Boundaries and Interfaces with localized Cubic Interactions in the O(N) model.

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Boundary and Defect CFTs

- Defects: localized interactions (line, surface, ...)
- Interface: co-dimension one defect
- Boundary: extra dimension only over half-space
- Describe many physical systems
  → ferromagnets with impurities, binary liquids, superfluids
- Both theoretically and experimentally

→ What about for the O(N) model?

Surface defects

[Metlitski; Trepanier; Giombi, Liu; Raviv-Moshe, Zhang]

$$S = \int d^d x \left(\frac{1}{2} \partial_\mu \varphi_a \partial^\mu \varphi_a + \frac{\lambda}{4!} (\varphi_a \varphi_a)^2 \right)$$

$$+ h \int d^2 x \, \varphi_a^2 .$$

- $1/N$ expansion in $d = 3$
- $4-\varepsilon$ expansion

Three universality classes:

- Ordinary: new surface exponent
- Special: multi-critical point
- Extraordinary: breaking of $O(N)$ to $O(N-1)$, log

Boundary: extraordinary up to N_c

Surface defect: extraordinary for all $N \geq 1$.

Co-dimension one defects

$$S = \int d^{d+1}x \left[\frac{1}{2} \partial_\alpha \varphi \partial^\alpha \varphi + \lambda_n \varphi^n \right] + \int d^d x \, \lambda_p \varphi^p$$

- UV dimension $\Delta_\varphi = \frac{d-1}{2}$
- Defect interaction marginal for $d_c = \frac{p}{p-2}$
- In $d_c + 1 = \frac{2p-2}{p-2}$
  $\varphi^{2(p-1)}$ marginal in the bulk

Co-dimension one defects

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d_c	Bulk	Defect
3	$m = 4$	$p = 3$
2	$m = 6$	$p = 4$[1]
5/3	$m = 8$	$p = 5$

[1] [Söderberg-Rousu 2304.05786]

$\Longrightarrow$ Here: cubic interactions with $d_c = 3 - \varepsilon$ expansion

Outline

1. Interface with cubic interactions

2. Boundary with cubic interactions

Action

$$S = \int d^{d+1}x \left[\frac{1}{2} \partial_\mu \varphi_I \partial^\mu \varphi_I + \frac{\lambda_4}{4!} (\varphi_I \varphi_I)^2 \right]$$

$$+ \int d^d x \left[\frac{\lambda_1}{2} \varphi_N \varphi_a \varphi_a + \frac{\lambda_2}{3!} \varphi_N^3 \right].$$

$I = 1, \dots, N \qquad a = 1, \dots, N-1$

$\Longrightarrow$ Symmetry broken to $O(N-1)$

Free bulk propagator:

$$\langle \varphi_I(x_1) \varphi_J(x_2) \rangle = \delta_{IJ} \underbrace{\frac{\Gamma\left(\frac{d-1}{2}\right)}{4\pi^{\frac{d+1}{2}}}}_{C_\varphi} \frac{1}{|x_{12}|^{d-1}}$$

Propagators

→ Fourier transform along the defect direction

$$\langle \varphi_I(p_1, y_1)\, \varphi_J(p_2, y_2) \rangle = \delta_{IJ}\, C_\varphi \int d^d x_1\, d^d x_2 \frac{e^{i p_1 \cdot x_1 + i p_2 \cdot x_2}}{\left(x_{12}^2 + y_{12}^2\right)^{\frac{d-1}{2}}}$$

$$= (2\pi)^d\, \delta^d(p_1 + p_2)\, \delta_{IJ} \frac{e^{-|p_1|\,|y_{12}|}}{2|p_1|}$$

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$$= (2\pi)^d\, \delta^d(p_1 + p_2)\, \delta_{IJ}\, \frac{e^{-|p_1||y_{12}|}}{2|p_1|}$$

- Interface to bulk $y_1 = 0$

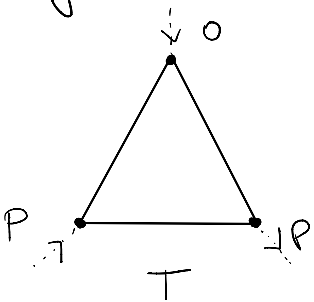
$$K_{IJ}(p, y) = \frac{e^{-|p||y|}}{2|p|}\, \delta_{IJ}$$

- Interface $y_1 = y_2 = 0$

$$G_{IJ}(p) = \frac{\delta_{IJ}}{2|p|}$$

One loop contributions

<u>Only cubic couplings:</u>



$\longrightarrow$ Standard computation in momentum space with interface propagators.

