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Norwegian Particle, Astroparticle & Cosmology Theory network

Next-to-leading power resummation in slepton pair production

Tore Klungland

N-PACT meeting 2024

June 19

- Lasse Lorentz Braseth, Tore Klungland, Are Raklev, *Slepton pair production to next-to-leading power* [in preparation]
- Christopher Chang et al, *Smoking* [in preparation]

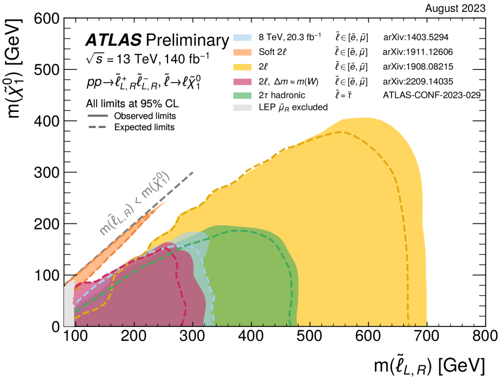
Outline

1. Background: Large logarithms in perturbative QCD
2. NLP resummation of large logarithms
3. Numerical impact
4. Conclusions and outlook

Background: Large logarithms in perturbative QCD

Motivation: The need for precision theory predictions

- $X\%$ CL exclusion based on combined theoretical and experimental uncertainty
- Large theoretical uncertainties smear out the likelihood $\Rightarrow$ less stringent exclusion
- High-precision theoretical calculations needed, *and an accurate estimate of the theory uncertainty*
- Initial attempt: Perturbation theory



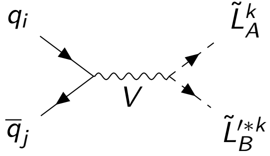
Slepton pair production in perturbative QCD

Writing the cross-section differential in the slepton-pair invariant mass Q^2 (for center-of-mass energy $\sqrt{s}$):

$$\frac{d\sigma}{dQ^2} = \sum_{ij} \int dx_1 dx_2 f_i\left(x_1, \mu_F\right) f_j\left(x_2, \mu_F\right) \frac{d\hat{\sigma}_{ij}}{dQ^2}\left(z = \frac{Q^2}{x_1 x_2 s}, Q^2, \mu_R, \mu_F\right)$$

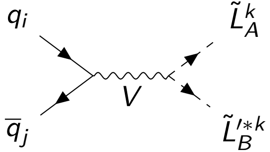
$\Rightarrow$ Partonic cross-sections $\frac{d\hat{\sigma}_{ij}}{dQ^2}$ (in principle) computable by order in α_s

Leading-order partonic diagram:

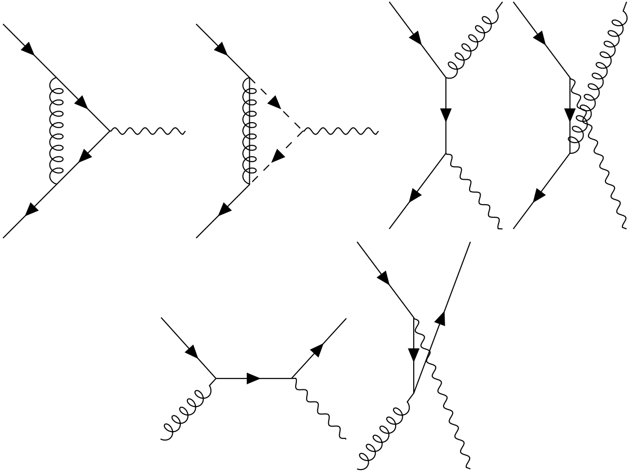


Partonic contributions to $\mathcal{O}(\alpha_s)$

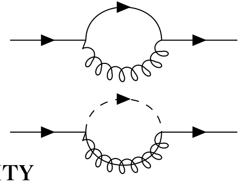
Tree-level



One-loop level (slepton part unchanged)



Counterterms



Threshold limit

$z \to 1$ ($Q^2 \to x_1 x_2 s$): Nearly all available energy goes to the sleptons, radiation becomes soft. In this limit,

$$\frac{d\hat{\sigma}^{\text{NLO}}_{q_i\bar{q}_j}}{dQ^2} \sim \frac{C_F \alpha_s}{\pi} \left\{ 4 \left[\frac{\log(1-z)}{1-z} \right]_+ - 2 \log \frac{\mu_F^2}{Q^2} \frac{1}{[1-z]_+} - 4 \log(1-z) + c_1^{(\delta)} \delta(1-z) \right\}$$

$$\frac{d\hat{\sigma}^{\text{NLO}}_{q_i g}}{dQ^2} \sim \frac{T_F \alpha_s}{\pi} \log(1-z)$$

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$$\frac{d\hat{\sigma}^{\mathrm{NLO}}_{q_i g}}{dQ^2} \sim \frac{T_F\alpha_s}{\pi}\underline{\log(1-z)}$$

These terms may dominate in this limit and spoil the stability of the result!

