Kerr binary dynamics from minimal coupling and double copy

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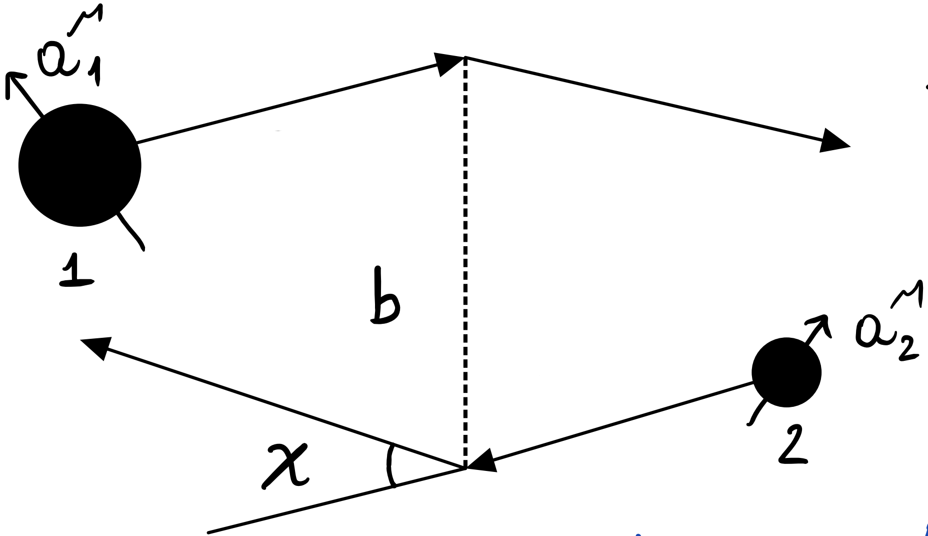
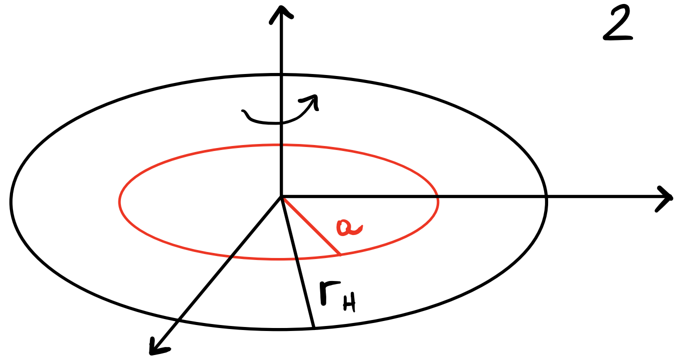
Kerr black-hole scattering in the PM regime:



ring radius: $a^\mu = \frac{S^\mu}{m}$

event horizon: $r_H = \frac{r_s + \sqrt{r_s^2 - 4a^2}}{2}$



"small G"

PM regime: $b \gg r_H \gg \frac{\hbar}{m}$ classical limit

Spin multipoles: $b \gg a \gg \frac{\hbar}{m}$ classical limit

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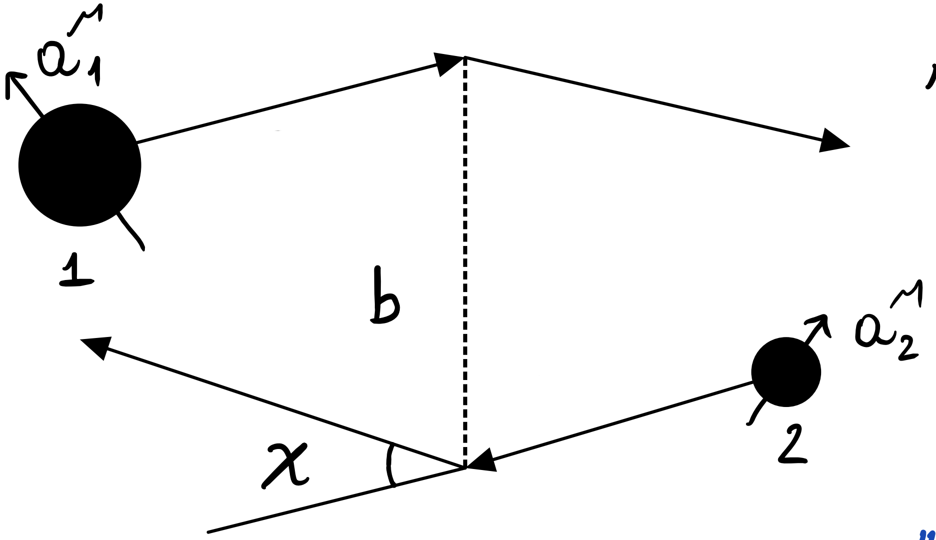

