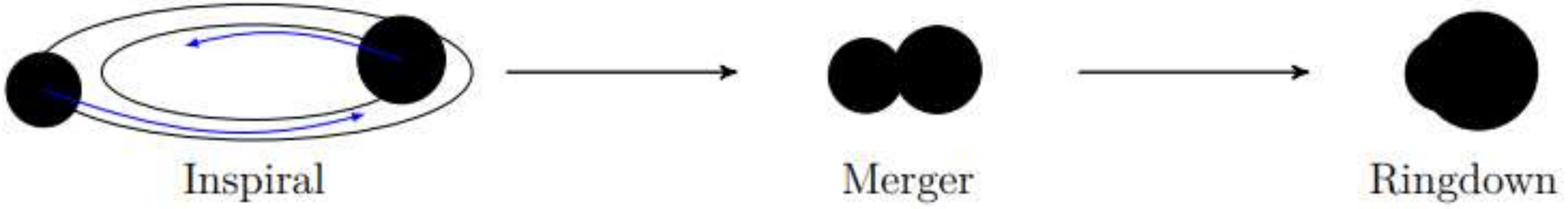


Black Hole Ringdowns in a Non-Kerr Geometry

Zaryab Ahmed

University of Stavanger

Universitetet i Stavanger

32nd Nordic Network Meeting on "Strings, Fields and Branes"

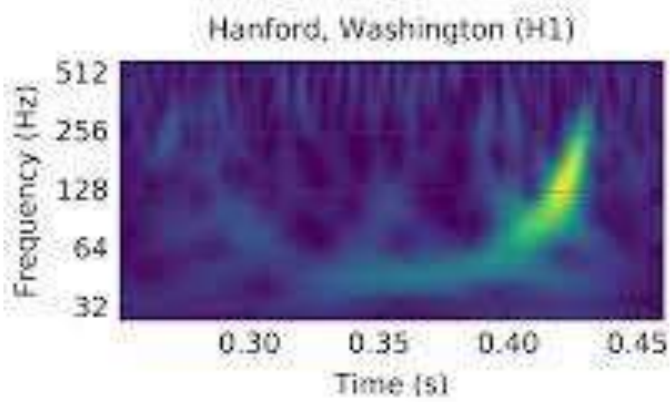
6/12/23

Norwegian Particle, Astroparticle & Cosmology Theory network

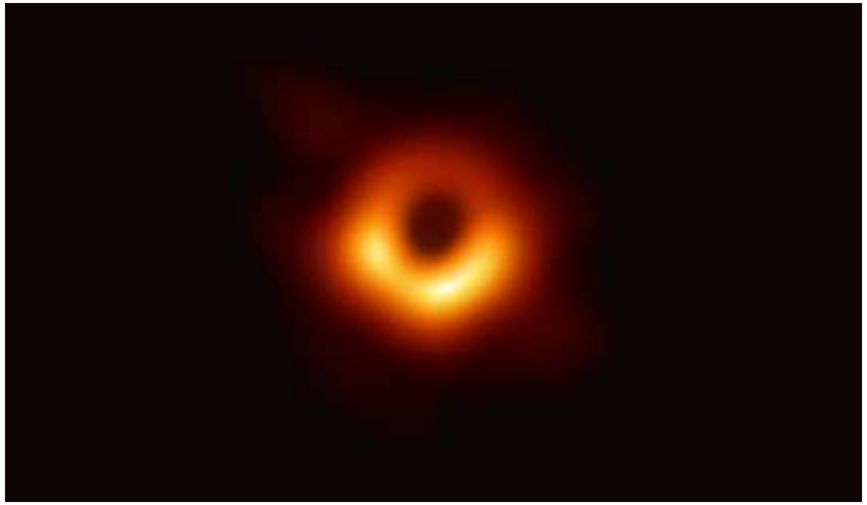
Probing Strong-Regime Gravity

- Einstein's Field Equations (2015)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu}$$



- LIGO observation GW150914 (2015), first gravitational waves
- Event Horizon Telescope, Black hole image captured (2017)



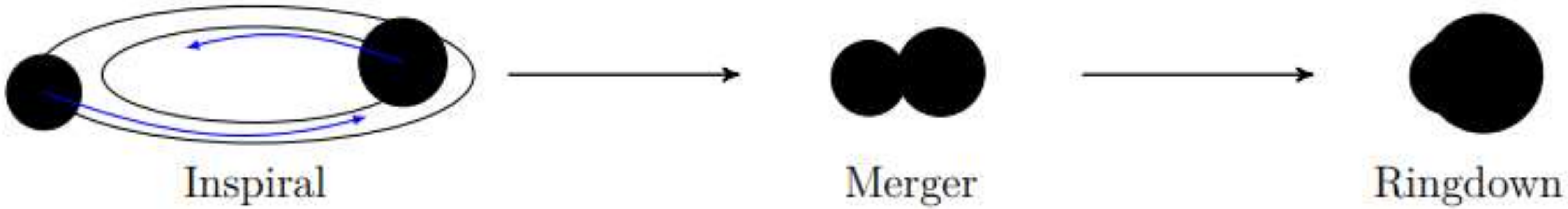
- Current observations consistent with General Relativity
- Novel constraints to non-GR models

1. https://www.ligo.org/science/Publication-GW150914/
2. https://eventhorizontelescope.org/blog/astronomers-reveal-first-image-black-hole-heart-our-galaxy

Our Ringdown Analysis with GW190521

Z. Ahmed (UiS), A. Nielsen (UiS), S. Kastha (NBI) (arxiv: xxxx.xxxx)

- Inspiral-Merger-Ringdown
- Model ringdown with some non-Kerr metric
- Confront Ringdown data with non-Kerr model



Introduction

- We are interested in the ringdown of black holes.
- We like to fit damped sinusoids (QNMs).
- Five parameters for each mode: Starting time, amplitude, phase, frequency, damping time.
- Perturb around a background BH (Kerr in GR), to find frequency and damping time (2 parameters -> 2 parameters in standard GR).
- Starting time, amplitude, phase determined by inspiral.

Ringdown of a Perturbed Black Hole

- The decay of a linearly perturbed BH is characterized by Quasi-Normal Modes
- A perturbed BH emits energy in the form of GW as it stabilizes
- A QNM has a real and imaginary part, such that it is given by damped sinusoids