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Large logarithmic terms: Classification

Such terms appear at all orders; generically we have

$$\frac{d\sigma}{dQ^2} \sim \sigma_0 \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \left[\sum_{m=0}^{2n-1} \left[\underbrace{c_{nm}^{(-1)} \left(\frac{\ln^m(1-z)}{1-z}\right)_+}_{\text{Leading power (LP) terms}} + \underbrace{c_{nm}^{(0)} \ln^m(1-z)}_{\text{NLP terms}} + \dots \right] + c_n^{(\delta)} \delta(1-z) \right]$$

$$m = \begin{cases} 2n-1: & \text{Leading logarithmic (LL) terms} \\ 2n-2: & \text{Next-to-leading logarithmic (NLL) terms} \\ 2n-k: & \text{N}^{k-1}\text{LL terms} \end{cases}$$

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- Perturbation theory is insufficient for precise predictions
- Resummation methods needed to properly capture threshold effects

NLP resummation of large logarithms

Factorization and exponentiation in QCD

In Mellin space ($\mathcal{M}(f) = \int_0^1 dx x^{N-1} f(x)$) near threshold, the (unrenormalized) cross-section factorizes:

$$\mathcal{M}\left(\frac{d\tilde{\sigma}_{ij}}{dQ^2}\right) = \underbrace{H_{ij}\left(Q^2, \mu_F, \mu_R\right)}_{\text{Hard-scattering effects}} \underbrace{\prod_{k=i,j} J_k\left(N, Q^2, \mu_F, \mu_R\right)}_{\text{Collinear effects}} \underbrace{S_{ij}\left(N, Q^2, \mu_F, \mu_R\right)}_{\text{(Next-to-)soft effects}}$$

Further, the infrared effects exponentiate, leaving

$$\mathcal{M}\left(\frac{d\hat{\sigma}_{ij}}{dQ^2}\right) = H_{ij}\left(Q^2, \mu_F, \mu_R^2\right) \exp\left(W_{ij}\left(N, Q^2, \mu_F, \mu_R\right)\right)$$

Resummation of $q\bar{q}$ contribution

The next-to-soft $q\bar{q}$ function can be constructed from the vacuum expectation value of *generalized Wilson lines* $F(p)$, representing the incoming partons:

$$S \sim \frac{1}{N_C} \sum_n \mathrm{Tr}\left[\left\langle 0 \left| F^\dagger(p_1) F(p_2) \right| n \right\rangle \left\langle n \left| F^\dagger(p_2) F(p_1) \right| 0 \right\rangle\right],$$

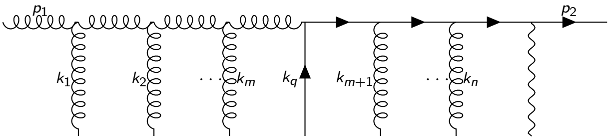
with $F(p) =$

$$P \exp\left[i g_s \mathbf{T}^a \int \frac{d^d k}{(2\pi)^d} \tilde{A}^a_\mu(k) \left(\underbrace{\frac{p^\mu}{p \cdot k}}_{\text{LP (straight line)}} \underbrace{- \frac{k^\mu}{2p \cdot k} + k^2 \frac{p^\mu}{2(p \cdot k)^2} + i k_\nu \frac{S^{\nu\mu}}{p \cdot k}}_{\text{NLP}} + \ldots \right) \right]$$

$\Rightarrow$ Evaluating with two-loop α_s and Wilson line renormalization yields LP at NLL, NLP at LL