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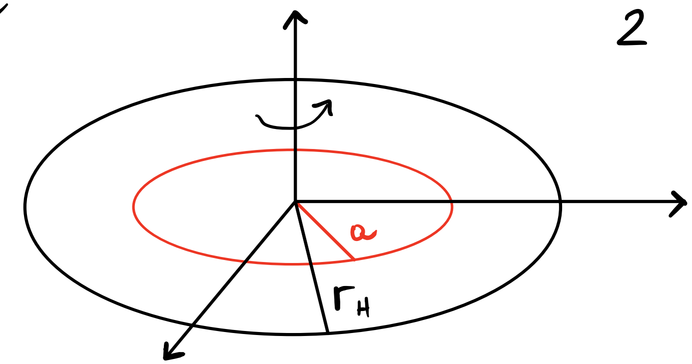

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- From quantum mechanics: $S^2 = S_\mu S^\mu = \hbar^2 s(s+1)$, with $\hbar \to 0$, $s \to \infty$
- Kerr black holes: $\lim_{s \to \infty}$ (massive s fields)

[Arkani-Hamed, Huang, Huang, Bern, Luna, Roiban, Sen, Zeng, Guevara, Ochirov, Vines, Aoude, Haddad, Helset, Chung, Kim, Chen, Chiodaroli, Pichini, Johansson, Jakobsen, Mogull, Plefka, Steinhoff, ...]

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Black holes $\longleftrightarrow$ elementary particles

- Schwarzschild
- Kerr

- minimally coupled scalars
- minimal coupling?

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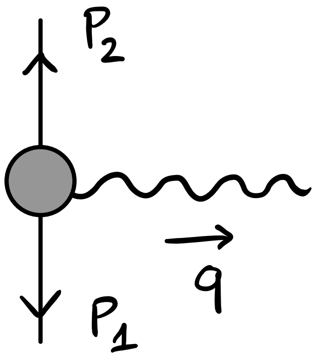
What can GR teach us about QFT?

M: A simple Lagrangian would allow to increase the precision in analytical computations (e.g. nPM $n>1$)

3-point amplitudes

- $p = p_1 - p_2$
- $\Lambda^{\rho}{}_{\mu}(a,q) = \exp\left\{ i \epsilon^{\rho}{}_{\mu\alpha\beta}\, a^{\alpha} q^{\beta} \right\}$

$a^{\mu} = 0$



$$= \begin{cases} i g\, p^{\mu} & \text{Gauge theory (scalar QED)} \\ \\ \kappa_N\, p^{(\mu} p^{\nu)} & \text{Gravity (Schwarzschild)} \end{cases}$$

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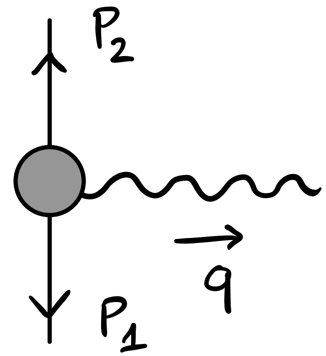