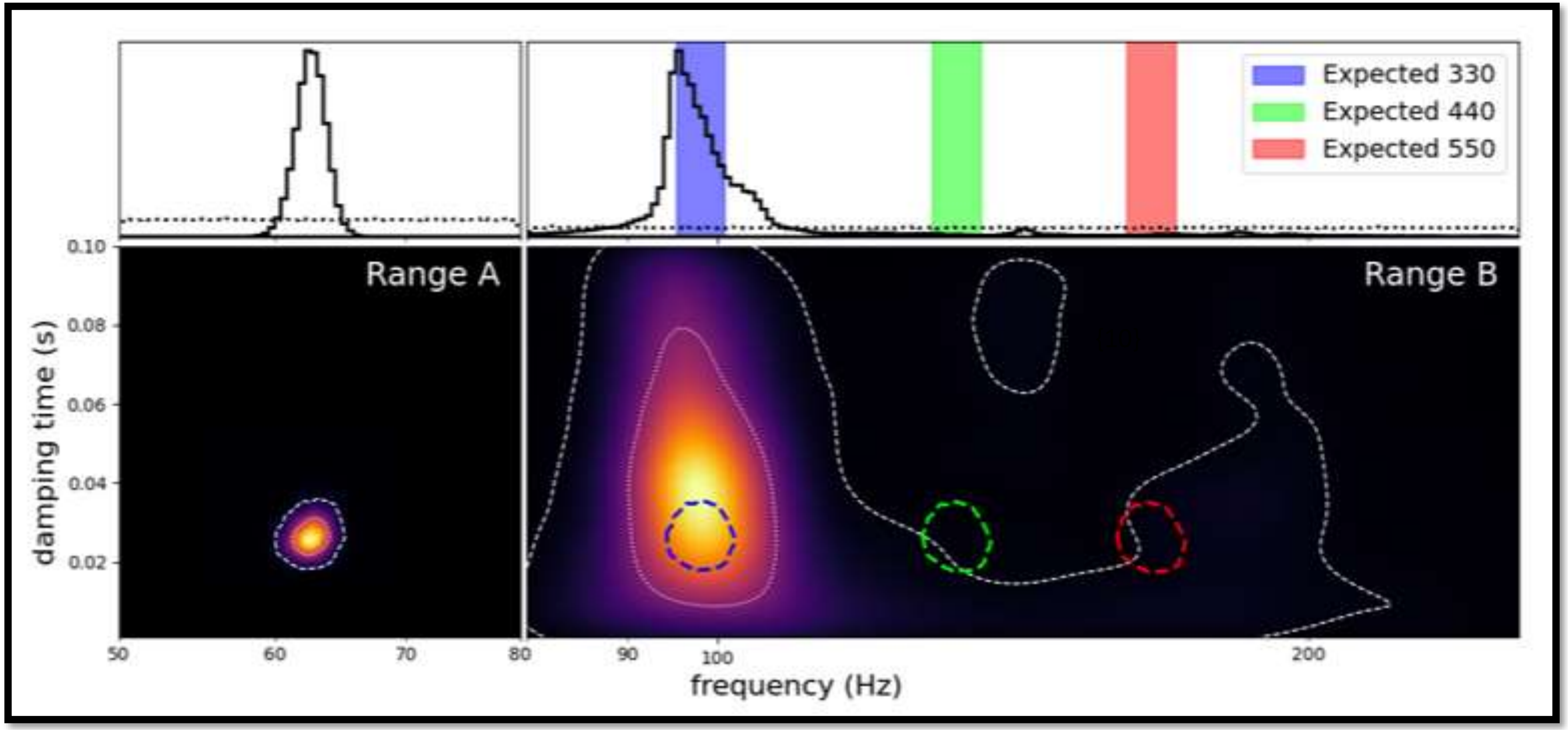
$$f(t) \propto e^{i(\omega t+\phi)} e^{-t/\tau}$$

- The modes are characterized by values of l, m and n

$$h_+ + \imath h_\times = \frac{M}{r} \sum_{lmn} A_{lmn} e^{\imath(\omega_{lmn} t + \phi_{lmn})} e^{-t/\tau_{lmn}} S_{lmn}$$

Multi-mode observation: GW190521

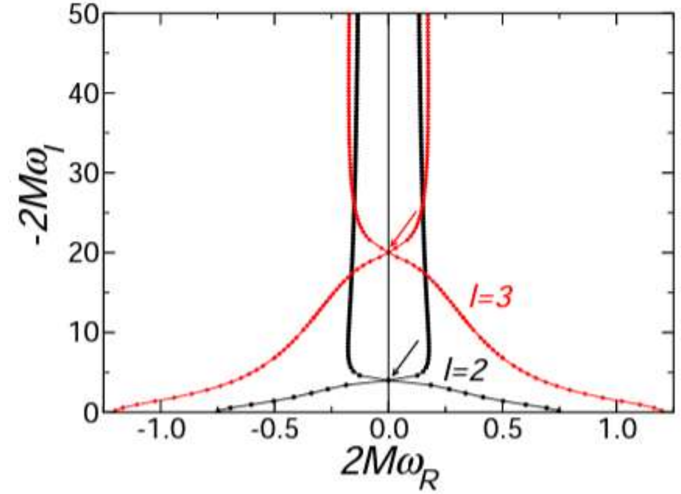
{3}



{3} Multimode Quasinormal Spectrum from a Perturbed Black Hole https://doi.org/10.1103/PhysRevLett.131.221402

Computation of Quasi-Normal Modes

- QNMs normally computed with linearized metric perturbations (Teukolsky master equation)



{4}

- QNMs can also be obtained from geodesics at the lightring $\omega = l\Omega - \iota\left(n + \frac{1}{2}\right)\gamma$ {5}
  - $Re(\omega)$: Depends on the orbital frequency of a graviton at the light ring
  - $Im(\omega)$: Depends on the characteristic timescale to escape the orbit

{4} QNMs of black holes and black branes https://iopscience.iop.org/article/10.1088/0264-9381/26/16/163001

{5} Stability of charged rotating black holes in eikonal limit 10.1103/PhysRevD.31.290

General Stationary, Axisymmetric Metrics may have more than 2 parameters

{6}

$$ds^2 = -[1+h(r,\theta)](1-\frac{2Mr}{\Sigma})dt^2 - [1+h(r,\theta)]\frac{4aMr\sin^2\theta}{\Sigma}dtd\phi + \frac{\Sigma[1+h(r,\theta)]}{\Delta + a^2\sin^2\theta h(r,\theta}) dr^2 + \Sigma d\theta^2 + [\sin^2\theta(r^2+a^2+\frac{2a^2Mr\sin^2\theta}{\Sigma}) + h(r,\theta)\frac{a^2(\Sigma+2Mr)\sin^4\theta}{\Sigma}]d\phi^2$$

where; $h(r,\theta) = \sum_{k=0}^{\infty}(\epsilon_{2k} + \epsilon_{2k+1}\frac{Mr}{\Sigma})(\frac{M^2}{\Sigma})^k$

$$g_{tt}^{JP} = -(1-\frac{2Mr}{\Sigma}) - \epsilon_3\frac{M^3(r-2M)}{r^4}$$

$$g_{rr}^{JP} = \frac{\Sigma}{\Delta} + \epsilon_3\frac{M^3(r-2M)}{\Delta^2}$$

$$g_{\theta\theta}^{JP} = \Sigma$$

$$g_{\phi\phi}^{JP} = (r^2 + a^2 + \frac{2Ma^2r\sin^2\theta}{\Sigma})\sin^2\theta + \epsilon_3\frac{a^2M^3(r+2M)}{r^3}$$

$$g_{t\phi}^{JP} = -\frac{2Mar\sin^2\theta}{\Sigma} - \epsilon_3\frac{2aM^4}{r^4}$$

- Non-GR (not a vacuum solution), parametric deviation, reduces to Kerr
- ϵ_0= ϵ_1=0, for the metric to be asymptotically flat
- ϵ_2 = 4.6x10^{-4} , Lunar Laser Ranging experiment
- ϵ_3 , first unconstrained parameter $\rightarrow h(r,\theta) = \epsilon_3\frac{M^3 r}{\Sigma^2}$
- Real and imaginary parts of the QNM spectrum for equatorial orbits in Johanssen-Psaltis geometry up to linear order in ϵ_3:

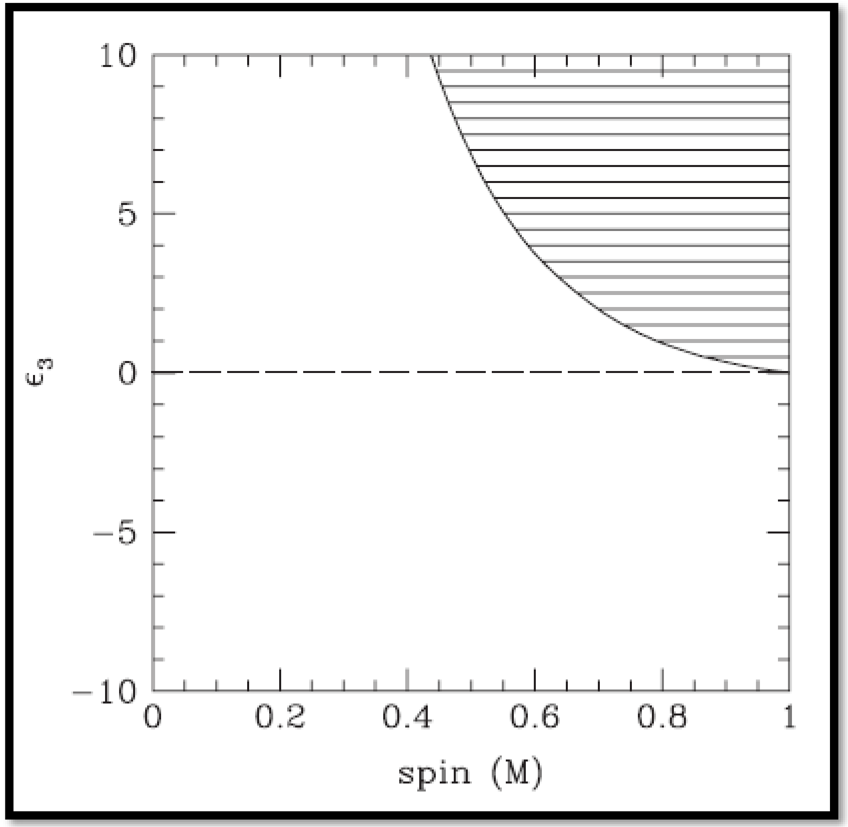
{7}

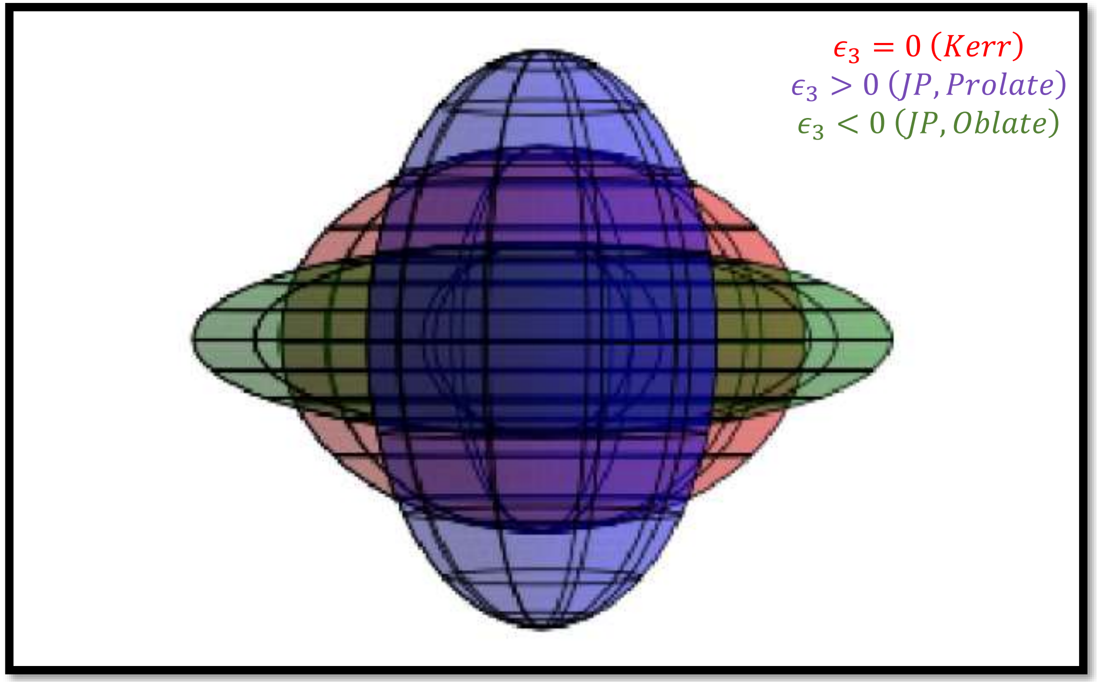
$$\omega_R^{JP} = \omega_R^K + \epsilon_3(\frac{1}{81\sqrt{3}M} + \frac{10}{729M}\chi + \frac{47}{1458\sqrt{3}M}\chi^2)$$

$$\omega_I^{JP} = \omega_I^K - \epsilon_3(\frac{1}{486M}\chi + \frac{16}{2187\sqrt{3}M}\chi^2)$$

{6} A metric for rapidly spinning black holes suitable for strong-field test of the no-hair theorem https://link.aps.org/doi/10.1103/PhysRevD.83.124015
{7} Probing beyond-Kerr spacetimes with inspiral-ringdown corrections to gravitational waves https://link.aps.org/doi/10.1103/PhysRevD.101.084050

