Viscous Fingering and Interfacial Instability Growth for Power-Law Fluids in Hele-Shaw Cells

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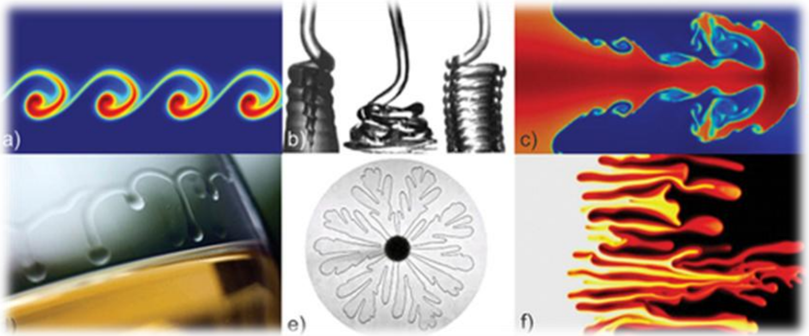
Why Study Interfacial Instabilities?

Interfacial instabilities occur in many multiphase flow systems, especially in porous media and confined geometries.

- Classic example
  - Saffman-Taylor instability: viscous fingering when *a less viscous fluid displaces a more viscous* one.

- These instabilities can **drastically reduce displacement efficiency**, critical in oil recovery, groundwater remediation, and microfluidics.
- Understanding and predicting interface growth is essential to improve flow control.



Beyond Newtonian Fluids: Added Complexity

- **Newtonian fluids** ➔ *Constant viscosity, well-understood*
  - Classical theory ➔ well predicted onset of fingering
  - CFD ➔ resolve full fingering patterns
- **Shear-thinning fluids** ➔ *more complex interface dynamics, harder to predict*
  - Theoretical power-law fluids:
    - Mora & Manna's [1] criteria ➔ linear regime
    - CFD ➔ onset of instability & detailed patterns
  - Realistic fluids with polymers:
    - Adding polymer: Shear-dependent viscosity (shear-thinning/thickening), develop normal stresses, elastic deformation (viscoelasticity)
    - Viscosity field varies spatially

    ➔ *Power-law model does not capture these effects, very limited comparison*

[1] S. Mora and M. Manna. "Saffman-Taylor instability for generalized Newtonian fluids". In: Phys. Rev. E 80 (1 July 2009), p. 016308. doi: 10 . 1103 / PhysRevE . 80 . 016308. url: https://link.aps.org/doi/10.1103/PhysRevE.80.016308.

Research Objectives

- Quantify **the growth rate of perturbations** in two-fluid displacement inside a Hele-Shaw cell.
- Analyze both Newtonian and power-law (shear-thinning) fluid systems.
- **Evaluate stability criteria** using a developed **2D model** in OpenFOAM
- **2D CFD simulations vs. linear stability theory** :
  - Friction pressure gradients
  - Influence of *interfacial tension*
  - The effect of rheological parameters (k, n from power-law fluids)

Under what conditions do perturbations grow, decay, or stabilize?

How accurately can we predict them with the 2D model?

Theoretical Framework: From Newtonian to Power-Law

- Mora & Manna [1]'s general linear stability:

$$\boxed{M = \frac{(G_2 - G_1 - \gamma k^2)k}{\sqrt{\frac{G_2}{V_2}\frac{dG_2}{dV_2}} + \sqrt{\frac{G_1}{V_1}\frac{dG_1}{dV_1}}}}$$

- M: Perturbation growth rate
- G_i: Unperturbed friction pressure gradient
- γ : interfacial tension
- k : wavenumber

- For Newtonian Fluids:

$$M = \frac{1}{\eta_2 + \eta_1} \cdot \frac{kh^2}{12} \cdot (G_2 - G_1 - \gamma k^2)$$

$$G_i = -\frac{12\eta_i U}{h^2}.$$

Non-Dimensional

The classic growth rate depends on ***viscosity contrast*** *&* ***interfacial tension***

$$\frac{Mh}{U} = M^* = \frac{kh}{\eta_2 + \eta_1}\left[(\eta_1 - \eta_2) - \frac{\gamma k^2 h^2}{12U}\right]$$

[1] S. Mora and M. Manna. "Saffman-Taylor instability for generalized Newtonian fluids". In: Phys. Rev. E 80 (1 July 2009), p. 016308. doi: 10 . 1103 / PhysRevE . 80 . 016308. url: https://link.aps.org/doi/10.1103/PhysRevE.80.016308.

