

Kerr binary dynamics
from
minimal coupling and double copy

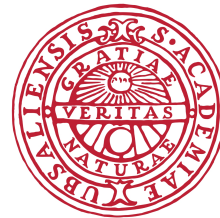
Francesco Alessio

2203.13272 F. A., Di Vecchia

2303.12784 F. A.



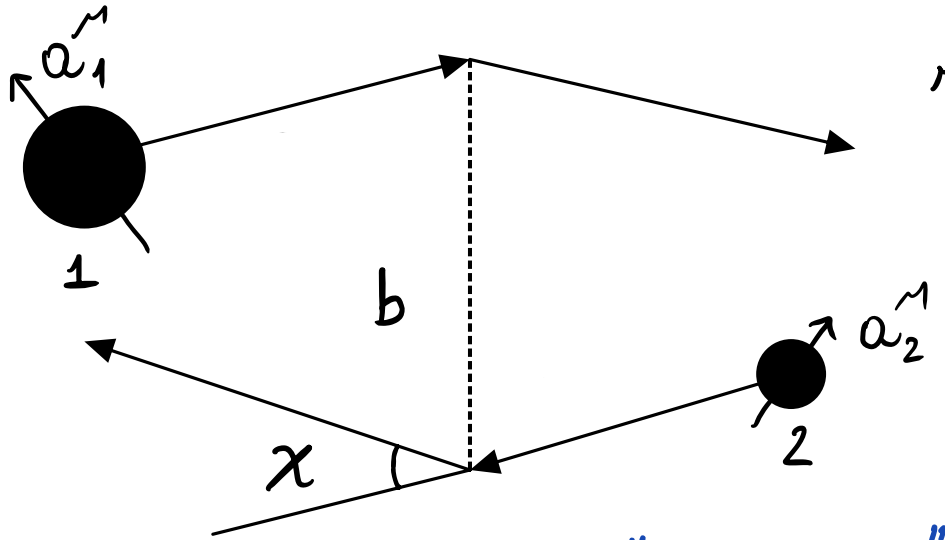
NORDITA
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32nd Nordic Meeting 4-6/12/23 - University of Stavanger

Kerr black-hole scattering in the PM regime:



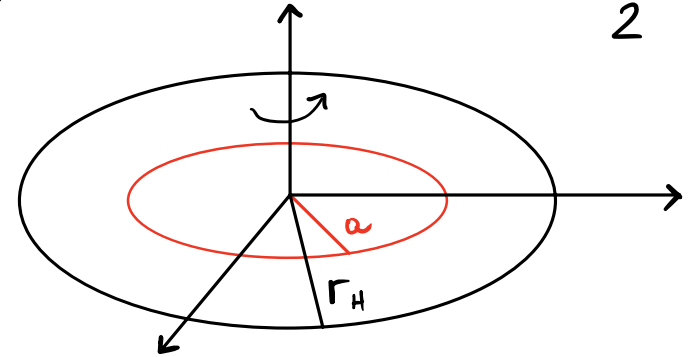
"small G"

ring radius:

$$a^m = \frac{S^m}{m}$$

event horizon:

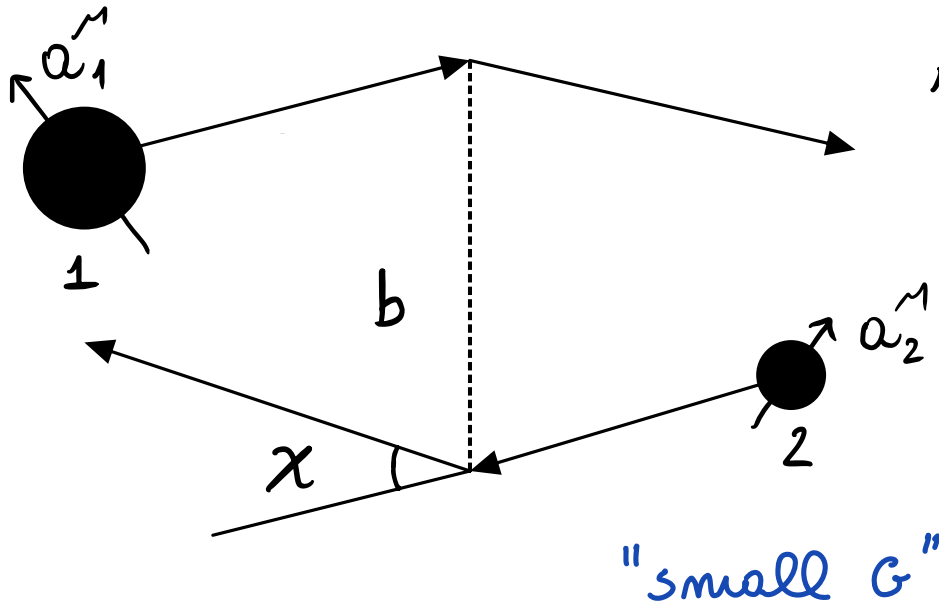
$$r_H = r_s + \frac{\sqrt{r_s^2 - 4a^2}}{2}$$



