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# Toward the Design of Chlorine Soft Sensors via Stepwise Safe Switching Observers for a Primary Water Distribution Network

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# Contribution

- Design, for the first time, of **chlorine soft sensors for primary WDNs**, using a safe switching observer approach.
- The primary WDN is modeled by a **set of nonlinear hyperbolic partial differential equations** (PDEs) that govern fluid dynamics within the pipe network, alongside additional PDEs representing the advection, dispersion, and decay of chlorine.
- The model also accounts for **user water demand**.
- The PDE-based model **is approximated by a nonlinear system of ordinary differential equations** (ODEs).
- **Linearized models** are then generated around selected operating points, forming the **basis for a bank of switching linear observers** tasked with estimating chlorine concentration at designated locations within the network.
- The parameters of these observers are **optimized using a metaheuristic algorithm**.
- **Data-driven, rule-based switching mechanism** is employed to select the appropriate observer based on the current operating condition.
- The **effectiveness** of the proposed observer design is validated through **computational experiments**, demonstrating its satisfactory performance.

# Advantages of the Proposed Approach

## Compared to Linear Observers based upon a Single Operating Point

- **More effectivve and accurate** in the sense that single operating point observers are in general poor at describing dynamic behavior when the system moves away from the operating point.

## Compared to Nonlinear Observers

- The switching approach of linear observers is **less sensitive to parameter changes and noise**, as well as more resilient towards model uncertainties.
- **Nonlinear observers**, although they are theoretically capable to estimate system dynamics, are subject to **requiring precise information** of the system and are prone to divergence when exposed to actual-world disturbances or model errors.

## Compared to Machine Learning Estimation Methods

- The method at hand provides **interpretability, convergence guarantees, and reliability**.
- Machine learning methods operate in general as **black box methods** and their results **depend upon the training data set**.
- Machine learning methods typically **require significant computational resources**.

# Water Distribution Network Modeling (1)

## Assumptions

- Each pipe is **straight** and free of fittings or slope.
- The fluid exhibits **slight compressibility**.
- The duct walls are **slightly flexible**.
- Variations in **convective velocity are negligible**.
- The duct maintains a **constant cross-sectional area**.
- The fluid **density and viscosity remain constant**.

## Flow and Pressure in a Conduit

$$\frac{\partial H(z,t)}{\partial t} + \frac{b^2}{gA}\frac{\partial Q(z,t)}{\partial z} = 0$$

$$\frac{\partial Q(z,t)}{\partial t} + gA\frac{\partial H(z,t)}{\partial z} + \frac{f(Q,D,\varepsilon)}{2DA}Q(z,t)\left|Q(z,t)\right| = 0$$

$Q$: Volumetric flow rate

$H$: Pressure head

$z$: Spatial coordinate

$t$: Time

$g$: Gravitational acceleration

$A$: Cross-sectional area of the conduit

$b$: Pressure wave speed

$D$: Inner conduit diameter

$f$: Friction coefficient

$\varepsilon$: Relative roughness of the pipe.

# Water Distribution Network Modeling (2)

## Friction Coefficient

Types of flow (depending upon the **Reynolds number - Re**):

- Laminar flow,
- Transient flow,
- **Turbulent flow** (typical for WDNs).

$$f\left(Q,D,\varepsilon\right)=\lambda_1\log_{10}\left(\frac{\lambda_2}{\mathrm{Re}}+\left(\varepsilon/\lambda_3\right)^{\lambda_4}\right)^{-2}\quad;\quad\lambda_j\in\mathbb{R}^+\quad\left(j=1,\ldots,4\right)$$

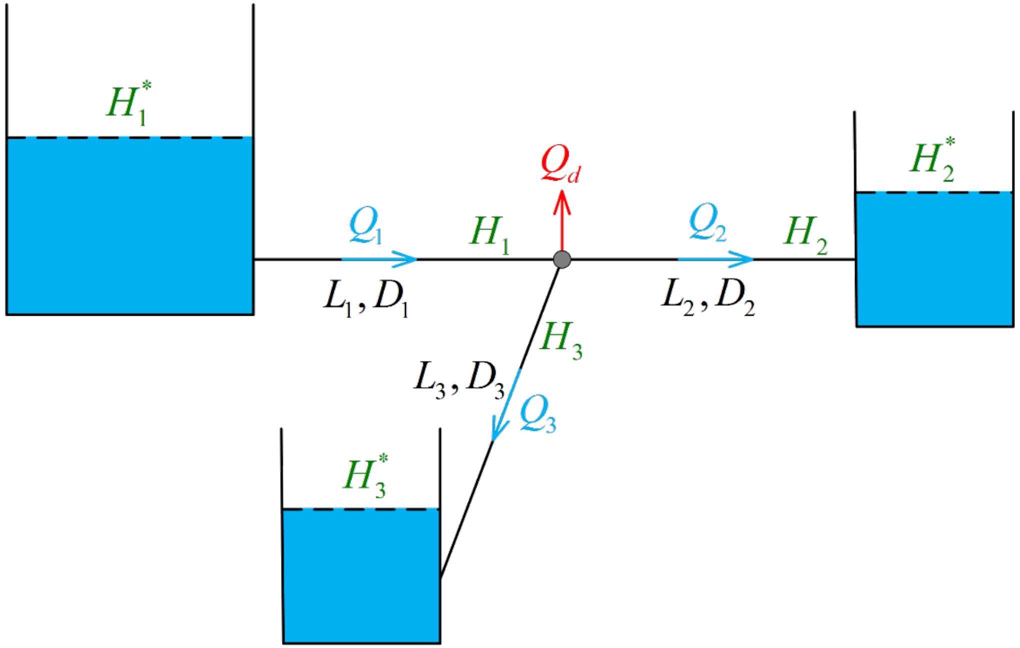
## Chlorine Concentration

$$\frac{\partial c\left(z,t\right)}{\partial t}+\frac{4Q\left(z,t\right)}{\pi D^2}\frac{\partial c\left(z,t\right)}{\partial z}+k_1c\left(z,t\right)=0$$

$c$: Constituent concentration.

