Strongly interacting dark matter at next-to-leading order

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Content

1. SIMP Dark Matter
2. QCD-like theories as models of SIMP
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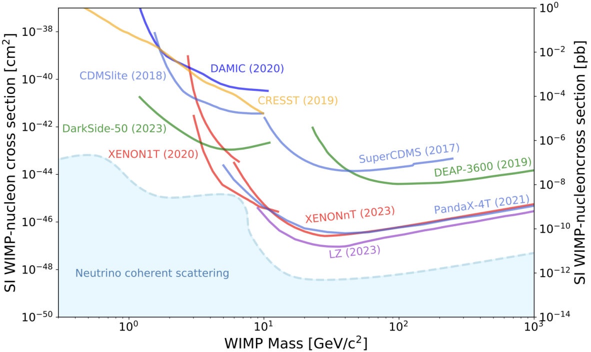
WIMP paradigm

WIMP relic abundance is set by $2 \to 2$ annihilation into SM. This leads to

$$m_\chi \sim \alpha_{\rm eff}\sqrt{T_{\rm eq} M_{\rm Pl}} \sim \alpha_{\rm eff} \times (30\ \text{TeV}) \sim 300\ \text{GeV}\ \ \left(\text{for } \alpha_{\rm eff} \sim 10^{-2}\right)$$

WIMP miracle!

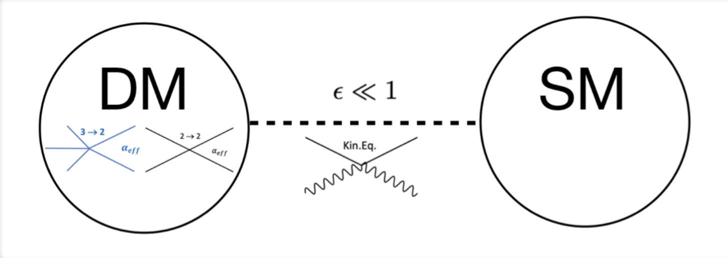


Particle Data Group, Dark matter (2023)

SIMP paradigm

Consider a scenario in which the relic abundance is set by $3 \to 2$ self-interaction (1402.5143v2).

- $\frac{\Gamma_{cool}}{\Gamma_{3\to 2}}(T_{FO}) \gtrsim 1, \quad \frac{\Gamma_{ann}}{\Gamma_{3\to 2}}(T_{FO}) \lesssim 1$



Reproduced from 1402.5143

Bolzmann equation

Bolzmann equation defines the evolution of the yield dark matter particle density $n(t)$:

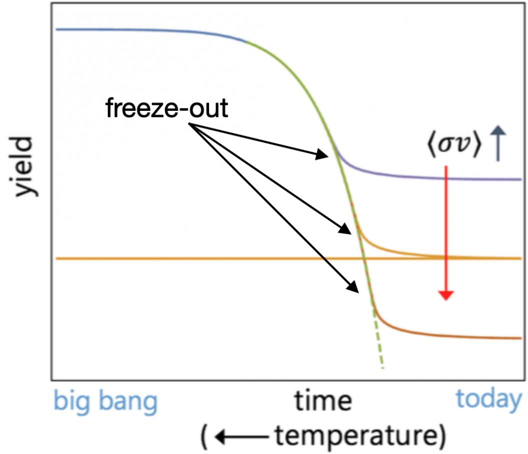
$$\dot{n} + 3Hn = -\left(n^3 - n^2 n^{\text{eq}}\right)\left\langle \sigma v^2 \right\rangle_{3\to 2}$$

where

- n^{eq} is the equilibrium particle density (with the temperature of the thermal bath),
- $\left\langle \sigma v^2 \right\rangle_{3\to 2}$ is the thermally averaged cross-section,
- H is the Hubble parameter.

Bolzmann equation

Schematic solutions for the yield, $Y = n(T)/s(T)$, where $s(T)$ is the entropy of the thermal bath:



SciPost Phys.Lect.Notes 59 (2022)

Freezeout

While the Universe expands the gas of particles becomes more diluted so that at some point the interaction almost stops.

Hamov criterion:

$$\Gamma_{3\to 2} \sim n^2\left(T_{FO}\right)\left\langle \sigma v^2 \right\rangle_{3\to 2} \sim H\left(T_{FO}\right),$$

leads to the SIMP miracle:

$$m \sim \alpha_{\text{eff}}\left(T_{\text{eq}}^2 M_{\text{Pl}}\right)^{1/3} \sim \alpha_{\text{eff}} \times (100\ MeV) \sim \text{strong scale (for } \alpha_{\text{eff}} \sim O(1))$$

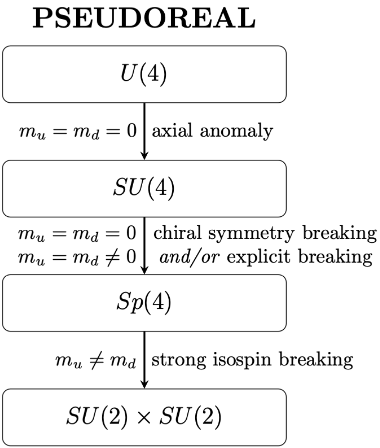
We got lighted DM candidates!