PM regime: $b \gg r_H \gg \frac{\hbar}{m}$ *classical limit*

Spin multipoles: $b \gg a \gg \frac{\hbar}{m}$ *classical limit*

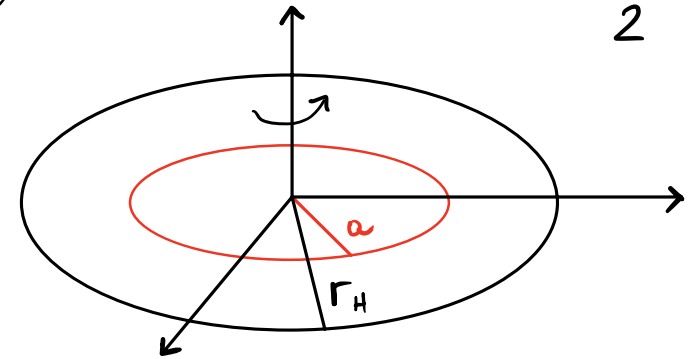
Kerr black-hole scattering in the PM regime:



ring radius: event horizon:

$$a^M = \frac{S^M}{m}$$

$$r_H = \frac{r_s + \sqrt{r_s^2 - 4a^2}}{2}$$



PM regime: $b \gg r_H \gg \frac{\hbar}{m}$ classical limit

Spin multipoles: $b \gg a \gg \frac{\hbar}{m}$ classical limit

• From quantum mechanics: $S^2 = S_M S^M = \hbar^2 s(s+1)$

\downarrow 0 \downarrow ∞

• Kerr black holes: $\lim_{s \rightarrow \infty}$ (massive s fields)

[Arkani-Hamed, Huang, Huang, Bern, Luna, Roiban, Sen, Zeng, Guervara, Ochirov, Vines, Arude, Haddad, Helset, Chung, Kim, Chen, Chiodaroli, Pichini Johansson, Jakobsen, Mogull, Plefka, Steinhoff, ...]

Q: Is it possible to describe Kerr black holes with an effective Lagrangian depending on the classical spin a^m and therefore carrying, as a built-in feature, an infinite spin-multipole expansion?

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Black holes \longleftrightarrow elementary particles

- Schwarzschild
- Kerr
- minimally coupled scalars
- minimal coupling?

What can GR teach us about QFT?

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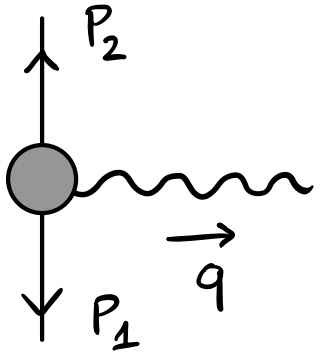
M: A simple Lagrangian would allow to increase the precision in analytical computations (e.g. n PM $n > 1$)

3-point amplitudes

- $P = P_1 - P_2$

- $\Lambda^P_\mu(a, q) = \exp\{i \epsilon^{\mu\alpha\beta} a^\alpha q^\beta\}$

$$a^\mu = 0$$



A Feynman diagram showing a 3-point vertex represented by a grey circle. An incoming fermion line from the left enters the vertex, with momentum P_1 pointing downwards. An outgoing fermion line from the vertex goes upwards, with momentum P_2 pointing upwards. A wavy boson line extends to the right from the vertex, with momentum q pointing to the right.