Dimensionless Growth Rate for Power-Law Fluids

- For power-law fluids:

$$U = V_i(G_i) = \frac{(h/2)^{1+1/n_i}}{2+1/n_i}\frac{G_i^{1/n_i}}{k_i^{1/n_i}}$$

$$G_i = cV_i^{n_i}$$

$$\frac{G_i}{V_i}\frac{dG_i}{dV_i} = n_i\left(\frac{G_i}{V_i}\right)^2 = n_i\left(\frac{G_i}{U}\right)^2$$

$$\boxed{\frac{Mh}{U} = M^* = \frac{(1-\lambda)hk - (hk)^3/Ca}{\lambda\sqrt{n_1} + \sqrt{n_2}}}$$

- M*: non-dimensional perturbation growth rate
- λ = G_1/G_2: ratio of friction pressure gradients
- Ca = $G_2 h^2/\gamma$: capillary number
- n_i : flow behavior index (shear-thinning <1)
- k : wavenumber
- γ : interfacial tension

- **λ** governs balance between destabilizing and stabilizing forces
- **Capillary force** suppress high-wave number growth
- **Rheology** enters through n1 and n2: controls denominator (growth damping)

Fast and Robust 2D Simulation Framework:

Gap-Averaged Model for Hele-Shaw flow of power-law and Newtonian fluids

- 2D gap-averaged model [2] developed and implemented in OpenFOAM
  - Hele-Shaw approximation:
    - Flow confined to the plane (*no velocity component* in the gap direction)
    - *Fully developed* velocity profile in the gap for power-law fluids
  - Momentum equation integrated over the gap width to reduce dimensionality
  - Captures essential physics at ~200x less cost than 3D models
- Governing equations:
  - Gap-averaged continuity and momentum

$$\rho_i \left(\partial_t \mathbf{u}_0 + \frac{2(m_i+2)}{2m_i+3} \mathbf{u}_0 \cdot \nabla_0 \mathbf{u}_0 \right) = -\nabla_0 p(z,x) + \nabla_0 \cdot \tau - k_i \left(\frac{2}{h}\right)^{n_i+1} (m_i+2)^{n_i} \|\mathbf{u}_0\|^{n_i-1} \mathbf{u}_0 + \rho_i \mathbf{g}, \quad (9)$$

  - Shear-rate approximate by its gap-wise contribution

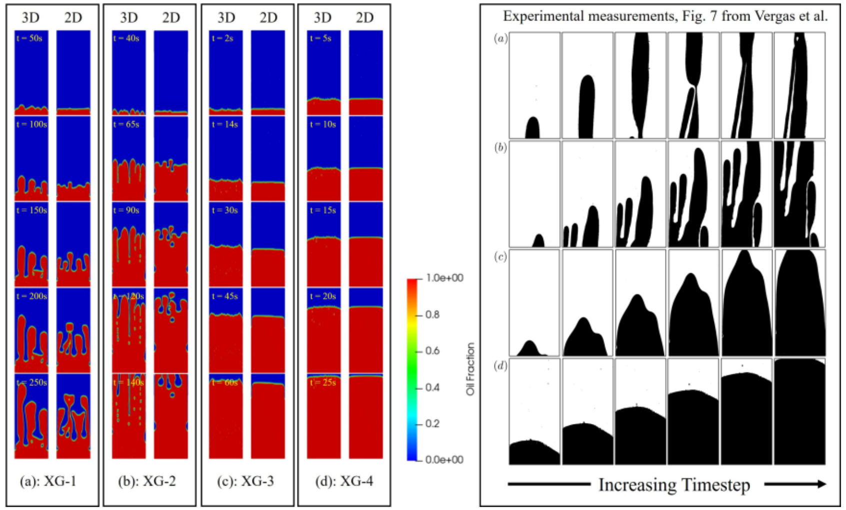
$$\dot{\gamma} = \frac{2(m_i+2)}{h} \left| \frac{2y}{h} \right|^{m_i} \|\mathbf{u}_0\|.$$