$k_1$: First order reaction rate (considered to be constant).

# A Benchmark Branched Water Distribution Network (1)




## Characteristics

- **Main line** with **one branch**.
- **Three reservoirs** providing **variable / actuatable head pressure** to the network
- **Water demand** to unmodelled parts of the network, acting as **measurable disturbance**.

# A Benchmark Branched Water Distribution Network (2)

## PDE Model of the Network

$$\frac{\partial H_j(z_j,t)}{\partial t} + \frac{b^2}{gA_j}\frac{\partial Q_j(z_j,t)}{\partial z_j} = 0$$

$$\frac{\partial Q_j(z_j,t)}{\partial t} + gA_j\frac{\partial H_j(z_j,t)}{\partial z_j} + \frac{f(Q_j,D_j,\varepsilon_j)}{2D_jA_j}Q_j(z_j,t)\left|Q_j(z_j,t)\right| = 0$$

$$\frac{\partial c_j(z_j,t)}{\partial t} + \frac{4Q_j(z_j,t)}{\pi D_j^2}\frac{\partial c_j(z_j,t)}{\partial z_j} + k_1 c_j(z_j,t) = 0$$

- $Q_j$, $H_j$ and $c_j$ $(j=1,2,3)$ are the volumetric flow rates, pressures and chlorine concentrations in the respective conduits.

Appropriate **algebraic constraints and boundary conditions** must be imposed for the above set of PDEs to accurately represent the water distribution network.

# Approximation through a System of ODEs

## Assumptions

- The PDEs describing the network will be discretized in space using a finite different approximation.
- The friction coefficient follows formula presented in Slide 5.
- The flow in the conduits is slowly varying and the flow variables can be discretized in space using a single step.
- The PDEs describing chlorine concentrations are divided into $n_{cl}$ sections.
- The flows in the conduit do not change directions.

## Nonlinear ODE Model in State Space Form

$$\frac{dx}{dt} = \Gamma(x) + Z(x)u(t)$$

$$x = \begin{bmatrix} x_1 & \cdots & x_{13} \end{bmatrix}^T = \begin{bmatrix} Q_{1,1} & Q_{2,1} & Q_{3,1} & H_n^* & c_{1,1} & c_{1,2} & c_{1,3} & c_{2,1} & c_{2,2} & c_{2,3} \end{bmatrix}$$

$$\begin{bmatrix} c_{3,1} & c_{3,2} & c_{3,3} \end{bmatrix}^T, \; u = \begin{bmatrix} u_1 & \cdots & u_5 \end{bmatrix}^T = \begin{bmatrix} H_1^* & H_2^* & H_3^* & Q_d & c_{1,0} \end{bmatrix}^T$$

$$\Gamma(x) = \left[\gamma_j(x)\right] \in \mathbb{R}^{13\times 1}, \; Z(x) = \left[z_{i,j}(x)\right] \in \mathbb{R}^{13\times 5}$$

# Operating Points (1)

## Definitions

$\bar{u}_j \ (j = 1,\ldots,5)$: Nominal values of the inputs of the system.

$\bar{x}_i \ (i = 1,\ldots,13)$: Nominal values of the state variables.

$\bar{u}$: Vector of nominal values of the inputs.

$\bar{x}$: Vector of nominal values of the state variables.

## Solution

$$\Theta(\bar{x},\bar{u}) = 0$$
$$\Theta(\bar{x},\bar{u}) = \Gamma(\bar{x}) + Z(\bar{x})\bar{u}$$

## Constraints

$$(\bar{x}_1 \geq 0) \wedge (\bar{x}_2 \geq 0) \wedge (\bar{x}_3 \geq 0) \wedge (\bar{u}_5 \geq 0)$$

# Operating Points (2)

## Solvability with Respect to the Nominal Values of the State Vector

$$\operatorname{rank}\left\{J_s\left(\overline{x},\overline{u}\right)\right\}=13$$
$$J_s\left(\overline{x},\overline{u}\right)=\partial\Theta\left(\overline{x},\overline{u}\right)/\partial\overline{x}\,.$$

- $J_s\left(\overline{x},\overline{u}\right)=\left[\left(j_s\right)_{i,j}\right]\in\mathbb{R}^{13\times13}$

## Solvability Conditions (through Implicit Function Theorem)

$$\overline{x}_j\neq-\frac{D_j^2k_jL_j\pi}{4n_{cl}}\quad;\quad j=1,2,3$$
$$\frac{1}{L_1^2}\left(j_s\right)_{2,2}\left(j_s\right)_{3,3}+\frac{A_2}{A_1L_1L_2}\left(j_s\right)_{1,1}\left(j_s\right)_{3,3}+\frac{A_3}{A_1L_1L_3}\left(j_s\right)_{1,1}\left(j_s\right)_{2,2}\neq0$$

- The **first condition is true**, as the nominal values of the **flow rates and physical parameters are positive**.
- Regarding the second condition, it is observed that **in the present case**, namely the case of turbulent flow, the parameters $\left(j_s\right)_{j,j}\left(j=1,2,3\right)$ **are negative** and hence **the condition holds true**.

