Do $T\overline{T}$ Deformations reproduce 2D gravity? An Operator Algebra approach

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32nd Nordic Network Meeting on “Strings, Fields and Branes”

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Why Operator Algebras?

- What: algebra of bounded observables in a quantum theory
- Natural setting for calculating Entanglement entropy and related quantities
- Formalism for answering questions like:
  - Structure of states in the Hilbert space?
  - When can we factorize a Hilbert space?
  - What are the implications of causality and locality?
- Example: Entanglement entropy diverges for local relativistic QFTs $\Longrightarrow$ Hilbert space of a QFT can not be factorized into subregions

[Murray, von Neumann, Araki, Powers, Fredenhagen, Yngvason, Driesler, Bisognano, Wichmann etc]

Type Classification of von Neumann factors

- von Neumann algebra: $\mathcal{A}'' = \mathcal{A}$, where $'$ is the commutant
- von Neumann factor: $\mathcal{A} \cap \mathcal{A}' \sim \mathbb{C}\mathbb{1}$.
- Classification of factors:
  - Type I: Minimal Projections, pure states, finite trace functional
  - Type II: No Minimal Projections, no pure states, renormalizable trace functional
  - Type III: No Minimal Projections, no pure states, no trace functional
- No minimal projections $\implies$ no irreps of a sub-Hilbert space, and therefore by Schur's lemma there can be no sub-Hilbert space
- Modular hamiltonian K generates automorphisms in a von Neumann algebra: $e^{-isK}\mathcal{A}e^{isK} \subseteq \mathcal{A}$
- Spectrum of modular hamiltonian characterizes the type of von Neumann factor

Examples of different types of algebras

- Type I_d: Finite dimensional quantum mechanical systems
- Type I_∞: Infinite dimensional quantum mechanical systems
- Type III_1: any local relativistic QFT [Bisognano, Wichmann '75, '76]
- Type III_λ: specific $N \to \infty$ limit of N pairs of Bell pairs [Powers '67]
- Type II_1: de Sitter with an observer [Chandrasekaran, Longo, Penington, Witten '22]
- Type II_∞: JT gravity coupled with matter [Penington, Witten '23]

$T\overline{T}$ Deformations

- Well defined for 2D translationally invariant QFT

$$\partial_\lambda S^{(\lambda)} = -\int d^2x \mathcal{O}_{T\overline{T}}, \quad \mathcal{O}_{T\overline{T}} \equiv \det T^{(\lambda)}$$

- Irrelevant deformation, however it is UV finite and integrable [Smirnov, Zamolodchikov '16]
- $\mathcal{O}_{T\overline{T}}$ factorizes: $\langle n|\mathcal{O}_{T\overline{T}}|n\rangle = \langle n|T|n\rangle\ \langle n|\overline{T}|n\rangle - \langle n|\Theta|n\rangle^2$, which implies deformed spectrum on the cylinder:

$$E_n^\lambda = \frac{1}{2\lambda}\left(1 - \sqrt{1 - 4\frac{\lambda}{R}E_n + 16\frac{\lambda^2}{R^2}P_n^2}\right)$$

  Can compute deformed finite sized spectrum from factorizability of $\mathcal{O}_{T\overline{T}}$ operator [Zamolodchikov '04].
- Flow equation of torus partition function follows ($\mu \equiv \lambda/R^2$) [Jiang, Datta '18]

$$\partial_\mu Z(\tau, \bar{\tau}|\mu) = \left[\tau_2 \partial_\tau \partial_{\bar{\tau}} + \frac{1}{2}\left(i(\partial_\tau - \partial_{\bar{\tau}}) - \frac{1}{\tau 2}\right)\mu\partial_\mu\right] Z(\tau, \bar{\tau}|\mu)$$

Deformed partition functions

- Perturbative expansion: $Z(\tau,\bar\tau|\mu) = \sum_{p=0}^{\infty} \mu^p Z_p(\tau,\bar\tau)$, where $Z_0(\tau,\bar\tau) = Z(\tau,\bar\tau)$ is the undeformed partition function
- $Z(\tau,\bar\tau|\mu)$ is modular invariant, μ transforms as a $(-1,-1)$ modular form, therefore $Z_p(\tau,\bar\tau)$ is a (p,p) modular form.
- $E_{8,1}$ WZW model with $c=8$: $Z(\tau,\bar\tau) = |j(\tau)^{\frac{1}{3}}|^2$. First few terms of the expansion:

$$Z_1 = \frac{\tau_2}{9|j|^{4/3}} j' \bar{j}'$$

$$Z_2 = \frac{\tau_2}{162|j|^{10/3}} \left(3 j j'' \left(3\bar{j}\tau_2\bar{j}'' - 2\tau_2 \left(\bar{j}'\right)^2 + 3i\bar{j}\bar{j}'\right) + (j')^2 \left(-6\bar{j}\tau_2\bar{j}'' + 4\tau_2\left(\bar{j}'\right)^2 - 6i\bar{j}\bar{j}'\right) + 3i j j' \left(2\left(\bar{j}'\right)^2 - 3\bar{j}\bar{j}''\right)\right)$$

