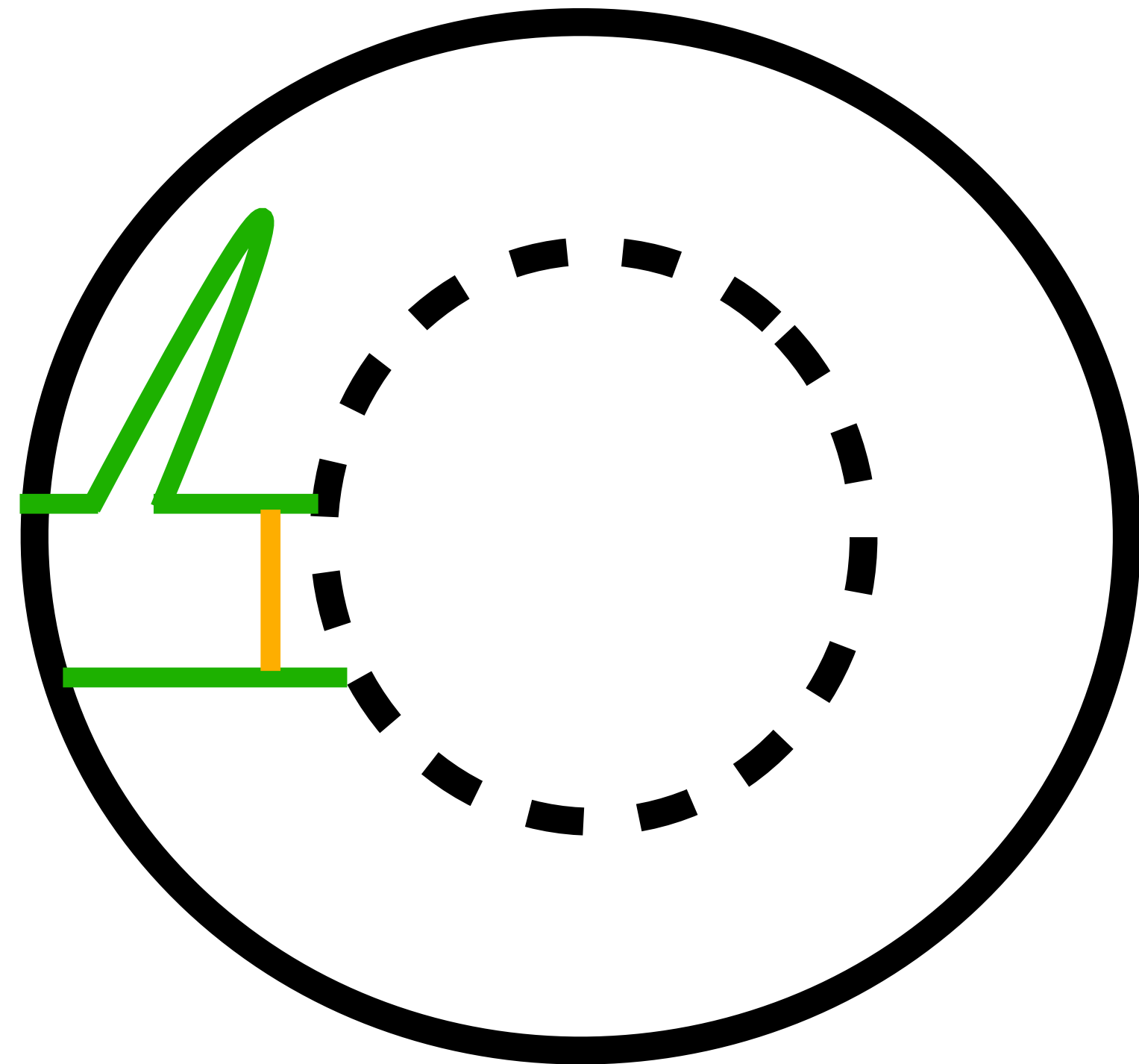
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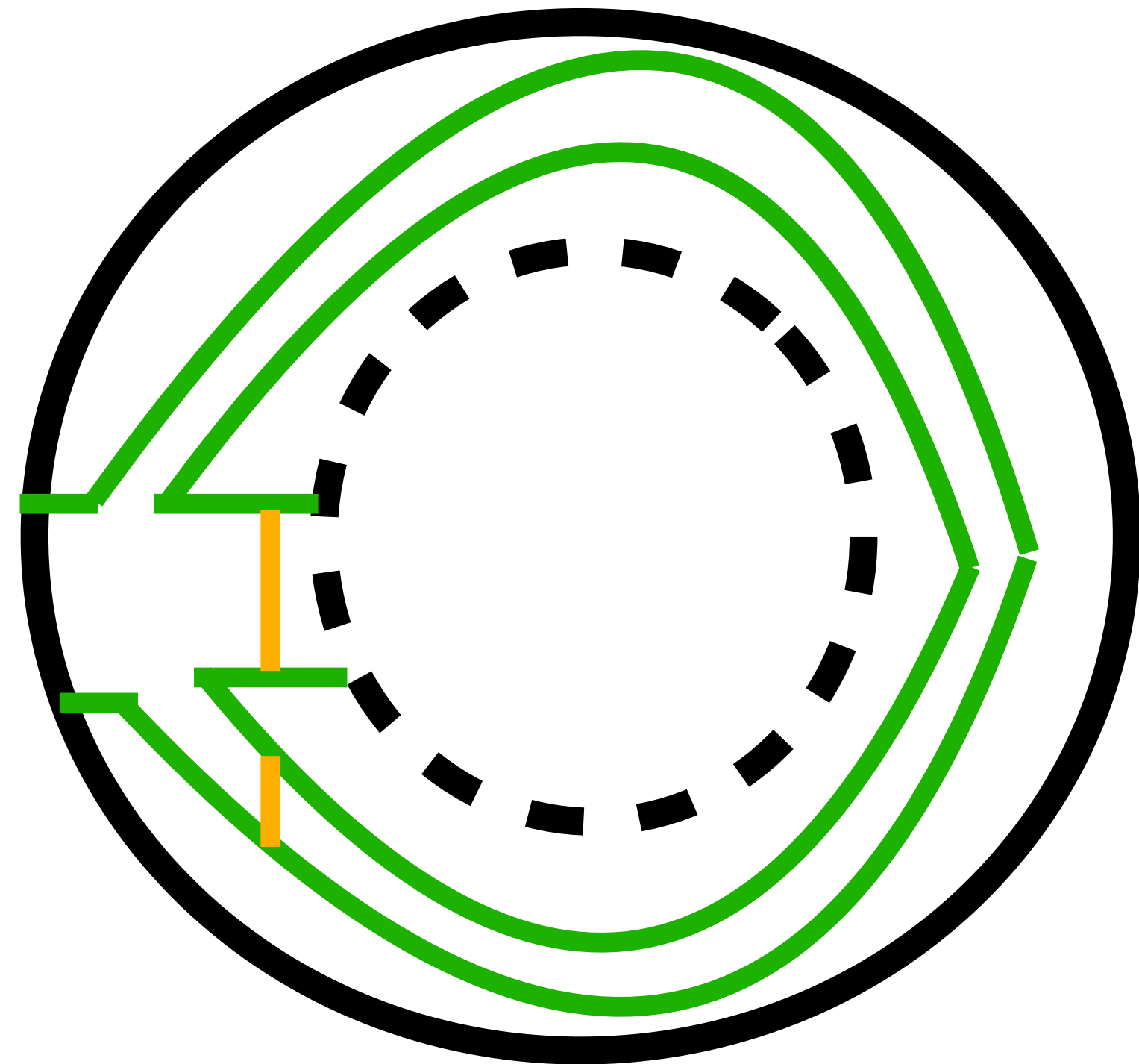