- Interface tracked using Volume of Fluid method
- Simulations run for 20 s, data sampled every 1 s

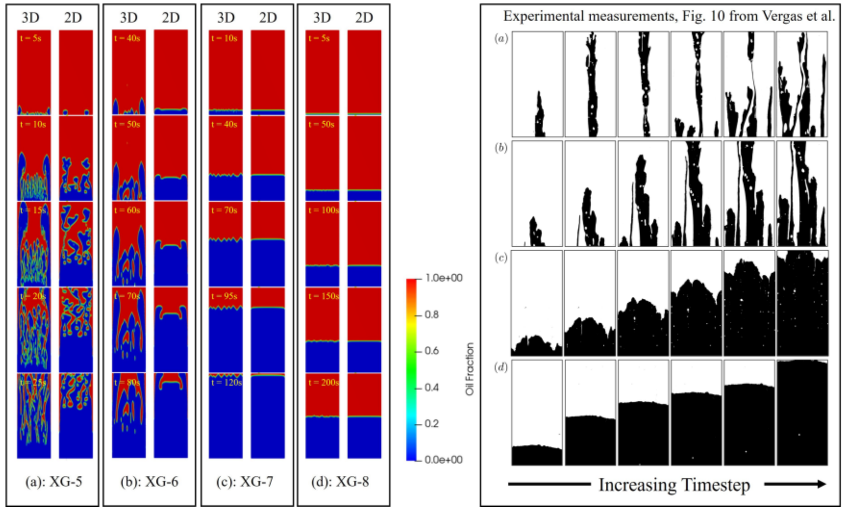
- 0.05 m x 0.2 m x 0.001 m
- 128 x 512 cells, 1 cell in gap

[2] Yao Zhang et al. "Numerical modeling of fluid displacement in Hele-Shaw cells: a gap-averaged approach for power-law and Newtonian fluids". Rheologica Acta (2025), pp. 1–16.

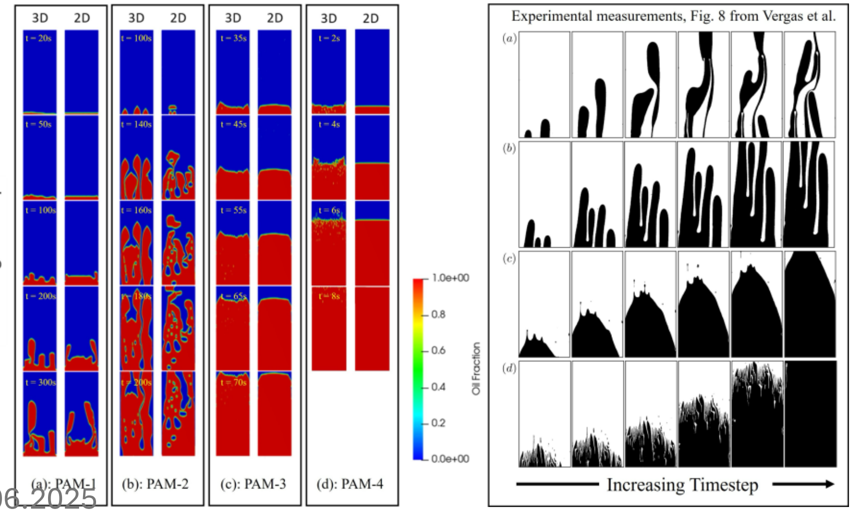
Interfacial Evolution from (2D vs 3D vs Experiment [2])



