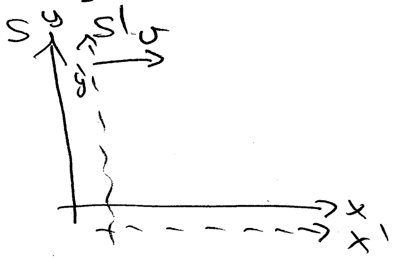
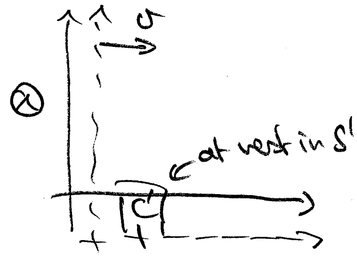
Special Relativity   Space-time diagrams



Re Collect: $x' = \frac{x - vt}{\sqrt{1-\beta^2}}$ $\quad \beta = v/c \quad$ $t' = \frac{t-(v/c^2)x}{\sqrt{1-\beta^2}}$

$y' = y \qquad z' = z$

① Moving clocks appear to tick slower "time dilation":

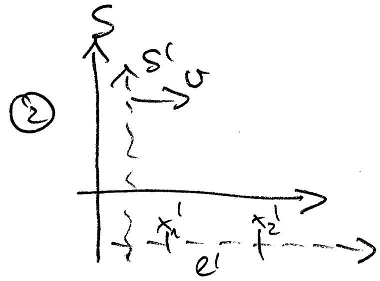


$$t_1 = \frac{t' + (v/c^2)x'}{\sqrt{1-\beta^2}} \qquad t_2 = \frac{(t'+\Delta t') + (v/c^2)x'}{\sqrt{1-\beta^2}}$$

first tick observed in S

tick of clock in S' "proper time"

$$\Delta t = t_2 - t_1 = \frac{\Delta t'}{\sqrt{1-\beta^2}} \qquad \Delta t > \Delta t'$$



② Moving measurement sticks appear shortened "length contraction"

length is measured at same time instant in S!

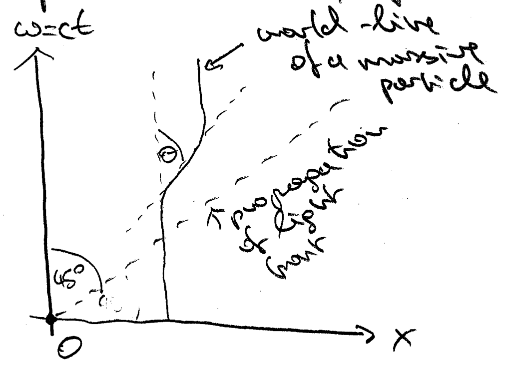
$$x_2' = \frac{x_2 - vt_2}{\sqrt{1-\beta^2}} \qquad x_1' = \frac{x_1 - vt_1}{\sqrt{1-\beta^2}}$$

$$l' = (x_2' - x_1') = \frac{x_2 - x_1}{\sqrt{1-\beta^2}} = \frac{l}{\sqrt{1-\beta^2}} \Rightarrow l = l'\sqrt{1-\beta^2}$$

proper length

$$l < l'$$

Very useful graphical way to analyze these problems: Space-time diagrams.



Since c is highest possible speed

$$\frac{dx}{dw} = \frac{1}{c}\frac{dx}{dt} = \frac{1}{c}u < 1$$

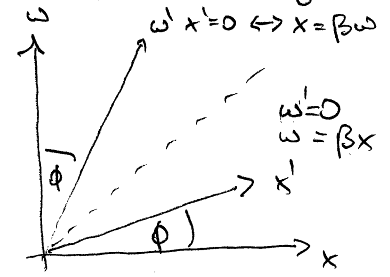
$$\tan\theta = \frac{dx}{dw} \qquad \theta \le 45°$$

Why highest speed: velocity addition formula

$$u_x = \frac{u_x' + v}{1}$$

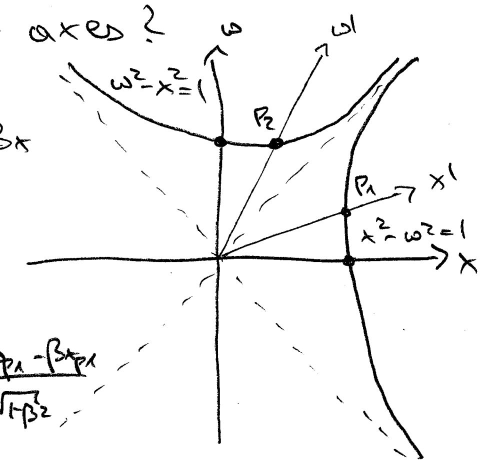
How to represent events in frame S' moving relative to S in x-dir?