Deformed partition functions

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- $c = 24$ Meromorphic CFT (eg. Monster CFT): $Z(\tau,\bar\tau) = |j(\tau) - \mathcal{N}|^2$, $\mathcal{N} \in \mathbb{Z}_{\leq 744}$ [Schellekens '92]. First few terms of the expansion:

$$Z_1 = \tau_2 j' \bar{j}'$$

$$Z_2 = \frac{\tau_2}{2}\left(i(j''\bar{j}' - j'\bar{j}'') + \tau_2 j''\bar{j}''\right)$$

$$Z_3 = \frac{\tau_2}{12}\left(9j''\bar{j}'' - 3j'''(\bar{j}' - 2i\tau_2\bar{j}'') - 3\bar{j}'''(j' + 2i\tau_2 j'') + 2\tau_2^2 j'''\bar{j}'''\right)$$

- Note that the value of $\mathcal{N}$ ($744-$number of spin 1 currents) does not affect the deformed partition function ($T\overline{T}$ does not see currents like momenta)

Deformed partition functions

- Resumming these series require the use of the remarkable identity [Mahler '69]:

$$\frac{j'''(\tau)}{j'(\tau)} - \frac{3}{2}\left(\frac{j''(\tau)}{j'(\tau)}\right)^2 + \frac{j(\tau)^2 - 1968 j(\tau) + 2654208}{2 j(\tau)^2 (j(\tau) - 1728)^2} j'(\tau)^2 = 0$$

- Another useful tool is writing the solution as an integral equation

$$Z\left(\tau, \bar{\tau} | \mu\right) = \frac{\tau_2}{\mu} \int \frac{d^2\sigma}{\sigma_2^2}\, e^{-\pi \frac{\mu}{\sigma_2} |\tau - \sigma|^2} Z(\sigma, \bar{\sigma} | 0)$$

- The integration kernel can be rewritten as a coupling to "flat JT" [Dubovsky, Gorbenko, Mirbabayi '17] [Dubovksy, Gorbenko Hérnandez-Chifflet '18]

$$S_{T\overline{T}} = S_0(\phi, g) + \int \sqrt{-g} \left(\varphi R + \frac{1}{\lambda}\right)$$

Holographic bulk dual to $T\overline{T}$-deformed CFT

What does $T\overline{T}$ deformation do in the bulk?

- Move the CFT into the bulk [McGough, Mezei, Verlinde '16]
  Radial Wheeler-DeWitt equation reproduces $T\overline{T}$ flow, only for $\lambda < 0$
- Mixed boundary conditions [Guica, Monten '19]
  $\delta\partial_\lambda S = \partial_\lambda \delta S$ imply flow equations for the metric and stress tensor and interpret them as mixed boundary conditions from the bulk, reproduce radial cutoff for empty AdS, $\lambda < 0$
- Glue on AdS [Apolo, Hao, Lai, Song '23]
  For $\lambda > 0$, attach auxilliary AdS to the boundary, reproduce mixed boundary conditions

Can a $T\overline{T}$ deformation change the von Neumann factor?

Evidence for why a $T\overline{T}$ deformed CFT has a von Neumann factor of type II_∞

- Irrelevant deformation drastically changes UV behaviour
- Can be expressed as a coupling to flat JT gravity [Dubovsky, Gorbenko, Mirbabayi, Hérnandez-Chifflet '17, '18]
- Holographic entanlgement entropy: RT surface has finite area [Chen, Chen, Hao '18] [Lewkowycz, Liu, Silverstein, Torroba '19]
- Renyi entropy on the sphere: UV finite entropy [Donnelly, Shyam '18]
- Finite Entanglement entropy in string theory [Dabholkar, Moitra '23]
- Breakdown of the split property, therefore not type I [Asrat, Kudlur-Flam '20]
- Locality is required for type III

Thank You!