NORDITA

UPPSALA UNIVERSITET

Leading-order gravitational radiation to all spin orders

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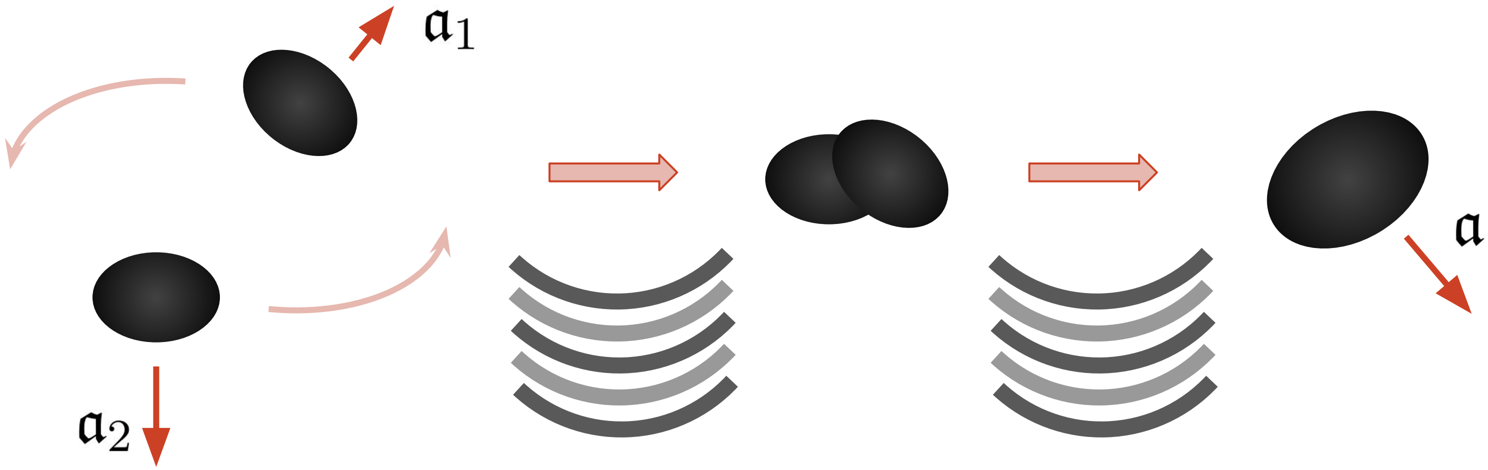
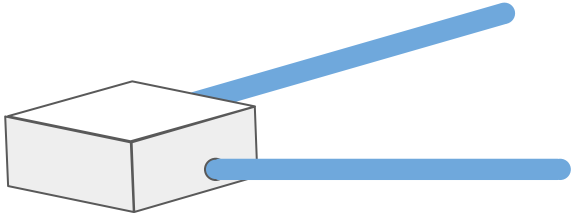
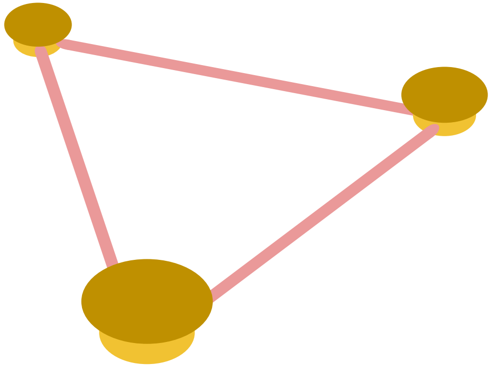
[arXiv:2310.05832] w/ Rafael Aoude, Carlo Heissenberg, Andreas Helset