Consider origin $x'=0$, seen from S as $x = vt = \beta\omega$



New coordinates are NOT orthogonal!

How to calibrate the new axes?



Consider P1: both on $\omega = \beta x$ and $x^2-\omega^2=1$

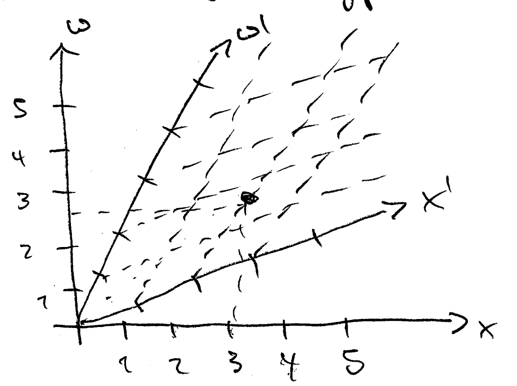
$$\Rightarrow \quad \omega_{P1} = \frac{\beta}{\sqrt{1-\beta^2}} \qquad x_{P1} = \frac{1}{\sqrt{1-\beta^2}}$$

Use Lorentz transform: $x' = \dfrac{x_{P1} - \beta\omega_{P1}}{\sqrt{1-\beta^2}} = 1 \qquad \omega' = \dfrac{\omega_{P1} - \beta x_{P1}}{\sqrt{1-\beta^2}} = 0$

For P_2 we get $x'=0$, $\omega'=1$

The upper hyperbola denotes UNIT TIME on the clock at rest in S'. Similarly right hyperbola denotes UNIT length.

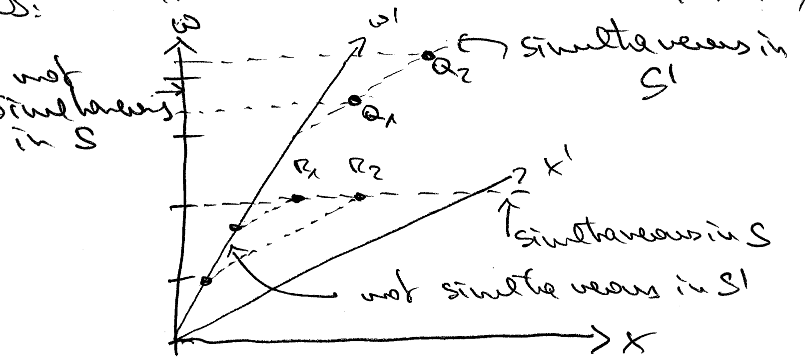
How to read off coordinates:



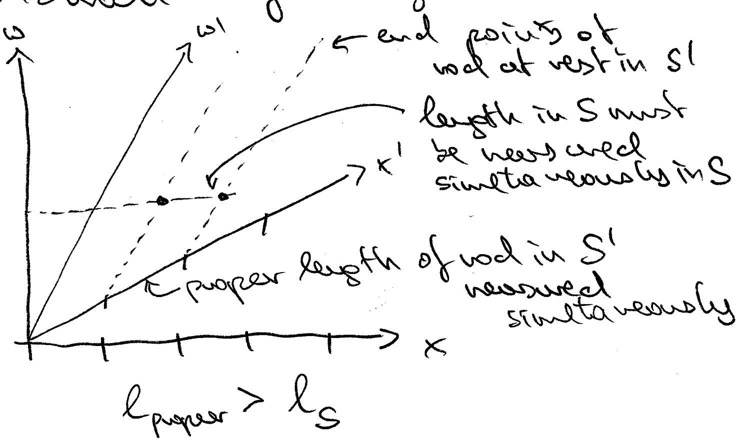
$\beta=0{,}5 \qquad x=3 \ \& \ \omega=2{,}5$