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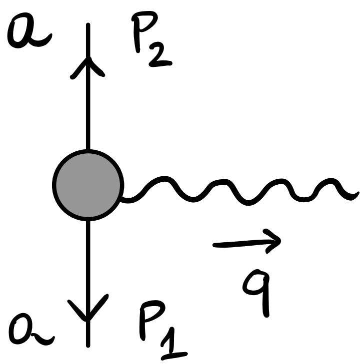
$a^{\mu} \neq 0$



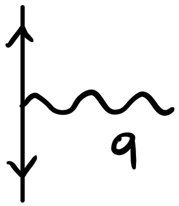
$$= \begin{cases} i g\, \Lambda^{\mu\sigma}(a,q)\, p_{\sigma} & \text{Gauge theory } (\sqrt{\text{Kerr}}) \\ \kappa_N\, p_{\sigma} \Lambda^{\sigma\,(\mu}(a,q)\, p^{\nu)} & \text{Gravity (Kerr)} \quad \text{[Vines, 2017]} \end{cases}$$

"Minimal coupling" [Arkani-Hamed, Huang, Huang, 2017]

Minimal coupling with classical spin

$$\mathcal{L}_{free} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_{\alpha=1,2} (\partial_\mu \phi_\alpha)^* (\partial^\mu \phi_\alpha) - m_\alpha^2 \phi_\alpha^* \phi_\alpha$$

- Assuming minimal coupling (QFT-like)



$$= i g \Lambda^{\mu\sigma}(a,q) p_\sigma \quad \Longrightarrow \quad D_\mu^{(a)} = \partial_\mu - i g \exp\left\{\epsilon_{\mu\sigma\alpha\beta} a^\alpha \partial^\beta\right\} A^\sigma$$

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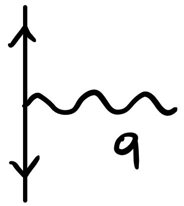
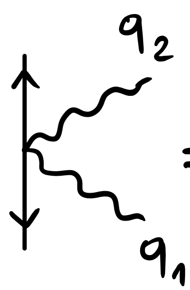


$$= i g \, \Lambda^{\mu\sigma}(a,q) \, p_\sigma \implies D_\mu^{(a)} = \partial_\mu - i g \exp\{\epsilon_{\mu\sigma\alpha\beta} a^\alpha \partial^\beta\} A^\sigma$$

- It is a "good" covariant derivative: $D_\mu^{(a)} \phi_\alpha \longrightarrow \mathcal{U} D_\mu^{(a)} \phi_\alpha$!!!

$D_\mu^{(a)} \equiv$ classical spin connection $\rightarrow \mathcal{L}^{(a)} = \mathcal{L}_{free}(\partial_\mu \longrightarrow D_\mu^{(a)})$

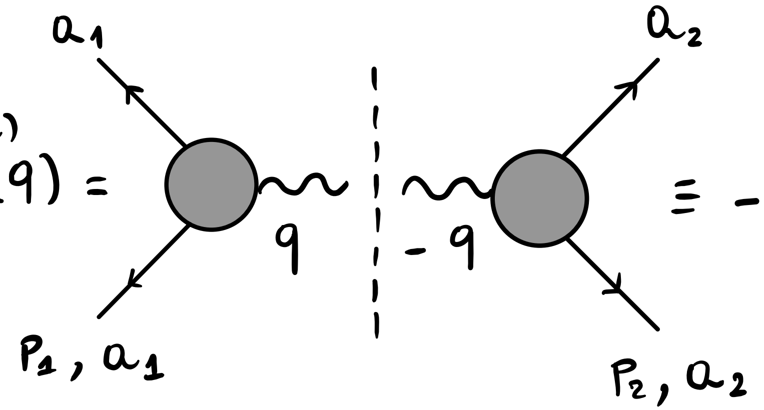


$$= i g^2 \Lambda^\nu{}_\rho (a, q_1) \Lambda^{\mu\rho}(a, q_2)$$

Contact terms are generated by the same mechanism that selects them in the spinless case. Schwarzschild is minimal.