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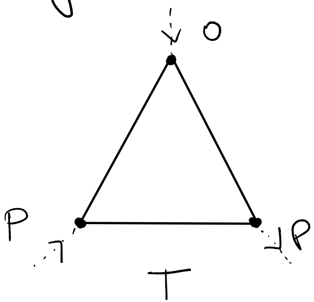
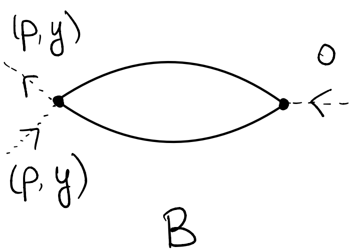


$\longrightarrow$ Standard computation in momentum space with interface propagators.

Both cubic and quartic couplings:



$\longrightarrow$ Mixed representation

$$I_B = \mu^{\varepsilon} \int \frac{d^d k}{(2\pi)^d} \int_{\mathbb{R}} dy \left(\frac{1}{2|k|} \right)^2 e^{-2(|p|+|k|)|y|}$$

$$= \frac{2}{(4\pi)^2 \varepsilon} + \mathcal{O}(1)$$

Beta functions

Rescale the couplings as $g_{1,2} = \frac{\lambda_{1,2}}{2\pi}$

$$\boxed{\begin{aligned} \beta_1 &= -\frac{\varepsilon}{2} g_1 - \frac{g_1^2}{4}(g_1+g_2) + \frac{g_4}{3}\left((N+5)g_1 + g_2\right) \\ \beta_2 &= -\frac{\varepsilon}{2} g_2 - \frac{1}{4}\left((N-1)g_1^3 + g_2^3\right) + g_4\left((N-1)g_1 + 3g_2\right) \end{aligned}}$$

Quartic coupling: not affected by the interface

$$\beta_4 = -\varepsilon g_4 + \frac{N+8}{3} g_4^2 \qquad g_4 = \frac{\lambda_4}{(4\pi)^2}$$

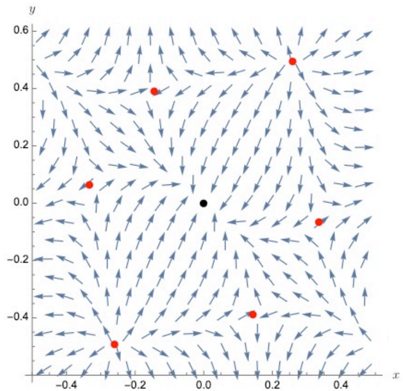
Interacting bulk

	$N=2$	$N=3$	$N=4$	$5 \leq N \leq 7$	$N \geq 8$
Real	6	6	4	4	0
Imaginary	0	2	2	4	4
Complex	0	0	0	0	4

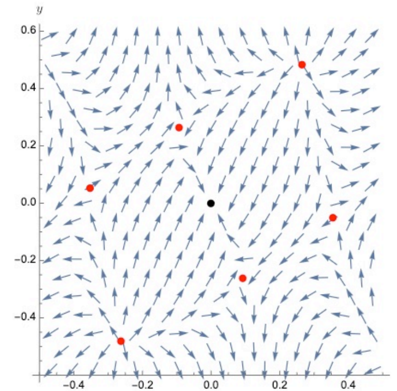
- Reminiscent of surface defects breaking $O(N)$ to $O(N-1)$
- Fine-tuning of several relevant operators

$\Longrightarrow$ Multi-critical version of extraordinary universality class ?

