Cosmological drift of redshift and its imprint on gravitational wave forms

Asta Heinesen

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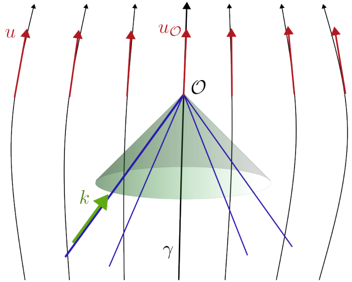
CARLSBERG FOUNDATION

The Niels Bohr International Academy

Outline

Real-time measurements in cosmology

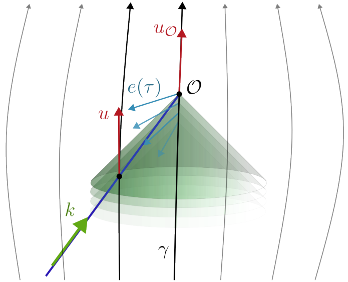
Usually durations of experiments on earth are considered a point in time in a cosmological context



Lightcone of an observer. γ represents the wordline of the observer, with 4-velocity $u_{\mathcal{O}}$ at the point of observation $\mathcal{O}$. A null congruence generated by k with vertex at $\mathcal{O}$ forms the past null cone of the observer.
M. Korzynski and AH (paper in progress)

Real-time measurements in cosmology

For precise enough measurements the extension of the experiment in time can provide new cosmological information



Family of lightcones of an observer with vertices along a small segment of the wordline γ around the original event $\mathcal{O}$. The position of a source, e, as viewed by the observer is drifting.
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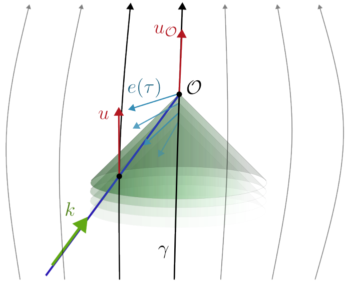

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Drift of cosmic observables: redshift, position, distance, polarization

Redshift drift

Redshift drift: time change of redshift, z, to a an emitter as viewed from the observer (Earth)

In a general FLRW space-time: $\frac{dz}{d\tau}_{\mathcal{O}} = (1+z)H_{\mathcal{O}} - H_{\mathcal{E}}$

H: Hubble parameter; $\mathcal{E}$: event of the emitter

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First proposed as an observable by Sandage in the 1960s, but not feasible measurement to perform at the time

$H_{\mathcal{O}} \sim 10^{-10}$/year, thus immense precision is required in the measurements of z

Possible for EM signals with near-future telescopes (SKA and ELT) for observation times ~ 10 years

Redshift drift in gravitational waveforms

Consider the leading-order Newtonian frequency for the inspiral in the emitter frame $\mathcal{E}$

$$\frac{\frac{df_{\mathcal{E}}}{dt}}{f_{\mathcal{E}}} = \frac{96}{5}\pi^{8/3} f_{\mathcal{E}}^{8/3} \mathcal{M}_{\mathcal{E}}^{5/3} \,,$$

When the signal travels through an expanding space the frequency is modified on its path to the observer $f_{\mathcal{O}} = f_{\mathcal{E}}/(1+z)$, similarly for the proper time measure $d\tau = (1+z)dt$

$$\frac{\frac{df_{\mathcal{O}}}{d\tau}}{f_{\mathcal{O}}} = \frac{96}{5}\pi^{8/3} f_{\mathcal{O}}^{8/3} (1+z)^{5/3} \mathcal{M}_{\mathcal{E}}^{5/3} - \frac{\frac{dz}{d\tau}}{(1+z)} \,,$$

For a non-constant redshift to the emitter over the duration of the signal, this leaves an imprint on the observed waveform

Redshift drift in gravitational waveforms

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Translates into an effective -4th order PN term in the phase:

$$\psi(f) = 2\pi f \tau_c - \frac{\pi}{4} - \phi_c + \frac{1}{(\pi\mathcal{M}f)^{5/3}}\frac{3}{128}\left(1 + \frac{1}{26}\kappa(\pi\mathcal{M}f)^{-8/3}\right).$$

$$\kappa \equiv \frac{5}{96}\mathcal{M}\frac{\frac{dz}{d\tau}}{(1+z)}.$$

Drift is expected to be very small: $\frac{\frac{dz}{d\tau}}{(1+z)}_{\mathcal{O}} \sim H_{\mathcal{O}} z \sim \frac{10^{-10}}{\text{year}} z$ for $z \lesssim 1$.

Redshift drift in gravitational waveforms

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Compare to signal/noise for extreme mass ratio inspiral in LISA with $f_{\mathcal{O}} \sim 0.01Hz \sim 10^5/\text{year}$; observation time $\Delta\tau \sim 1$ year: $\Delta\frac{\frac{df_{\mathcal{O}}}{d\tau}}{f_{\mathcal{O}}} \sim \frac{1}{f_{\mathcal{O}}\Delta\tau^2} \sim \frac{10^{-5}}{\text{year}}$.

Looking for a factor of 10^5 amplification of the redshift drift signal

Redshift drift in gravitational waveforms

Indeed the few papers that mention redshift drift in the literature judge that it is not detectable in the next generation surveys.

C. Bonvin et al. (2016) [1609.08093] & N. Tamanini et al. (2019) [1907.02018]: *"Our results confirm that the first component [redshift drift] is not detectable with LISA"*

Are there ways to overcome the low amplitude of the signal?

Redshift drift in gravitational waveforms

Are there ways to overcome the low amplitude of the signal?

- ⋆ Ideal sources pushing the number of cycles and time-duration in band. For instance $f_{\mathcal{O}} \sim 0.1Hz$ in band for 10 years (number of cycles $N \sim 10^7$) yields $\Delta \frac{\frac{df_{\mathcal{O}}}{d\tau}}{f_{\mathcal{O}}} \sim \frac{1}{f_{\mathcal{O}} \Delta\tau^2} \sim \frac{10^{-8}}{\text{year}}$

- ⋆ Stacking of large number of sources (effect does not average out)

- ⋆ Mergers at extreme redshifts? Primordial black holes in the radiation epoch:

$$\frac{\frac{dz}{d\tau}}{1+z}_{\mathcal{O}} = H_{\mathcal{O}} - \frac{H_{\mathcal{E}}}{1+z} \approx -\frac{H_{\mathcal{E}}}{1+z} \approx -H_{\mathcal{O}}\Omega_{r\mathcal{O}}(1+z)$$

At beginning of radiation dominated epoch $T \sim 10^9 K$; $z \sim 10^9$. With $\Omega_{r\mathcal{O}} \sim 10^{-5}$, we have $\frac{\frac{dz}{d\tau}}{1+z}_{\mathcal{O}} \sim -10^4 H_{\mathcal{O}} \sim -\frac{10^{-6}}{\text{year}}$

Discussion

Cosmic drift effects are interesting upcoming probes of the Universe now becoming detectable through electromagnetic waves

Could one imagine redshift drift becoming detectable with graviational waves?

Difficulty: mergers are not standardised in terms of their inspiral physics. Differences in frequency evolution may arise due to environmental effects

Physics entering at effective -4PN order will be degenerate with the redshift drift signal. Example: In modified gravity theories variation of Newtons constant enters at -4PN order