Q : Are Kerr black-holes minimal ?

Kerr three-point and tree-level (1PM)

$A_0^{(a)}(q) =$ 

$$\equiv -i\, A^3_\mu \frac{\Pi^{\mu\nu}_{(a)}}{q^2} A^3_\nu$$

- Double copy: $\text{Kerr}(a) = \sqrt{\text{Kerr}}(a) \otimes \sqrt{\text{Kerr}}(a=0)$

$$\mathcal{M}_0(q) = \frac{\kappa_N^2}{q^2} m_1^2 m_2^2 \sigma^2 \sum_{\pm} (1 \pm v)^2 \exp\left\{ \pm i \frac{\epsilon_{\mu\nu\rho\sigma} p_1^\mu p_2^\nu a^\rho q^\sigma}{m_1 m_2 \sigma v} \right\}$$

[Guevara, Ochirov, Vines, 2018]

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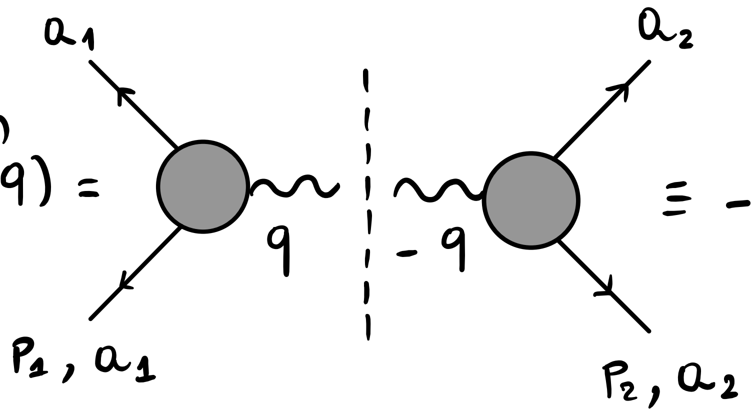
$A_0^{(a)}(q) =$ 

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[Guevara, Ochirov, Vines, 2018]

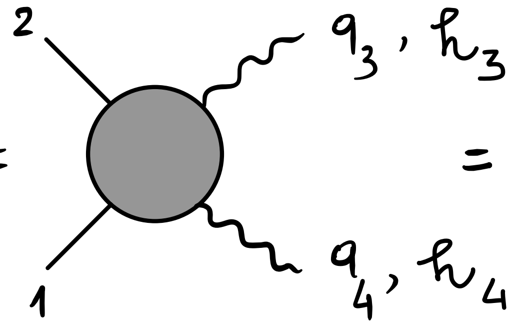
- Observables → 1PM eikonal

$$2\delta_0(b) = \frac{1}{4E|p|} \int \frac{d^{2-2\epsilon} q}{(2\pi)^{2-2\epsilon}} \mathcal{M}(q)\, e^{i q \cdot b} \propto \sum_{\pm} \frac{1}{|b \pm i a|^{-2\epsilon}}$$

Newman-Janis shift

Kerr Compton amplitude & 1-loop amplitude (2PM)

$$\mathcal{M}^{(a)}_{(h_3,h_4)} =$$



$$= e^{-a\cdot(h_3 q_3 + h_4 q_4)}\, \mathcal{M}^{(a=0)}_{(h_3,h_4)}$$

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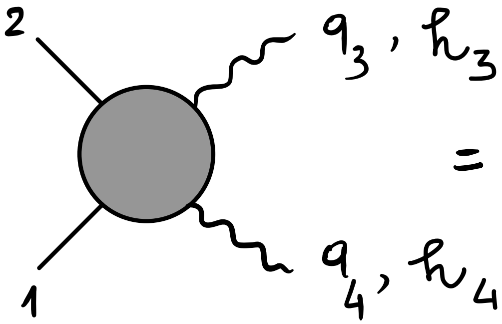