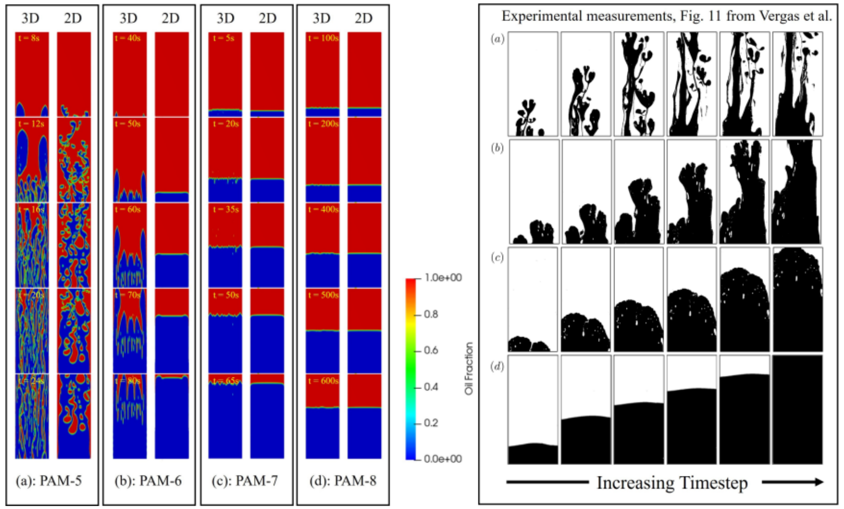
Oil displacing Xanthan gum at different flow rate



Xanthan gum displacing Oil at different flow rate



Oil displacing Polyacrylamide at different flow rate



Polyacrylamide displacing Oil at different flow rate

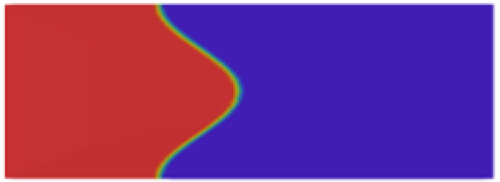
[2] Yao Zhang et al. "Numerical modeling of fluid displacement in Hele-Shaw cells: a gap-averaged approach for power-law and Newtonian fluids". Rheologica Acta (2025), pp. 1–16.

Controlled Onset of Viscous Fingering

- **Controlled Initial interface** given a *single sinusoidal perturbation*

$$y(x) = Asin(kx) + y_0$$

- Amplitude A = 0.004 m
- Wavenumber k = 2π/0.05m^{-1}
- Offset $y_0 = 0.002\ m$



- To **ensure reproducible, well-defined growth conditions**
- **Avoids random perturbations** from numerical noise
- *Finger length tracked overtime* ➔ *fitted to an exponential curve to extract growth rate*

Fluids, Parameters, and Simulation Setup

Newtonian Cases

- **Displacing fluid:** water-based solution (density 998 kg/m^3)
- **Displaced fluid:** mineral oil (viscosity 0.133 Pa·s, density 887.6 kg/m^3)
- **Imposed velocity varied**
- **Interfacial tension**: varied from 10^{-10} to 0.09 mN/m

[3] PR Varges et al. "Immiscible liquid-liquid displacement flows in a Hele-Shaw cell including shear thinning effects". Physics of Fluids 32.1 (2020)
[4] Maduranga Amaratunga et al. "Predicting rheological properties of water-based polymer mixtures from their component properties – Poly-Anionic Cellulose and Xanthan gum". ANNUAL TRANSACTIONS OF THE NORDIC RHEOLOGY SOCIETY, VOL. 26. 2018.

