Holographic superconductors with Born-Infeld electrodynamics and noncommutative geometries

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December 5, 2023

- Material which has the property that its electrical resistance decreases (~ 0) below a temperature T_c.
- In presence of H, it can not penetrate them. If a H_c is applied below T_c, the system loses its superconductor properties.

Minimal ingredients to describe a holographic superconductor

- Charged black hole $\leftrightarrow$ Introduce another scale (finite chemical potential μ)
- System that admits hairy solutions at low temperatures and not at high ones.
- Gauge field A_μ (global $U(1)$ symmetry) $\leftrightarrow$ Conserved current J^μ
- Charged field ψ $\leftrightarrow$ Charged operator $\mathcal{O}$ (to break $U(1)$ symmetry spontaneously)

Backreacted BI superconductor

Lagrangian

$$\mathcal{L} = \frac{1}{16\pi}\left(G^{-1}(R - 2\Lambda) + \mathcal{L}_B\right) - \frac{m^2}{L^2}\psi^2 - |\nabla\psi - iqA\psi|^2$$

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)n(r)}dr^2 + r^2 d\Omega_2$$

$$\mathcal{L}_B = b^2\left(1 - \sqrt{1 + \frac{F^{\mu\nu}F_{\mu\nu}}{2b^2}}\right)$$

Equations

$$16\pi G q^2 r\psi\psi^*\phi^2 + f^2\left(n' + 16\pi G r n \psi' \psi^{*'}\right) = 0$$

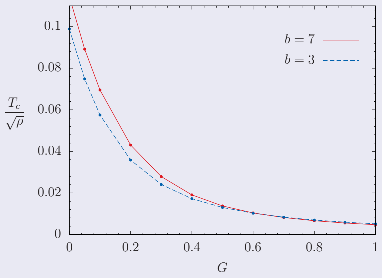
$$\frac{4b^2 r^2 G}{\sqrt{1 - \frac{n\phi'^2}{b^2}}} - 2n(rf' + f) - rfn' - 16\pi G m^2 r^2 \psi\psi^* - 2\Lambda r^2 + 4b^2 r^2 G = 0$$

$$8\pi q^2 r^2 \psi\psi^*\phi - f\sqrt{n}\left(\frac{r^2\sqrt{n}\phi'}{\sqrt{1 - \frac{n\phi'^2}{b^2}}}\right)' = 0$$

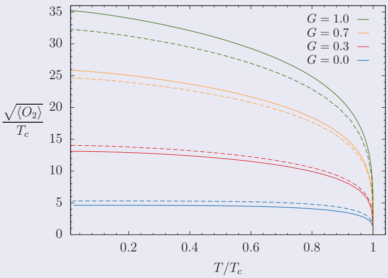
$$(q^2\phi^2 - m^2 f)r^2\psi + f\sqrt{n}\left(r^2 f\sqrt{n}\psi'\right)' = 0$$

Asymptotic behaviour $r \to \infty$

$$\psi(r) = \frac{\mathcal{O}_1}{r} + \frac{\mathcal{O}_2}{\sqrt{2}r^2} + \cdots \qquad \phi(r) = \mu - \frac{\rho}{r} + \cdots$$



(a) $T_c/\sqrt{\rho}$ vs G



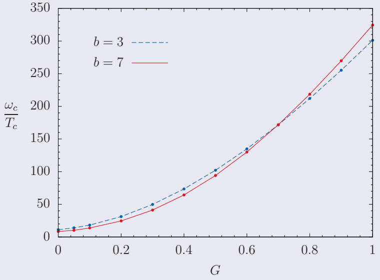
(b) $\sqrt{\langle \mathcal{O}_2 \rangle}/T_c$ vs T/T_c

Perturbation equation

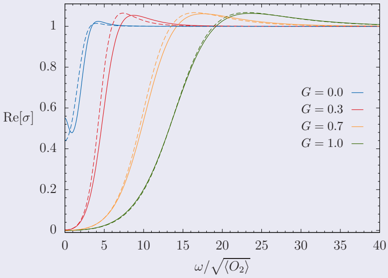
$$A_x'' + \left(\frac{2f' + 16\pi q^2 \psi^2 \phi \phi' \sqrt{1 - n\phi'^2}}{f} + \frac{n'}{n} - \frac{4n\phi'^2}{r} \right) \frac{A_x'}{2}$$

$$+ \left(\frac{\omega^2}{fn} - \frac{4G\phi'^2}{\sqrt{1 - n\phi'^2}} - \frac{8\pi q^2 \psi^2 \sqrt{1 - n\phi'^2}}{n} \right) \frac{A_x}{f} = 0$$

$$A_x(r) = -\frac{iE}{\omega} + \frac{J}{r} + \cdots$$



(c) ω_c / T_c vs G



(d) $Re(\sigma)$ vs $\omega/\sqrt{\langle \mathcal{O}_2 \rangle}$

Noncommutative distributions of mass and charge / Born-Infeld

$$\rho_m(r) = \frac{M}{(4\pi\theta)^{3/2}} e^{-r^2/4\theta} \qquad \rho_Q = \frac{Q}{(4\pi\theta)^{3/2}} e^{-r^2/4\theta}$$

$$\mathcal{L}_{BI} = b^2 \left(1 - \sqrt{1 + \frac{F^{\mu\nu} F_{\mu\nu}}{2b^2}} \right)$$