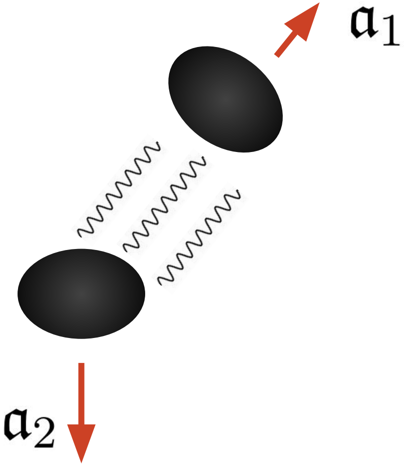




cartoon LIGO



cartoon LISA



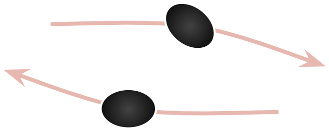
weak gravity

$$\frac{GM}{R} \ll 1$$

non-relativistic

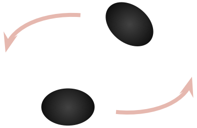
$$v \ll 1$$

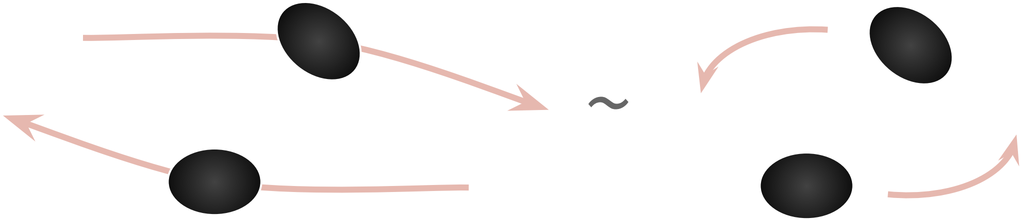
weak gravity ⟹ post-Minkowskian (PM) expansion

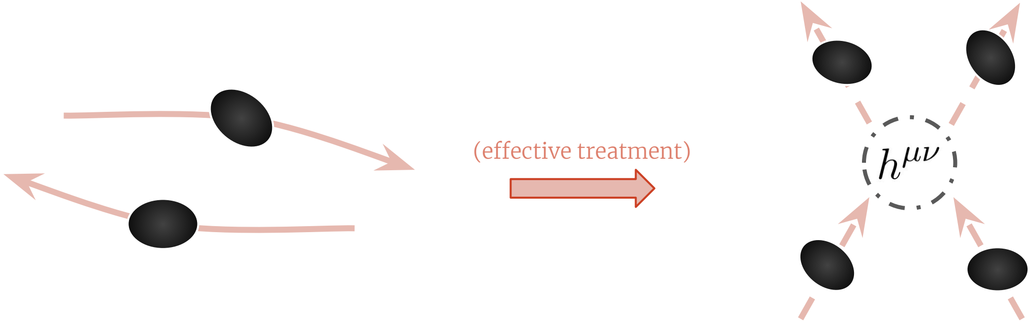


weak gravity
non-relativistic
virial theorem

} ⟹ post-Newtonian (PN) expansion



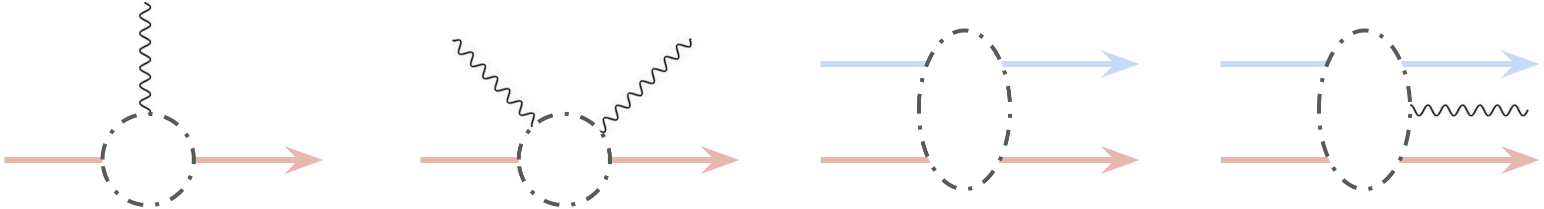




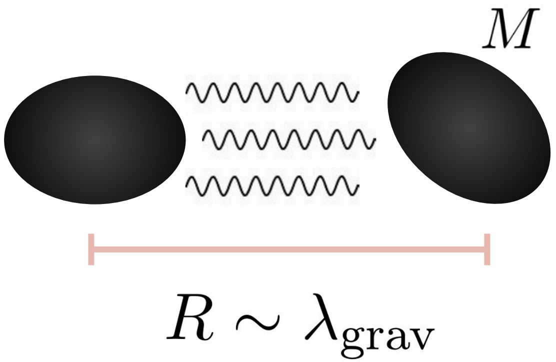
classical scattering (GR)