Power-Law Cases

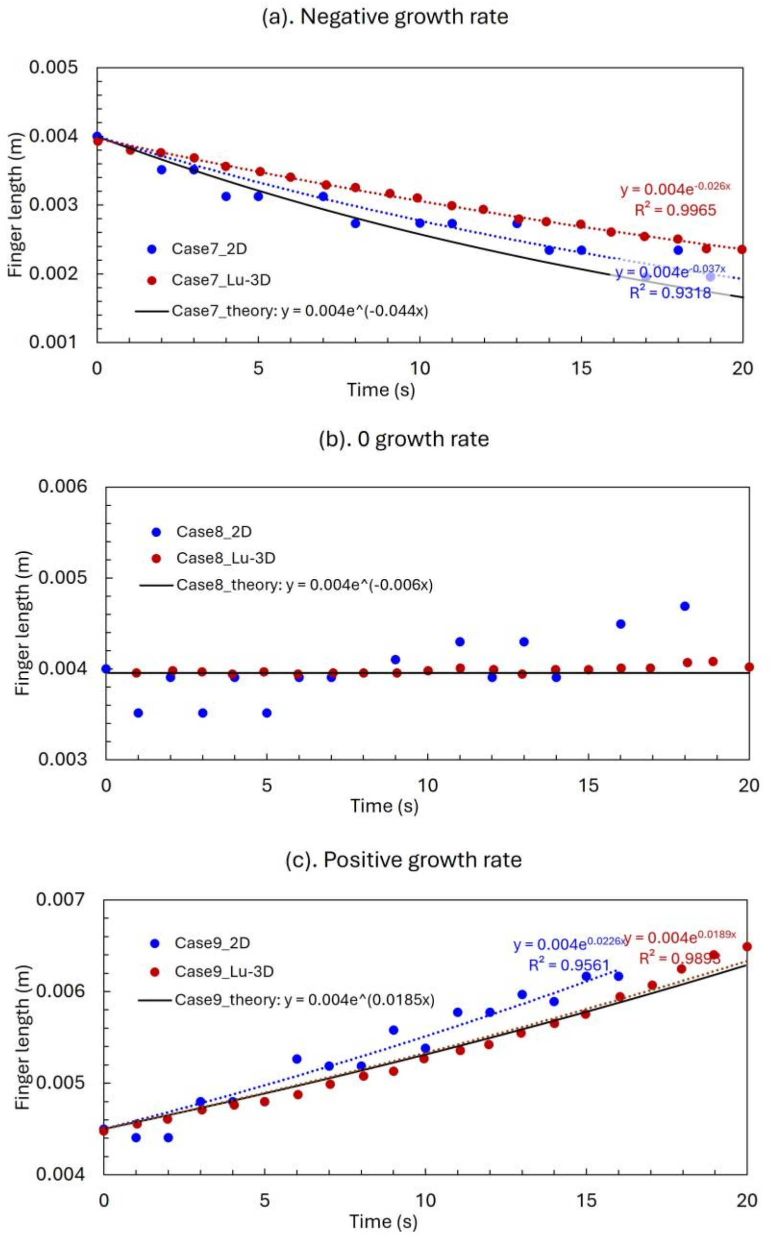
- **Displacing & displaced fluid:** either mineral oil or power-law fluid
  - **Real power-law fluids:** Xanthan gum & polyacrylamide solutions
  - **Rheological parameters** k and n from experiments in literature, → e.g., *Varges et al.*[3], *Amaratunga et al.*[4]
- Broad parametric ranges explored:
  - Friction pressure gradient: $\lambda = G_1/G_2 \in [0.42, 12.87]$
  - Capillary number: $Ca \in [0.0125, 1.071]$

Newtonian Case:
2D Simulation vs. 3D DNS & Theory

- Validated against results from *Lu et al. (2020)* [5]:

→ 3D simulations & linear stability theory

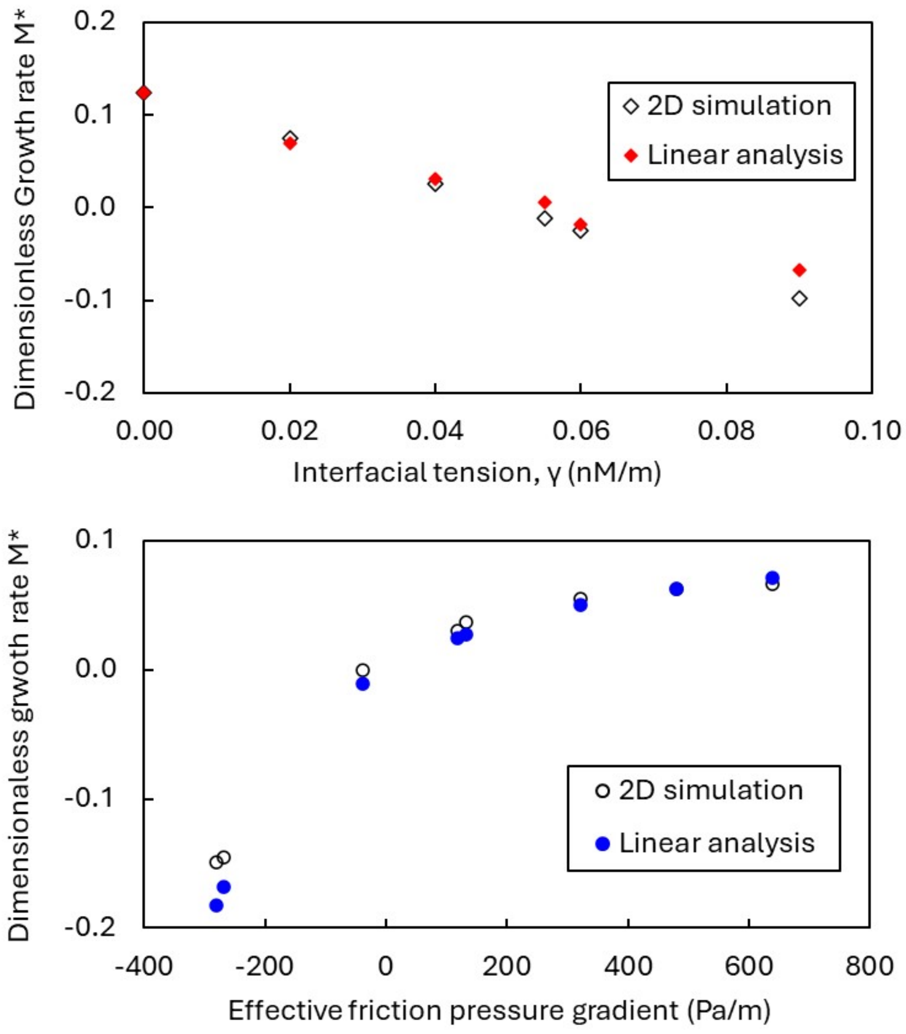
- 3 benchmark cases with:
  - **Negative, zero,** and **positive** growth rates.
- 2D simulation results match:
  - Theoretical predictions
  - 3D DNS evolution of finger length over time



[5] Daihui Lu, Federico Municchi, and Ivan C Christov. "Computational analysis of interfacial dynamics in angled Hele-Shaw cells: instability regimes". Transport in Porous Media 131.3 (2020), pp. 907–934.

Influence of Surface Tension and Pressure Gradient

- **Effect of Interfacial Tension** (γ):
  - Higher γ → lower growth rate M*
  - Capillary term γk^2 stabilizes interface
  - Especially effective at suppressing high-frequency perturbations
- **Effect of Effective Pressure Gradient** $(G_2 - G_2 - \gamma k^2)$:
  - **Strong** positive gradient → destabilization (finger growth)
  - **Low or negative** gradient → stabilization (finger decay)



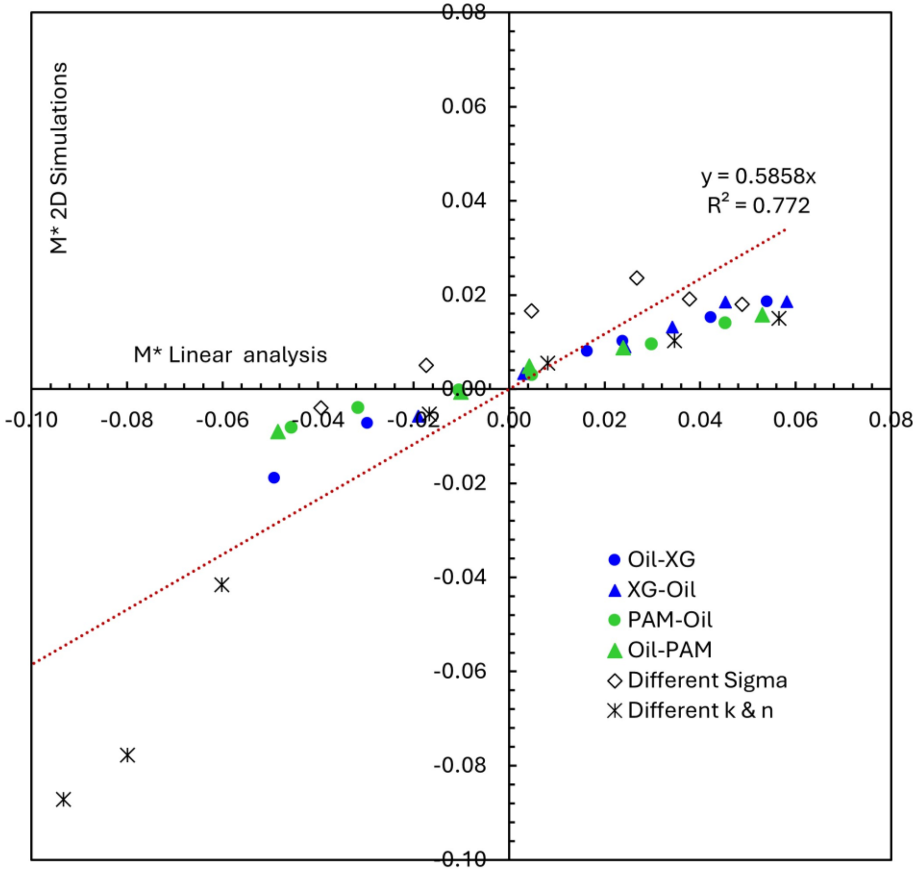
Growth rate vs. γ and vs. effective pressure gradient