# Linear Approximant

## Linear Approximant Model

$$\frac{d}{dt}\delta x = J_s\left(\overline{x},\overline{u}\right)\delta x + Z\left(\overline{x}\right)\delta u\left(t\right)$$

- $\delta x$ is the response of the above linear system for $\delta u = \Delta u = u - \overline{u}$, that approximates $\Delta x = x - \overline{x}$ around the operating point $\overline{o} = \left(\overline{u},\overline{x}\right)$.

## Linear Approximant System Matrices Reformulation

Since the vector $\overline{x}$ can be determined given the vector $\overline{u}$, there exist a nonlinear vector function, mapping the nominal values of the inputs and the nominal values of the states.

$$\overline{x} = \sigma\left(\overline{u}\right) \Rightarrow \begin{array}{c} A\left(\overline{u}\right) = J_s\left(\sigma\left(\overline{u}\right),\overline{u}\right) \\ B\left(\overline{u}\right) = Z\left(\sigma\left(\overline{u}\right)\right) \end{array}$$

$$A\left(\overline{u}\right) = \begin{bmatrix} A_{1,1}\left(\overline{u}\right) & 0_{4\times 9} \\ A_{2,1}\left(\overline{u}\right) & A_{2,2}\left(\overline{u}\right) \end{bmatrix},\ B\left(\overline{u}\right) = \begin{bmatrix} \underset{i=1,2,3,4,5}{\operatorname{diag}}\left\{b_{i,i}\left(\overline{u}\right)\right\} \\ 0_{8\times 5} \end{bmatrix},\ A_{1,1}\left(\overline{u}\right) \in \mathbb{R}^{4\times 4},\ A_{2,1}\left(\overline{u}\right) \in \mathbb{R}^{9\times 4},\ A_{2,2}\left(\overline{u}\right) \in \mathbb{R}^{9\times 9}.$$

# A Luenberger type Full order Observer for the WDN

## Measurable Variables

i. Volumetric flow rates downstream the node.
ii. Chlorine concentrations at the entrance of reservoirs 2 and 3

$$y_m = C_m x \ ; \ C_m = \left[ (c_m)_{i,j} \right] \in \mathbb{R}^{4\times 13}$$

- $y_m$: Vector of measurable variables
- $(c_m)_{1,3} = (c_m)_{2,4} = (c_m)_{3,10} = (c_m)_{4,13} = 1$

## Full order Observer

$$\mathfrak{I}: \frac{d}{dt}\delta\hat{x}(t) = F(\bar{u})\delta\hat{x}(t) + G(\bar{u})\Delta y_m(t) + B(\bar{u})\Delta u(t), \delta\hat{x}(0-) = \delta\hat{x}_0$$

- $F(\bar{u}) \in \mathbb{R}^{13\times 13}$, $G(\bar{u}) \in \mathbb{R}^{13\times 4}$

$$F(\bar{u}) = A(\bar{u}) - G(\bar{u})C_m$$

# Observer Design (1)

## Linear Approximant Characteristic Polynomial

$$p(s) = p_f(s)\,p_c(s)$$

- $p_f(s)$ is a fourth order polynomial, depending upon the parameters of the fluid dynamics
- $p_c(s)$ is a ninth order polynomial, depending upon the chlorine concentration parameters.

## Special form of the observer Linear Approximant Characteristic Polynomial

$$G(\bar{u}) = \begin{bmatrix} G_{1,1}(\bar{u}) & 0_{4\times 2} \\ 0_{9\times 2} & G_{2,2}(\bar{u}) \end{bmatrix} \;;\; G_{1,1}(\bar{u}) = \left[\left(g_{1,1}\right)_{i,j}\right] \in \mathbb{R}^{4\times 2},\; G_{2,2}(\bar{u}) = \left[\left(g_{2,2}\right)_{i,j}\right] \in \mathbb{R}^{9\times 2}$$

## Observer Characteristic Polynomial

$$p_o(s) = \det\left(sI_{13} - F(\bar{u})\right) = p_{o,f}(s)\,p_{o,c}(s)$$

$$p_{o,f}(s) = \det\left(sI_4 - A_{1,1}(\bar{u}) + G_{1,1}(\bar{u})C_{m,1}\right),\quad p_{o,c}(s) = \det\left(sI_9 - A_{2,2}(\bar{u}) + G_{2,2}(\bar{u})C_{m,2}\right),\quad C_m = \begin{bmatrix} C_{m,1} & 0_{2\times 9} \\ 0_{2\times 4} & C_{m,2} \end{bmatrix}$$

# Observer Design (2)

## Assumptions - Definitions

- Let $\pi_{f,j}$ $(j=1,\ldots,4)$ are the roots of $p_f(s)$ and $\pi_{c,i}$ $(i=1,\ldots,9)$ are the roots of $p_c(s)$.
- Without loss of generality, assume that $\left|\mathrm{Re}\left(\pi_{f,j}\right)\right| \leq \left|\mathrm{Re}\left(\pi_{f,j+1}\right)\right|$ $(j=1,2,3)$ and that $\left|\mathrm{Re}\left(\pi_{c,i}\right)\right| \leq \left|\mathrm{Re}\left(\pi_{c,i+1}\right)\right|$ $(i=1,\ldots,8)$.