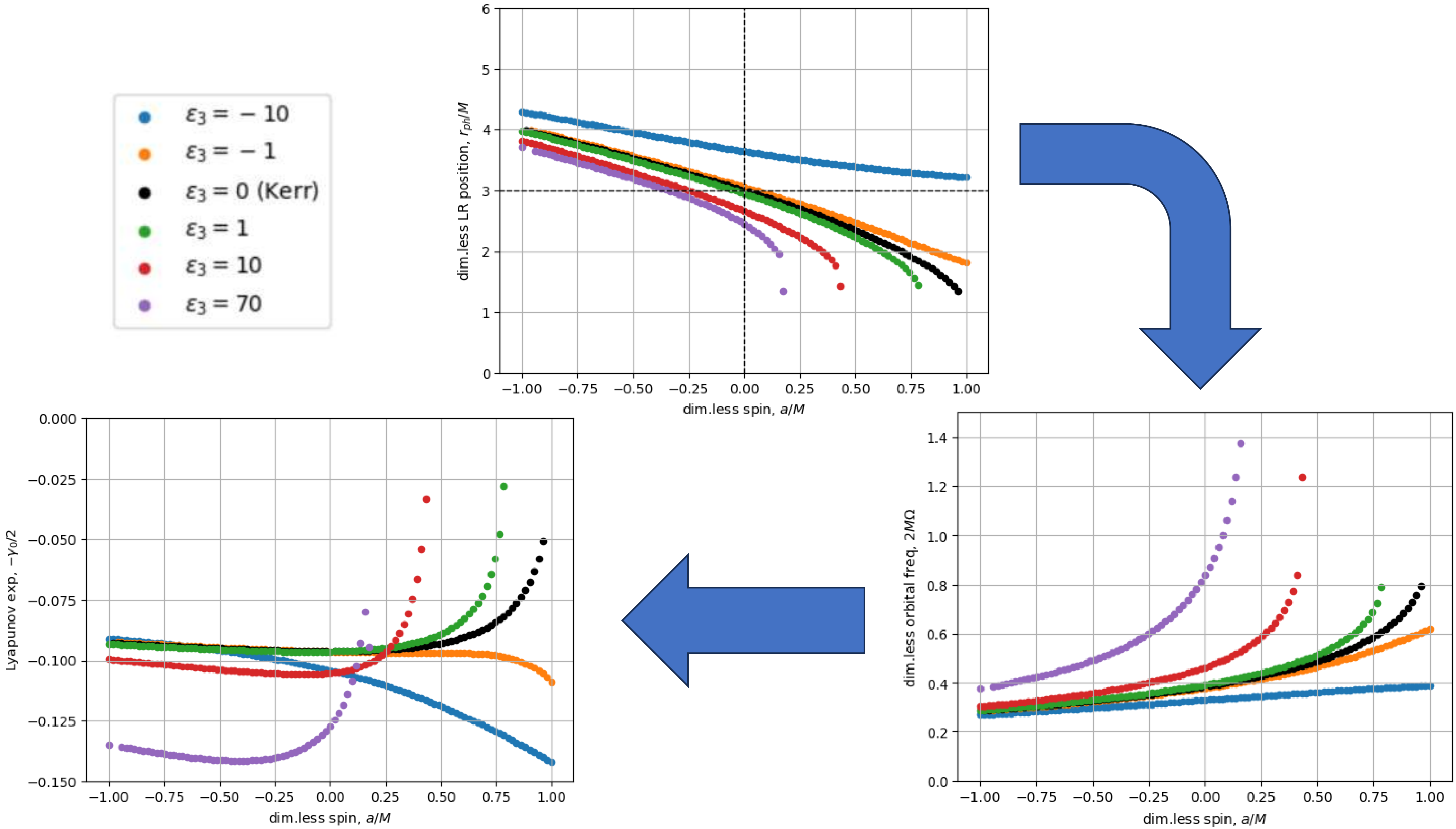
{6}





A visual representation of how ϵ_3 affects the ergospheres of Kerr and JP spacetimes.

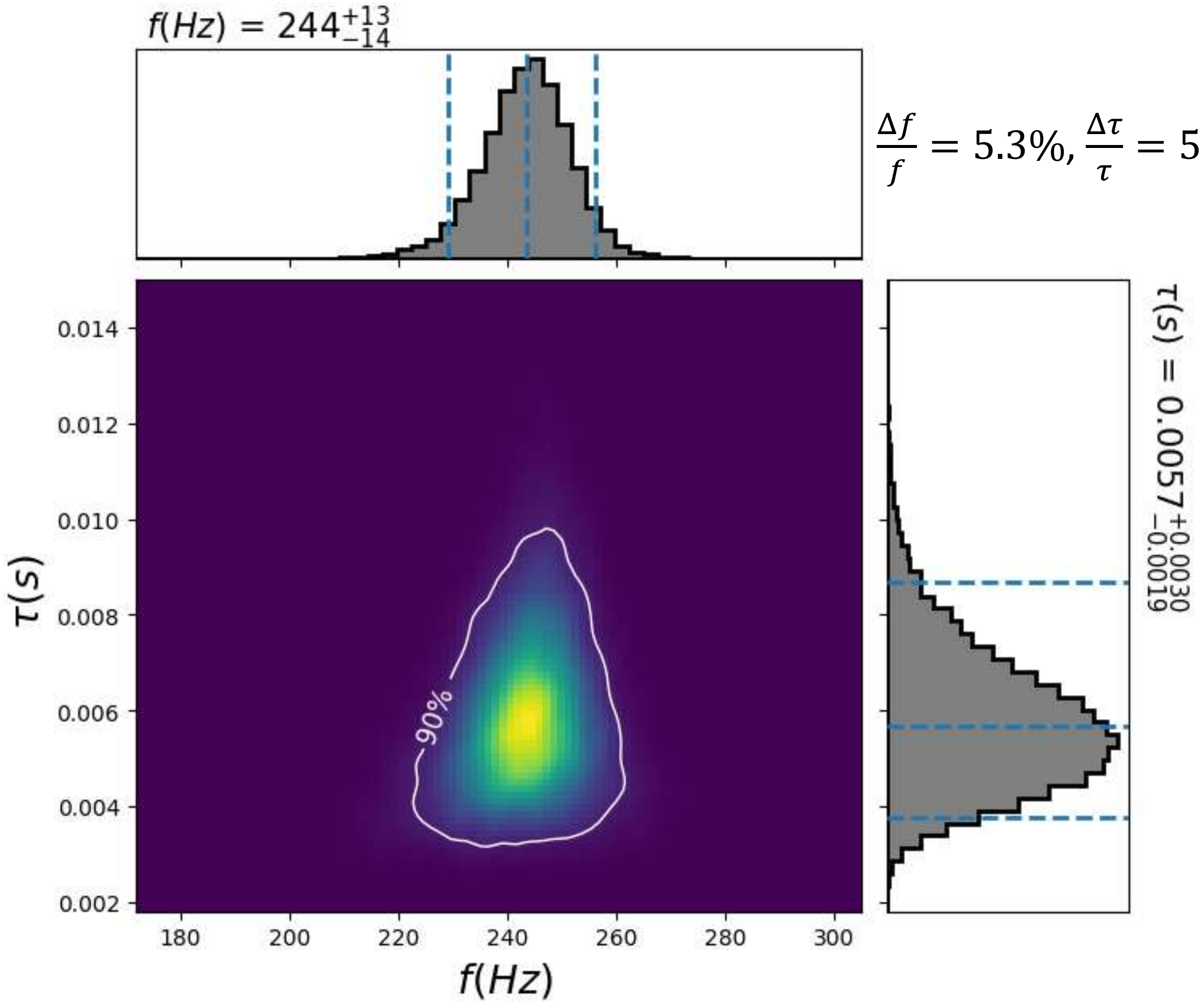
{6} A metric for rapidly spinning black holes suitable for strong-field test of the no-hair theorem https://link.aps.org/doi/10.1103/PhysRevD.83.124015



$\varepsilon_3 = -10$
$\varepsilon_3 = -1$
$\varepsilon_3 = 0$ (Kerr)
$\varepsilon_3 = 1$
$\varepsilon_3 = 10$
$\varepsilon_3 = 70$
dim.less LR position, r_{ph}/M
dim.less spin, a/M
dim.less orbital freq, $2M\Omega$
dim.less spin, a/M
Lyapunov exp, $-\gamma_0/2$
dim.less spin, a/M