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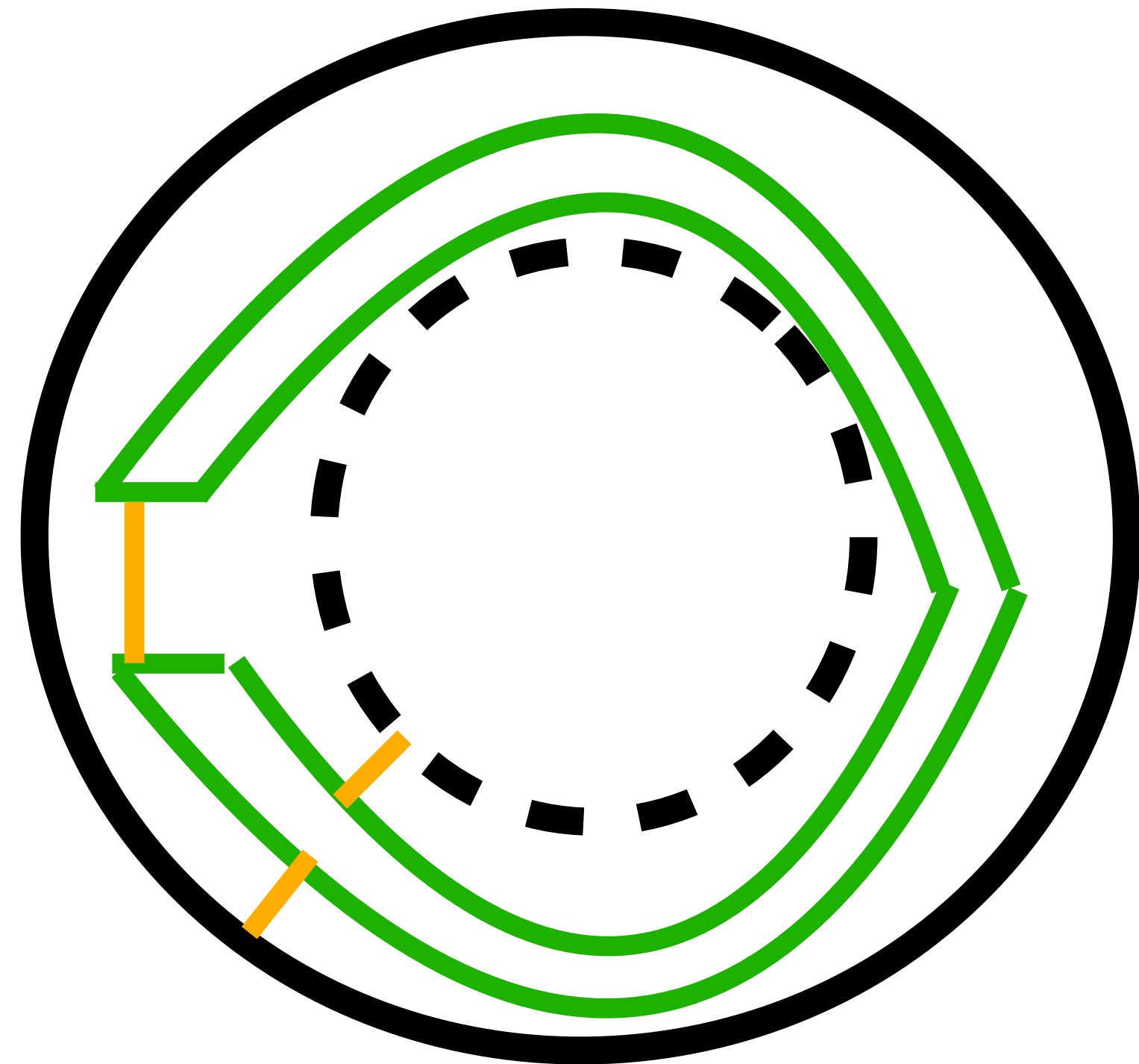