## Constraints

- The roots of $p_{o,f}(s)$ and $p_{o,c}(s)$ are real and negative, i.e., $p_{o,f}(s)=\prod_{j=1}^{4}\left(s-(\pi_{o,f})_j\right)$ and $p_{o,c}(s)=\prod_{i=1}^{9}\left(s-(\pi_{o,c})_i\right)$, where $(\pi_{o,f})_j<0$ $(j=1,\ldots,4)$ and $(\pi_{o,c})_i<0$ $(i=1,\ldots,9)$.
- The roots of $p_{o,f}(s)$ and $p_{o,c}(s)$ are ordered and have a minimum distance between them being equal to $\gamma$ i.e. it holds that $\left|(\pi_{o,f})_{j+1}\right|-\left|(\pi_{o,f})_j\right|>\gamma$ $(j=1,2,3)$ and $\left|(\pi_{o,c})_{i+1}\right|-\left|(\pi_{o,c})_i\right|>\gamma$.
- Regional per pole stability is achieved, i.e. it holds that $\left|\left(\pi_{o,f}\right)_j\right|>\lambda\left|\mathrm{Re}\left(\pi_{f,j}\right)\right|$ and $\left|\left(\pi_{o,c}\right)_i\right|>\lambda\left|\mathrm{Re}\left(\pi_{c,i}\right)\right|$ where $\lambda>0$.

## Toward Determination of the Observer Degrees of Freedom

- The parameters $\gamma$ and $\lambda$ are to be selected by the observer designer.
- The pole placement problem will be solved using the observer degrees of freedom appearing in the first columns of $G_{1,1}(\overline{u})$ and $G_{2,2}(\overline{u})$.

# A Heuristic Approach toward Determination of the Observer Degrees of Freedom (1)

## Observer Frequency Response Dynamics

$$\delta\hat{X}(s)=\Phi(s)\left[G(\bar{u})\Delta Y_m(s)+M(\bar{u})\Delta U(s)\right]+\Phi(s)\delta\hat{x}_0$$

- $\Phi(s)=\left(sI_{13}-F(\bar{u})\right)^{-1}=\left[\varphi_{i,j}(s)\right]\in\mathbb{R}(s)^{13\times 13}$ is the observer resolvent matrix

## Free Observer Parameter Vector

$$\chi=\left[\chi_1 \quad \cdots \quad \chi_{26}\right]^T=\Big[(\pi_{o,f})_1 \quad \cdots \quad (\pi_{o,f})_4 \;\vdots\; (\pi_{o,c})_1 \quad \cdots \quad (\pi_{o,c})_9 \;\vdots$$
$$\vdots\; (g_{1,1})_{1,2} \quad \cdots \quad (g_{1,1})_{4,2} \;\vdots\; (g_{2,2})_{1,2} \quad \cdots \quad (g_{2,2})_{9,2}\Big]^T.$$

The elements of the free observer parameter vector will be determined using a **metaheuristic algorithm** so that the **influence of the free response is attenuated**.

# A Heuristic Approach toward Determination of the Observer Degrees of Freedom (2)

## Cost Criterion to be Minimized

$$J(\chi,\bar{u}) = \max_{i=1,\ldots,13} \left\{ \int_0^\infty \sum_{j=1}^{13} \left| h_{i,j}(t) \right| dt \right\}$$

- $h_{i,j}(t) = \mathcal{L}^{-1}\{\varphi_{i,j}(s)\}$ is the $\{i, j\}$ element of the transition matrix of the observer dynamics.
- $\mathcal{L}^{-1}\{\bullet\}$ denotes the inverse Laplace transform of the argument transfer function.
- The optimization procedure must be executed **separately at each operating point**.

The main idea of the algorithm* is to define a search area for the observer parameters and after series of computations to contract to suboptimal values, satisfying the observer constraints.

* Drosou, T.C., Kouvakas, N.D., Koumboulis, F.N., Tzamtzi, M.P.: A Mixed Analytic/Metaheuristic Dual Stage Control Scheme Toward I/O Decoupling for a Differential Drive Mobile Robot. In: Farmanbar, M., Tzamtzi, M., Verma, A.K., Chakravorty, A. (eds) Frontiers of Artificial Intelligence, Ethics, and Multidisciplinary Applications. FAIEMA 2023. Springer, Singapore, pp. 197–214 (2024).

  Kouvakas, N.D., Koumboulis, F.N., Sigalas, J.: A Two Stage Nonlinear I/O Decoupling and Partially Wireless Controller for Differential Drive Mobile Robots. Robotics 13(2), 26 (2024).