$\qquad\qquad x'=2 \qquad \omega'=1{,}5$

How to visualize SIMULTANEITY

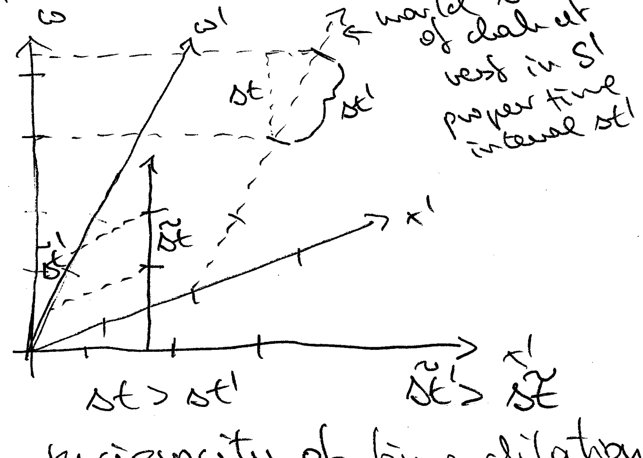


Visualizing length contraction



$l_{proper} > l_S$

Time dilation



$\Delta t > \Delta t' \qquad \Delta\tilde{t}' > \Delta\tilde{t}$

reciprocity of time dilation

Using space-time diagrams to analyze SR problems:

"Pole-in-a-barn": horizontal pole of proper length 15 m moves at $u = 0.75c$ towards a barn. Barn has front & rear doors. Observer at rest can open & close barn doors simultaneously by remote control. When the pole is in the barn the observer momentarily closes and opens barn doors.

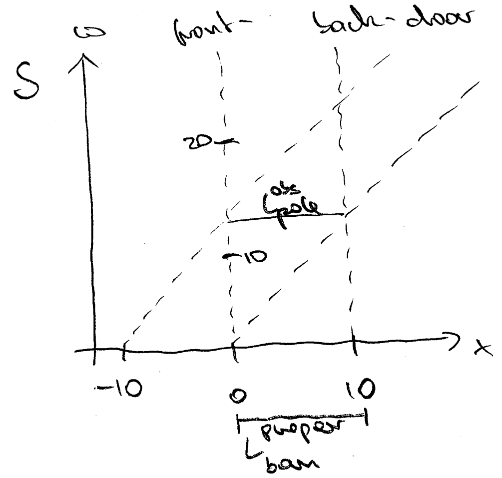
Is the rod destroyed during this scenario?

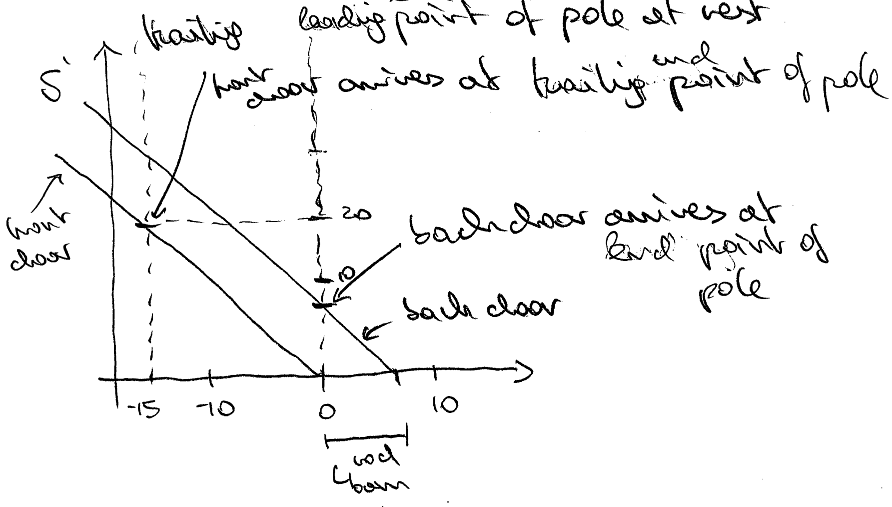
$$L^{obs}_{pole} = L^{proper}_{pole}\sqrt{1-\beta^2}$$
$$= 15\,m\sqrt{1-0.75^2} = 9.9\,m$$

seems reasonable that the pole fits into barn!