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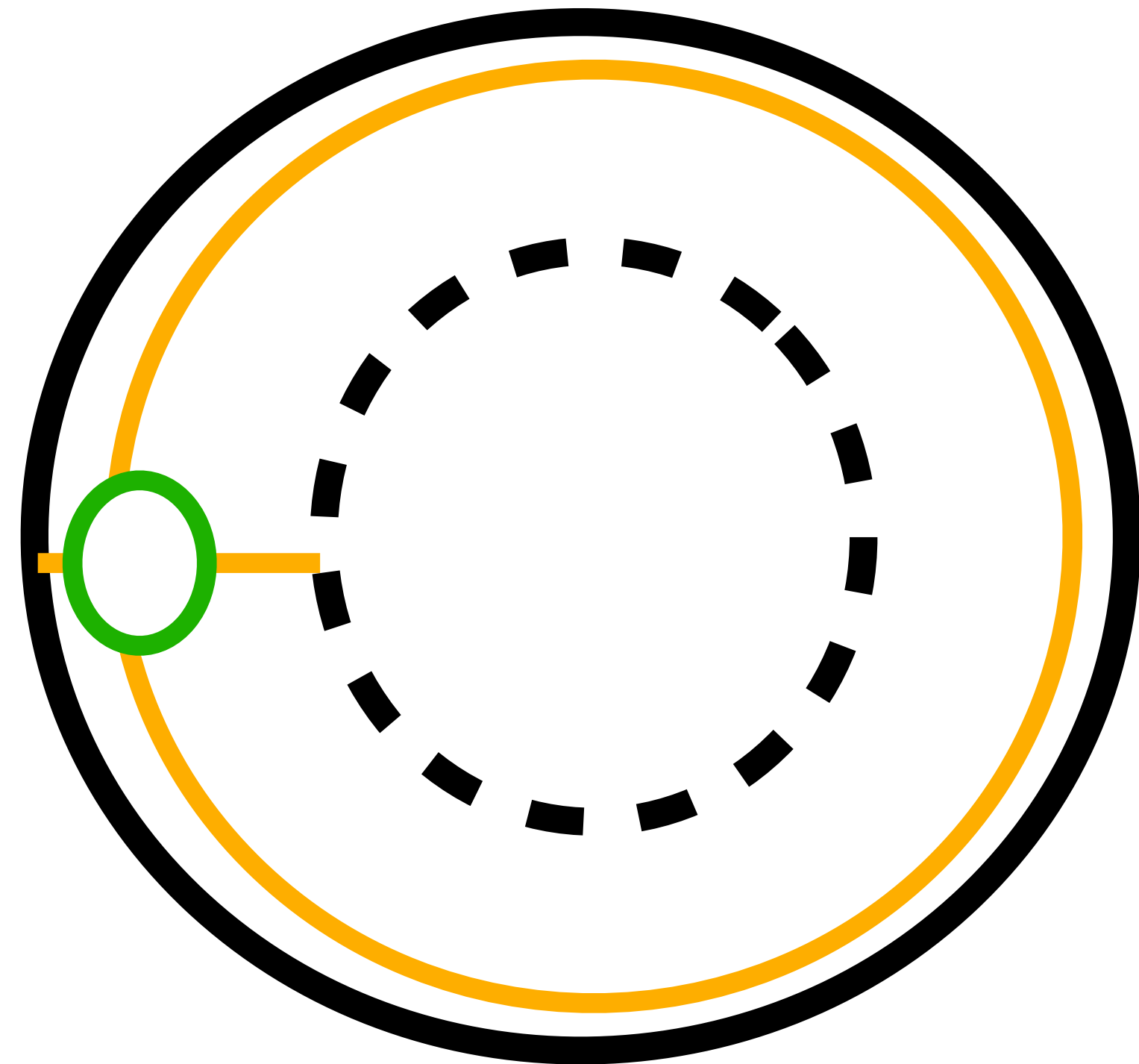
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