# Target Operating Areas (1)

## Definitions

- $\tilde{u}_j$ denotes the steady state value of $u_j$ $(j = 1,\ldots,5)$ during a step wise transition.
- $\bar{u}_j$ denotes the nominal values of the inputs.
- $\tilde{x}_i$ $(i = 1,\ldots,13)$ denotes the steady state value of the state variable $x_i$ corresponding to $\tilde{u}_j (j = 1,\ldots,5)$.
- $\tilde{u}$ and $\tilde{x}$ denote the steady state input and state variable vectors corresponding to the above transition.
- $\tilde{x}_e$ denotes the steady state of the observer

$$\tilde{x}_e = \bar{x} - F(\bar{u})^{-1}\left[G(\bar{u})C_m(\tilde{x} - \bar{x}) + M(\bar{u})(\tilde{u} - \bar{u})\right]$$

- Five-dimensional spheroid or radius $R$

$$\sum_{j=1}^{5}\left(\frac{\tilde{u}_j - \bar{u}_j}{\bar{u}_j}\right)^2 = R^2$$

# Target Operating Areas (2)

## Normalized estimation steady state error metric

$$\varepsilon_{ss} = \sqrt{\frac{\left[W\left(\tilde{\zeta}-\tilde{\zeta}_e\right)\right]^T \left[W\left(\tilde{\zeta}-\tilde{\zeta}_e\right)\right]}{\left[W\left(\tilde{\zeta}-\bar{\zeta}\right)\right]^T \left[W\left(\tilde{\zeta}-\bar{\zeta}\right)\right]}} \times 100\%$$

$$\tilde{\zeta} = \begin{bmatrix} \tilde{x}^T & \tilde{u}^T \end{bmatrix}^T,\ \bar{\zeta} = \begin{bmatrix} \bar{x}^T & \bar{u}^T \end{bmatrix}^T,\ \tilde{\zeta}_e = \begin{bmatrix} \tilde{x}_e^T & \tilde{u}^T \end{bmatrix}^T,\ W = \begin{bmatrix} \mathrm{diag}\left\{\bar{x}_j^{-1}\right\} & 0_{13\times 5} \\ 0_{5\times 13} & \mathrm{diag}\left\{\bar{u}_i^{-1}\right\} \end{bmatrix}$$

The **target area for the operating point** $\bar{o} = \left(\bar{u}, \bar{x}\right)$ is defined as the **maximum radius of the spheroid**, such that all input transitions from the operating point to a new steady state value inside the spheroid, result in $\varepsilon_{ss} < \varepsilon_{ss,\max}$, where $\varepsilon_{ss,\max}$ is a positive parameters selected by the observer designer.

- This procedure must be repeated for a **sufficiently large number of points** to ensure that the desired area is covered by target operating regions that **satisfy the dense web principle**.

# Switching Between Observers

- For proper operation it is evident that **a switching mechanism** that appropriately enables the operation of appropriately chosen observer of the bank **is necessary**.
- Considering that the performance outputs of the system are measurable in real time and that the trajectory of the nonlinear process is known, an approach based upon the convergence of the measurable variable of the system to their target values is proposed.

## Convergence metric

$$\varepsilon_c(t) = \sqrt{\frac{\left(y_m(t) - C_m\tilde{x}\right)^T \left(y_m(t) - C_m\tilde{x}\right)}{\left[C_m\left(\tilde{x} - \overline{x}\right)\right]^T \left[C_m\left(\tilde{x} - \overline{x}\right)\right]}} \times 100\%$$

- Switching between observers will take place whenever the convergence metric reaches a threshold $\varepsilon_s \in \mathbb{R}^+$, i.e. when $\varepsilon_c(t) = \varepsilon_s$.

# Simulation Results (1)

## Convergence metric

$$A_1 = 0.0079\left[\text{m}^2\right],\ A_2 = 0.0079\left[\text{m}^2\right],\ A_3 = 0.0028\left[\text{m}^2\right],\ L_1 = 62[\text{m}],\ L_2 = 124[\text{m}],\ L_3 = 80[\text{m}],\ D_1 = 0.1[\text{m}],\ D_2 = 0.1[\text{m}],$$

$$D_3 = 0.06[\text{m}],\ \varepsilon_1 = 0.0035[-],\ \varepsilon_2 = 0.0035[-],\ \varepsilon_3 = 0.0058[-],\ g = \left[\text{m/s}^2\right],\ \rho = \left[\text{Kg/m}^3\right],\ \mu = 0.0011[\text{Pa s}],\ n_{cl} = 3[-]\ ,$$

$$k_1 = 0.1\left[\text{h}^{-1}\right],\ b = 1200[\text{m/s}],\ \lambda_1 = 0.308642[-],\ \lambda_2 = 6.9[-],\ \lambda_3 = 3.7[-],\ \lambda_4 = 1.11[-].$$