QCD-like theories and flavour symmetries

Gauge group	$SU(N_c)$	$Sp(N_c)$	$O(N_c)$
Dark quarks in fundamental representation	Complex	Pseudoreal	Real
Chiral symmetry breaking pattern	$SU(N_f) \times SU(N_f)/SU(N_f)$	$SU(2N_f)/Sp(2N_f)$	$SU(2N_f)/SO(2N_f)$
Number of dark pions	$N_f^2 - 1$	$(2N_f + 1)(N_f - 1)$	$\frac{1}{2}(N_f + 2)(N_f - 1)$

$SU(4)/Sp(4)$ theory

- $Sp(2N_f) \subset SU(2N_f)$
- $U^T J_A U = J_A, \quad U \in Sp(4)$
- $J_A = \begin{pmatrix} 0 & -I_{2\times2} \\ I_{2\times2} & 0 \end{pmatrix}$
- Goldstone theorem: number of (pseudo-)Goldstone bosons is $N_\pi = \dim(SU(4)) - \dim(Sp(4)) = 5$
- Broken generators: $(X^a)^T J_A - J_A X^a = 0$
- Unbroken generators: $(T^a)^T J_A + J_A T^a = 0$



From 2202.05191v2

Effective theory for $SU(4)/Sp(4)$ theory

Chiral perturbation theory describes the low-energy dynamics of the dark pions which are the (pseudo-)Nambu-Goldstone bosons due to the broken chiral symmetry.

Pions parameterize the coset space G/H: $u = \exp\left(\frac{i}{\sqrt{2}F}\pi^a X^a\right)$

Building blocks for the EFT:

$$u_\mu = i\left(u^\dagger\left(\partial_\mu - iV_\mu\right)u - u\left(\partial_\mu + iJ_A V_\mu^T J_A^T\right)u^\dagger\right),$$

$$\chi_\pm = u^\dagger \chi J_A^T u^\dagger \pm u J_A \chi^\dagger u,$$

$$\chi = 2B_0\hat{\mathcal{M}} = 2B_0 \begin{pmatrix} 0 & 0 & -m_u & 0 \\ 0 & 0 & 0 & -m_d \\ m_u & 0 & 0 & 0 \\ 0 & m_d & 0 & 0 \end{pmatrix}$$

Leading order Lagrangian, $O(p^2)$

The leading order Lagrangian generates infinite number of (even) terms:

$$\begin{aligned}\mathcal{L}_2 &= \frac{F}{2}\left\langle u_\mu u^\nu + \chi_+ \right\rangle \\ &= \frac{1}{2}\sum_{i=1}^{N_\pi=5}\left(\partial_\mu \pi^i\right)^2 - \frac{M^2}{2}\sum_{i=1}^{N_\pi=5}\left(\pi^i\right)^2 + O\left(\pi^4\right).\end{aligned}$$

All pions have the same mass at leading order:

$$M^2 = B_0\left(m_u + m_d\right)$$

Self-scattering cross-section at leading order in non-relativistic limit:

$$\sigma_{\text{scatter}} = \frac{225 M^2}{512\pi^2 F^4}\frac{1}{N_\pi^2}$$

Wess-Zumino-Witten term

The topological Wess-Zumino-Witten term generates 5-point interaction:

$$\mathcal{L}_{WZW} = \frac{N_c}{15\pi^2 F^5} \epsilon^{\mu\nu\rho\sigma} \mathrm{Tr} \left[\pi \partial_\mu \pi \partial_\nu \pi \partial_\rho \pi \partial_\sigma \pi \right]$$

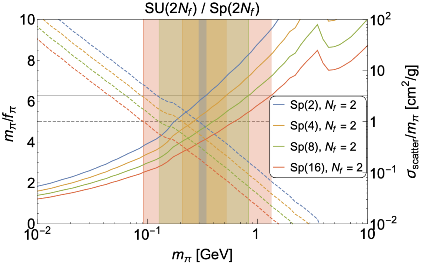
In $SU(3) \times SU(3)/SU(3)$ QCD this terms allows for observed process

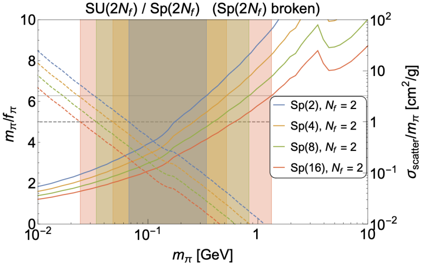
$$K^+ K^- \to \pi^+ \pi^- \pi^0.$$

Wess-Zumino-Witten term is responsible for $3 \to 2$ annihilation in the dark sector:

$$\left\langle \sigma v^2 \right\rangle_{3\to 2} = \frac{75\sqrt{5}}{512\pi^5} \frac{N_c^2}{N_\pi^3} \frac{m_\pi^5}{F^{10}} \frac{1}{x^2}, \quad x = \frac{M}{T}$$