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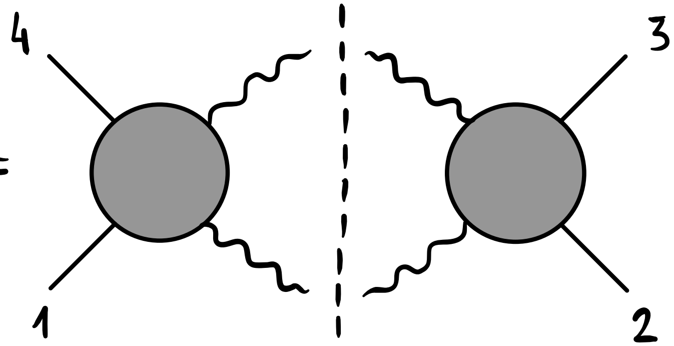


$$= \mathcal{M}^{(\text{even})}_{1\text{-loop}} + \mathcal{M}^{(\text{odd})}_{1\text{-loop}}$$

$$\mathcal{M}^{(\text{even})}_{1\text{-loop}} = \frac{\kappa^4}{16}\,\frac{1}{32\sqrt{-q^2}}\Bigg[4m_1\left(m_1^4 + m_1^2(m_2^2 - 2s) + (m_2^2 - s)^2\right) I_0\big(\mathcal{F}(a,q)\big)$$
$$+ 2m_1\left(m_1^2 m_2^2 - \frac{(s - m_1^2 - m_2^2)}{4}\right)\frac{I_1\big(\mathcal{F}(a,q)\big)}{\mathcal{F}(a,q)} + (m_1 \leftrightarrow m_2)\Bigg]$$

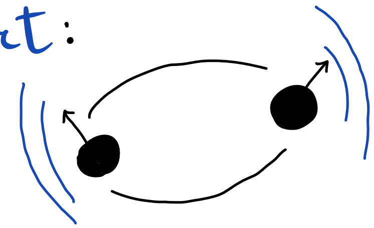
$$\mathcal{M}^{(\text{odd})}_{1\text{-loop}} = -\frac{\kappa^4}{16}\,\frac{\sqrt{\sigma^2-1}}{32\sqrt{-q^2}}\Big[m_1(m_1^2 + m_2^2 - s)\, I_1\big(\mathcal{F}(a,q)\big) + (m_1 \leftrightarrow m_2)\Big]$$

$\mathcal{F}(a,q) = \sqrt{(a\cdot q)^2 - a^2 q^2}$ is shift-symmetric [2203.06197]

Radiation reaction (RR) effects

- At 3PM the eikonal develops an imaginary part:

$$2\delta_2(b) = \mathrm{Re}\,2\delta_2(b) + i\,\mathrm{Im}\,2\delta_2(b) \quad \leftrightarrow$$



  - $\mathrm{Re}\,2\delta_2(b) = 2\delta_2^{\mathrm{con}}(b) + 2\delta_2^{\mathrm{rr}}(b)$
  - $\mathrm{Im}\,2\delta_2(b) = -\frac{1}{\pi\epsilon}\,2\delta_2^{\mathrm{rr}}(b) + O(\epsilon^0)$

[Bern et al, 1908.01493]
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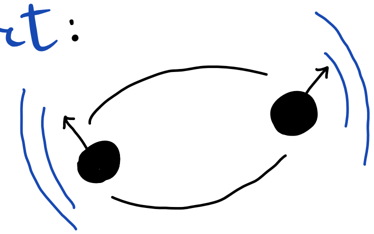