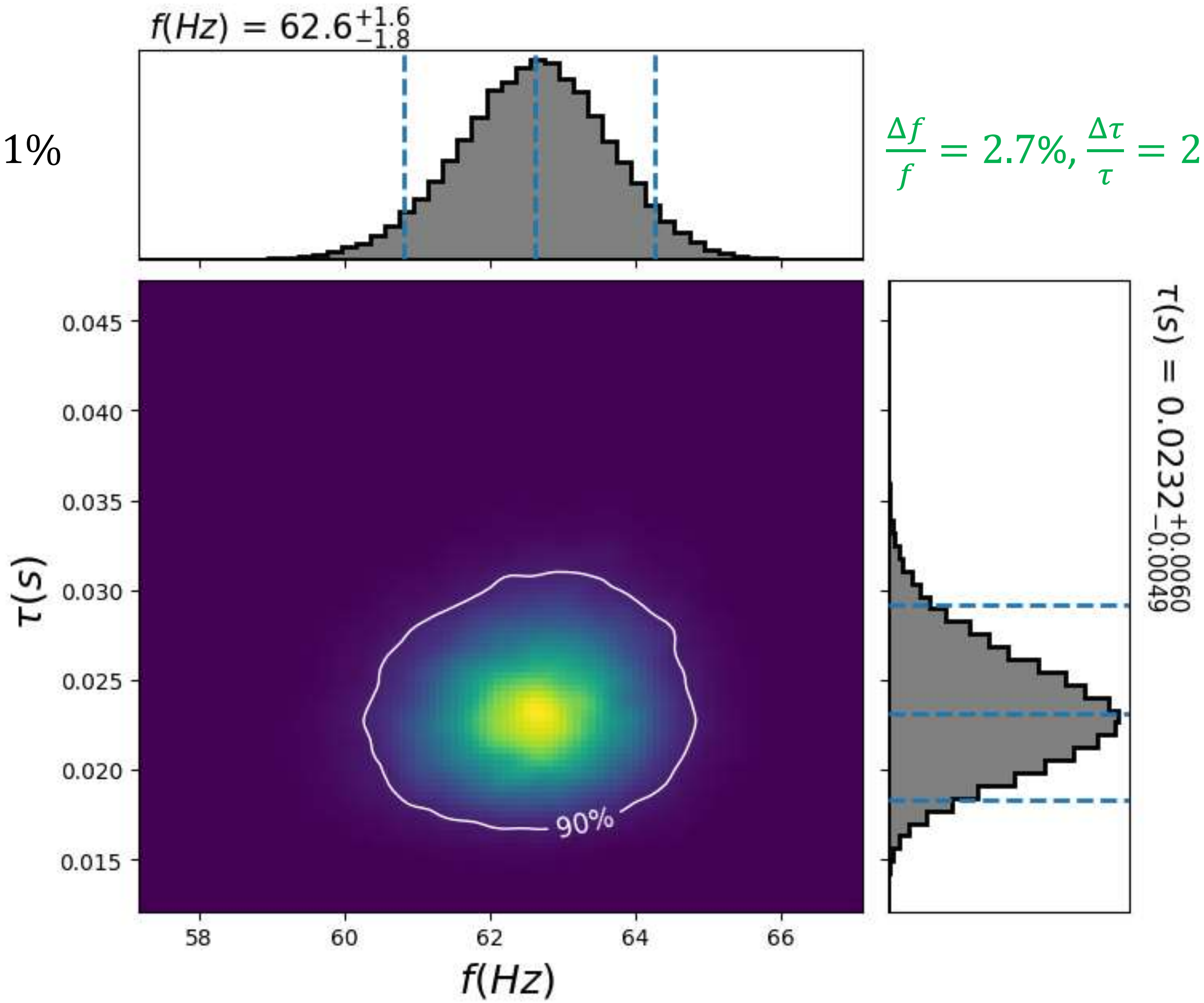
Frequency-Tau Distribution

- GW150914



$\frac{\Delta f}{f} = 5.3\%, \frac{\Delta \tau}{\tau} = 51\%$

- GW190521



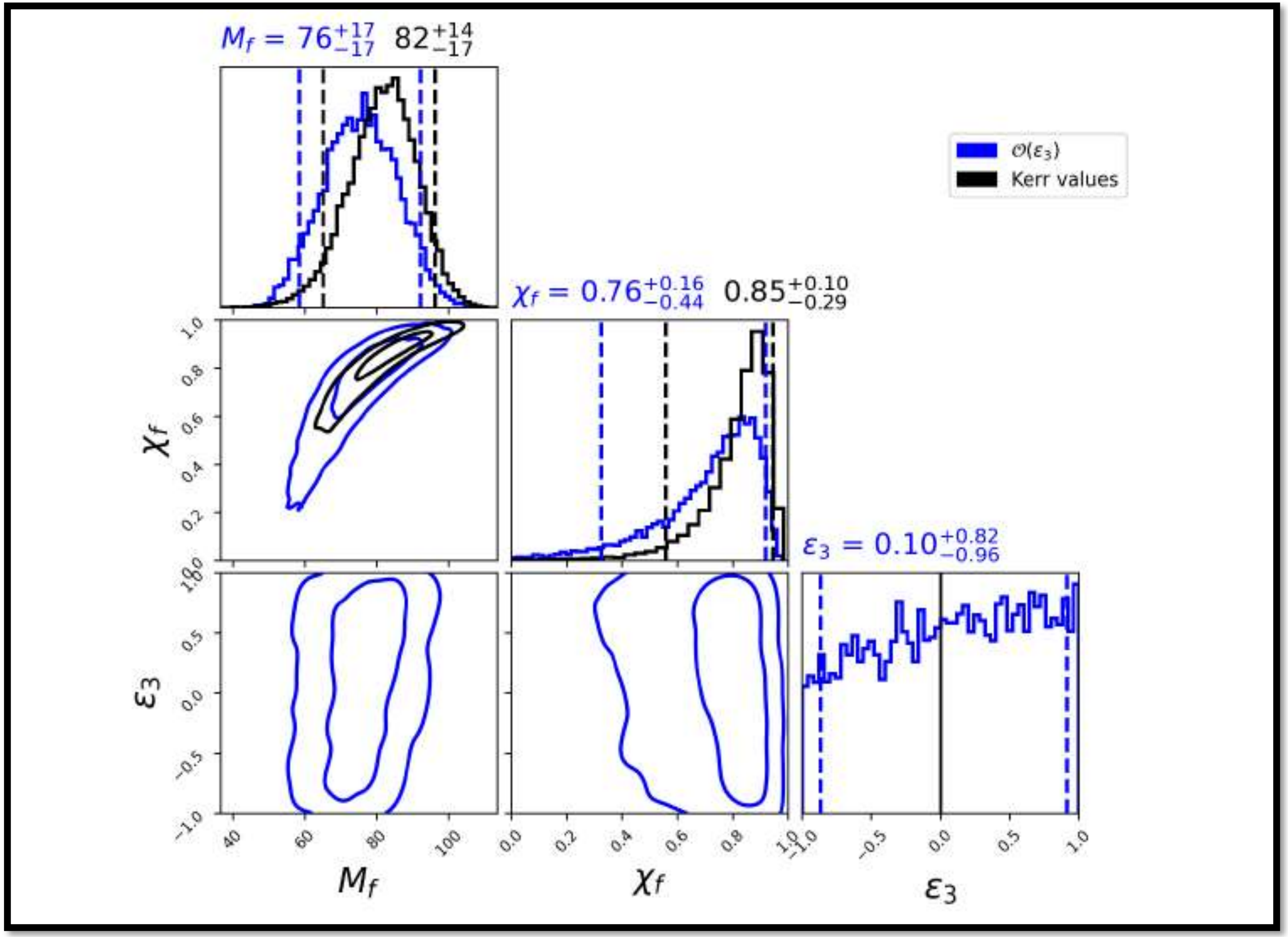
$\frac{\Delta f}{f} = 2.7\%, \frac{\Delta \tau}{\tau} = 23\%$

