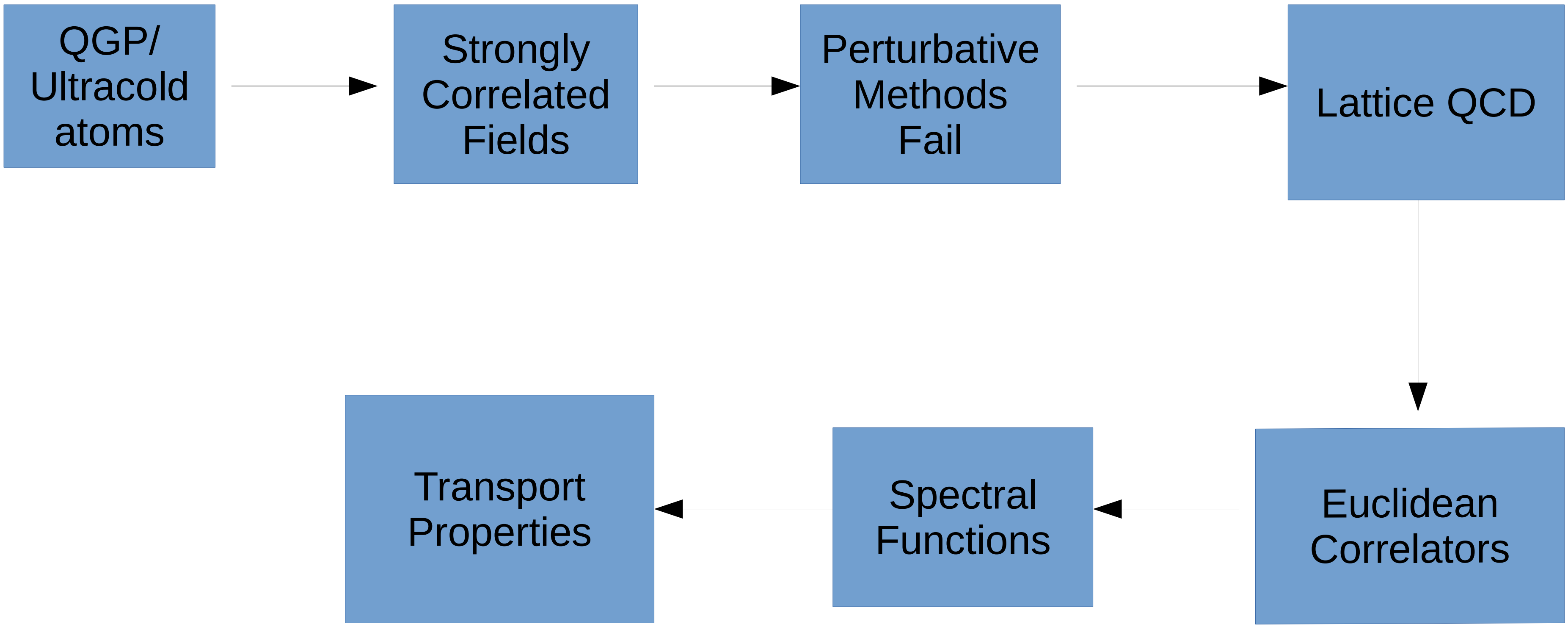
Spectal Reconstruction and Real-Time simulations

NPACT 2020
Gaurang Parkar
gparkar@ux.uis.no

Outline

- Bayesian Methods for spectral reconstruction: State of the art methods(MEM and BR method).
- Non local priors: Gaussian Prior
- Some other ideas of real time properties.

Why Spectral Reconstruction ?



- Relevant properties like Conductivity, shear and bulk viscocity are computed from ‘Spectral Functions’(ρ) which are computed using solving an inverse problem.

$$D(\tau) = \int d\omega K(\omega, \tau)\rho(\omega)$$

- We are mainly interested in the case when

$$K(\omega, \tau) = e^{-\omega\tau}$$

- Inversion is an ill-posed problem.
- We adopt probabilistic techniques.
- Find an approximate solution instead of exact.
- Use bayesian statistics.