(classical limit of)
QFT scattering amplitudes

metric | gw/bh scattering | interaction Hamiltonian | gravitational waveform



KMOC formalism for classical observables [Kosower, Maybee, O'Connell, '18]



calculate

$$\langle \text{out}|\mathcal{O}|\text{out}\rangle = \langle \text{in}|S^\dagger \mathcal{O} S|\text{in}\rangle$$

where quantum properties of "in" state unresolvable: $\lambda_{\text{grav}} \gg \ell_w \gg \lambda_{\text{DB}}$

KMOC formalism for gravitational waveform [Cristofoli, Gonzo, Kosower, O'Connell, '21]

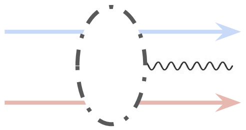
waveform operator:

$$\mathbb{h}_{\mu\nu}(x) = \int d\Phi(k) \sum_h \left[e^{-ik\cdot x} a_h(k) \varepsilon_\mu^{(h)*}(k) \varepsilon_\nu^{(h)*}(k) + e^{ik\cdot x} a_h^\dagger(k) \varepsilon_\mu^{(h)}(k) \varepsilon_\nu^{(h)}(k) \right]$$

with expectation value (leading order in coupling, observed at a distant position)

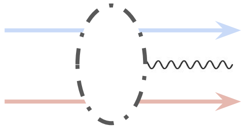
$$\kappa h_{\mu\nu}(x) \equiv \kappa \langle \text{out} | \mathbb{h}_{\mu\nu}(x) | \text{out} \rangle$$

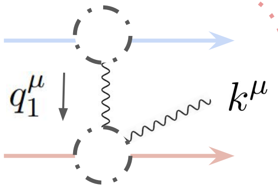
$$= \frac{\kappa}{4\pi|\boldsymbol{x}|} \sum_h \varepsilon_\mu^{(h)*} \varepsilon_\nu^{(h)*} \int_{\omega, q_1, q_2} \hat{\delta}(2p_1 \cdot q_1) \hat{\delta}(2p_2 \cdot q_2) \hat{\delta}^D(q_1 + q_2 - k) e^{i(q_1\cdot b_1 + q_2 \cdot b_2 - k\cdot x)}$$

$\times$ 

gravitational waveform for Kerr scattering

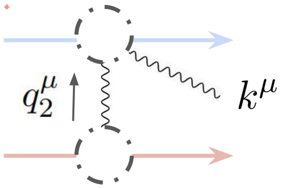
$$\kappa h_{\mu\nu}(x) = \frac{\kappa}{4\pi|x|}\sum_h \varepsilon_\mu^{(h)*}\varepsilon_\nu^{(h)*}\int_{\omega,q_1,q_2}\hat{\delta}(2p_1\cdot q_1)\hat{\delta}(2p_2\cdot q_2)\hat{\delta}^D(q_1+q_2-k)e^{i(q_1\cdot b_1+q_2\cdot b_2-k\cdot x)}$$

$\times$ 



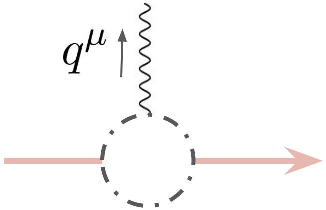
$$\frac{i}{q_1^2}\sum_h \mathcal{M}_3(p_1,-q_1^h)\mathcal{M}_4(p_2,q_1^h,-k^{h_k})$$