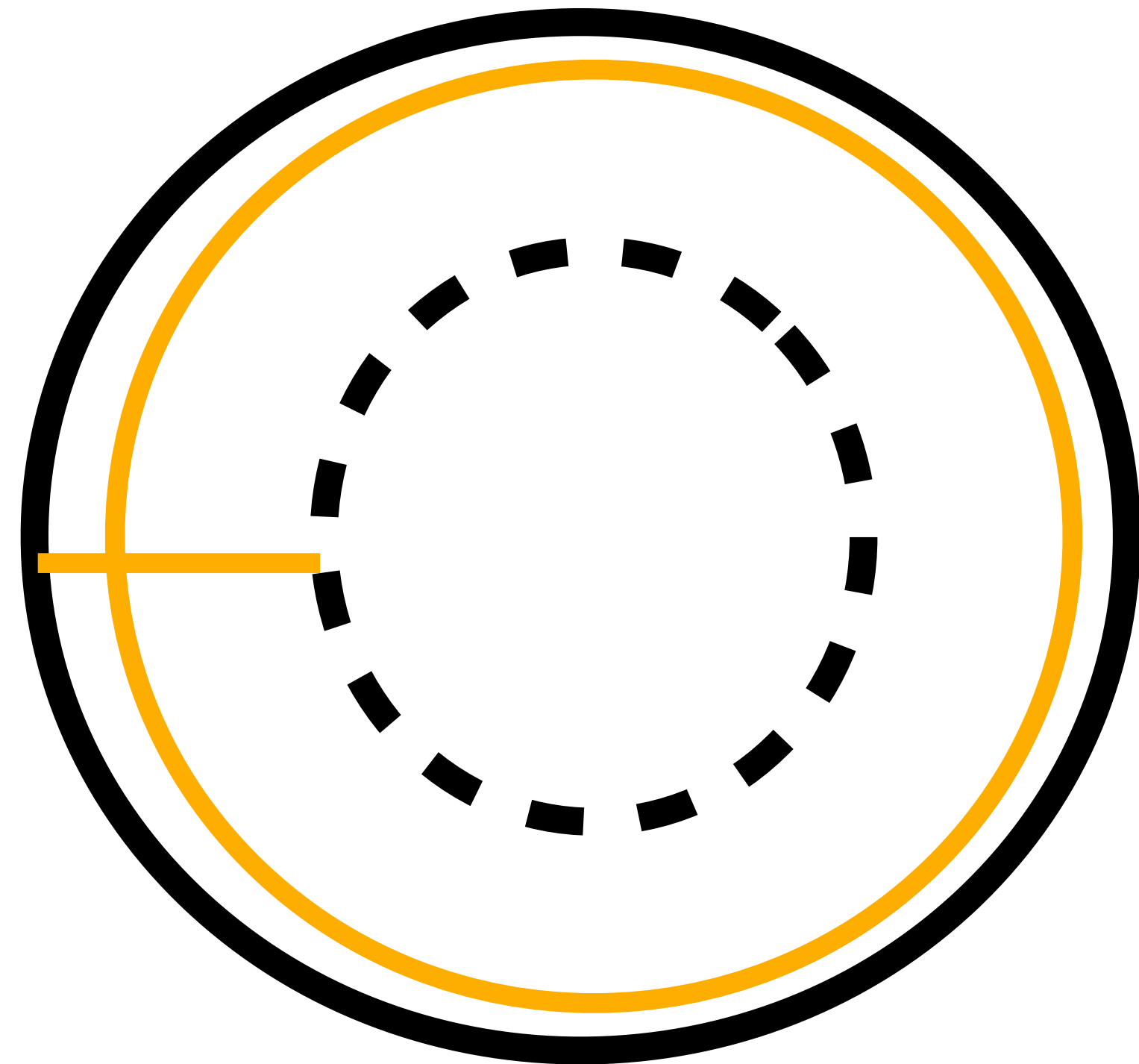
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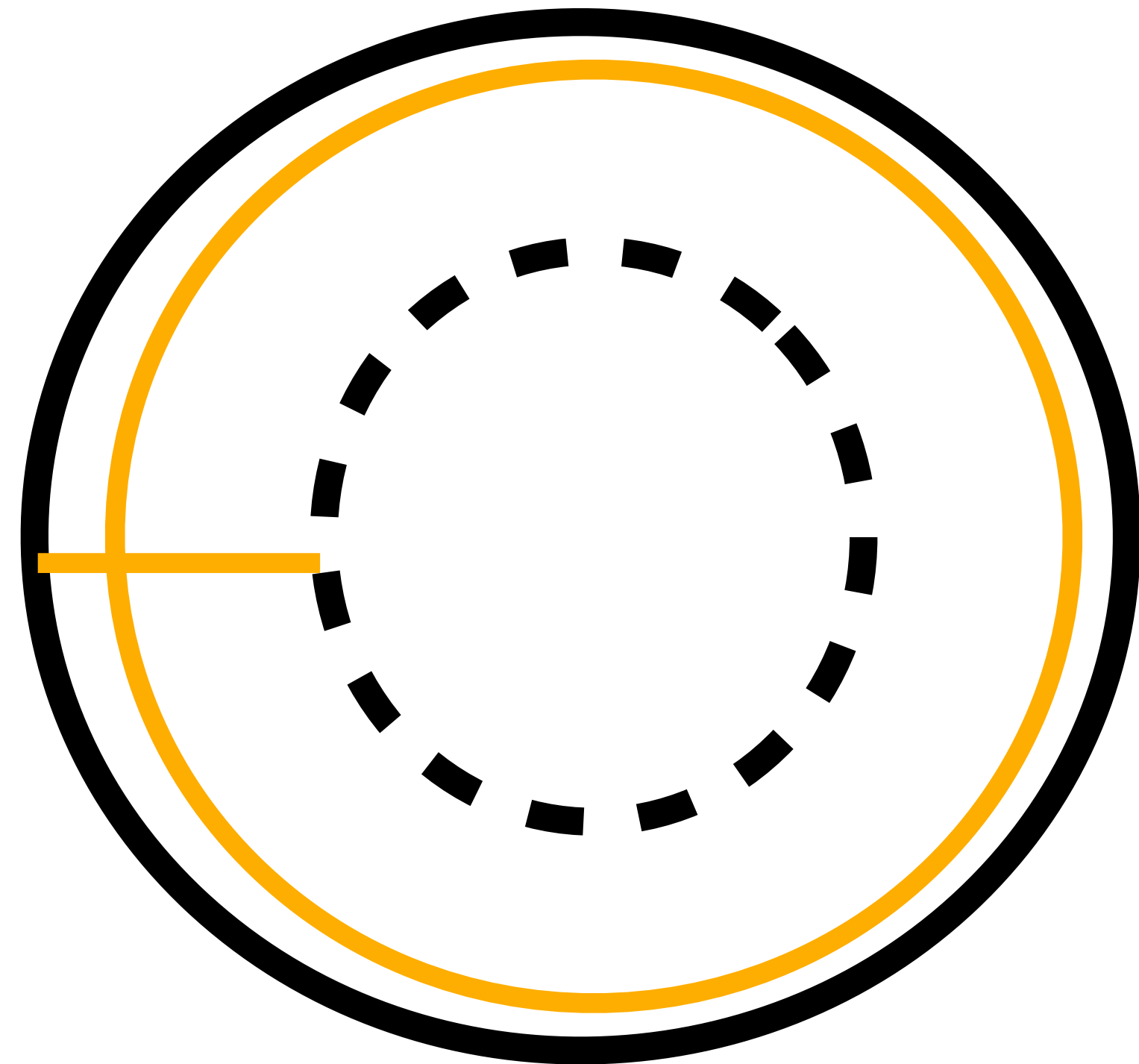
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Hopf link correlator of lines in $\mathcal{A}^{N,p}$

