Amplification of impact loading

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Abstract. During an impact, a load interacts with a structure during the time interval t_d. Depending on the impact characteristics and the load rise time, t_r, the amplification of the load can be found as a function of the parameters: t_d/T_0 and t_r/T_0 where T_0 is the eigenperiod of the structure. The paper will investigate different impact characteristics and summarize the amplification due to impact loading in a one-degree-of-freedom system. The near static case, where the impact rise time is long, will give minimal amplification, while a rapid impact will result in an amplification factor closer to 2. Practical examples from snap-loading in a wire, explosions, and collisions will be discussed.

Keywords: Impact Loading, Dynamic Amplification, Load Duration, Load Rise Time, Snap Loading, Explosion, Collision.

1 Introduction

1.1 Background to the study of the amplification of impact loading

The study of the transfer of a load to structures is important as the load rise time and the loading duration are important parameters. A very rapid load transfer causes dynamic effects that represent dynamic amplification of the loading. In practical examples, such rapid loading is present during explosions and collisions. In the marine environment, impact loading occurs when a wave breaks against a structure.

Note that impact loading is characterized by:

- a single principal impulse
- a relatively short time duration.

During equipment lifting, a very rapid load transfer, a situation called snap loading, occurs when the load in the wire suddenly changes, for example, due to rapid loss of buoyancy when the structure gets out of the water. A criterion for good lifting practice is to avoid rapid lifting of equipment out of the water [1] and [2].

The response to the impact loading, is, however, to a large degree depending on the eigenperiod of the structure being impacted. An impact on a soft structure having a long eigenperiod gives rise to a lower response than an impact on a very stiff structure with a short eigenperiod. One can compare this to hitting a sandbag versus hitting a log. The sandbag will delay the effect of the impact. Thus, the amplification of the impact must be considered relative to the load duration time t_d, and the load rise time, t_r versus the eigenperiod T_0 of the structure being impacted; t_d/T_0 and t_r/T_0, [3].

It is of interest to investigate the response to loading caused by collisions, explosions, and wave impacts, in particular from breaking waves, [5], [6], [7]. The paper is organized as follows: In Chapter 2 several types of impact/ impulse loadings are discussed and in Chapter 3 specific loading scenarios and the associated dynamic response are discussed.

2 The response to an impulse load

2.1 The impulse-response method

A simple way to investigate the response to an impact loading is to study a one-degree-of-freedom system with eigenperiod T_0 exposed to an impact with duration t_d and rise time t_r. It should be noted that an impact load could be expressed in a Fourier series as any periodic loading function F(t) can be written as a Fourier series.

The response to this loading is the sum of the contributions from the harmonic terms. In the case of a linear dynamic system, we have:

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F(t) \tag{1}$$

The standard method to find the response due to a non-periodic loading on a single-degree-of-freedom linear system is, however, to use ***the impulse-response method*****.** The response is found by dividing the loading into rectangular impulses, Figure 1. We calculate the response from each rectangular impulse and the total response at time t_i is the sum of the responses to the loading at time t_i.

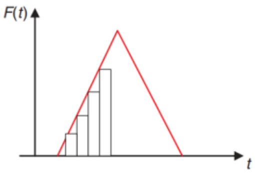

Figure 1. The Impulse loading can be regarded as a sum of rectangular impulses.

An impulse I_i is generated by a force $F_i(t)$ that has a large value over a short period $\Delta t = t_2 - t_1$:

$$I_i = F_i \Delta t = m\frac{dx_2}{dt} - m\frac{dx_1}{dt} \tag{2}$$

Here,

I_i = Impulse = $F_i(t) \cdot \Delta t$

$\frac{dx_2}{dt}$ = Velocity after the impulse, at time t_2

$\frac{dx_1}{dt}$ = Velocity before the impulse, at time t_1

$m\frac{dx}{dt}$ = Mass-movement

We then define a unit impulse:

$$\tilde{I}_i = \lim_{\Delta t \to 0} \int_t^{t+\Delta t} F_i\,(t)dt \tag{3}$$

As Δt → 0, a unit value of 1 for the integral is obtained when $F_i(t) \to \infty$.

We then define the δ-function $F_\delta(t)$ = δ(t-τ) that is zero except for at t = τ.