## Operating point trajectory

| # | $\bar{u}_1$ [m] | $\bar{u}_2$ [m] | $\bar{u}_3$ [m] | $\bar{u}_4$ [l/min] | $\bar{u}_5$ [mg/l] |
|---|---|---|---|---|---|
| 1 | 20.0000 | 17.0000 | 14.0000 | 360.0000 | 2.0000 |
| 2 | 20.5263 | 16.8947 | 14.1053 | 356.8421 | 1.9737 |
| 3 | 21.0526 | 16.7895 | 14.2105 | 353.6842 | 1.9474 |
| 4 | 21.5789 | 16.6842 | 14.3158 | 350.5263 | 1.9211 |
| 5 | 22.1053 | 16.5789 | 14.4211 | 347.3684 | 1.8947 |
| 6 | 22.6316 | 16.4737 | 14.5263 | 344.2105 | 1.8684 |
| 7 | 23.1579 | 16.3684 | 14.6316 | 341.0526 | 1.8421 |
| 8 | 23.6842 | 16.2632 | 14.7368 | 337.8947 | 1.8158 |
| 9 | 24.2105 | 16.1579 | 14.8421 | 334.7368 | 1.7895 |
| 10 | 24.7368 | 16.0526 | 14.9474 | 331.5789 | 1.7632 |
| 11 | 25.2632 | 15.9474 | 15.0526 | 328.4211 | 1.7368 |
| 12 | 25.7895 | 15.8421 | 15.1579 | 325.2632 | 1.7105 |
| 13 | 26.3158 | 15.7368 | 15.2632 | 322.1053 | 1.6842 |
| 14 | 26.8421 | 15.6316 | 15.3684 | 318.9474 | 1.6579 |
| 15 | 27.3684 | 15.5263 | 15.4737 | 315.7895 | 1.6316 |
| 16 | 27.8947 | 15.4211 | 15.5789 | 312.6316 | 1.6053 |
| 17 | 28.4211 | 15.3158 | 15.6842 | 309.4737 | 1.5789 |
| 18 | 28.9474 | 15.2105 | 15.7895 | 306.3158 | 1.5526 |
| 19 | 29.4737 | 15.1053 | 15.8947 | 303.1579 | 1.5263 |
| 20 | 30.0000 | 15.0000 | 16.0000 | 300.0000 | 1.5000 |

# Simulation Results (2)

## Metaheuristic Algorithm Parameters

$$\gamma = 0.01,\ \lambda = 1.1,\ \varepsilon = 0.01,\ n_{loop} = 1000,\ n_{rep} = 30,\ n_{total} = 10^8,\ \left(\chi_j\right)_c = \frac{\lambda+5}{2}\left|\mathrm{Re}\left(\pi_{f,j}\right)\right| \text{ and } \left(\chi_j\right)_w = \frac{3\lambda+5}{2}\left|\mathrm{Re}\left(\pi_{f,j}\right)\right| \text{ for } j = 1,\dots,4$$

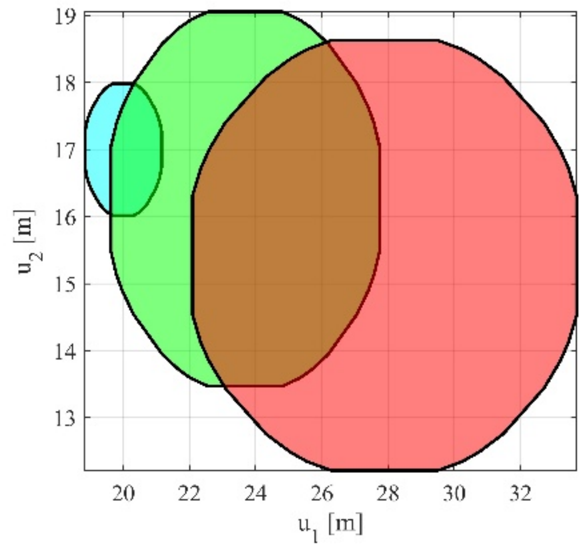
$$\left(\chi_j\right)_c = \frac{\lambda+5}{2}\left|\mathrm{Re}\left(\pi_{c,j-4}\right)\right| \text{ and } \left(\chi_j\right)_w = \frac{3\lambda+5}{2}\left|\mathrm{Re}\left(\pi_{c,j-4}\right)\right| \text{ for } j = 5,\dots,13,\ \left(\chi_j\right)_c = 0 \text{ and } \left(\chi_j\right)_w = 50 \text{ for } j = 14,\dots,26,\ \varepsilon_{ss,\max} = 5\%$$

## Spheroid Radii per Operating Point

| # | $R$ | # | $R$ |
|---|---|---|---|
| 1 | 0.0603 | 11 | 0.2184 |
| 2 | 0.0833 | 12 | 0.2065 |
| 3 | 0.1043 | 13 | 0.1837 |
| 4 | 0.1292 | 14 | 0.2080 |
| 5 | 0.1466 | 15 | 0.2121 |
| 6 | 0.1696 | 16 | 0.2169 |
| 7 | 0.1828 | 17 | 0.2191 |
| 8 | 0.1789 | 18 | 0.2209 |
| 9 | 0.1838 | 19 | 0.2493 |
| 10 | 0.2452 | 20 | 0.2541 |

**The target areas are overlapping** and consequently, to implement an observer switching scheme as described previously, **not all points need to be used**.

Indicative Projection of the Target area for points 1, 8 and 16 (1 – cyan, 8 – green, 16-red)



# Simulation Results (3)

## Transitions Definition

i. Initial Point: $\bar{u}_1 = 19.7556[\text{m}]$, $\bar{u}_2 = 17.0289[\text{m}]$, $\bar{u}_3 = 14.0429[\text{m}]$, $\bar{u}_4 = 360[\text{l/min}]$, $\bar{u}_5 = 2.0039[\text{mg/lt}]$