$$\mathcal{Z}_{S^2 \times S^2} = \frac{1}{N} \sum_{b,c \in \mathbb{Z}_N} \frac{\left\langle L^b \text{ (Hopf link) } L^c \right\rangle_{\mathcal{A}^{N,p}}}{\langle \cdot \rangle_{\mathcal{A}^{N,p}}} \mathcal{Z}[b,c]_{S^2 \times S^2}$$



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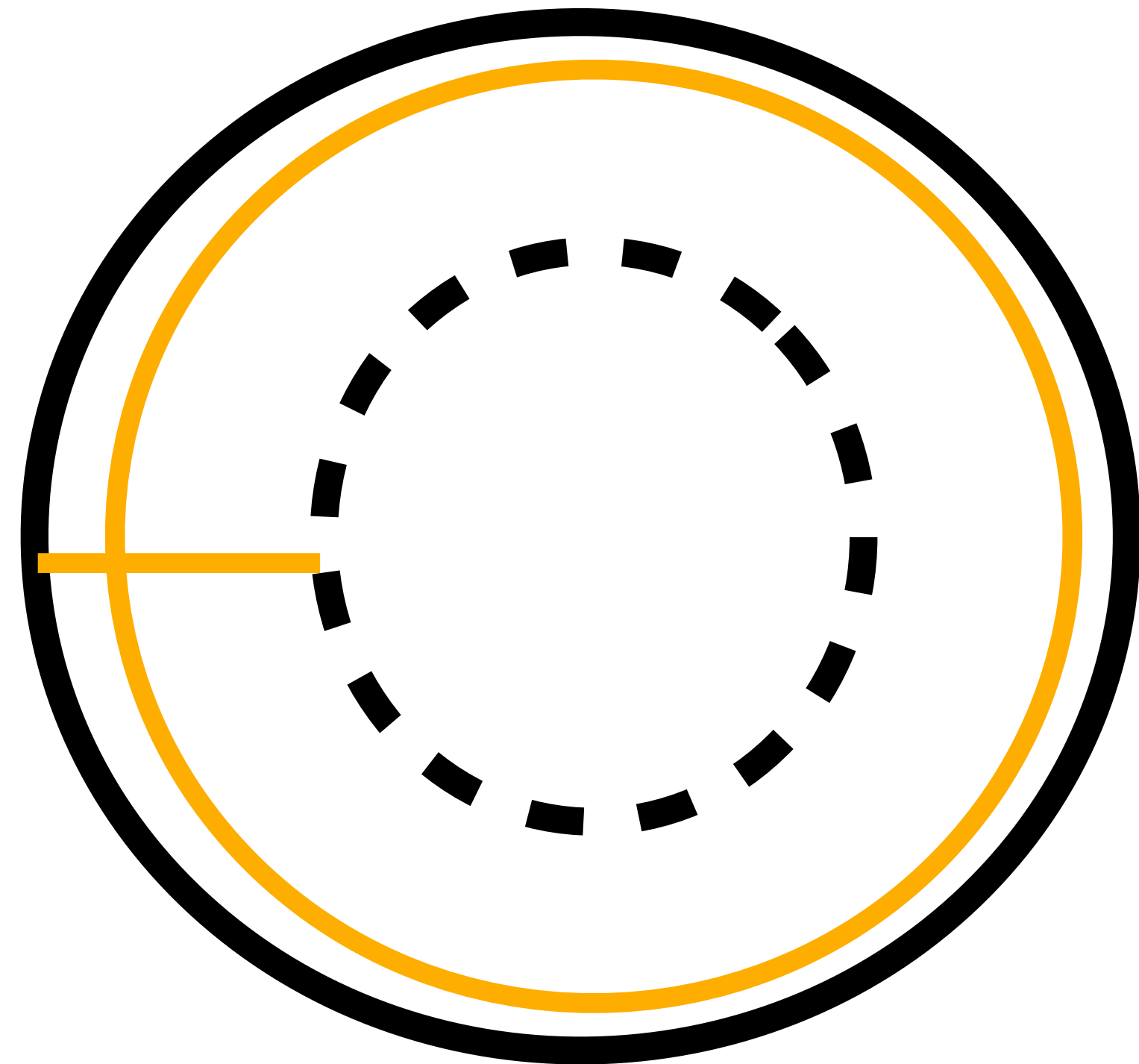
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Easy to generalize thanks to Kirby description of 4-manifolds!