If we give the system given by equation (2) a unit impulse when the displacement x at t = 0 is 0, and the velocity at time t = 0^- = 0 (zero motion before the impulse), the velocity at t = 0^+ is $\frac{dx}{dt}$ (t = 0^+) = $\dot{x}_0$ and the displacement at t = 0^+ (following the impact) is given by:

$$x(t) = e^{-\lambda\omega_0 t}\frac{1}{m\omega_d}\sin(\omega_d t) \tag{4}$$

Here,

ω_0 is the eigenfrequency of the motion = $\sqrt{\frac{k}{m}}$

ω_d is the damped frequency = $\omega_0\sqrt{1-\lambda^2}$

λ is the relative damping; λ = $\frac{c}{2m\omega_0}$

The response is a damped oscillatory motion with frequency ω_d. In the case of an arbitrary impulse load I, the displacement must be multiplied by the value I.

As any arbitrary loading function can be divided into rectangular impulses acting at the time t = τ, the response at the time t from a rectangular impulse I (τ) acting at time t = τ, is

$$\Delta x(t) = I(\tau)e^{-\lambda\omega_0(t-\tau}\frac{1}{m\omega_d}\sin(\omega_d(t-\tau)) = I(\tau)h(t-\tau) \tag{5}$$

The total response at time t is the sum of the response from the individual impulses

$$\text{x(t)} = \frac{1}{m\omega_d}\int_0^t F(\tau)\,e^{-\lambda\omega_0(t-\tau)}\sin(\omega_d(t-\tau))\text{d}\tau \tag{6}$$

This is the “convolution integral” or Duhamel integral.

In the case of no damping, the exponential part is equal to 1 as $\lambda = 0$, and $\omega_d = \omega_0$

The total displacement in the case of no damping must include the initial conditions, and by adding the solution of the homogeneous equation (2) when F(t) = 0, we obtain the total solution as

$$x(t) = x_0\cos(\omega_0 t) \;+\; \frac{1}{\omega_0}\frac{dx_0}{dt}\sin\omega_0\, t \;+\; \frac{1}{m\omega_0}\int_0^t F(\tau)\sin\left(\omega_0(t-\tau)\right)d\tau \qquad (7)$$

Here,

x_0 is the value of the displacement at t = 0

$\frac{dx_0}{dt}$ is the value of the velocity at t = 0

2.2 Immediate release of a mass-spring system.

The immediate release of a mass-spring system is equivalent to a situation when a sudden load appears in the wire (the problem of slack wire).

In this case, the initial conditions are as follows:

$F_1(t) = 0$ for $t < 0$ and $F_1 = mg$, for $t \geq 0$

Neglecting damping, $\lambda = 0$ and $\omega_d = \omega_0$

Then,

$$x(t) = \frac{F_1}{m\omega_0}\int_0^t sin\omega_0(t-\tau)d\tau = \frac{F_0}{k}\{1-\cos(\omega_0 t)\} \qquad (8)$$

The dynamic amplification is

$$DLF = 1-\cos(\omega_0 t) \qquad (9)$$

Neglecting damping, we see that the maximum displacement is twice the static displacement $x(t) = \frac{F_1}{k}$ at $\cos\omega_0 t = -1$, i.e. at $\omega_0 t = \pi$, i.e., at the time $t = T_0/2$. The maximum elongation of the spring (the dynamic amplification) is thus twice the elongation compared to the case of static loading; as if the impact load was twice the static load. The situation resembles a snap load. Introducing damping, the displacement would be less.

2.3 Rectangular pulse load with finite duration, t_d

In the case of a constant rectangular pulse with value F_1 and finite duration, t_d, the displacement for $t \leq td$ is given by (9), while the value of the load F_1 is equal to 0 at $t \geq t_d$.

In this case, only the homogeneous part remains for $t \geq t_d$:

$$x(t) = \frac{F_1}{k}\{\cos\omega_0\,(t-t_d) - \cos(\omega_0 t)\} \qquad (10)$$

If $t_d > \frac{T_0}{2}$, the maximum dynamic amplification is 2. For short values of t_d, the value of the term $\cos(\omega_0 t)$ during the interval from t = 0 to $t = t_d$ is small, and the dynamic amplification, DLF, is less, see Figure 2, [3].