Solution of Bolzmann equation





From 1411.3727

Next-to-leading order Lagrangian, $O(p^4)$

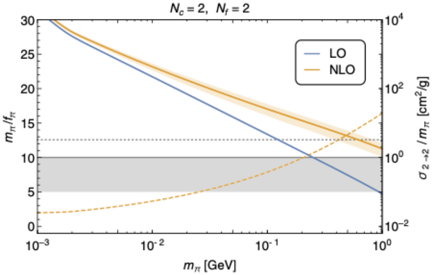
WZW term is, in fact, next-to-leading order. Consistent treatment requires going to next-to-leading order in all the observables (1507.01590)!

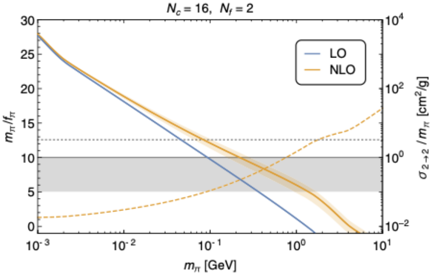
$$\begin{aligned}\mathcal{L}_4 = & L_0 \langle u^\mu u^\nu u_\mu u_\nu \rangle + L_1 \langle u^\mu u_\mu \rangle \langle u^\nu u_\nu \rangle + L_2 \langle u^\mu u^\nu \rangle \langle u_\mu u_\nu \rangle \\ & + L_3 \langle u^\mu u_\mu u^\nu u_\nu \rangle + L_4 \langle u^\mu u_\mu \rangle \langle \chi_+ \rangle + L_5 \langle u^\mu u_\mu \chi_+ \rangle + L_6 \langle \chi_+ \rangle^2 \\ & + L_7 \langle \chi_- \rangle^2 + \frac{1}{2} L_8 \left\langle \chi_+^2 + \chi_-^2 \right\rangle - i L_9 \langle f_{+\mu\nu} u^\mu u^\nu \rangle + \frac{1}{4} L_{10} \left\langle f_+^2 - f_-^2 \right\rangle\end{aligned}$$

Low-energy constants (LECs) $L_0 ... L_{10}$ are not known for Sp(4)!

For mass-degenerate case the mass, decay constants and self-scattering amplitudes are calculated in 0910.5424, 1102.0172 and 1507.01590.

SIMPs at NLO





From 1507.01590

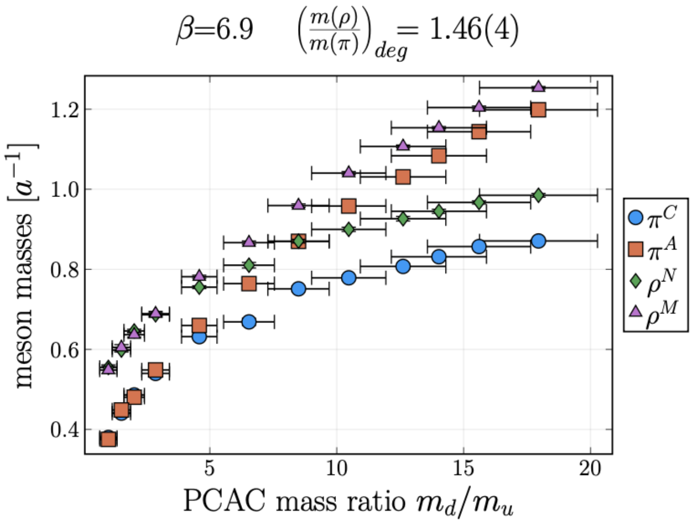
Non-degenerate case

For $m_u \neq m_d$ the pions get different mass at next-to-leading order:

$$M_{4,i}^2 = M^2 \left(1 + \frac{M^2}{F^2}\left[-32L_4^r - 8L_5^r + 64L_6^r + 16L_8^r + \frac{3}{64\pi^2}\log\left(\frac{M^2}{\mu^2}\right)\right]\right),$$
$$i = 1, 2, 4, 5$$

$$M_{4,3}^2 = M^2 \left(1 + \frac{M^2}{F^2}\left[-32L_4^r - 8L_5^r + 64L_6^r + 64L_7^r \frac{(m_d - m_u)^2}{(m_d + m_u)^2} + 32L_8^r \frac{m_d^2 + m_u^2}{(m_d + m_u)^2} + \frac{3}{64\pi^2}\log\left(\frac{M^2}{\mu^2}\right)\right]\right)$$

Non-degenerate case



From 2202.05191v2

What's next?

- Phenomenology of non-degenerate SIMPs,

- Defining LECs from new lattice results (2202.05191v2, 2405.06506)

Thank you for attention!