Overview of MEM

- The Maximum Entropy Method (arXiv 0011040) tries to find an approximate solution using Bayes' theorem.

$$P(\rho|DH) = \frac{P(D|\rho H)P(\rho|H)}{P(D|H)}$$

- D = Monte carlo Data
- H = Prior knowledge about the spectral function
- ρ = Spectral Function

- The likelyhood function is defined by

$$P(D|\rho H) = \frac{1}{Z_L} e^{-L}$$

$$L = \frac{1}{2} \sum_{i,j} (D(\tau_i) - D_A(\tau_i)) C_{ij}^{-1} (D(\tau_j) - D_A(\tau_j))$$

- Here C is the covariance matrix.

- The MEM uses Shannon-Jaynes entropy as the prior

$$P(\rho|H) = \frac{1}{Z_S} e^{\alpha S}$$

$$S = \sum_{l=1}^{N_\omega} (A_l - m_l - \rho_l \log \frac{\rho_l}{m_l})$$

- α is a parameter and 'm' is called default model.

- The most probable solution is now given by

$$\frac{\delta Q}{\delta \rho(\omega)} = 0 \qquad\qquad Q = \alpha S - L$$

- Optimisation is done by LBFGS algorithm.

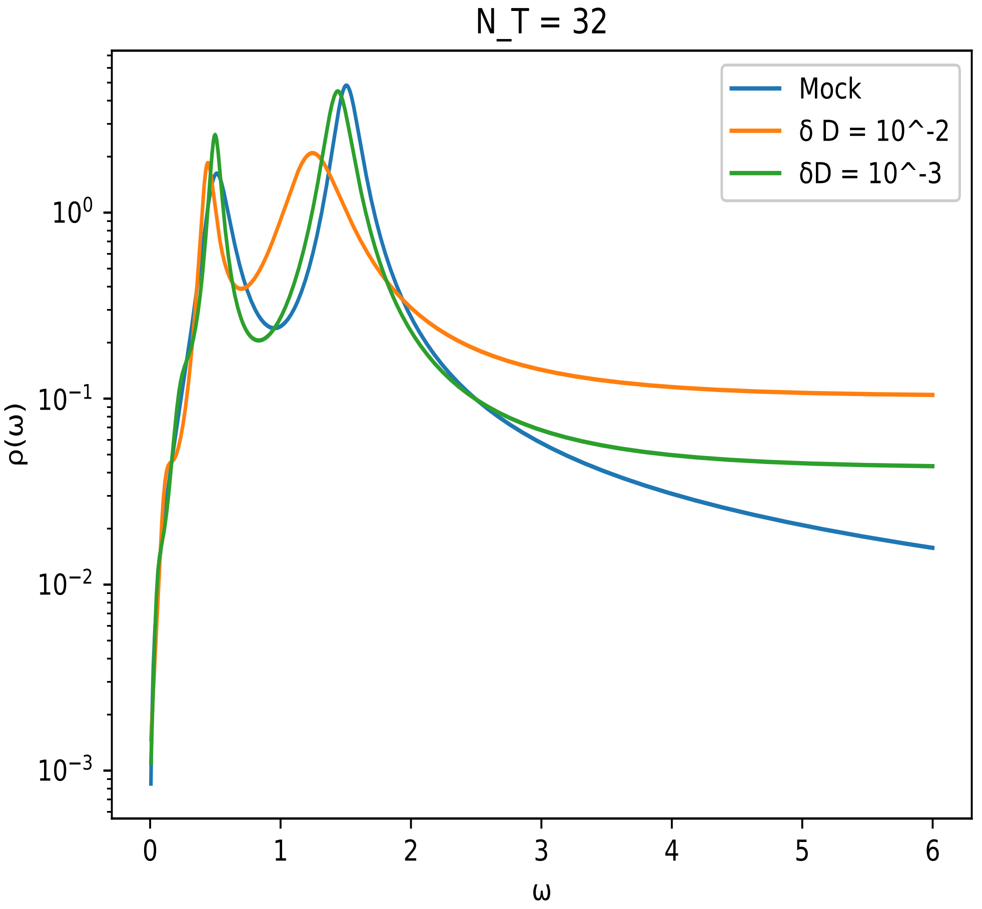
BR Method (arXiv:1307.6106)