$+$



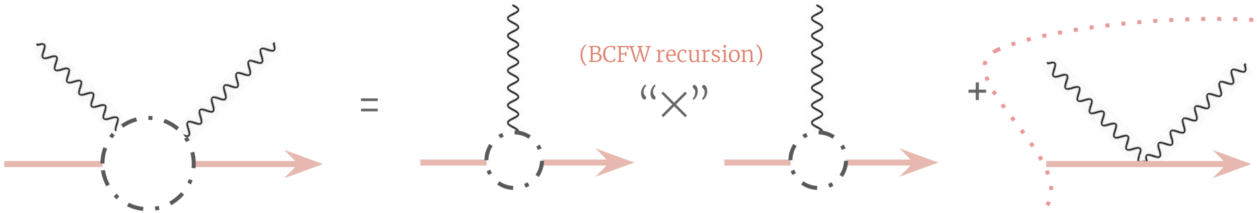
$$\frac{i}{q_2^2}\sum_h \mathcal{M}_3(p_2,-q_2^h)\mathcal{M}_4(p_1,q_2^h,-k^{h_k})$$

gravitational waveform for Kerr scattering



$$\mathcal{M}_3(p, -q^h) = -\kappa \left[p \cdot \varepsilon_h(q)\right]^2 \exp\left(hq \cdot \mathfrak{a}\right)$$

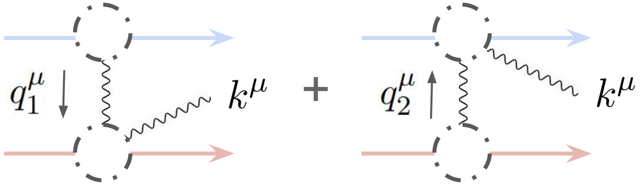
[Levi, Steinhoff, ‘15; Vines, ‘17; Guevara, Ochirov, Vines, ‘18; Chung, Huang, Kim, Lee, ‘18]



candidate up to $\mathcal{O}(\mathfrak{a}^6)$
[Bautista, Guevara, Kavanagh, Vines, ‘22]

[Arkani-Hamed, Huang, Huang, ‘17; Chung, Huang, Kim, Lee, ‘18; Chiodaroli, Johansson, Pichini, ‘20; Aoude, **KH**, Helset, ‘22; Bern, Kosmopoulos, Luna, Roiban, Teng, ‘22; Cangemi, Chiodaroli, Johansson, Ochirov, Pichini, Skvortsov, ‘22+‘23; Bautista, Guevara, Kavanagh, Vines, ‘22; Bjerrum-Bohr, Chen, Skowronek, ‘23; Scheopner, Vines, ‘23]

gravitational waveform for Kerr scattering



$$\mathcal{M}_5^{\rm cut}(k^-) = -\frac{\kappa^3}{8q_2^2} q_{2\mu} q_{2\nu} \sum_{h=\pm}\sum_{n=0}^{4} \frac{r^{h,\mu\nu}_{(1),n}}{2^n} e^{-hq_2\cdot\mathfrak{a}_2}\left[e^{(-hq_2-k)\cdot\mathfrak{a}_1} F_4^{(n)} + C_4^{(n)}\right] + (1\leftrightarrow 2)$$

KMOC integration

$$\kappa h(x) = -\frac{\pi G^2}{|\boldsymbol{x}| m_1 m_2}\left[h_f(x) + h_c(x)\right]$$

$$h_f(x) = \frac{1}{(p_1\cdot\rho)^2}\Bigg[\tilde{r}^{-,\mu_1\mu_2}_{(1),0}\mathcal{I}_{(1),\mu_1\mu_2}(b_{(1),-}) + \sum_{s=0}^{\infty}\frac{1}{s!}\mathcal{L}^{\mu_1\ldots\mu_{s+2}}_{(1),s}\mathcal{I}_{(1),\mu_1\ldots\mu_{s+2}}(b_{(1),+})\Bigg] + (1\leftrightarrow 2)$$