GW150914

- Prior ranges:

$$M = [20,200]$$
$$\chi = [0,1]$$
$$\epsilon_3 = [-1,1]$$

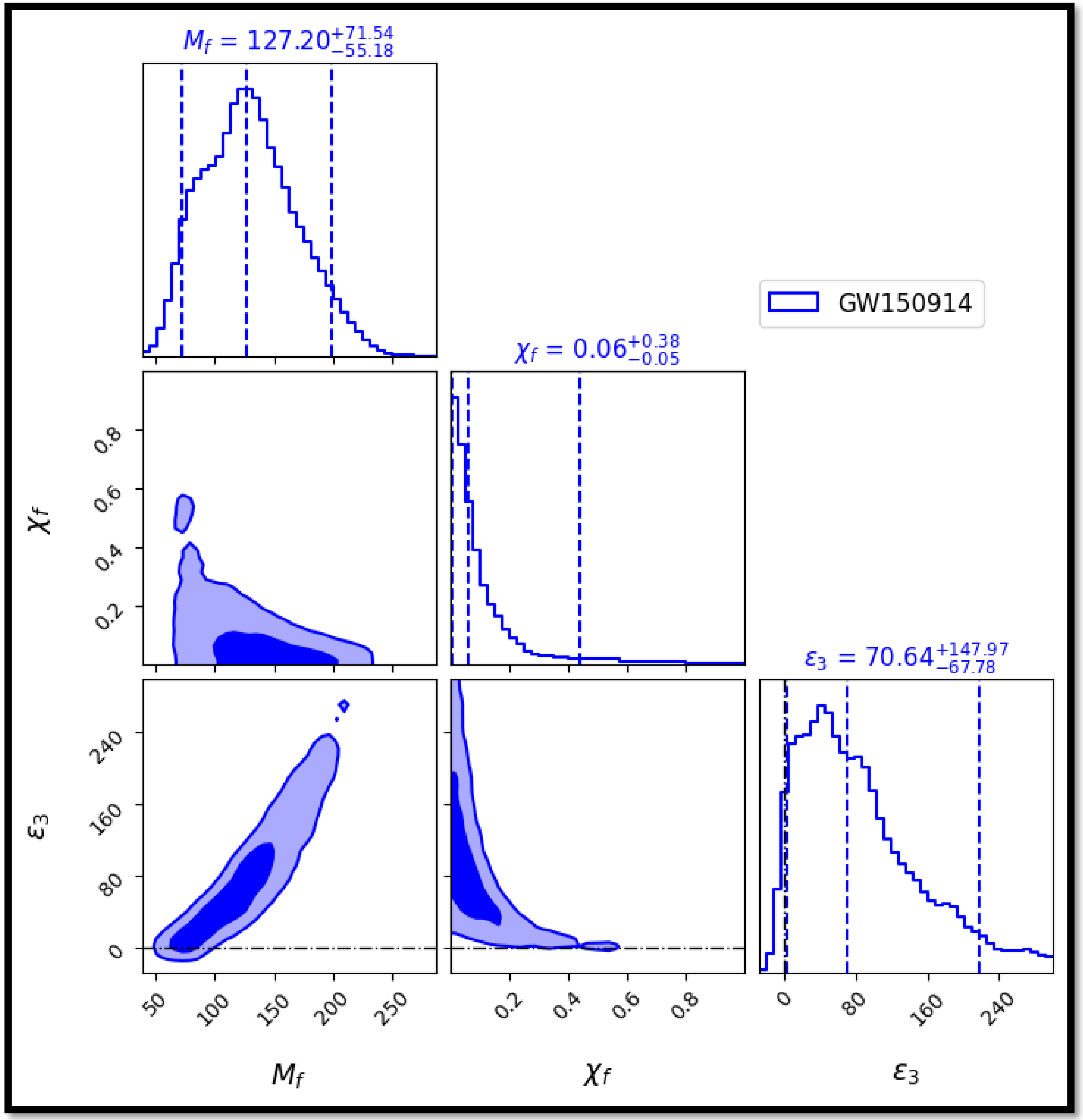
- ϵ_3 posterior almost flat, more room for ϵ_3.



GW150914

- Prior ranges:

$$M = [20,300]$$
$$\chi = [0,1]$$
$$\epsilon_3 = [-30,300]$$



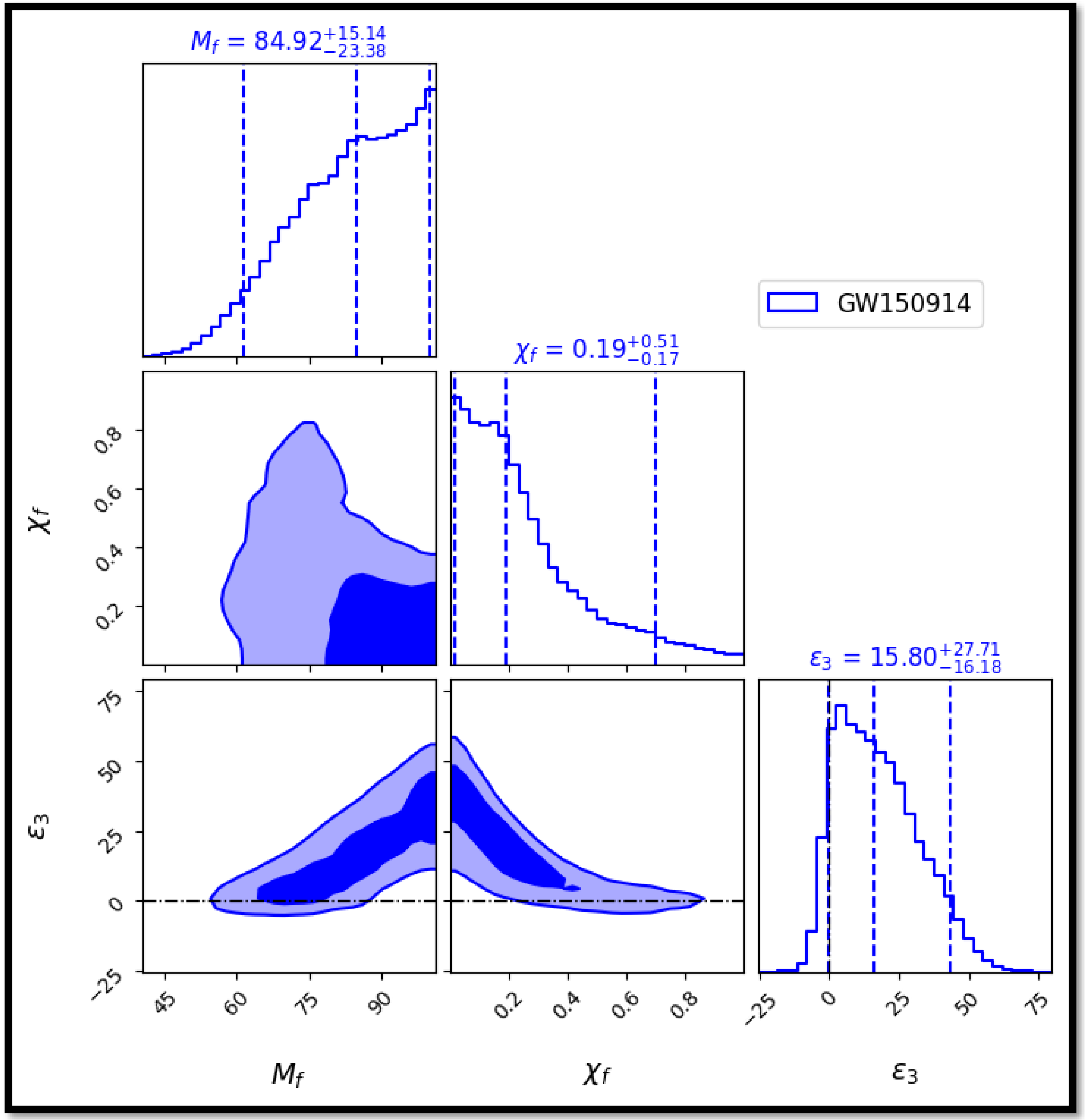
GW150914

- Maximum likelihood of Mass from IMR

$$ML = 67M_o$$

- IMR-informed prior ranges:

$$M = [0.5ML, 1.5ML]$$
$$\chi = [0,1]$$
$$\epsilon_3 = [-30,300]$$



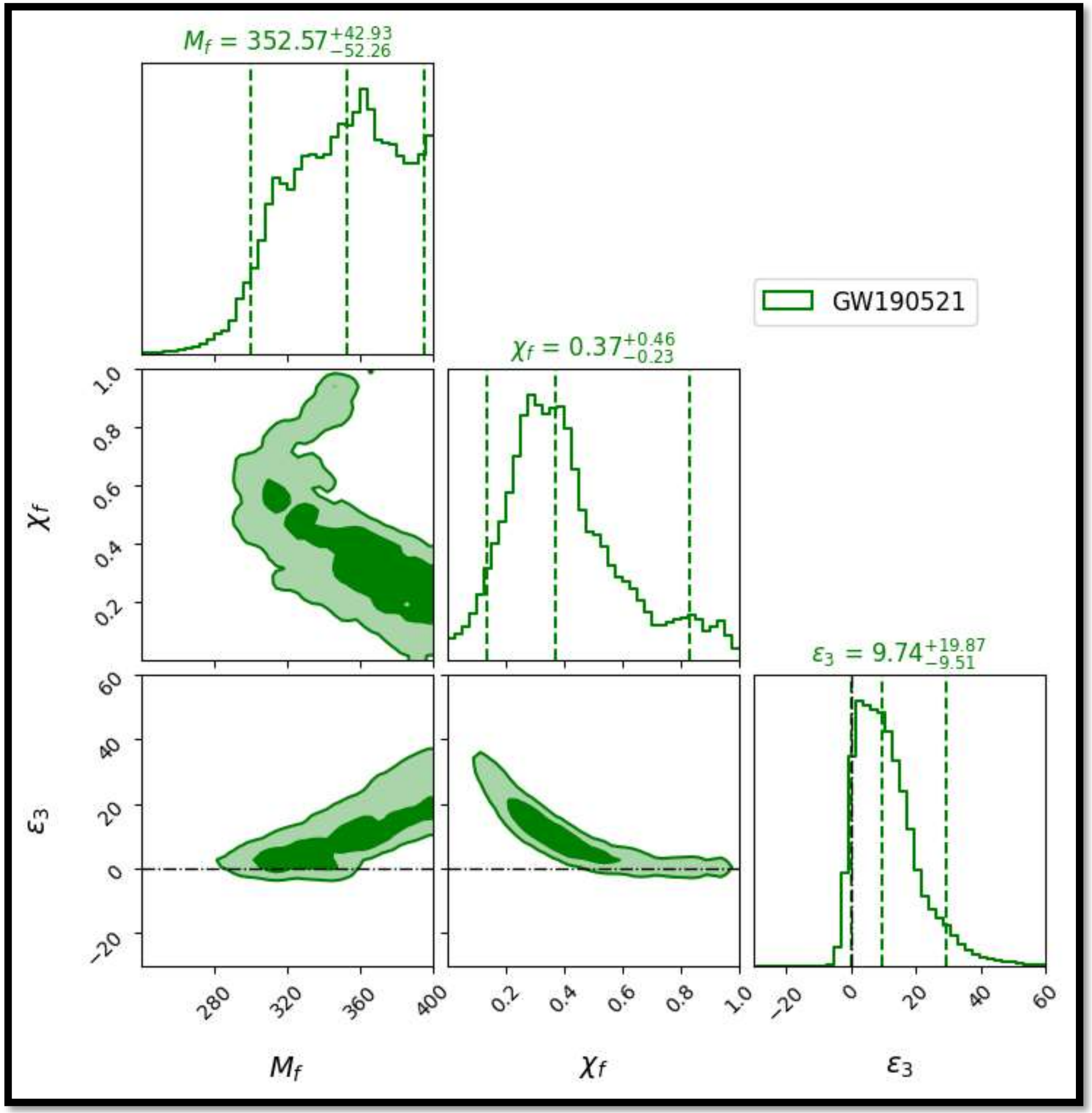
GW190521

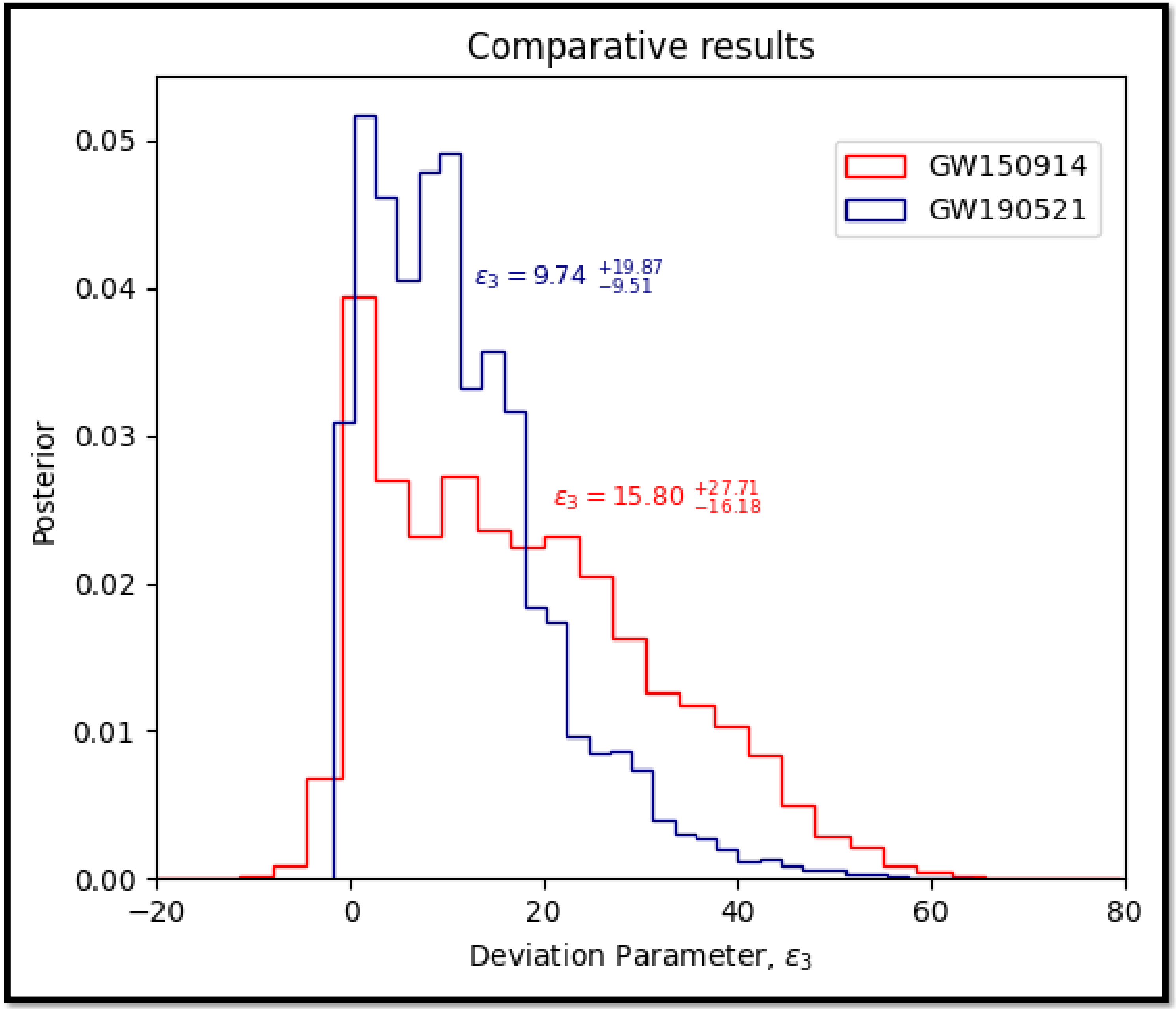
- Maximum likelihood of Mass from IMR

$$ML = 266M_o$$

- IMR-informed prior ranges:

$$M = [0.5ML, 1.5ML]$$
$$\chi = [0,1]$$
$$\epsilon_3 = [-30,300]$$





Comparative results
GW150914
GW190521
$\varepsilon_3 = 9.74\ ^{+19.87}_{-9.51}$
$\varepsilon_3 = 15.80\ ^{+27.71}_{-16.18}$
Posterior
0.05
0.04
0.03
0.02
0.01
0.00
−20
0
20
40
60
80
Deviation Parameter, ε_3

Conclusions

- Ringdown is sensitive to post-merger metric.
- 2 -> 3 problem
- Spin is anti-correlated with ϵ_3 in the ringdown; influences the prior choices.
- We find a tighter constraint $\epsilon_3 = 9.74^{19.87}_{-9.51}$ using GW190521 220 mode ringdown, relative to existing constraints from GW150914 220 mode ringdown.[8]
- GW190521 ringdown (SNR=12.2) is a better candidate than GW150914 (SNR = 6.3) ringdown for non-Kerr parameter estimation.
- Higher SNR results in tighter constraints.
- Future detectors like LISA and CE and/or multimode observations like GW190521 are expected to provide better quality of data and therefore improve the constraint on deviation parameters like ϵ_3.[9]

{8} Measuring deviations from the Kerr geometry with black hole ringdown. (https://arxiv.org/abs/2212.10725)
{9} Probing beyond-kerr spacetimes with inspiral-ringdown corrections to gravitational waves. (https://journals.aps.org/prd/abstract/10.1103/PhysRevD.101.084050)

THANK YOU!

Post-Kerr QNM Validity[5]