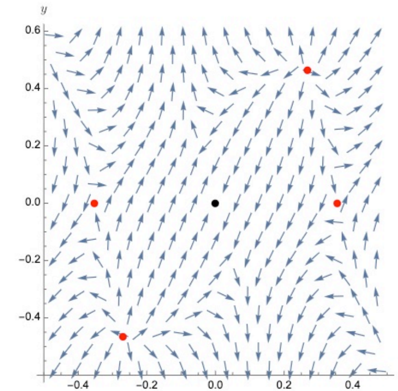
RG Flows



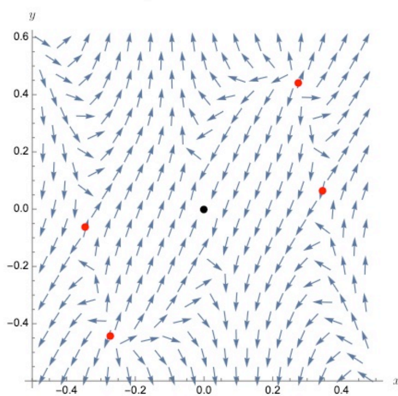
(a) $N = 2$.



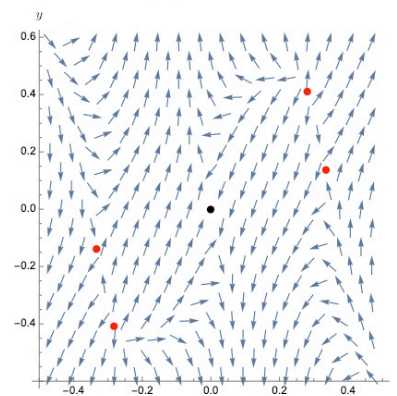
(b) $N = 3$.



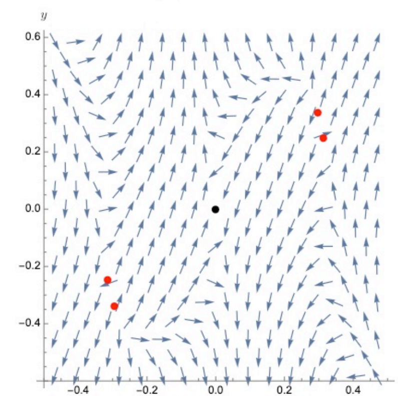
(c) $N = 4$.



(d) $N = 5$.



(e) $N = 6$.



(f) $N = 7$.

Large-N behavior

Purely imaginary stable fixed points:

$$x^* = \pm i\left(\frac{1}{N} - \frac{3}{N^2}\right) + \mathcal{O}(N^{-3}) \quad , \quad g_1 = \sqrt{8\varepsilon}\, x$$

$$y^* = \mp i\left(\frac{1}{2} - \frac{3}{2N} + \frac{35}{4N^2}\right) + \mathcal{O}(N^{-3}) \quad , \quad g_2 = \sqrt{8\varepsilon}\, y$$

Critical exponents:

$$\omega_+ = \varepsilon\left(1 + \frac{6}{N} + \dots\dots\right) \qquad \omega_- = \varepsilon\left(\frac{1}{2} - \frac{11}{N} + \dots\dots\right)$$

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Quadratic operators:

- Real dimensions
- Expect non-unitarity but well-defined path integral.

Boundary with cubic interactions

$$S = \int_{y\geq 0} d^d x \, dy \left[\frac{1}{2} \partial_\mu \varphi_I \partial^\mu \varphi_I + \frac{\lambda_4}{4!} (\varphi_I \varphi_I)^2 \right]$$

$$+ \int d^d x \left[\frac{\lambda_1}{2} \varphi_N \varphi_a \varphi_a + \frac{\lambda_2}{3!} \varphi_N^3 \right].$$

- Neumann boundary conditions
- Two terms in the free bulk propagator

$$\langle \varphi_I(x_1, y_1) \varphi_J(x_2, y_2) \rangle =$$

$$\delta_{IJ} C_\varphi \left(\frac{1}{(x_{12}^2 + y_{12}^2)^{\frac{d-1}{2}}} + \frac{1}{(x_{12}^2 + (y_1+y_2)^2)^{\frac{d-1}{2}}} \right)$$

Propagators

Boundary to bulk and boundary propagators

$$K_{IJ}^{(B)}(p,y) = \frac{e^{-|p|y}}{|p|}\,\delta_{IJ} \,, \qquad G_{IJ}^{(B)}(p) = \frac{1}{|p|}$$

- Pure boundary graphs : factor 2 for each propagator
- Boundary to bulk graphs :
  - → factor 2 for each propagator
  - → extra-dim° integrated only over half-space

Propagators

Boundary to bulk and boundary propagators

$$K_{IJ}^{(B)}(p,y) = \frac{e^{-|p|y}}{|p|}\,\delta_{IJ} \qquad G_{IJ}^{(B)}(p) = \frac{1}{|p|}$$

- Pure boundary graphs : factor 2 for each propagator
- Boundary to bulk graphs :
  - $\rightarrow$ factor 2 for each propagator
  - $\rightarrow$ extra-dim° integrated only over half-space

$\Longrightarrow$ Not a simple rescaling !