$$= \begin{cases} i g P^\mu \\ \kappa_N P^{(\mu} P^{\nu)} \end{cases}$$

Gauge theory (scalar QED)

Gravity (Schwarzschild)

3-point amplitudes

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Gauge theory (scalar QED)

Gravity (Schwarzschild)

$a^\mu \neq 0$

$$= \begin{cases} i g \Lambda^{\mu\sigma}(a, q) P_\sigma \\ \kappa_N P_\sigma \Lambda^\sigma{}^{(\mu}(a, q) P^{\nu)} \end{cases}$$

Gauge theory ($\sqrt{\text{Keur}}$)

Gravity (Keur) [Vines, 2017]

"Minimal coupling" [Arkani-Hamed, Huang, Huang, 2017]

Minimal coupling with classical spin

$$\mathcal{L}_{\text{free}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_{\alpha=1,2} (\partial_{\mu} \phi_{\alpha})^* (\partial^{\mu} \phi_{\alpha}) - m^2_{\alpha} \phi_{\alpha}^* \phi_{\alpha}$$

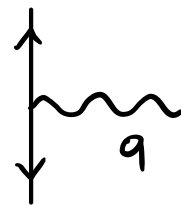
- Assuming minimal coupling (QFT-like)

$$\begin{array}{c} \uparrow \\ \text{wavy line} \\ \downarrow \\ q \end{array} = ig \Lambda^{\mu\sigma}(a, q) P_{\sigma} \quad \Rightarrow \quad D_{\mu}^{(a)} = \partial_{\mu} - ig \exp\left\{ \epsilon_{\mu\sigma\alpha\beta} a^{\alpha} \partial^{\beta} \right\} A^{\sigma}$$

Minimal coupling with classical spin

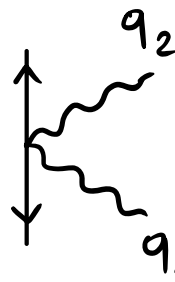
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- Assuming minimal coupling (QFT-like)


$$= ig \Lambda^{\mu\sigma}(a, q) P_{\sigma} \Rightarrow D_{\mu}^{(\omega)} = \partial_{\mu} - ig \exp\left\{ \epsilon_{\mu\sigma\alpha\beta} a^{\alpha} \partial^{\beta} \right\} A^{\sigma}$$

- It is a "good" covariant derivative: $D_{\mu}^{(\omega)} \phi_{\alpha} \rightarrow \mathcal{U} D_{\mu}^{(\omega)} \phi_{\alpha} !!!$

$$D_{\mu}^{(\omega)} \equiv \text{classical spin connection} \rightarrow \mathcal{L}^{(\omega)} = \mathcal{L}_{\text{free}} (\partial_{\mu} \rightarrow D_{\mu}^{(\omega)})$$


$$= ig^2 \Lambda^{\nu\rho}(a, q_1) \Lambda^{\mu\sigma}(a, q_2)$$

Contact terms are generated by the same mechanism that selects them in the spinless case. Schwarzschild is minimal.

Q: Are Kerr black-holes minimal?

Kerr three-point and tree-level (1PM)

$$A_0^{(a)}(q) = \text{Diagram} \equiv -i A_{\mu}^3 \frac{\pi^{\mu\nu}}{q^2} A_{\nu}^3$$

- Double copy: $\text{Kerr}(a) = \sqrt{\text{Kerr}(a)} \otimes \sqrt{\text{Kerr}(a=0)}$

$$\mathcal{M}_0(q) = \frac{\kappa_N^2}{q^2} m_1^2 m_2^2 \sigma^2 \sum_{\pm} (1 \pm v)^2 \exp \left\{ \pm i \frac{\epsilon_{\mu\nu\rho\sigma} p_1^{\mu} p_2^{\nu} a^{\rho} q^{\sigma}}{m_1 m_2 \sigma v} \right\}$$

[Guevara, Ochirov, Vines, 2018]