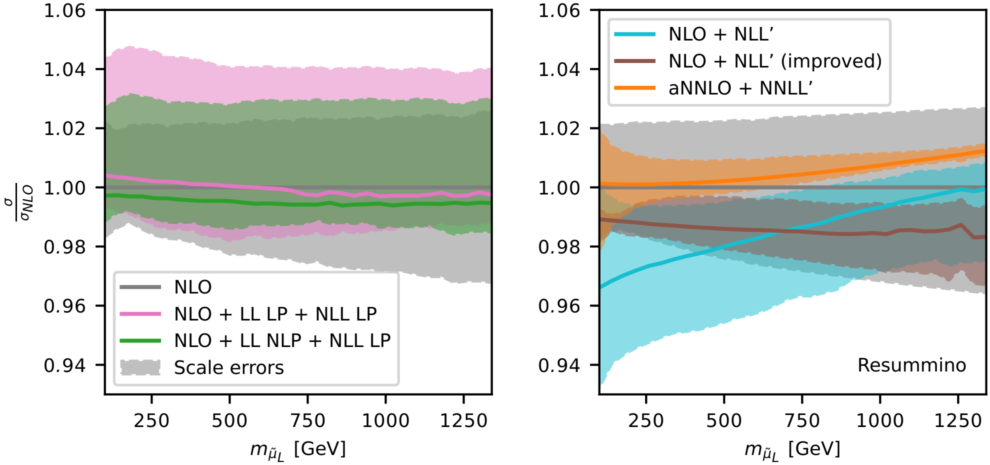
Resummation of *qg* contribution

- Emission of a low-energy quark results in a factor $\sum_s u^s(k_q)\,\overline{u}^s(k_q) = \not{k}_q$
- Only need to consider further radiation of soft gluons, with the approximate vertex $\propto g_s \mathbf{T}_R \frac{p_i^\mu}{p_i \cdot k}$
- LL NLP cross-section obtained through diagrammatic methods and factorization arguments

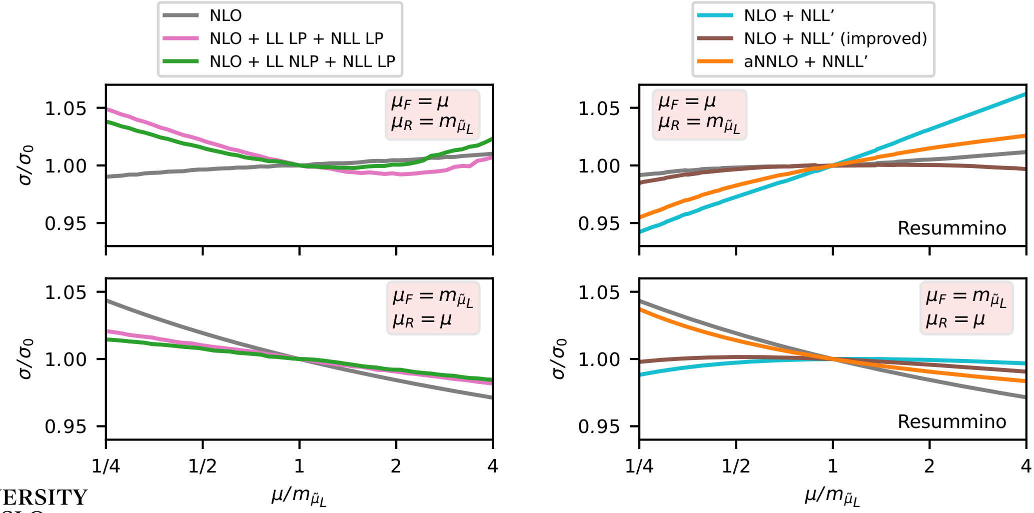


Numerical impact

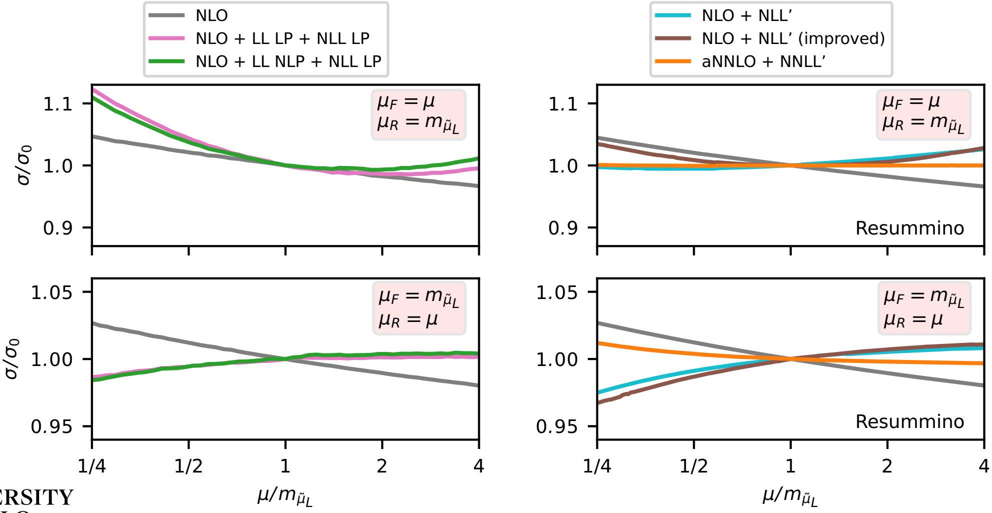
Impact and scale uncertainty



Scale dependence ($m_{\tilde{\mu}_L} = 120$ GeV)