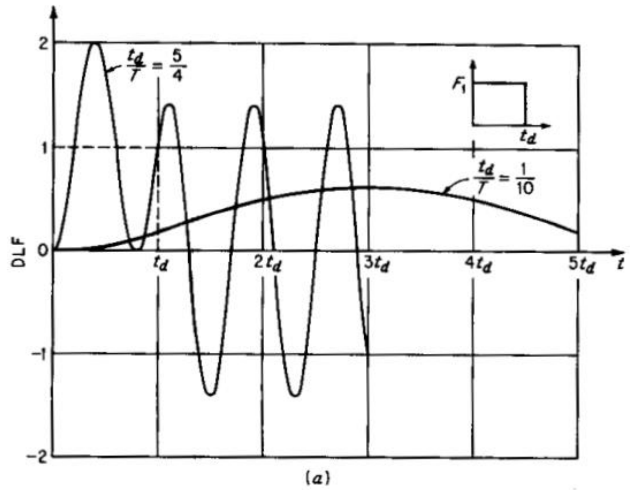

Figure 2. Typical responses of one-degree-of-freedom systems exposed to a rectangular load F_1 with duration t_d, from [3].

For an immediate loading, omitting damping to identify the maximum dynamic effect is acceptable, as the damping effect would not be activated quickly compared to the load application.

2.4 Constant impact load with a finite rise time, t_r

All impact loading will be transferred over a finite time, although the rise time might be very short in certain cases. It is of interest to find the effect of the rise time, t_r:

The impact loading is given as:

$$F(t) = (F_1 \cdot \frac{t}{t_r}), \text{ for } t \leq t_r \tag{11a}$$

$$F(t) = F_1, \text{ for } t \geq t_r \tag{11b}$$

In this case, the dynamic amplification, DLF is given as:

$$DLF = \frac{1}{t_r}\left(t - \frac{\sin \omega_0 t}{\omega_0}\right), \text{ for } t \leq t_r \tag{12a}$$

$$DLF = 1 + \frac{1}{\omega_0 t_r}\{sin\omega_0(t - t_r) - \sin\omega_0 t\}, \text{ for } t \geq t_r \tag{12b}$$

When the rise time t_r is short compared to T_0, $\frac{t_r}{T_0}$ is low, and the value of $(\omega_0\, t)$ gets large during the rise time. The displacement will reach the value $\frac{2F_1}{k}$. When the rise time is large compared to the eigenperiod T_0, the value of ω_0t is small during the rise time, and the response will continue to increase almost statically, as the load could be seen as just pushing onto the structure. Figure 3 (from [3]) shows examples of the dynamic amplification, DLF, for short and large values of $\frac{t_r}{T_0}$, while Figure 4 shows the maximum DLF for different values of $\frac{t_r}{T_0}$. It can be seen from this figure that if the rise time is less than ¼ of the eigenperiod, T, the max value of the DLF is close to the same effect as a sudden applied load.

It is possible to follow the displacement after the rise time of the impact, i.e., when the load has reached the constant value, F_1. In this case, we must use Equation (8) with the initial values of the displacement and velocity at the time t_r:

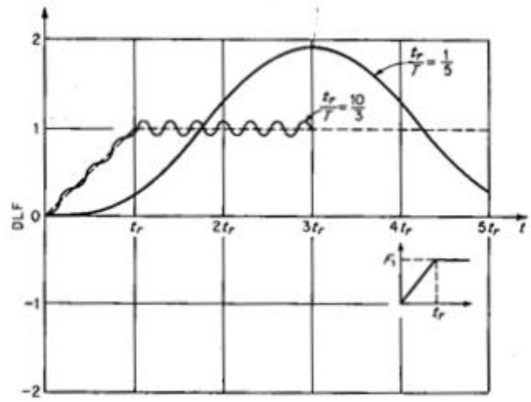

Figure 3. Typical responses of one-degree-of-freedom systems exposed to a constant load with finite rise time t_r, from [3].

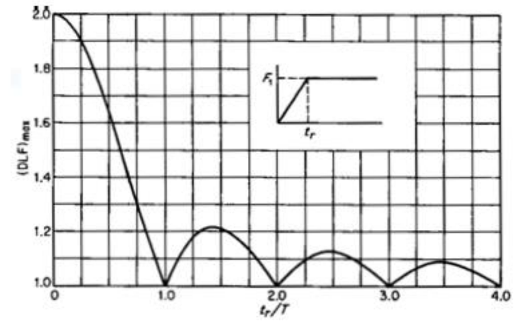

Figure 4. Maximum dynamic response of one-degree-of-freedom systems exposed to a constant load with finite rise time t_r. T is the eigenfrequency of the system, from [3].