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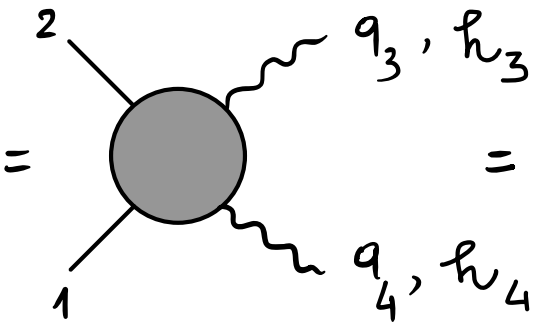
[Guevara, Ochirov, Vines, 2018]

- Observables \rightarrow 1PM eikonal

$$2\delta_0(b) = \frac{1}{4E|b|} \int \frac{d^4q}{(2\pi)^{2-2\epsilon}} \mathcal{M}(q) e^{iq \cdot b} \propto \sum_{\pm} \frac{1}{|b \pm ia|^{-2\epsilon}}$$

Newman-Janis shift

Kerr Compton amplitude & 1-loop amplitude (2PM)

$$\mathcal{M}^{(a)}(\hbar_3, \hbar_4) = \text{Diagram} =$$


$$= e^{-a \cdot (\hbar_3 q_3 + \hbar_4 q_4)} \mathcal{M}^{(a=0)}(\hbar_3, \hbar_4)$$

Keur Compton amplitude & 1-loop amplitude (2PM)

$$\mathcal{M}^{(a)}_{(h_3, h_4)} = \text{Diagram} = e^{-a \cdot (h_3 q_3 + h_4 q_4)} \mathcal{M}^{(a=0)}_{(h_3, h_4)}$$

Diagram: A central grey circle with four external lines. Line 1 (bottom-left) and line 2 (top-left) are straight lines. Line 3 (top-right) and line 4 (bottom-right) are wavy lines. Momenta \$q_3, h_3\$ and \$q_4, h_4\$ are labeled near lines 3 and 4 respectively.

$$\mathcal{M}_{1\text{-loop}} = \text{Diagram 1} + \text{Diagram 2} = \mathcal{M}_{1\text{-loop}}^{(\text{even})} + \mathcal{M}_{1\text{-loop}}^{(\text{odd})}$$

Diagram 1: A grey circle with four external lines. Line 1 (bottom-left) and line 4 (top-left) are straight lines. Line 3 (top-right) and line 2 (bottom-right) are wavy lines. A vertical dashed line is drawn between the two diagrams.

Diagram 2: A grey circle with four external lines. Line 3 (top-right) and line 2 (bottom-right) are straight lines. Line 4 (top-left) and line 1 (bottom-left) are wavy lines.

$$\mathcal{M}_{1\text{-loop}}^{(\text{even})} = \frac{\kappa^4}{16} \frac{1}{32 \sqrt{-q^2}} \left[4m_1 \left(m_1^4 + m_1^2 (m_2^2 - 2s) + (m_2^2 - s)^2 \right) \mathcal{I}_0(F(a, q)) \right. \\
 \left. + 2m_1 \left(m_1^2 m_2^2 - \frac{(s - m_1^2 - m_2^2)}{4} \right) \frac{\mathcal{I}_1(F(a, q))}{F(a, q)} + (m_1 \leftrightarrow m_2) \right]$$

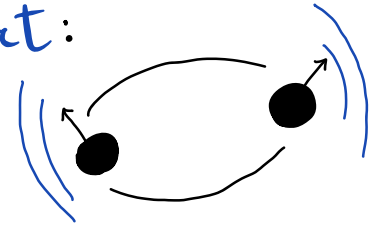
$$\mathcal{M}_{1\text{-loop}}^{(\text{odd})} = -\frac{\kappa^4}{16} \frac{\sqrt{\sigma^2 - 1}}{32 \sqrt{-q^2}} \left[m_1 (m_1^2 + m_2^2 - s) \mathcal{I}_1(F(a, q)) + (m_1 \leftrightarrow m_2) \right]$$

$F(a, q) = \sqrt{(a \cdot q)^2 - a^2 q^2}$ is shift-symmetric [2203.06197]

Radiation reaction (RR) effects

- At 3PM the eikonal develops an *imaginary part*:

$$2\delta_2(b) = \text{Re } 2\delta_2(b) + i \text{Im } 2\delta_2(b) \leftrightarrow$$