- The BR Method uses Bayesian statistics, but is based on different assumptions than that of MEM.
- It is based on the assumption of scale invariance and smoothness.
- It makes use of the BR prior instead of the Shannon Jaynes entropy.
- Avoids the asymptotic flatness of MEM and also introduces scale invariance.

$$S = \alpha \int d\omega \left(1 - \frac{\rho}{m} + \log \frac{\rho}{m}\right)$$

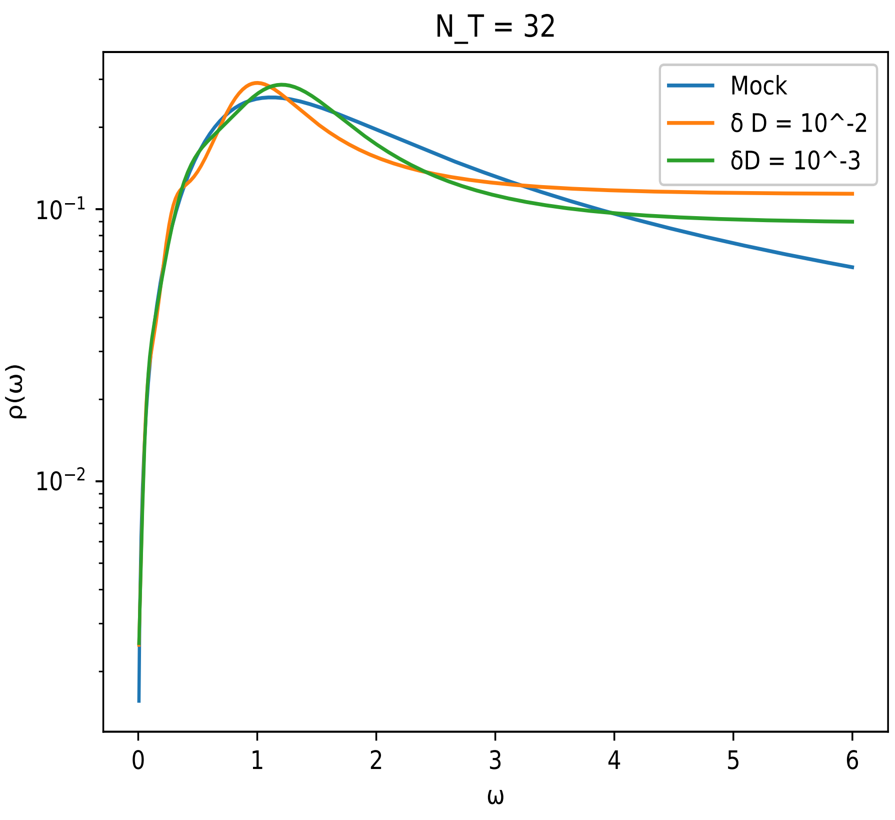
Where the BR method shines

- A mock spectral function is choosen.
- 10000 Configurations are generated usi;ng gaussian random noise to mock.