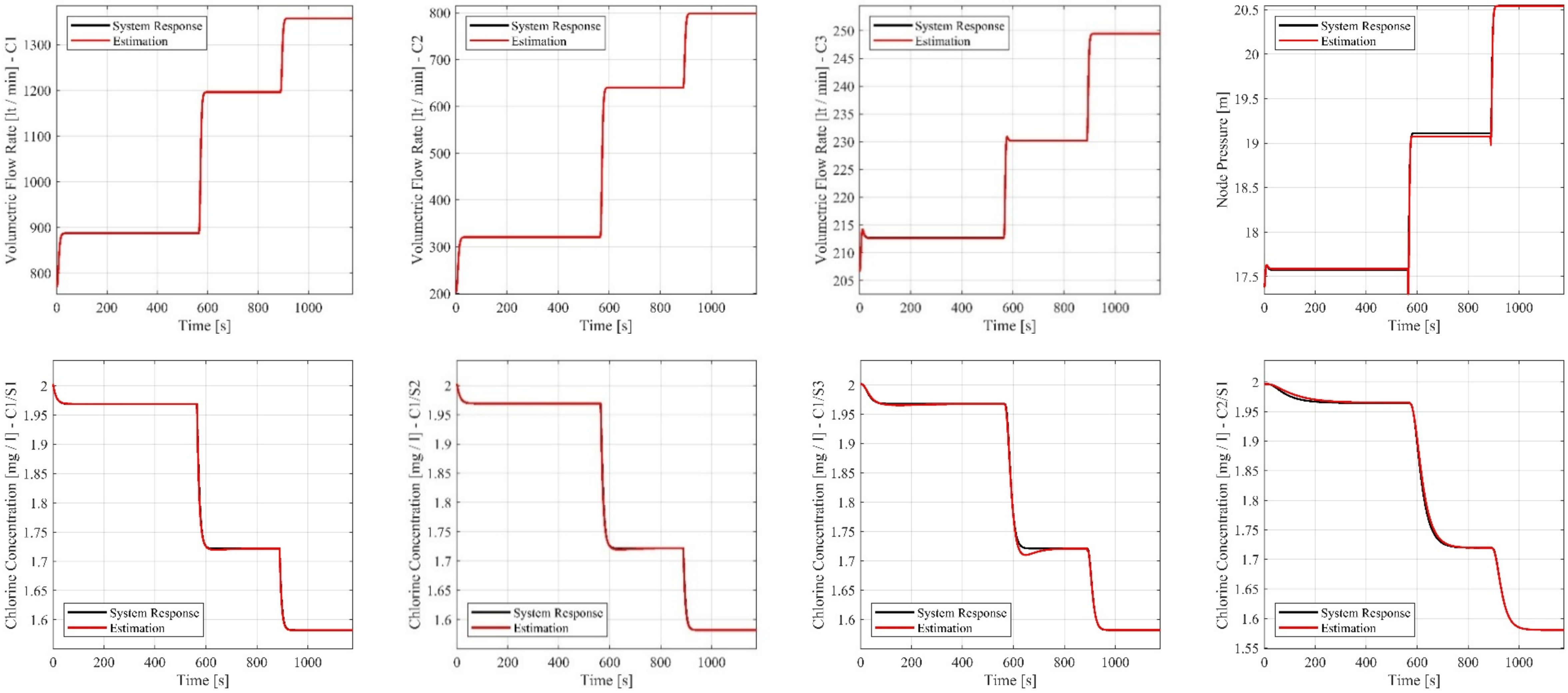
ii. Transition Point 1: $\bar{u}_1 = 20.7163[\text{m}]$, $\bar{u}_2 = 16.7226[\text{m}]$, $\bar{u}_3 = 14.0429[\text{m}]$, $\bar{u}_4 = 354[\text{l/min}]$, $\bar{u}_5 = 1.9697[\text{mg/lt}]$

iii. Transition Point 2: $\bar{u}_1 = 24.7883[\text{m}]$, $\bar{u}_2 = 15.8112[\text{m}]$, $\bar{u}_3 = 14.9878[\text{m}]$, $\bar{u}_4 = 324[\text{l/min}]$, $\bar{u}_5 = 1.7221[\text{mg/lt}]$

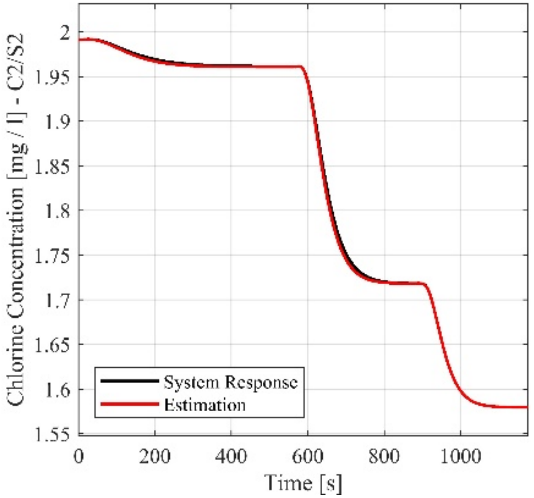
iv. Final Point: $\bar{u}_1 = 27.8408[\text{m}]$, $\bar{u}_2 = 15.4377[\text{m}]$, $\bar{u}_3 = 15.7097[\text{m}]$, $\bar{u}_4 = 312[\text{l/min}]$, $\bar{u}_5 = 1.5826[\text{mg/lt}]$

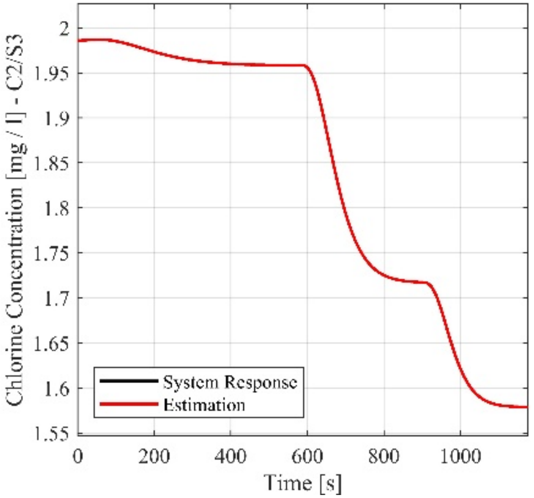
Transitions $u_1$ to $u_4$ from point to point will be assumed to take place **smoothly** and not in step form. This is a common approach in closed conduits and water distribution networks to prevent **water hammer effects**, which can potentially **damage infrastructure**.