2.5 Triangular symmetric impact load with duration t_d.

In many realistic situations, the load will be transferred over a certain rise time, t_r, and have a limited duration, t_d. It is, therefore appropriate to study the effect of such a realistic load transfer situation. To simplify, let us consider a symmetric triangular pulse reaching its maximum F_1 at half of the total duration, so: $t_r = ½\, t_d$:

F(t) = 2F$_1 \frac{t}{t_d}$, for $0 \le t \le ½ t_d$ (13a)

F(t) = 2F$_1(1-\frac{t}{t_r})$, for $½ t_d \le t \le t_d$ (13b)

F(t) = 0, for $t_d \le t$ (13c)

In this case, the dynamic amplification, DLF is given as:

DLF = $\frac{2}{t_d}$ (t - $\frac{\sin \omega_0 t}{\omega_0}$), for $0 \le t \le ½ t_d$ (14a)

DLF = $\frac{2}{t_d}$ {t_d - t + $\frac{1}{\omega_0}\{2sin\omega_0(t - \frac{t_d}{2})$ - sin$\omega_0 t\}\}$, for $½ t_d \le t \le t_d$ (14b)

DLF = $\frac{2}{\omega_0 t_d}$ $\{2sin\omega_0(t - \frac{t_d}{2})$ - sin$\omega_0 t$ - sin$\omega_0(t - t_d)$ for $t_d \le t$ (14c)

These equations give interesting results as shown in Figure 5 (from [3]). The maximum dynamic response is less than 2 and obtains its maximum of close to 1.5 when the duration is close to the eigenperiod of the structure.

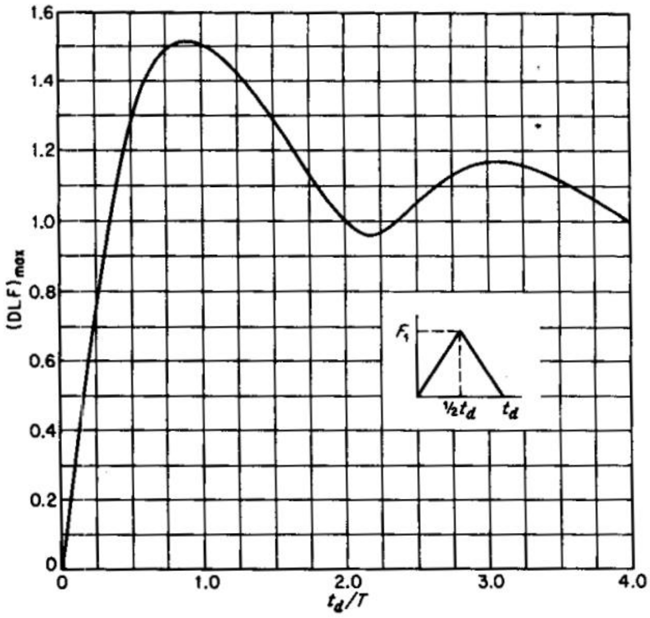

Figure 5 Maximum dynamic response of one-degree-of-freedom systems exposed to a symmetric triangular load with duration t_d and finite rise time ½ t_d. T is the eigenfrequency of the system, from [3].

2.5 Triangular sudden impact load with duration t_d.

An explosion is represented by a very rapid build-up of the load, followed by a decline of the loading. The load could be represented by a sudden triangular pulse with duration t_d.

The loading is represented by the equations:

$$F(t) = F_1(1 - \frac{t}{t_d}),\ \text{for } t \leq t_d \quad (15a)$$

$$F(t) = 0,\ \text{for } t \geq t_d \quad (15b)$$

In this case, the dynamic amplification, DLF is given as:

$$\text{DFL} = 1- \cos \omega_0 t + \frac{sin\omega_0 t}{\omega_0 t_d} - \frac{t}{t_d},\ \text{for } t \leq t_d \quad (16a)$$

$$\text{DLF} = \frac{1}{\omega_0 t_d}\{sin\omega_0 t - sin\omega_0(t - t_d)\} - \cos \omega_0 t,\ \text{for } t \geq t_d \quad (16b)$$

For long durations of the load compared with the eigenperiod of the structure, the dynamic amplification tends toward the value of 2 in the case of a triangular load. The dynamic amplification is, thus, large in the case of a very stiff structure with a short eigenperiod.

The response to a rectangular load pulse is, on the other hand, more rapid.
For a short duration of the load compared to the eigenperiod of the structure, the dynamic amplification is far less pronounced. The consequence is that an explosion will be amplified strongly in the case the load hits a very stiff structure while the dynamic amplification is limited for a softer structure. This finding should be used for the design of structures exposed to potential triangular explosion type of loading.