Fixed points

$N=1$ Two real fixed points, $\omega < 0$

$N=2$ Six real fixed points, at least one relevant direction

$N>2$ Four real and four complex fixed points
Always at least one relevant direction

	$N=2$	$N>2$
Real	6	4
Imaginary	0	0
Complex	0	4

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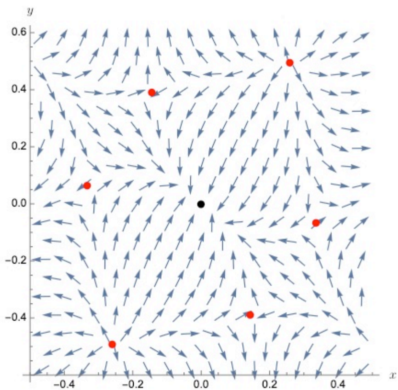

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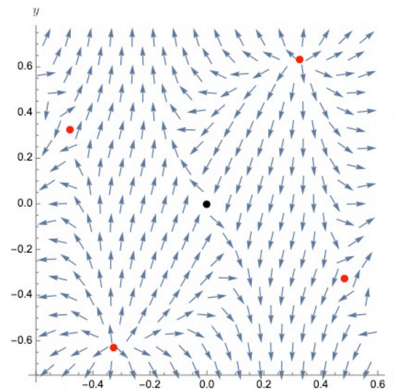

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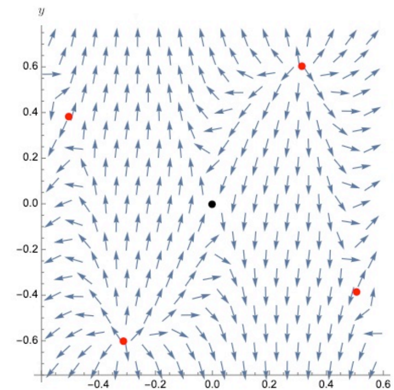

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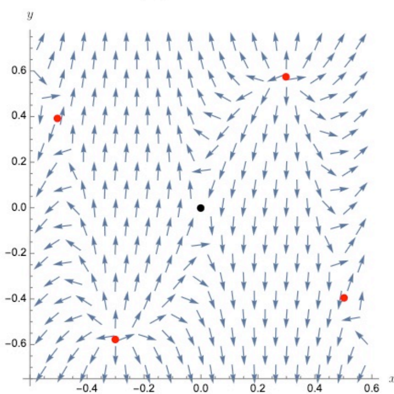

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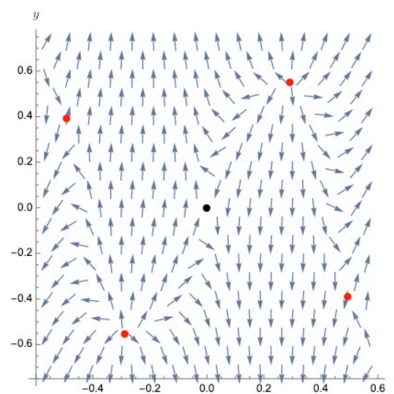

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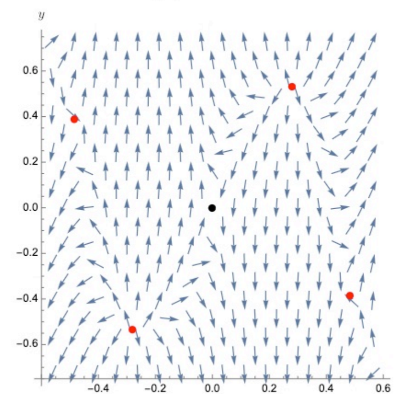

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Conclusion

Interface: Stable purely imaginary FP

No real FP for $N > N_{crit} \sim 7$

Boundary: Real FP for all N

Always at least one relevant direction

- Unitarity? Extraordinary class?
- $\varepsilon = 1$? Need higher-loop computations
- Monotonicity theorems?