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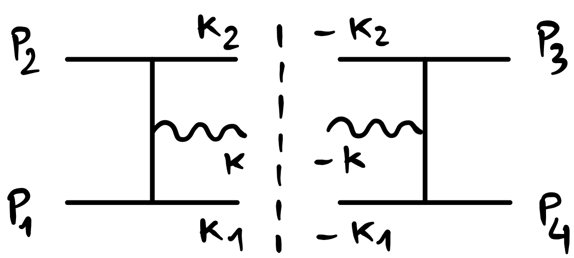

$$= \frac{1}{2}\int_{k,k_1,k_2} |\tilde{M}_5|^2 \overset{\epsilon\to 0}{=} \frac{1}{2}\int_{k,k_1,k_2} \left| \widetilde{k_\nu \sum_{i=1}^{4} \frac{p_i^\mu p_i^\nu}{p_i \cdot k}}\, M_0 \right|^2$$

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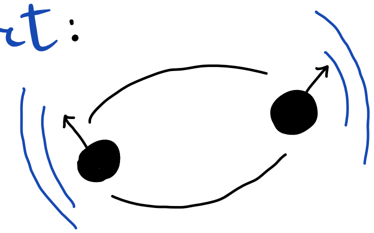

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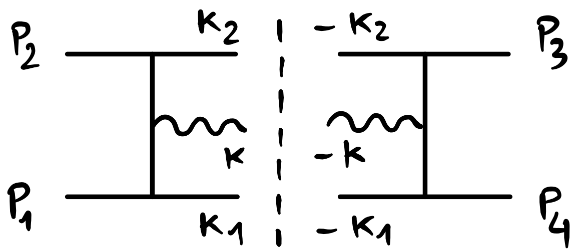

$$= \frac{1}{2}\int_{k,k_1,k_2} \left|\tilde{M}_5\right|^2 \overset{\epsilon\to 0}{=} \frac{1}{2}\int_{k,k_1,k_2} \left| \kappa \sum_{i=1}^{4} \frac{p_i^\mu p_i^\nu}{p_i\cdot k} M_0 \right|^2$$

- By knowing the 1PM amplitude we can compute $2\delta_2^{\mathrm{rr}}(b) \Rightarrow \chi_2^{\mathrm{rr}}(b)$

$$\chi_2^{\mathrm{rr}}(b) = -\frac{1}{p}\,\frac{\partial\,\mathrm{Re}\,2\delta_2^{\mathrm{rr}}(b)}{\partial b}$$

Radiation reaction (RR) effects for Kerr

- $2\delta_2^{rr}(b) = -\frac{\pi}{2} \lim_{\epsilon \to 0} \epsilon \int_{k,k_1,k_2} |\tilde{\mathcal{M}}_5|^2 = 2\delta_2^{rr}(b)\Big|_{a=0} f^2 , \quad f^\mu = \frac{\sigma^2 b}{2(2\sigma^2-1)} \sum_{\pm} \frac{(1 \pm v)^2 b_\pm^\mu}{b_\pm^2}$

where: $b_\pm = b \pm ia$

- $$\chi_2^{rr}(b) = \chi_2^{rr}(b)\Big|_{a=0} \frac{\left(1 + \frac{2\sigma\sqrt{\sigma^2-1}}{2\sigma^2-1}\frac{a}{b}\right)\left[1 + \frac{4\sigma\sqrt{\sigma^2-1}}{2\sigma^2-1}\frac{a}{b} + \left(\frac{a}{b}\right)^2\right]}{\left[1-\frac{a}{b}\right]^3}$$

[F.A. Di Vecchia 2203.13272]

$$= \chi_2^{rr}(b)\Big|_{a=0} \left[1 + \frac{6\sigma\sqrt{\sigma^2-1}}{2\sigma^2-1}\frac{a}{b} + \left(\frac{a}{b}\right)^2 + O(a^3)\right]$$

[Jakobsen, Mogull 2201.07778]

Conclusions:

- Using a notion of minimal coupling that incorporates classical spin it is possible to construct a consistent field theory that describes the dynamics of classically spinning compact objects and it can be used to perform precision computations.

Outlook:

- Add more contact terms and higher derivative corrections;
- Worldline description;
- Beyond 2PM in the conservative sector.