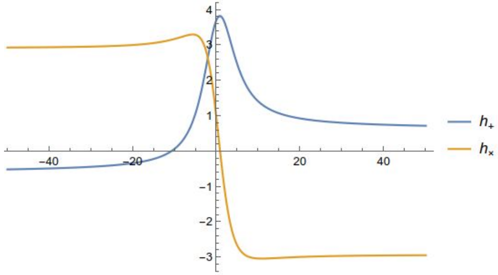
$$h_c(x) = \frac{32 m_1 v_1^{\mu_1} v_1^{\mu_2}}{(v_1\cdot\rho)^3}\left[\sum_{s=5}^{\infty} C_4^{(s),\mu_3\ldots\mu_s}(\mathfrak{a}_1)\mathcal{J}_{(1),\mu_1\ldots\mu_s}(b_{(1)})\right] + (1\leftrightarrow 2)$$

$$\mathcal{I}^{\mu_1\ldots\mu_n}_{(1)}(b) \equiv \int_{q_2}\hat{\delta}(v_2\cdot q_2)\frac{q_2^{\mu_1}\ldots q_2^{\mu_n} e^{iq_2\cdot b}}{q_2^2(q_2\cdot\rho)(v_1\cdot q_2)}$$

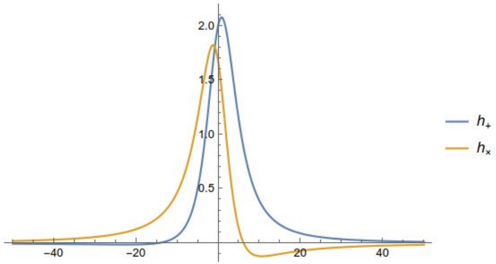
$$\mathcal{J}^{\mu_1\ldots\mu_n}_{(1)}(b) \equiv \int_{q_2}\hat{\delta}(v_2\cdot q_2)\frac{e^{iq_2\cdot b}}{q_2^2} q_2^{\mu_1}\ldots q_2^{\mu_n}$$

gravitational waveform for Kerr scattering (agreement with [De Angelis, Gonzo, Novichkov, ‘23; Brandhuber, Brown, Chen, Gowdy, Travaglini, ‘23])

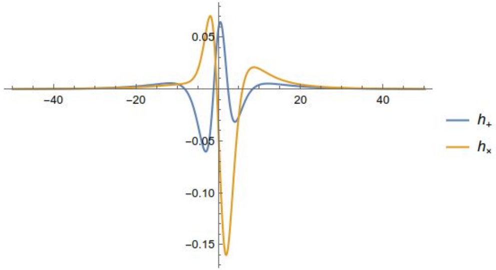
$\mathcal{O}(\mathfrak{a}^0)$



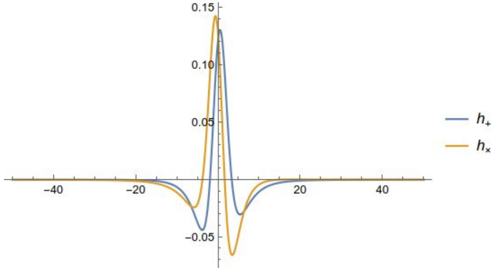
$\mathcal{O}(\mathfrak{a}^1)$



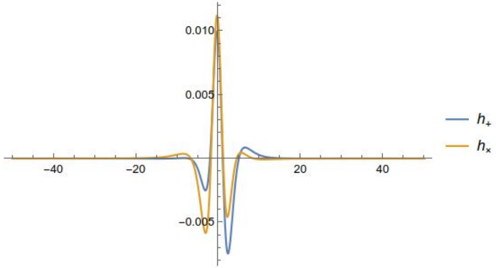
$\mathcal{O}(\mathfrak{a}^2)$



$\mathcal{O}(\mathfrak{a}^3)$



$\mathcal{O}(\mathfrak{a}^4)$



summary

scattering amplitudes powerful tools for computing classical observables

expression for leading-order scattering waveform to all spin orders

checks: agreement with concurrent calculations, agreement with classical large-retarded-time computations

future directions

better understanding of Kerr amplitudes

higher-order observables

relation to bound systems