Ward Identities and bordisms

Idea is very simple: two ways to look at a compact 4-manifold:

- Null bordism
- Handle decomposition

$$\emptyset \rightarrow \mathbb{S}^3 \xrightarrow{H_1} \cdots \xrightarrow{H_n} \emptyset$$

surgery diagram

$\mathbb{S}^2 \times \mathbb{S}^2$ example:

$$\emptyset \rightarrow \mathbb{S}^3 \xrightarrow{H_1} \mathbb{S}^1 \times \mathbb{S}^2 \xrightarrow{H_2} \mathbb{S}^3 \rightarrow \emptyset$$

Where we are only gluing 2-handles.

For all 4-manifolds with a handle decomposition with 2-handles only the surgery diagram we obtain is a link in \mathbb{S}^3 : the Ward identity we obtain is the expectation value of such a link, decorated with lines, in the $\mathcal{A}^{N,p}$ 3d TFT.

Executive Summary

Symmetries are a topological sector of the spectrum of operators

The latter is organized by a higher category

We have seen some applications of these ideas

This is just the beginning of a long story

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Hey, but what about **branes** and **strings**?

Maybe I can say something on the blackboard, but for sure I am already out of time...

Thank you for your attention!

Thank you for your attention!

But before I go let me mention:

Inauguration of the Centre for Geometry and Physics with lecture by honorary doctorate Nikita Nekrasov

Add to your calendar

- Date: 24 January, 13:00–15:00
- Location: Ångströmlaboratoriet, Lägerhyddsvägen 1 , lecture hall Eva von Bahr
- Lecturer: Nikita Nekrasov
- Organiser: Department of Mathematics and Department of Physics and Astronomy
- Contact person: [Tobias Ekholm](#)
- [Föreläsning](#)

Welcome to the inauguration of the Centre for Geometry and Physics. The centre starts 2024 based on grants from the Swedish Research Council's excellence initiative for projects with great potential for innovative research.

[Please register](#) to help us estimate how many people will be attending.