Scale dependence ($m_{\tilde{\mu}_L} = 1000$ GeV)



Conclusions and outlook

Conclusions and outlook

- Higher-power resummation an important complement to higher logarithmic accuracy
  - Reduced scale uncertainty compared to leading-power resummation
  - Larger shift in cross-section than the scale uncertainty from existing calculations
    $\Rightarrow$ may be underestimating impact of new corrections
- Implemented in the *Smoking* code (under development), initially for slepton pair production

References

- N. Bahjat-Abbas, D. Bonocore, J. Sinninghe Damsté, E. Laenen, L. Magnea, L. Vernazza and C.D. White, *Diagrammatic resummation of leading-logarithmic threshold effects at next-to-leading power*, *JHEP* **11** (2019) 002 [1905.13710].
- M. van Beekveld, L. Vernazza and C.D. White, *Threshold resummation of new partonic channels at next-to-leading power*, *JHEP* **12** (2021) 087 [2109.09752].

Extras

Threshold limit in Mellin space

Mellin transforming the $z \to 1$ expression, the large-N limit gives

$$\mathcal{M}\left(\frac{d\hat{\sigma}^{\text{NLO}}_{q_i\bar{q}_j}}{dQ^2}\right) \sim \frac{C_F\alpha_s}{\pi}\left\{2\log^2 N + 2\log\frac{\mu_F^2}{Q^2}\log N + 2\frac{\log N}{N} + C\right\}$$

$$\mathcal{M}\left(\frac{d\hat{\sigma}^{\text{NLO}}_{q_ig}}{dQ^2}\right) \sim -\frac{T_F\alpha_s}{\pi}\frac{\log N}{N}$$

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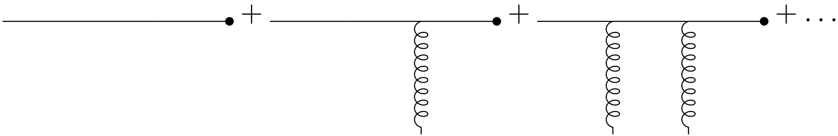
Diagrammatic expansion of Wilson lines

Wilson lines evaluated by expressing gluon field in momentum space, and expanding by order in g_s. Diagrammatically:



Resummation results

$$W^{\text{res}}_{q\bar{q}} = \exp\left\{\log \overline{N} g_1(\lambda) + g_2(\lambda) + h_1(\lambda, N)\right\},$$

with $(\overline{N} \equiv N e^{\gamma_E},\ \lambda \equiv b_0 \alpha_s \log \overline{N})$

$$g_1(\lambda) = \frac{\Gamma^1_{\text{cusp}}}{\pi b_0 \lambda}\left(2\lambda + (1-2\lambda)\log(1-2\lambda)\right),$$

$$\begin{aligned} g_2(\lambda) = &-\frac{\Gamma^2_{\text{cusp}}}{\pi^2 b_0^2}\left(2\lambda + \log(1-2\lambda)\right) + \frac{\Gamma^1_{\text{cusp}}}{\pi b_0^2}\left(2\lambda + \log(1-2\lambda) + \frac{1}{2}\log^2(1-2\lambda)\right) \\ &- \frac{\Gamma^1_{\text{cusp}}}{\pi b_0}\left(2\lambda + \log(1-2\lambda)\right)\log\frac{\mu_R^2}{Q^2} + \frac{2\Gamma^1_{\text{cusp}}\lambda}{\pi b_0}\log\frac{\mu_F^2}{Q^2}, \end{aligned}$$

$$h_1(\lambda, N) = -\frac{\Gamma^1_{\text{cusp}}}{\pi b_0}\frac{\log(1-2\lambda)}{N}$$

Resummation results

$$W_{qg}^{\text{res}} = \frac{T_F}{C_A - C_F} \frac{1}{2N \log N} \left[\exp\left\{ 2\frac{\alpha_s}{\pi} C_F \log^2 N \right\} \mathcal{B}_0 \left(\frac{\alpha_s}{\pi} (C_A - C_F) \log^2 N \right) - \exp\left\{ \frac{\alpha_s}{2\pi} (C_F + 3C_A) \log^2 N \right\} \right],$$

where

$$\mathcal{B}_0(x) \equiv 1 - \frac{x}{2} - \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n!)^2} |B_{2n}| x^{2n}$$