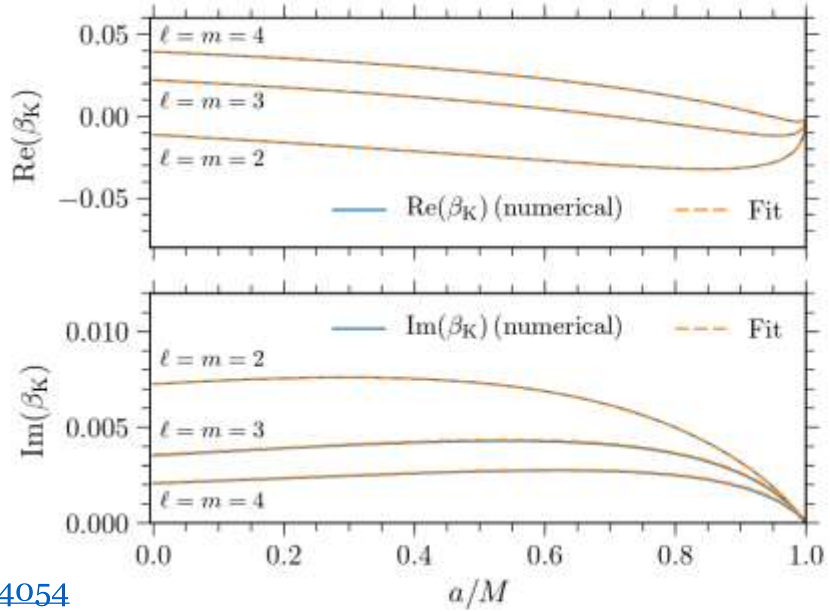
- QNM spectrum used for GW analysis based on Kerr metric
- Ensure that it is valid for a non-Kerr geometry
- Introduce an offset function, β_K

$$\omega_{\rm obs} = \sigma + \beta_{\rm K}$$

$$\omega_{\rm K} = \sigma_{\rm K} + \beta_{\rm K}$$

$$\omega_{\rm obs} - \omega_{\rm K} = \sigma - \sigma_{\rm K} \neq 0$$

$$\sigma = \sigma_{\rm K} + \delta\sigma$$



{5} Post-Kerr black hole spectroscopy https://link.aps.org/doi/10.1103/PhysRevD.96.064054

General Stationary, Axisymmetric Metrics have more than 2 parameters[1]

- A general metric, in Boyer-Lindquist coordinates, with principle null directions, ∂_t and ∂_ϕ, takes the form

$$g_{\mu\nu} = \begin{bmatrix} -g_{tt} & 0 & 0 & g_{t\phi} \\ 0 & g_{rr} & 0 & 0 \\ 0 & 0 & g_{\theta\theta} & 0 \\ g_{\phi t} & 0 & 0 & g_{\phi\phi} \end{bmatrix}$$

- In this scenario, the computation of the light ring position is given by finding the roots of:

$$g_{tt}(r_{ph})b_{ph}^2 + 2g_{t\phi}(r_{ph})b_{ph} + g_{\phi\phi}(r_{ph}) = 0$$
$$g'_{tt}(r_{ph})b_{ph}^2 + 2g'_{t\phi}(r_{ph})b_{ph} + g'_{\phi\phi}(r_{ph}) = 0$$

- And the orbital frequency and Lyapunov exponent (inverse of damping time) are given by:

$$\Omega_{ph} = b_{ph}^{-1} = \frac{g'_{tt}}{-g'_{t\phi} \pm \sqrt{g'^2_{t\phi} - g'_{tt}g'_{\phi\phi}}}$$

$$\kappa_{ph}^2 = \frac{1}{2}\frac{d^2 f}{dU^2}(U_{ph}) = \frac{f''(U_{ph})}{2U_{ph}^2}$$

$$\gamma_{ph} = |\kappa_{ph}\Omega_{ph}|$$

{1} Post-Kerr black hole spectroscopy https://link.aps.org/doi/10.1103/PhysRevD.96.064054