- $\text{Re } 2\delta_2(b) = 2\delta_2^{\text{con}}(b) + 2\delta_2^{\pi\pi}(b)$

[Bern et al, 1908.01493]

- $\text{Im } 2\delta_2(b) = -\frac{1}{\pi\epsilon} 2\delta_2^{\pi\pi}(b) + O(\epsilon^0)$

[Damour, 2010.01641]

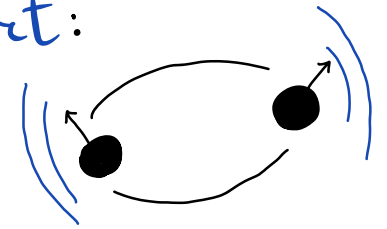
[Di Vecchia et al, 2101.05772]

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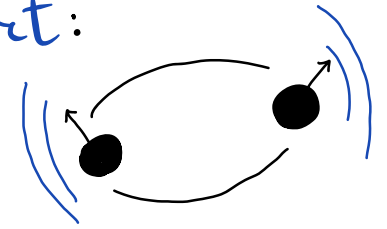
$$\text{Im } 2\delta_2(b) = \begin{array}{c} P_2 \\ \hline \text{---} K_2 \text{---} \\ | \\ \text{---} K \text{---} \\ | \\ \text{---} K_1 \text{---} \\ P_1 \end{array} \Big| \Big| \begin{array}{c} -K_2 \\ \hline \text{---} \\ | \\ \text{---} -K \text{---} \\ | \\ \text{---} -K_1 \text{---} \\ P_4 \end{array} P_3$$

$$= \frac{1}{2} \int_{K, K_1, K_2} |\tilde{M}_5|^2 \stackrel{\epsilon \rightarrow 0}{=} \frac{1}{2} \int_{K, K_1, K_2} \left| K_N \sum_{i=1}^4 \frac{\overbrace{P_i^M P_i^V}}{P_i \cdot K} M_0 \right|^2$$

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- By knowing the 1PM amplitude we can compute $2\delta_2^{\text{rr}}(b) \Rightarrow \chi_2^{\text{rr}}(b)$

$$\chi_2^{\text{rr}}(b) = -\frac{1}{P} \frac{\partial \text{Re } 2\delta_2^{\text{rr}}(b)}{\partial b}$$

Radiation reaction (RR) effects for Kerr

$$\bullet 2\delta_2^{\text{rr}}(b) = -\frac{\pi}{2} \lim_{\epsilon \rightarrow 0} \epsilon \int_{K, K_1, K_2} |\tilde{M}_5|^2 = 2\delta_2^{\text{rr}}(b) \Big|_{a=0}, \quad f^M = \frac{\sigma^2 b}{2(2\sigma^2 - 1)} \sum_{\pm} \frac{(1 \pm \sigma)^2}{b_{\pm}^2} b_{\pm}^M$$

where: $b_{\pm} = b \pm ia$

$$\bullet \chi_2^{\text{rr}}(b) = \chi_2^{\text{rr}}(b) \Big|_{a=0} \frac{\left(1 + \frac{2\sigma\sqrt{\sigma^2 - 1}}{2\sigma^2 - 1} \frac{a}{b}\right) \left[1 + \frac{4\sigma\sqrt{\sigma^2 - 1}}{2\sigma^2 - 1} \frac{a}{b} + \left(\frac{a}{b}\right)^2\right]}{\left[1 - \frac{a}{b}\right]^3} \quad [\text{F.A. Di Vecchia 2203.13272}]$$

$$= \chi_2^{\text{rr}}(b) \Big|_{a=0} \left[1 + \frac{6\sigma\sqrt{\sigma^2 - 1}}{2\sigma^2 - 1} \frac{a}{b} + \left(\frac{a}{b}\right)^2 + O(a^3)\right]$$

[Jakobsen, Mogull 2201.07778]

Conclusions:

- Using a notion of minimal coupling that incorporates classical spin it is possible to construct a consistent field theory that describes the dynamics of classically spinning compact objects and it can be used to perform precision computations.

Outlook:

- Add more contact terms and higher derivative corrections;
- Worldline description;
- Beyond 2PM in the conservative sector.