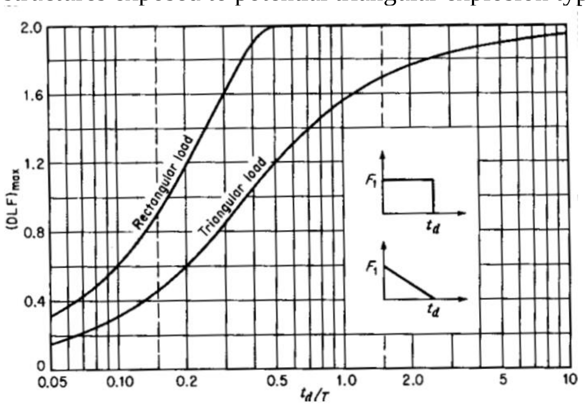

Figure 5 Maximum dynamic response of one-degree-of-freedom systems exposed to a triangular load with duration t_d. The results are compared with the solution of a one-degree-of-freedom system exposed to a rectangular load with duration t_d. T is the eigenfrequency of the system, from [3].

3. Applications and discussions

3.1 General considerations

A series of impact load functions are discussed in Chapter 2. Figures are taken from reference [3]. The chapter focuses on the dynamic amplification of the load while [3] also presents results for the time of the maximum response. However, for the design of structures to withstand the loading, the maximum dynamic effect is the most important.

For structural design, the load is multiplied by a load factor $\gamma_L$and the material capacity is reduced by a material factor γ_M. For loadings caused by the physical environment the value of the product of γ_L and γ_M is typically 1.5 [8]. In the case of a dynamic amplification factor of 2, as demonstrated in Chapter 2 for many impact loading scenarios, the inclusion of the DLF is most important, so the safety against damage is DLF·γ_L·γ_M. The value of DLF is critical for structural safety.

In Chapter 2 it was demonstrated that the DLF is dependent on the relative rise time and the relative duration of the load, i.e. by $\frac{t_r}{T_0}$ and $\frac{t_d}{T_0}$, where T_0 is the eigenperiod of the structure. A soft structure, i.e. a structure having a high eigenperiod compared to the rise time or the duration of the load, responds slower to the impact loading, and the dynamic effect is lower. Specific examples are considered below.

3.2 Collisions

During a collision, the impact loading could be modeled as a constant impact load with a finite rise time, t_r, Figure 4. The dynamic effect depends to a large degree on the stiffness of the structure, and a soft collision (with a structure having a high eigenperiod) has fewer consequences than a hard collision. Similarly, the rise time; the speed of the impact (the velocity during the impact) is important.

- Modern cars are designed to buckle (high eigenperiod) during a collision rather than to limit the damage to the body of the car.
- The bow of an icebreaker is very stiff (low eigenperiod). The damage during a collision with an icebreaker is more severe than a collision with a softer vessel.
- Similarly, the stern rollers of a supply ship are stiffer than the bow of traditional supply ships, and the damage caused by the rollers could be large in case the supply ship backs into a structure.

3.3 Explosions

During an explosion, the loading increases very rapidly and the load thereafter reduces quickly. The situation of triangular impact might describe the response. Softer structures (long value of the eigenperiod) give rise to lower dynamic amplifications than stiff structures.

- This effect could be taken into account for the design of equipment holding very explosive liquids, like hydrogen [4] or gasoline.

3.4 Breaking waves.

For the design of offshore structures or sea-going vessels, steep waves pose specific concerns to the designers. Breaking waves could be in the form of spilling breakers with a finite rise time or plunging breakers giving a sudden impact, i.e. the crest front steepness is an important parameter. The responses to breaking waves have been of concern to much research, see for example [5] to [7]. It is proposed in [7] that the impact loading caused by breaking waves is a linear function of the steepness (See Figure 6) of the waves. Considering Figure 4 related to constant impact load with a finite rise time, t_r, we see that the dynamic amplification is reduced almost linearly over a wide range of values of $\frac{t_r}{T_0}$, i.e. when the wave steepness is reduced.

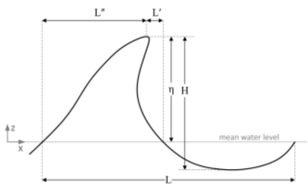

Figure 6 Wave steepness = $\frac{\eta}{L'}$, from [7]

3.5 Snap loading in a wire

The case of snap loading must be avoided as the response to such loading is a dynamic amplification of 2, see Chapter 2.2.

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