Power-Law Fluids: Agreement and Deviations

- 2D simulation growth rates M* vs. linear theory predictions $\frac{Mh}{U} = M^* = \frac{(1-\lambda)hk - (hk)^3/Ca}{\lambda\sqrt{n_1} + \sqrt{n_2}}$

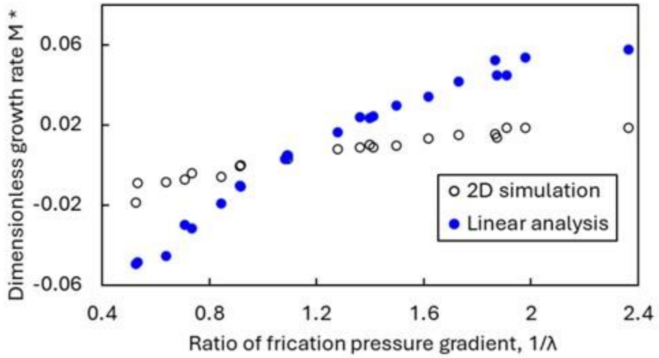
  ➔ *General linear trend observed, BUT with noticeable scatter*

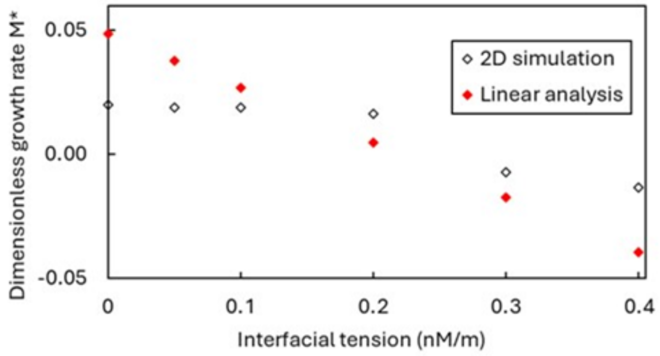
- **Deviations** linked to:
  - Strong shear-thinning effects (extreme values of n)
  - Varying interfacial tension (non-linear)
  - Known limitations in gap-averaged model (e.g. *underestimated shear Or higher effective viscosity* ➔ *reduced rate*)