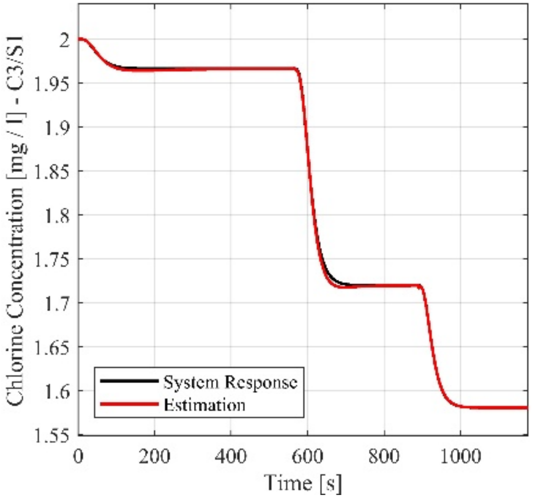
# Simulation Results (4)

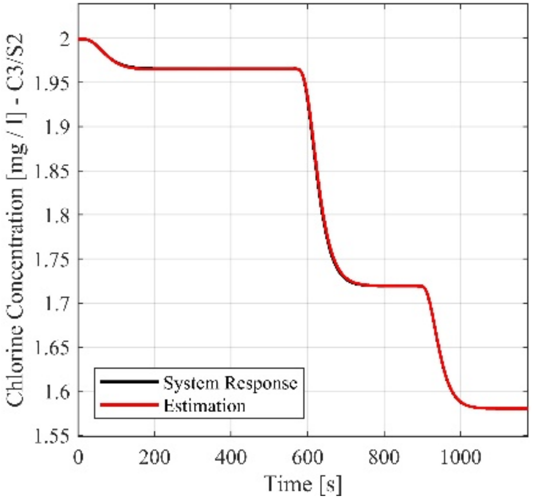


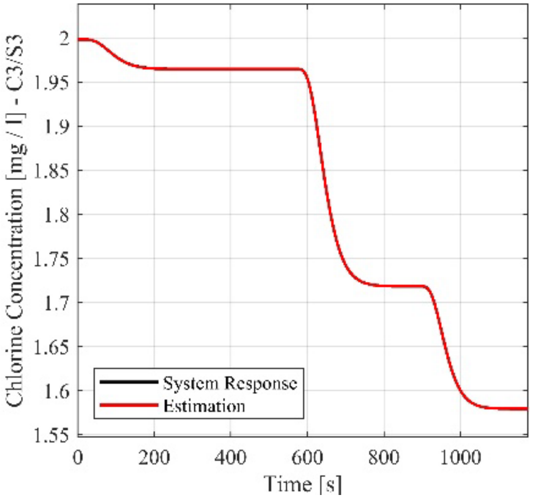
# Simulation Results (5)




Switching between observers took place at $t = 9.42[\mathrm{min}]$ and at $t = 14.82[\mathrm{min}]$.

# Simulation Results (6)

- The observer is **highly accurate with negligible deviations** that are visible in only a few instances.
- Regarding the non-measurable variables
  - The volumetric flow rate through conduit 1 shows a **nearly ideal match** between the system response and the observer estimate. The estimation curve **closely tracks the system curve**, both in steady-state operation as well as under sudden changes when switching takes place.
  - The head pressure estimation is **very close to tracing the system response**. However, there is a **temporary mismatch during the step changes**, where estimation is slightly behind the system response. Nevertheless, **the estimation approaches the correct value**, demonstrating that the observer is still reasonably accurate.
  - The estimations of chlorine concentrations show that the observer performs satisfactorily for all parts of the conduits being observed. The estimated concentrations follow the actual system responses accurately, following their steady-state levels and their dynamic transitions, with high accuracy. Minor deviations are observed in the steeply sloping (observer switch points)
- Regarding estimation of the measurable variables, they are practically identical to the respective measurements.

# Conclusions

- In the present paper, a **novel design approach for chlorine soft sensors** in primary WDNs, employing a **bank of linear safe switching observers** has been presented.
- The method has employed a nonlinear dynamic **approximation of the PDE-based fluid and chlorine transport dynamics** and a **linear approximant of the nonlinear approximation** about selected operating points.
- The observer parameters have been **optimized with a metaheuristic algorithm**.
- A **rule-based data-driven switching** has been adopted to switch observers in real time.
- Computational experiments showed that the proposed method provides **reliable and accurate chlorine concentration estimates** under various operating conditions.

➢ Future research will focus on enhancing the **observer's resilience**. Toward this goal, **adaptive learning mechanisms** and **machine learning techniques** to adjust observer parameters, in response to model uncertainties and unmodeled dynamics, will be developed.

➢ **Experimental validation** will be performed to evaluate the operational feasibility and scalability of the suggested solution within actual water networks.