AdS-Einstein-Born-Infeld black hole ($r^2 \gg 4\theta$)

$$ds^2 = -f dt^2 + \frac{1}{f} dr^2 + r^2 d\Omega_{(2)}$$

$$f = -\frac{2M}{r} + \frac{r^2}{L^2} + \frac{2}{3} b^2 r^2 \left(1 - \sqrt{1 + \frac{Q}{b^2 r^4}} \right) + \frac{4Q^2}{3r^2} {}_2F_1\left(\frac{1}{2}, \frac{1}{4}; \frac{5}{4}; -\frac{Q^2}{b^2 r^4} \right) + g^\theta(r) e^{-r^2/4\theta}$$

$$g^\theta(r) = \frac{2M}{\sqrt{\pi\theta}} + \frac{2Q^2}{3\sqrt{\pi\theta}} \frac{r}{\sqrt{r^4 + Q^2/b^2}} - (4\pi)^2 \frac{1}{3r} \frac{Q^2}{(4\pi\theta)^{3/2}} \frac{r^2}{\sqrt{r^4 + Q^2/b^2}} +$$

$$\frac{8Q^2}{3r} \sqrt{\frac{\theta}{\pi}} \frac{1}{\sqrt{r^4 + Q^2/b^2}} \left(-2 + \frac{Q^2}{b^2} \frac{1}{r^4 + Q^2/b^2} \right) - \frac{16\Gamma\left(\frac{5}{4}\right)^2}{3\sqrt{\pi}} \left| \frac{Q}{b} \right|^{3/2} \frac{b^2}{\sqrt{\pi\theta}}$$

Lagrangian

$$\mathcal{L}_M = -r^2\left(f\psi'^2 - \frac{q^2}{f}\phi^2\psi^2\right) - m^2r^2\psi^2 + r^2b^2\left(1-\sqrt{1-\frac{\phi'^2}{b^2}}\right)$$

Equations of motion ψ ϕ

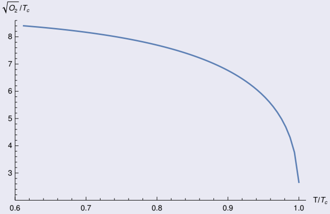
$$\phi'' + \frac{2}{r}\phi'\left(1-\frac{\phi'^2}{b^2}\right) - \frac{2q^2\psi^2}{f}\phi\left(1-\frac{\phi'^2}{b^2}\right)^{3/2} = 0$$

$$\psi'' + \left(\frac{f'}{f}+\frac{2}{r}\right)\psi' - \frac{m^2}{f}\psi + \frac{q^2\phi^2}{f^2}\psi = 0$$

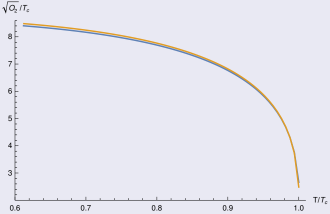
Asymptotic behaviour $r\to\infty$

$$\phi(r) = \mu - \frac{\rho}{r} + \cdots \qquad \psi(r) = \frac{\psi^-}{r} + \frac{\psi^+}{r^2} + \cdots$$

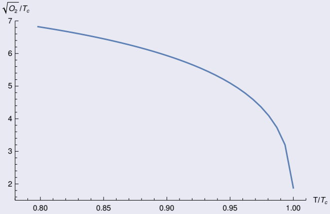
Changes in condensate with θ and Q



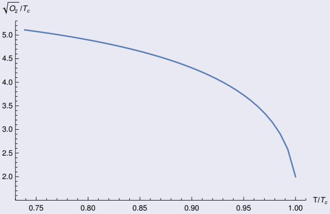
(e) $\sqrt{O_2}/T_c$ vs T/T_c ($\theta = 1/100$, $b = 7$)



(f) $\sqrt{O_2}/T_c$ vs T/T_c ($\theta = 1/50$, $b = 7$)



(g) $\sqrt{O_2}/T_c$ vs T/T_c ($Q = 1/2$, $\theta = 1/100$, $b = 7$)



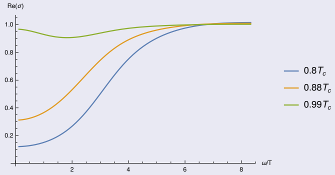
(h) $\sqrt{O_2}/T_c$ vs T/T_c ($Q = 4/10$, $\theta = 1/100$, $b = 7$)

Perturbation equation

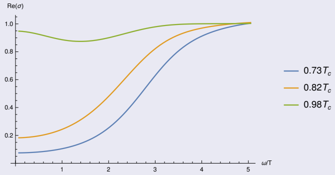
$$f^2 A'' + \left(ff' + \frac{\phi'\phi'' f^2}{b^2 - \phi'^2} \right) A' + \left(\omega^2 - 2f\psi^2 \left(1 - \phi'^2/b^2 \right)^{1/2} \right) A = 0$$

$$A_x = A_x^{(0)} + \frac{A_x^{(1)}}{r} + \cdots \qquad A_x = A_x^{(0)} \qquad \langle J_x \rangle = A_x^{(1)}$$

$$\sigma(\omega) = \frac{\langle J_x \rangle}{E_x} = -\frac{\langle J_x \rangle}{\dot{A}_x} = -\frac{i\langle J_x \rangle}{\omega A_x} = -\frac{iA_x^{(1)}}{\omega A_x^{(0)}}$$



(i) Re(σ) vs ω/T
($Q = 1/2, \theta = 1/100, b = 7$)



(j) Re(σ) vs ω/T
($Q = 4/10, \theta = 1/100, b = 7$)

Backreacted BI superconductor

- Increasing backreaction difficults superconducting condensate.
- Frequency gap increases with backreaction.
- Backreaction changes the behaviour with the variation of nonlinearity.

Noncommutative / Born-Infeld superconductor

- Increasing noncommutativity difficults condensate.
- Frequency gap changes with noncommutativity.