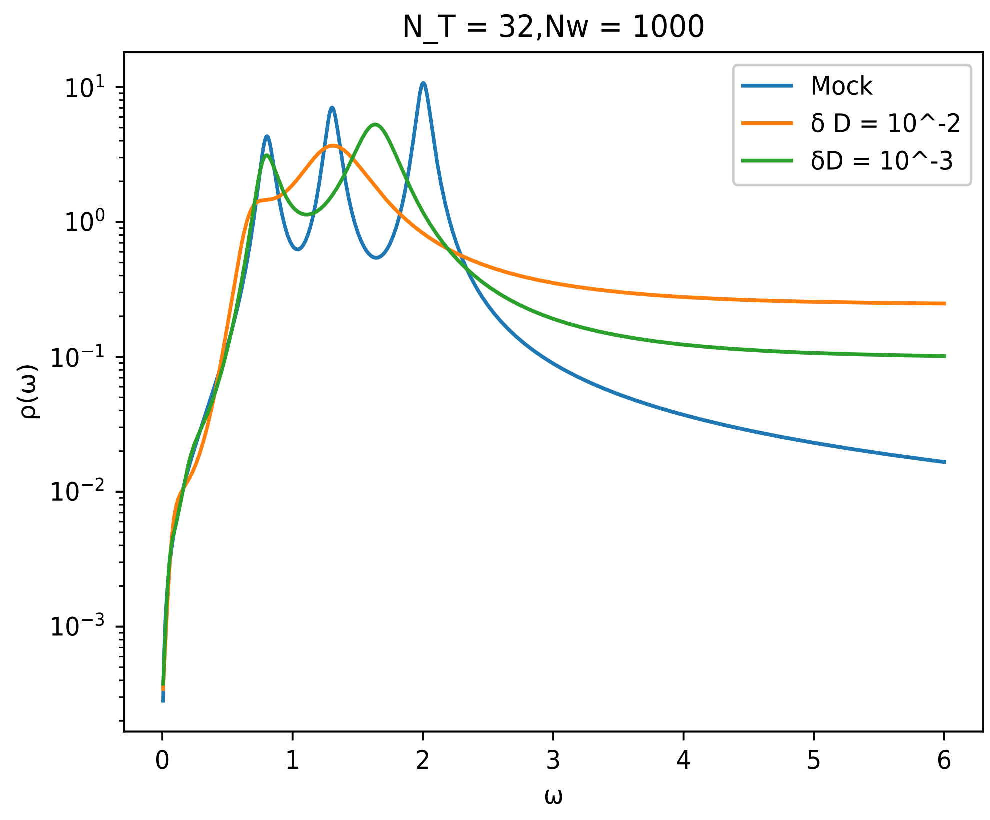


Where it does not do so well

Ringing Effects



Fails to reconstruct more than two peaks



A New idea: Non-local connections

- Try with gaussian prior

$$S = \sum_{ij} (\rho_i - m_i) \sigma_{ij} (\rho_j - m_j)$$

- Here the matrix σ is a general matrix that we try to learn.
- Start with a diagonal matrix with one constant parameter.