Reciprocity: length of barn also shrinks for rod.

$$L^{rod}_{barn} = L^{proper}_{barn}\sqrt{1-0.75^2} = 10\,m\sqrt{1-0.75^2} = 6.6\,m$$





At which time for observer is pole "in the barn"?

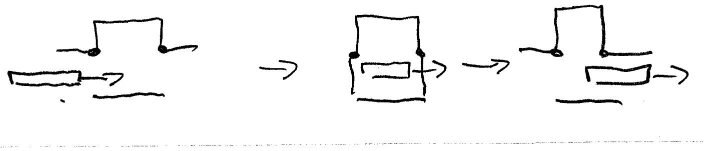
$$\Delta t = \frac{\Delta x}{v} = \frac{9.9\,m}{0.75c} = \frac{13.2\,m}{c} \quad \text{at } c\Delta t = 13.2\,m$$

From point of view of pole the "simultaneous" opening and closing of the barn doors IS NOT simultaneous

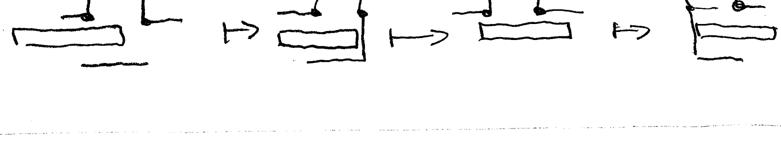
leading end leaves barn $\frac{\Delta x}{v} = \frac{6.6\,m}{0.75c} = \frac{8.8\,m}{c}$ $c\Delta t = 8.8\,m$

trailing end enters barn $\frac{\Delta x}{v} = \frac{15\,m}{0.75c} = \frac{20\,m}{c}$ $c\Delta t = 20\,m$

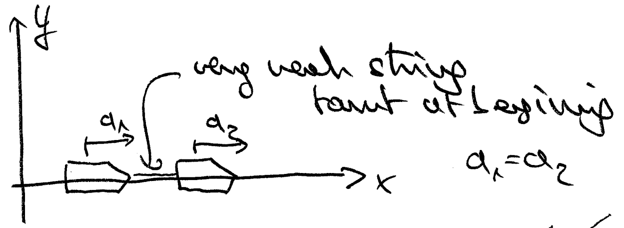
Point of view of obs



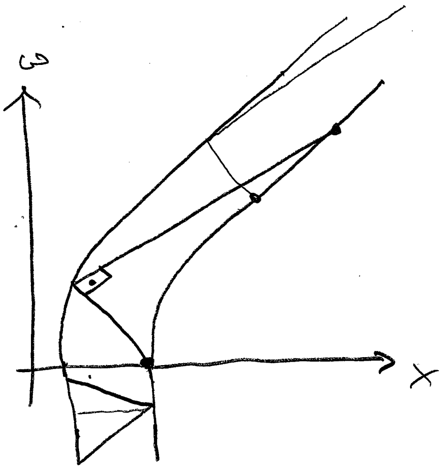
From rod:



String between space-ships



Given the situation on the right will the string relax, stay taut or will it snap?

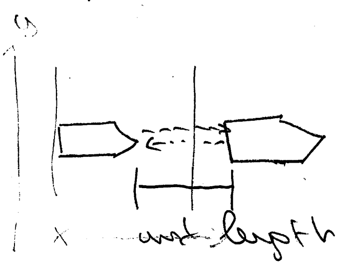


First impression: length contraction between ships makes string relax? Or string gets contracted and breaks?

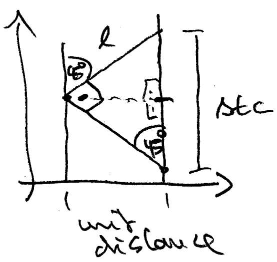
How to determine distance: "Radar distance"

Let light reflect of the other spaceship and measure time it took:

At rest:



$c\Delta t = 2$ in space-time diagram



$\sin(45) = 1/l \Rightarrow l = 1/\sin(45) = 1/\sqrt{1/2} = \sqrt{2}$

$|\Delta t c|^2 = 2 + 2$
$= \sqrt{4} = 2$

When ships have accelerated but cut off engines after "same" proper time has passed → leading ship will see that radar ping takes longer and longer. as if trailing ship falls behind.