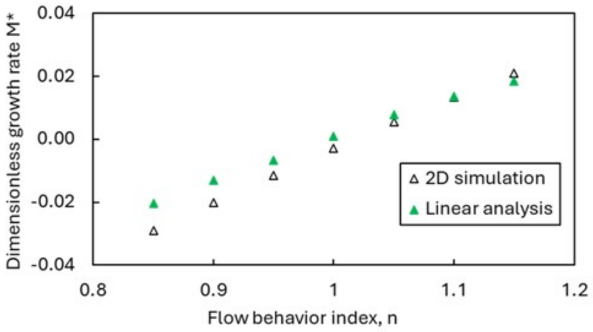


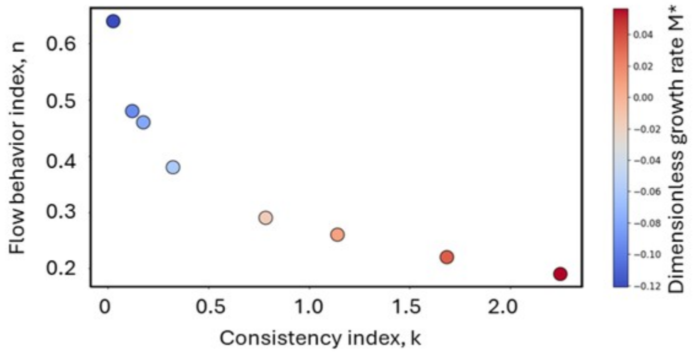
Stability in Power-Law Displacement

- Friction Gradient Ratio (1/λ)
  - M^* >0 when G2 > G1→ unstable
  - Transition around $\lambda \approx 1$
- Interfacial Tension (γ)
  - Higher γ → lower M^*, suppresses fingering
  - 2D model shows weaker dependence at low γ
- Flow Behavior Index (n)
  - Lower n (stronger shear-thinning) → stabilizes
  - Agreement improves as n → 1 (Newtonian)
- **Combined Effect (k and n)**
  - High k & low n → high M^*(strongest fingering)



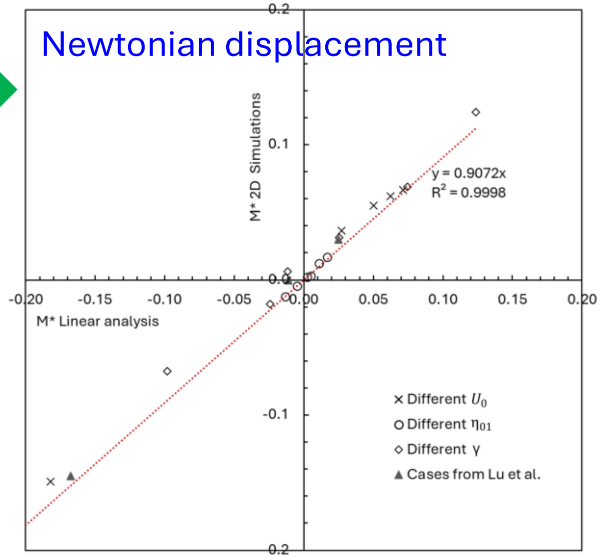






Newtonian vs. Power-Law

- **Newtonian Fluids**
  - ***Growth rate*** governed by friction pressure gradient (viscosity contrast) and interfacial tension
  - 2D simulations vs 3D DNS & linear theory, **almost perfectly**
  - **Transition** between stable and unstable regimes, **very clear**



- **Power-Law Fluids**
  - Same general trends, BUT:
    - *Rheology-dependent* damping/amplification (by k and n)
    - Discrepancies between theory & 2D models
  - **Limitations:**
    - **Cross-gap shear** not fully captured in 2D gap-averaged model
    - **Effective viscosity** may be overestimated → lower M* in simulation

Conclusion & Future Work

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Key Conclusions

- Growth rate of interfacial instabilities **well quantified** across Newtonian and power-law fluids
- 2D gap-averaged model shows:
  - *Excellent agreement* with theory for **Newtonian cases**
  - *Predictive but some discrepancies* for **power-law fluids**
- Critical parameters governs interfacial instability: λ, γ, n, k

Future Directions

- Refine shear-stress modeling to address cross-gap effects
- Explore 3D simulations or hybrid models for high-shear regimes
- Extend model to tapered or radial geometries

Acknowledgements & Open Access Model

- Research conducted at ***University of Stavanger***
- Supported by:
  - Supervisor: Prof. Hans Joakim Skadsem
  - Prof. Knut Erik Teigen Giljarhus
  - Department of Energy and Petroleum Engineering
- Computational resources: UiS HPC Cluster

- 2D Model and sample case for simulation available at: https://github.com/feebsssz/heleShawFoam

Thank you!
Any Questions?