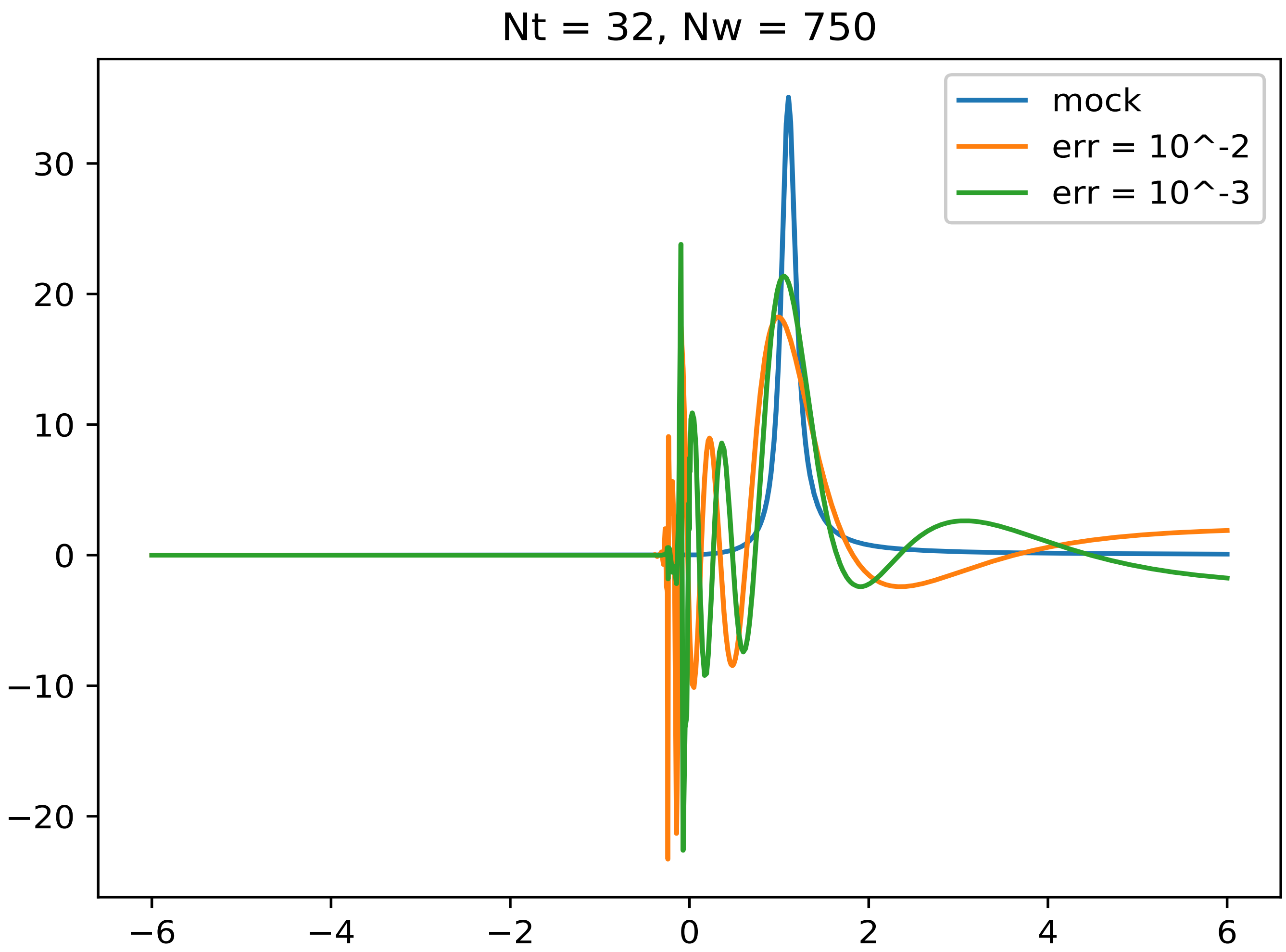
Advantages of using gaussian prior

- The solution for the reconstructed spectrum can be exactly written down in the form of a system of linear equations.
- Greatly increases speed : No numerical optimisation required
- Implementing high precision libraries is easy.

Some drawbacks

- No way to impose positivity in the spectral function.
- High condition number of the matrix.
- Oscilatory behavior of the reconstructed spectral function.

Some Analysis with Mock data



Future

- Correlated Gamma distribution: Extension of BR prior to include non-local effects.
- Try to approximate from an ensemble of solution:
- Machine learning techniques: There have been several ideas proposed.
- Direct image recognition does no better than MEM/ BR.

Real-Time Simulations

- Inversion technique
- Designing neural networks for sampling

- Observables in quantum fields are computed by doing a path integral

$$Z = \int D\phi e^{iS}$$

- The path integral is computed using acceptance sampling techniques like monte carlo.

- We try to seperate out the real and imaginary parts and sample them independently.

Inversion Technique

- It is based on the principle of universality of the uniform.
- It lets us generate a given distribution from uniform distribution provided we know the inverse.

Inversion Theorem

- Let F be a cumulative continuous distribution R with an inverse defined.

- If U is a uniform [0,1] random variable then $F^{-1}(U)$ has distribution F.

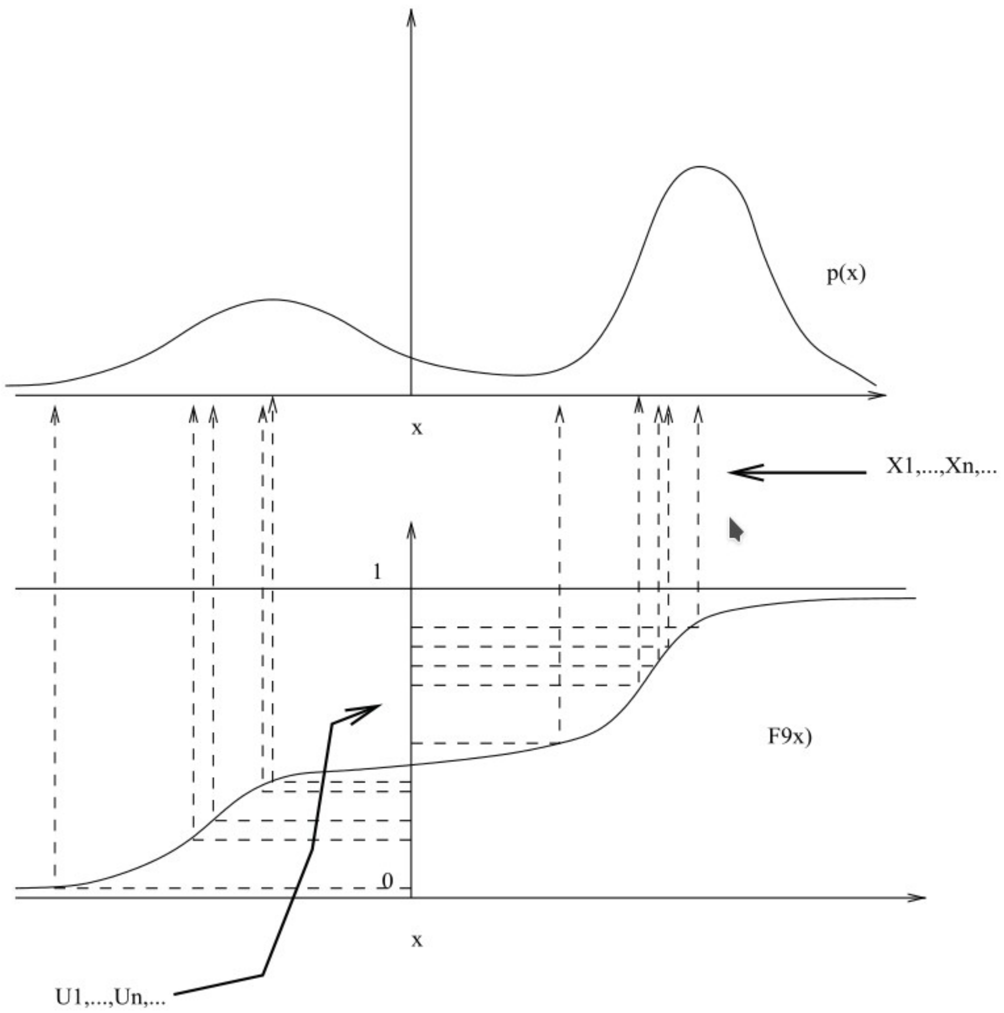


Image taken from ‘Stocastic Processes’ by Pierre Moral and Spiridon Penev

Some difficulties

- Inverse function : The inverse is generally not known.
- Cumulative distribution : We can compute e^{iS} fairly easiely, but computing the cumulative distribution function amounts to doing a path integral.

Neural Networks for sampling

- Neural networks can learn arbitary probability distributions.
- There are some papers trying to designed restricted boltzman machines to do this.

Thank You